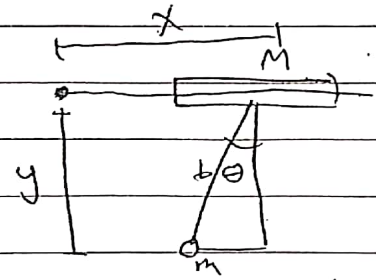
Q#1) a) $M(X, 0)$ ~~$m(l\cos\theta, X - l\sin\theta)$~~

$m(x_m, y_m)$

$\therefore$ ~~$T = \frac{1}{2} M X^2 + \ldots$~~ $m(X - l\sin\theta, l\cos\theta)$



$$\dot{y}_m = -l\sin\theta\,\dot{\theta} \qquad \dot{x}_m = \dot{X} - l\cos\theta\,\dot{\theta}$$

$$T = \frac{1}{2} M \dot{X}^2 + \frac{1}{2} m (\dot{x}_m^2 + \dot{y}_m^2)$$

$$T = \frac{1}{2} M \dot{X}^2 + \frac{1}{2} m\left[(\dot{X} - l\cos\theta\,\dot{\theta})^2 + l^2 \sin^2\theta\,\dot{\theta}^2\right]$$

$$= \frac{1}{2} M \dot{X}^2 + \frac{1}{2} m (\dot{X}^2 + l^2\cos^2\theta\,\dot{\theta}^2 - 2\dot{X} l\cos\theta\,\dot{\theta} + l^2\sin^2\theta\,\dot{\theta}^2)$$

$$= \frac{1}{2}(M+m)\dot{X}^2 + \frac{1}{2} m (l^2\dot{\theta}^2 - 2l\cos\theta\,\dot{X}\dot{\theta})$$

$$U = mg\,y_m = -mgl\cos\theta$$

$$\boxed{L = T - U = \frac{1}{2}(M+m)\dot{X}^2 + \frac{1}{2} m (l^2\dot{\theta}^2 - 2l\cos\theta\,\dot{X}\dot{\theta}) + mgl\cos\theta}$$

b) $$\boxed{\frac{\partial L}{\partial X} - \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{X}}\right) = 0}$$

$$\frac{\partial L}{\partial X} = 0$$

$$\frac{\partial L}{\partial \dot{X}} = (m+M)\dot{X} - ml\cos\theta\,\dot{\theta}$$

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$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}}\right) = (m+M)\ddot{x} + ml\sin\theta\dot{\theta}^2 - ml\cos\theta\ddot{\theta} = 0$$

$$\frac{\partial L}{\partial \theta} - \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\theta}}\right) = 0$$

$$\frac{\partial L}{\partial \theta} = -mgl\sin\theta \cdot \dot{x}$$

why not ?

$$\frac{\partial L}{\partial \dot{\theta}} = ml^2\dot{\theta} - ml\cos\theta\dot{x}$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\theta}}\right) = ml^2\ddot{\theta} + ml\sin\theta\dot{x}\dot{\theta} - ml\cos\theta\ddot{x}$$

$$-mgl\sin\theta - ml^2\ddot{\theta} - ml\sin\theta\dot{x}\dot{\theta} + ml\cos\theta\ddot{x} = 0$$

$$-g\sin\theta - l\ddot{\theta} - \sin\theta\dot{x}\dot{\theta} + \cos\theta\ddot{x} = 0$$

constants at . mind ?

-2

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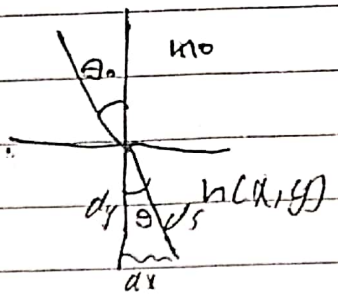
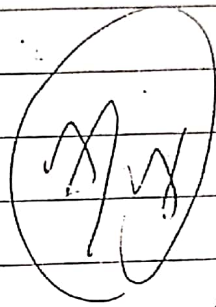
Q#2 $n(x,y) = n_0(1+Ky)$ $\qquad V = \frac{c}{n}$



$dt = \frac{ds}{V}$

$ds^2 = \sin\theta\, dx^2 + dy^2$

$ds = \sqrt{d^2x + d^2y}$



$dt = \frac{ds}{V} = \frac{\sqrt{d^2x + d^2y}}{V}$

$= \frac{n_0}{c}\sqrt{d^2x + d^2y}\,(1+Ky)$

$* \; V = \frac{c}{n}$

$= \frac{c}{n_0(1+Ky)}$

$dt = \frac{n_0}{c}\int (1+Ky)\sqrt{d^2x + d^2y}$

$\dot{x} = \frac{dx}{dy}$

$t = \frac{n_0}{c}\int (1+Ky)\sqrt{\dot{x}^2+1}\; dy$

$f = (1+Ky)\sqrt{\dot{x}^2+1}$

$\frac{\partial f}{\partial x} - \frac{d}{dy}\left(\frac{\partial f}{\partial \dot{x}}\right) = 0$

