

Introduction to Metamaterials

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Abstract

Recently, a new type of material, called metamaterial, has come out with interesting and strange properties that differ from the conventional materials. For example, it has a negative index of refraction, and the waves are reflected from it in the opposite direction. In this article, I will introduce this material and demonstrated the main different features such as the direction and the reverse Cherenkove radiation. After that I will mention how to make such a material with negative index of refraction n . Then, I will point on an experiment that can verify that this material has negative n . Finally, I mention two possible applications that metamaterials can be used in. It turns out that such a substance does not exists in nature, but the researchers made it in labs and used its interesting features in many applications.

1 Introduction

In literature, the index of refraction of a material is defined in terms of its permittivity and permeability, i.e,

$$n^2 = \epsilon_r \mu_r, \quad (1)$$

where $\epsilon_r = \epsilon/\epsilon_0$ is the relative permittivity (dielectric constant), and $\mu_r = \mu/\mu_0$ is the relative permeability of the substance.

Substances are affected by external electric and magnetic fields according to these well-known equations:

$$\vec{D} = \epsilon_0 \epsilon_r \vec{E} \quad (2)$$

$$\vec{B} = \mu_0 \mu_r \vec{H} \quad (3)$$

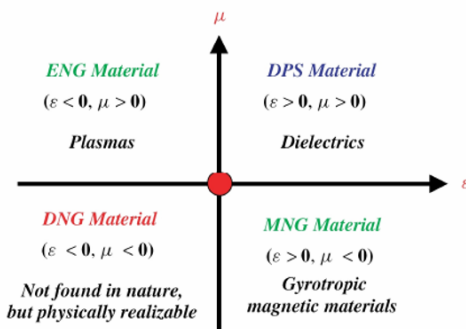


Figure 1: Classification of materials according to the index of refraction

There are 4 classifications of materials in terms of index of refraction. Figure(1) demonstrates the 4 types;

- **DPS Material:** It stands for Double Positive Material, which is as the name tells, a material with both the ϵ and μ are positive. Hence, the index of refraction of these materials is real and positive. Most known materials have this property. Some calls this type as a Right Handed Material (RHM) for reasons explained in the next sections.
- **ENG Material:** It stands for Epsilon Negative Material. In this case, n is imaginary. Many plasmas exhibit this characteristic.
- **MNG Material:** It stands for Mu Negative Material. n is also imaginary in this type. Famous examples of MNGs are ferromagnetic materials. So we can find this type in nature.
- **DNG Material:** It stands for Double Negative Material. As its name indicates, both ϵ and μ are negative. In this type, n would be real and negative. However, there is no existence of these materials in nature. Some calls it Left Handed Material (LHM). **This type is what we call: Metamaterial.**

Therefore, metamaterials cannot be found in nature. We knew that the index of refraction for vacuum is 1, and for glass is approximately 1.5, but we never hear that some substances in some where have negative index. So, where the 'meta' materials come from?

The beginning of the story is in 1968, when V.G.Veselago published an article discussing the possibility of existing a material with negative μ and negative ϵ . And he proved that if μ and ϵ are negative, then n is real and negative[1].

He theoretically investigated the electrodynamic consequences of a medium having such properties. He concluded that such a medium would have dramatically different propagation characteristics stemming from the sign change of the group velocity, including reversal of both the Doppler shift and Cherenkov radiation, anomalous refraction, and even reversal of radiation pressure to radiation tension. These effects could not be experimentally verified since, as Veselago pointed out, substances with negative μ were not available.

In this article, I will show some of the properties of such a material, how to make it with negative n , and how would we know that it has a negative n . Finally I mention some applications of metamaterials; it could be used as perfect lens and as a cloaking device.

2 New Phenomena & Different Features

Due to the negative sign of n , many different features appear, and new phenomena appear as reverse of the ordinary phenomena. Directions and radiations are two examples.

2.1 Direction & Propagation

The direction of waves in metamaterials has an interesting feature. Veselago showed that if

$$\mu < 0, \epsilon < 0 \rightarrow n < 0.$$

Now, if $n < 0$, then $v = c/n < 0$. The speed of the wave is negative, and hence, the wave somehow travels backwards toward the source.

