

**Phys332/Final Exam**  
**Saturday 23/1/2021**

1. (10 %) A square loop of wire with sides length  $a$ , lies in the first quadrant of the  $xy$ -plane, with one corner at the origin. In this region, there is a non-uniform time-dependent magnetic field  $\vec{B}(x, y, t) = 16x^3y^3t^2\hat{z}$ . Find the emf induced in the loop.
2. (15%) Consider an electromagnetic plane wave that is linearly polarized in the  $x$ -direction (i.e.  $\vec{E} = E_0 e^{i(kz - \omega t)} \hat{x}$ ). Using Maxwell's equations, show that (i)  $\vec{E} \perp \vec{B}$  (ii)  $E_0 = cB_0$ .
3. (15%) Consider a sphere with a total charge  $Q$  uniformly spread on its surface, with a radius  $R(t)$  which pulsates:  $R(t) = R_0 + R_1 \cos(\omega t)$ , where  $0 < R_1 < R_0$ , so  $R(t)$  is always positive. Show that this system does not radiate, even though the charges are accelerating.
4. (16%) If in an inertial system ( $\bar{S}$ ) that is moving with a speed  $v$  to the right relative to another inertial system  $S$ , show that for an electromagnetic wave that satisfies  $\vec{E} \cdot \vec{B} = 0$  and  $E^2 = cB^2$  in  $S$ , satisfies  $\vec{E} \cdot \vec{B} = 0$  and  $\bar{E}^2 = c\bar{B}^2$  in  $\bar{S}$ .
5. (10%) If the electric and magnetic fields are perpendicular at some point  $(x, y, z)$  in frame of reference  $S$ . Find the velocity  $v$  of  $\bar{S}$  relative to  $S$  in which  $\bar{B} = 0$  at  $(\bar{x}, \bar{y}, \bar{z})$ .
6. (10%) A frame  $\bar{S}$  moves with a constant velocity  $\vec{v}$  along  $x$  relative to  $S$ . If a rod in  $\bar{S}$  makes the angle  $\bar{\theta}$  with the direction of motion. What angle does the rod makes in  $S$ ?
7. (24%) Consider two identical charges separated by a vertical distance  $d$  and moving parallel to each other with a velocity  $\vec{v} = v_0 \hat{x}$  with respect to the lab frame of reference ( $S$ ). Find the force between the two charges in  $S$ .

Hint: This can be done by first finding the force between the two particles in a frame of reference ( $\bar{S}$ ) that move with a velocity  $\vec{v}$  with respect to  $S$  and in which the two charges at rest, and subsequently transforming the force appropriately to find in  $S$ .