$\frac{\partial f}{\partial x} = 0$

$\frac{d}{dy}\left(\frac{\partial f}{\partial \dot{x}}\right) = 0 \Rightarrow \frac{\partial f}{\partial \dot{x}} = \text{constant} = C$

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$$\frac{\partial f}{\partial \dot{\alpha}} = \frac{(1+\kappa y)\,\dot{x}}{\sqrt{1+\dot{x}^2}} = c$$

$$(1+\kappa y)^2 \dot{x}^2 = 1 + \dot{\alpha}^2$$

$$(1+\kappa y)^2 \dot{\alpha}^2 - \dot{\alpha}^2 = 1$$

$$\dot{\alpha}^2 = \frac{1}{(1+\kappa y)^2 - 1}$$

$$\dot{x} = \frac{1}{\sqrt{(1+\kappa y)^2 - 1}}$$

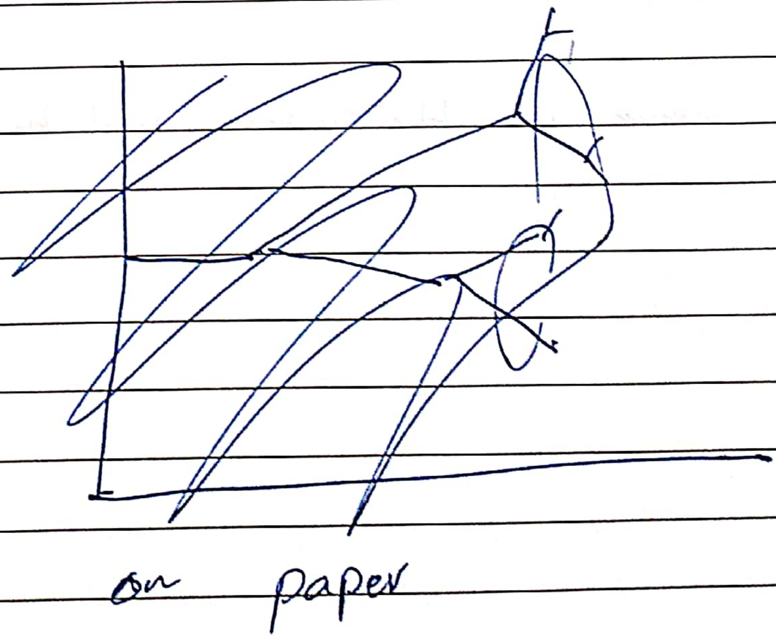
$$x = \int \frac{1}{\sqrt{(1+\kappa y)^2 - 1}}\, dy$$

Q1 $x_{n+1} = a - x_n^2$ if $x_1 = 0$

a =	0.5	1.476	2
x_2	0.5	1.476	2
3	0.25	-0.70225	2
4	0.4375	0.9823	2
5	0.308	0.5109	2
6	0.404	1.2149	2
7	0.3361	-0.00013	2
8	0.3869	1.47599	
⋮		-0.7025	
		⋮	
	0.366		
	0.366	0.9827	
	0.366	0.5109	
		-0.000140	

a = 1

$x_1 = 0$
$x_2 = 1$
$x_3 = 0$
$x_4 = 1$
$x_5 = 0$
$x_6 = 1$
$x_7 = 0$
$x_8 = 1$
$x_9 = 0$
⋮ 1
⋮ 0
⋮ 1
$x_{20} = 1$



on paper

~~...tion between successive iterations...~~

~~the first Graph explains how the system behaves when $\alpha = 0$~~

* when $a = 0.5$ the system will not be chaiotich and the successive iterations will not effect the system behavior hugly