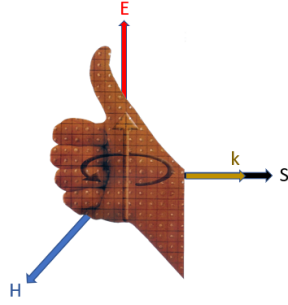


Figure 2: Right-Hand-Rule

The fields in metamaterials are determined by the left hand rule, so it is usually called Left Handed Materials (LHM), while determined by right hand rule for conventional materials (RHM). This is shown in figure(4). Also, for conventional material, the refracted waves are spreading away on entering and exiting the medium. For metamaterial, the waves are refracted so that it is produced a focus inside the material and then another focus outside.

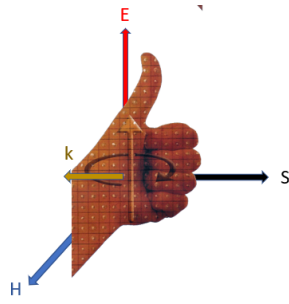


Figure 3: Left-Hand-Rule

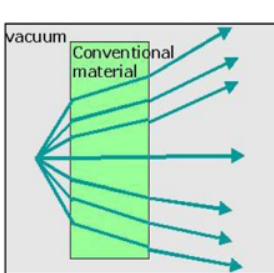


Figure 2.8: Refracted rays in conventional material

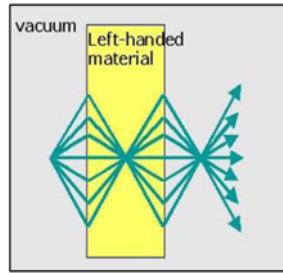


Figure 2.9: Refracted rays in Left-handed material

Figure 4: Refracted waves in LHM and RHM

Figure(4) shows an interesting behavior for refracted waves in LHM. This is due to inverting Snell's law. In metamaterial, for negative n , Snell's law tells us:

$$n_1 \sin(\theta_1) = -|n_2| \sin(\theta_2). \quad (4)$$

Since the left hand side of the equation is positive and $n_2 < 0$, it follows that $\theta_2 < 0$. This is illustrated in figure(5). This phenomenon is used as we will see, for making 'perfect lens'.

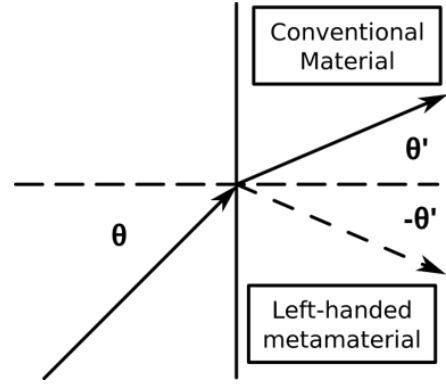


Figure 5: Snell's law in LHM

2.2 Reverse Cherenkov Radiation

Cherenkov radiation in normal materials is simply the result of charged particles (mostly electron) traveling faster than the phase velocity of light in that material.

The speed of light in vacuum is c , and the speed of light in water is $0.75c$. Matter can accelerate beyond this speed during nuclear reactions in particle accelerators, such that,

$$0.75c < v_{particle} < c.$$

Anyway, when Cherenkov radiation is being in a metamaterial, it is called reverse Cherenkov radiation. It seems that as a phenomenon occurs in a metamaterial, it becomes 'reversed' phenomenon.

In 2011, the reversed Cherenkov radiation is studied, and the results are in figure(6). The figure shows radiation power in each angle in the negative refraction band (solid line) and positive refraction band (dashed line)[2]. Note that the power

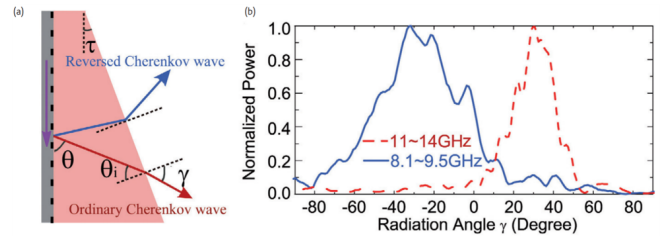


Figure 6: Normalized power VS radiation angle($^{\circ}C$) in reversed Cherenkov radiation

observed in the region of negative angles.

