BIRZEIT UNIVERSITY

Problem 1 # electromagnetic

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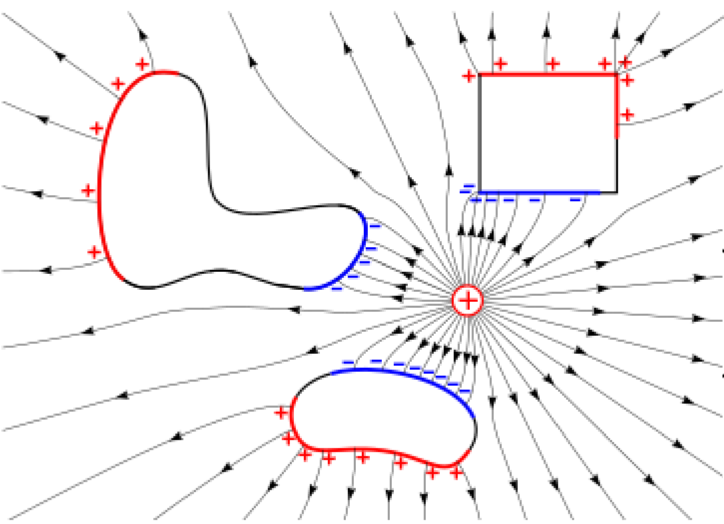
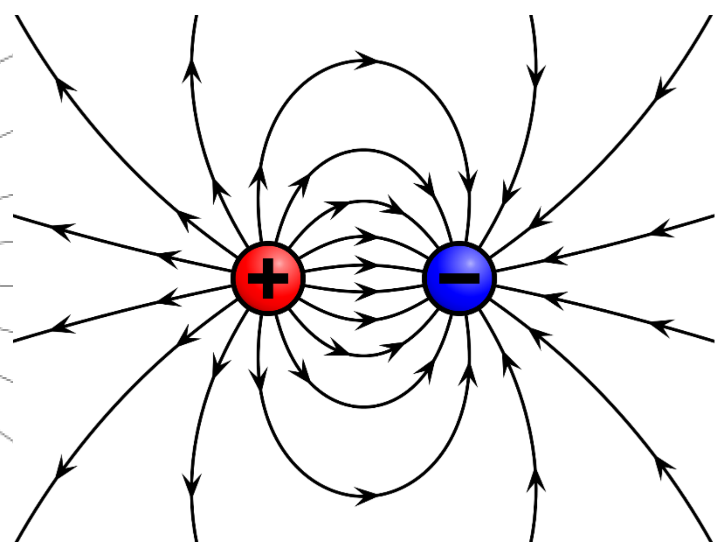
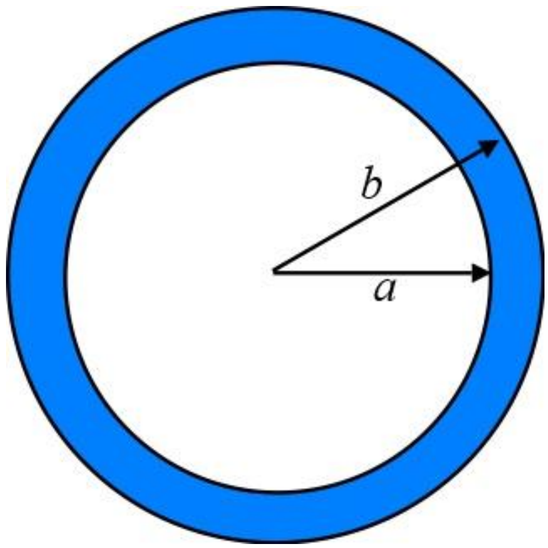




#Problem: Find the potential everywhere for a spherical shell of inner radius a and outer radius b with a uniform volume charge of total q