The reversed Cherenkov radiation has many possible applications, such as novel vacuum electronic devices, particle detectors, accelerators and new types of plasmonic couplers.

3 How to Make Metamaterials

We cannot find metamaterial in nature. It is a human made, modified and designed material.

There are multiple designs, but the famous one is by using Split Ring Resonator (SRR).

Split-ring resonators (SRR) are one of the most common elements used to fabricate metamaterials [7]. Split-ring resonators are non-magnetic materials. The first of which were usually fabricated from circuit board material to create metamaterials [3].

As figure(7) shows, it can be seen that at first a single SRR looks like an object with a two square perimeters, with each

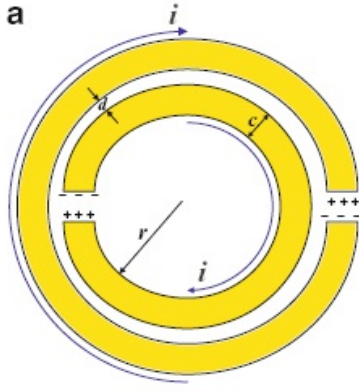


Figure 7: Split ring resonator with radius r , and dimensions d and c , while i is the produced current

perimeter having a small section removed. This results in square "C" shapes on fiberglass printed circuit board material. In this type of configuration it is actually two concentric bands of non-magnetic conductor material. There is one gap in each band placed 180° relative to each other. The gap in each band gives it the distinctive "C" shape, rather than a totally circular or square shape.

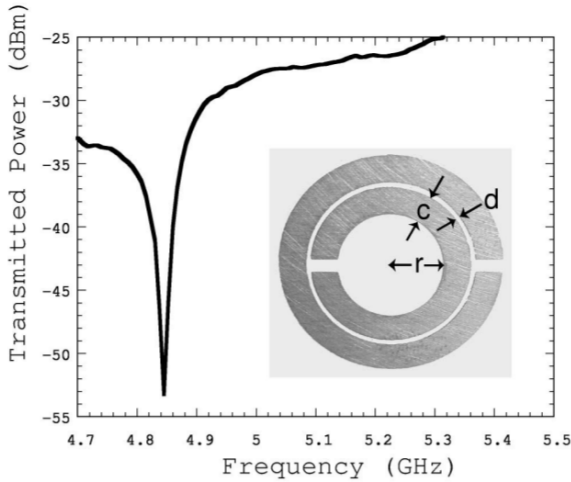


Figure 8: Resonance curve of an actual copper split ring resonator (SRR). $c = 0.8\text{mm}$, $d = 0.2\text{mm}$, and $r = 1.5\text{mm}$. The SRR has its resonance at about 4.845 GHz . [8]

The gaps produce a capacitance which produces a resonance that amplifies the magnetic field. When a time varying magnetic field is applied on it, a produced current produces magnetic field that may either oppose or enhance the incident field. The magnetic field in SRRs is dipolar. This allows accumulation of charges on the opposite gaps. The charges create a capacitance that lowers the resonant frequency and concentrates the electric field. This processes make the index of refraction negative. In figure(8), a graph of transmitted power versus frequency for a material having SRRs [8].

However, SRR alone doesn't have a negative index. SRR is nothing but a block building for metamaterials. Periodic structures and many cells of SRRs on averaging produce metamaterial with negative n as a macroscopic effect. This structure is shown in figure(9).

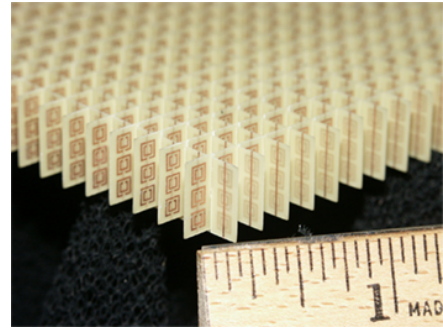


Figure 9: A split-ring resonator array is configured as a material that produces negative index of refraction. It was constructed of copper split-ring resonators and wires mounted on interlocking sheets of fiberglass circuit board. The total array consists of $3 \times 20 \times 20$ unit cells with overall dimensions of $10 \times 100 \times 100\text{ mm}$ [9][8].

4 Experimental Verification of a Negative Index of Refraction

Now, after making a metamaterial, how do you guarantee that this material has a negative index of refraction?

As we know, DNS materials show inverse Snell's law; the refracted angle is negative. If a detector is constructed so that it detects the refracted waves from a material and measures the refracted angle, then the material has negative n if the angles are negative, i.e, the waves refract in the opposite direction. This experiment was done in 2001 by a group of researchers [3].

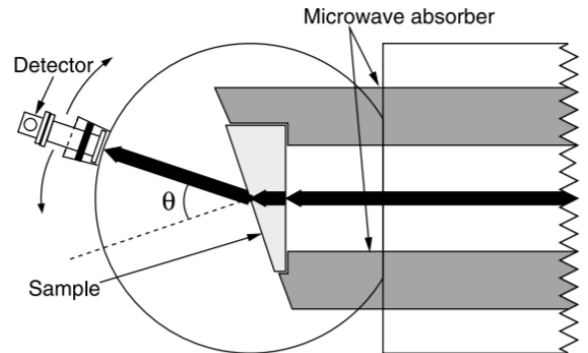


Figure 10: The detector used in the experiment that detects the refracted waves from a material [3]

It was presented experimental scattering data at microwave frequencies on a structured metamaterial that exhibits a frequency band where the effective index of refraction n is negative.

The material consists of a two-dimensional array of repeated unit cells of copper strips and split ring resonators on interlocking strips of standard circuit board material.

By measuring the scattering angle of the transmitted beam through a prism fabricated from this material, the effective n was determined, appropriate to Snell's law.

The results is shown in figure(11). Note that the detected angles was negative at frequency 10.5 GHz . However, the material didn't reflect negative angles at any range of frequencies. So it is metamaterial at specific range of frequency. In this experiment, the microwaves were used (at $\approx 10.5\text{ GHz}$). This is shown in figure(12).

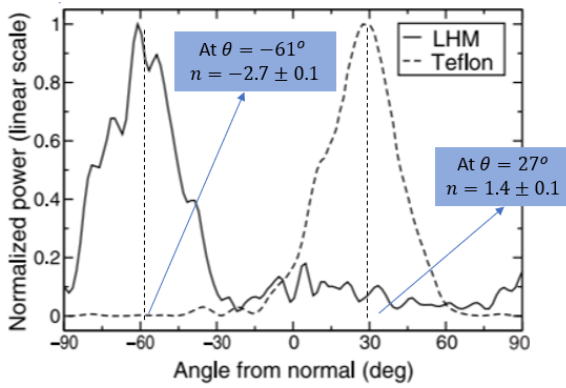


Figure 11: Transmitted power at 10.5 GHz as a function of refraction angle for both a Teflon sample (dashed curve) and a LHM sample (solid curve). [3]

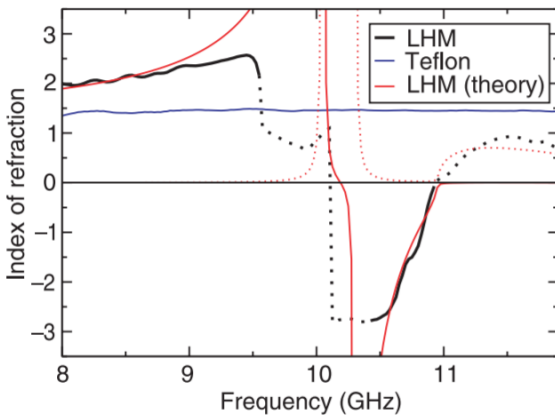


Figure 12: Index of refraction versus frequency. The blue curve: data from the Teflon sample. Black curve: the LHM data. The dotted black curve: regions where the index is expected to be either outside the limit of detection ($|n| > 3$) or dominated by the imaginary component and therefore could not be reliably determined experimentally. The solid red curve: the real component of the theoretical expression. The dotted red curve: the imaginary component of the theoretical expression. [3]