-a uniform volume charge=ρ

-there are three region

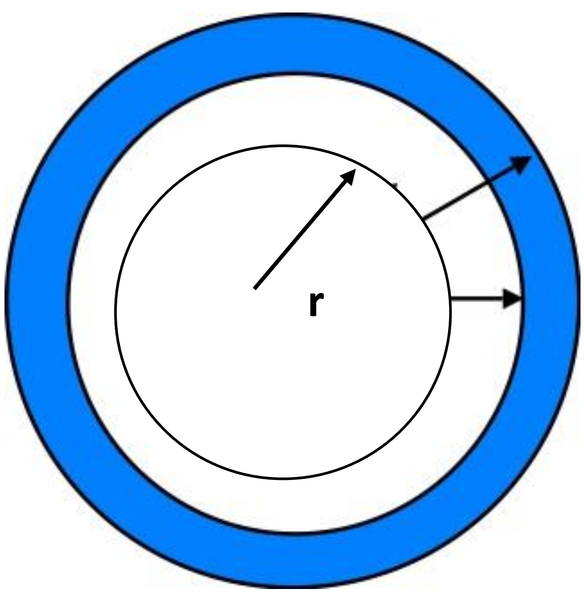
- r < a
- a < r <b
- r > b

***Find electric field by Guess law.**

-let the Gaussian surface is sphere

****At region r < a**

At this region no charge enclose so the electric field =zero



At region a < r <b

$$\oint E.da = \frac{q_{enc}}{\varepsilon_0}$$

$$q_{enc} = \int \rho.d\tau$$

$q_{enc} = \int \rho.r^2 \sin\theta \; d\theta \; dr \; d\varphi.$

Where boundary (θ) 0 --π, (r) a -- r,

(φ) 0 -- 2π

$$q_{enc} = \rho \; \frac{4}{3} \; \pi(r^3 - a^3)$$

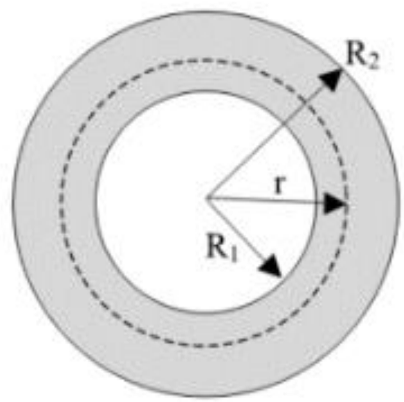


Figure 1: where R1is a and R2 is b

Because symmetry:

$$|E| \oint da = \frac{\rho \; \frac{4}{3} \; \pi \; (r^3 - a^3)}{\varepsilon_0}$$

$$|E|.\, 4\pi r^2 = \frac{\rho \; \frac{4}{3} \; \pi \; (r^3 - a^3)}{\varepsilon_0}$$

$$\vec{E} = \frac{\rho \; (r^3 - a^3)}{3\varepsilon_0 r^2}\hat{r}$$

$$\vec{E} = \frac{\rho}{3\varepsilon_0}(r - \frac{a^3}{r^2})\hat{r}$$

At region r>b

$$\oint E.da = \frac{q_{enc}}{\varepsilon_0}$$

$$q_{enc} = \int \rho . d\tau$$

$q_{enc} = \int \rho . r^2 \sin\theta \, d\theta \, dr \, d\varphi.$

Where boundary (θ) 0 --π, (r) a -- r, (φ) 0 -- 2π

$q_{enc} = \rho \, \frac{4}{3} \, \pi(b^3 - a^3)$

$$|E| \oint da = \frac{\rho \, \frac{4}{3} \, \pi \, (b^3 - a^3)}{\varepsilon_0}$$

$$|E|. 4\pi r^2 = \frac{\rho \, \frac{4}{3} \, \pi(b^3 - a^3)}{\varepsilon_0}$$

$$\vec{E} = \frac{\rho \, (b^3 - a^3)}{3\varepsilon_0 r^2} \hat{r}$$

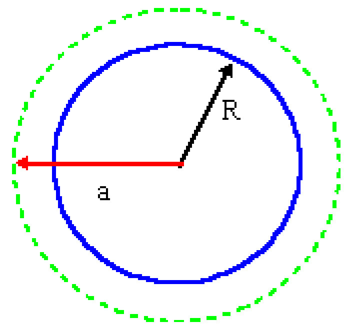


Figure 2(Gaussian surface outside sphere)

Electric field:

- At r=0 , E=0
- At a<r<b, $\vec{E} = \frac{\rho}{3\varepsilon_0}(r - \frac{a^3}{r^2})\hat{r}$
- At r>b,$\vec{E} = \frac{\rho\ (b^3 - a^3)}{3\varepsilon_0 r^2}\hat{r}$

The electrical potential:

****At r=0, V(r) =0, because charge equal zero**

**** At a<r<b:**

$$V(r) = -\int_{\partial}^{r} \vec{E}\,.dr = -\int_{\infty}^{r} \vec{E}\,.dr$$

$$V(r) = -\int_{\infty}^{b} \vec{E}\,.dr - \int_{b}^{r} \vec{E}\,.dr$$

$$V(r) = -\int_{\infty}^{b} \frac{\rho\ (b^3 - a^3)}{3\varepsilon_0 r^2}.dr - \int_{b}^{r} \frac{\rho}{3\varepsilon_0}(r - \frac{a^3}{r^2}).dr$$

$$V(r) = -\frac{\rho\ (b^3 - a^3)}{3\varepsilon_0}\int_{\infty}^{b} \frac{1}{r^2}.dr - \frac{\rho}{3\varepsilon_0}\int_{b}^{r} (r - \frac{a^3}{r^2}).dr$$

$$V(r) = -\frac{\rho\ (b^3 - a^3)}{3\varepsilon_0}\int_{\infty}^{b} \frac{1}{r^2}.dr - \frac{\rho}{3\varepsilon_0}\int_{b}^{r} r\ dr + \frac{\rho}{3\varepsilon_0}\int_{b}^{r} \frac{a^3}{r^2}.dr$$

$$V(r) = -\frac{\rho\ (b^3 - a^3)}{3\varepsilon_0}\left(-\frac{1}{b}\right) - \frac{\rho}{3\varepsilon_0}\left(\frac{r^2}{2} - \frac{b^2}{2}\right) + \frac{\rho}{3\varepsilon_0}\left(-\frac{a^3}{r} + \frac{a^3}{b}\right)$$

$$V(r) = \frac{\rho\ (b^3 - a^3)}{3\varepsilon_0 b} - \frac{\rho r^2}{6\varepsilon_0} + \frac{\rho b^2}{6\varepsilon_0} - \frac{\rho a^3}{3\varepsilon_0 r} + \frac{\rho}{3\varepsilon_0}\frac{a^3}{b}$$

$$V(r)=\frac{\rho\ \ b^3}{3\varepsilon_0 b}-\frac{\rho\ \ a^3}{3\varepsilon_0 b}-\frac{\rho r^2}{6\varepsilon_0}+\frac{\rho b^2}{6\varepsilon_0}-\frac{\rho a^3}{3\varepsilon_0 r}+\frac{\rho}{3\varepsilon_0}\frac{a^3}{b}$$

After some calculation:

$$V(r)=\frac{\rho b^2}{2\varepsilon_0}-\frac{\rho}{3\varepsilon_0}\left(\frac{r^2}{2}+\frac{a^3}{r}\right)$$

Where $\frac{q_{enc}}{\frac{4}{3}\pi(r^3-a^3)}=\rho$

$$V(r)=\frac{q_{enc}}{\frac{4}{3}\pi(r^3-a^3)}\left(\frac{b^2}{2\varepsilon_0}-\frac{1}{3\varepsilon_0}\left(\frac{r^2}{2}+\frac{a^3}{r}\right)\right)$$

**** At r>b**

$$V(r)=-\int_{\partial}^{r}\overrightarrow{E}.dr=-\int_{\infty}^{r}\overrightarrow{E}.dr$$

$$V(r)=-\int_{\infty}^{r}\frac{\rho\ \ (b^3-a^3)}{3\varepsilon_0 r^2}.dr$$

$$V(r)=-\frac{\rho\ \ (b^3-a^3)}{3\varepsilon_0}\int_{\infty}^{r}\frac{1}{r^2}.dr$$

$$V(r)=\frac{\rho\ \ (b^3-a^3)}{3\varepsilon_0 r}$$

Where $\frac{q_{enc}}{\frac{4}{3}\pi(b^3-a^3)}=\rho$

$$V(r)=\frac{q_{enc}}{4\pi\varepsilon_0 r}$$

The potential outside sphere the same as point charge.