This confirmed the predictions of Maxwell's equations that n is given by the negative square root of $\epsilon \cdot \mu$ for the frequencies where both the permittivity (ϵ) and the permeability (μ) are negative.

5 Applications

Any new phenomenon or a different property appears in metamaterial, the researchers make advantage of it and use it as a new application. There are many applications. In this article I will talk about lenses and cloaking.

5.1 Metamaterials as a Perfect Lens

The first article mentioned the use of metamaterials as lens was published in 2000 by Pendry [4]. He called it 'perfect lens'.

As shown in figure(13), in a DNS material the refracted waves are scattered and then focused in a point. This allows something called subwavelength imaging. Subwavelength imaging is an optical microscopy with the ability to

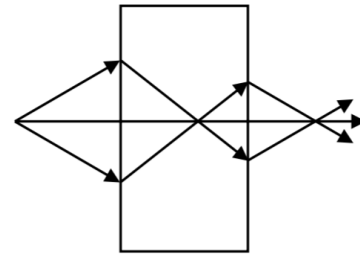


Figure 13: Representation of refracted waves in a DNS material

see details of an object or organism below the wavelength of visible light. Therefore, using metamaterial makes high resolution. In other words, to have the capability to observe, in real time, below 200 nanometers. For example: observing a whole living cell, observing cellular processes, such as how proteins and fats move in and out of cells. And In the technology domain, it is used in manufacturing small computer chips.

After Pendry, many experiments have been motivated by this idea, and several authors have reported observing perfect lens effects in DNM for small frequency intervals [5] [6].

Nevertheless, there are several limitations in these lenses. Such as: the limited frequency range of effect, the difficulty to construct an ideal perfect lens with isotropic DNM properties, and facing attenuation of signals.

So, Pendry predicted that double negative metamaterials (DNG) with a refractive index of $n = -1$, can act, at least in principle, as a "perfect lens" allowing imaging resolution which is limited not by the wavelength, but rather by material quality.

5.2 Cloaking & Invisibility

Surprisingly, metamaterial can be used as a cloaking device. Figure(14) illustrates the cross section of a PEC cylinder subject to a plane wave (only the electric field component of the wave is shown). At the left, the field is scattered, while at the right, the field remains unchanged outside the cloak and the cylinder is invisible electromagnetically. Note the special distortion pattern of the field inside the cloak. It represents a circular cloak, designed using transformation optics methods, and it is used to cloak the cylinder.

Metamaterial cloaking is based on 'transformation optics'. It describes the process of shielding something from view by controlling electromagnetic radiation. Objects in the defined location are still present, but incident waves are guided around them without being affected by the object itself.

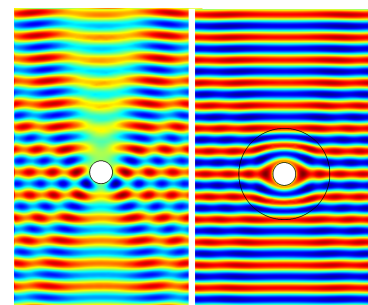


Figure 14: The cross section of a PEC cylinder subject to a plane wave

6 Conclusion

Negative index of refraction (DNG) material was predicted theoretically to exist in 1968 by Veselago and recently experimentally made by researchers.

There are many interesting effects that characterize metamaterials, such as: reverse Cherenkov radiation, reverse Doppler effect, Inverse Snell's law, etc.

Metamaterial can be made by using (for example) many cells of SRRs.

There are many applications for metamaterials. It can be used as a perfect lens, or as a cloaking device.

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