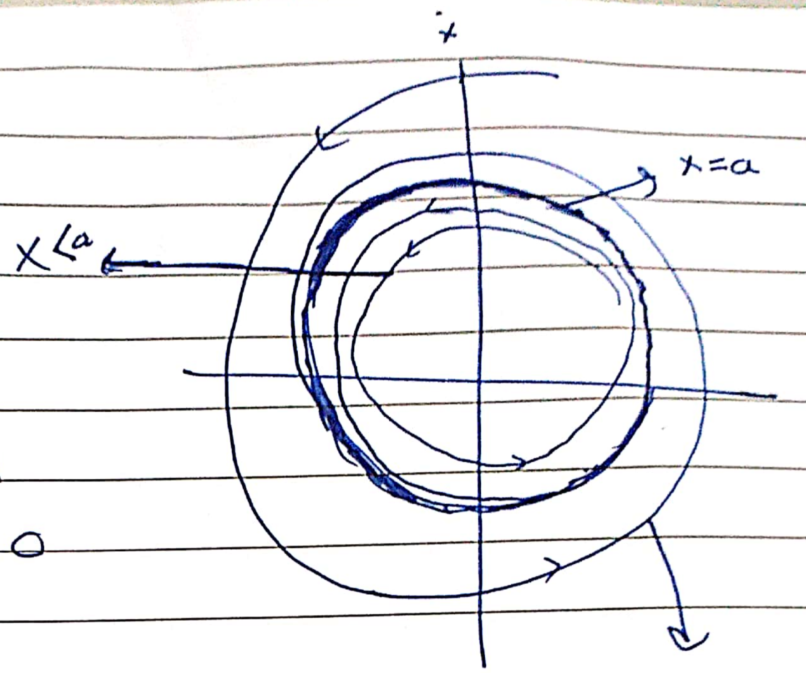
* when $a = 1$ or from ≈ 1.7 to 1.25 the system will shift betwen tow position

* when a around 1.47 the system will have 7 states and shifts between them

* above 1.5 the system will be chaiotich

$x > a$

if $x > a$ the system will lose energy until ~~the~~ $x = a$ and the the equation will change to $\ddot{x} + \omega_0^2 x = 0$



if $x < a$ the system will gain energe until $x = a$ and then the equation will became $\ddot{x} + \omega_0^2 x = 0$

$x > a$

6/6

Q2 $T = \frac{1}{2} m \left[\dot{r}^2 + r^2 \dot{\phi}^2 \right]$ $y = r\sin\phi$

$U = + mgr\sin\phi + \frac{A}{\alpha} r^{\alpha}$

$L = \frac{1}{2} m \left[\dot{r}^2 + r^2 \dot{\phi}^2 \right] + mgr\sin\phi - \frac{A}{\alpha} r^{\alpha}$

$\frac{dL}{dr} - \frac{d}{dt}\frac{dL}{d\dot{r}} = 0$

$F = -\nabla U$

$U = -\int F dx$

$mr\dot{\phi}^2 + mg\sin\phi - A r^{\alpha-1} - \frac{d}{dt} m\dot{r} = 0$

$\ddot{r} - r\dot{\phi}^2 + g\sin\phi + \frac{A}{m} r^{\alpha-1} = 0$ ①

$U = -\int -A r^{\alpha-1} dr$

$= + \frac{A}{\alpha} r^{\alpha}$

$dr + r d\theta$

$dr =$

$\frac{dL}{d\phi} - \frac{d}{dt}\frac{dL}{d\dot{\phi}} = 0$

$+ mgr\cos\phi - \frac{d}{dt} mr^2\dot{\phi} = 0$

$mr^2\ddot{\phi} + 2mr\dot{r}\dot{\phi} + mgr\cos\phi = 0$

$\ddot{\phi} + \frac{2}{r}\dot{r}\dot{\phi} - \frac{g}{r}\cos\phi = 0$ ②

c) $P_\theta = \frac{\partial L}{\partial \dot{\phi}} = mr^2\dot{\phi}$

$\dot{P}_\theta = -\frac{dH}{d\theta} = -\frac{d(K+U)}{d\theta}$

d) Yes it is since U dont depend on velocity and k is t independent -1

shud show that h is const. function → h = TAU

e) $H = U + K = \frac{1}{2}m\dot{r}^2 + \frac{1}{2}mr^2\dot{\phi}^2 + \frac{A}{\alpha}r^{\alpha} - mgr\sin\phi$

This is not $H(p_r, p_\phi, r, \phi)$ no $\dot{\phi}, \dot{r}$ X

~~$\dot{P}_\phi = -\frac{\partial H}{\partial \phi} = \frac{1}{2}mr^2\dot{\phi}$~~

c) $\dot{P}_\phi = +mgr\cos\phi \neq 0$

∴ Angular momentum is not conserved ✓

f) $\dot{P}_\phi = +mgr\cos\phi$

$\dot{P}_r = -\frac{\partial H}{\partial r} = -mr\dot{\phi}^2 + -Ar^{\alpha-1} + mg\sin\phi$

$P_\phi = \frac{\partial L}{\partial \dot{\phi}} = mr^2\dot{\phi}$

$P_r = \frac{\partial L}{\partial \dot{r}} = \frac{m\dot{r}^2}{2} = m\dot{r}$

7/12

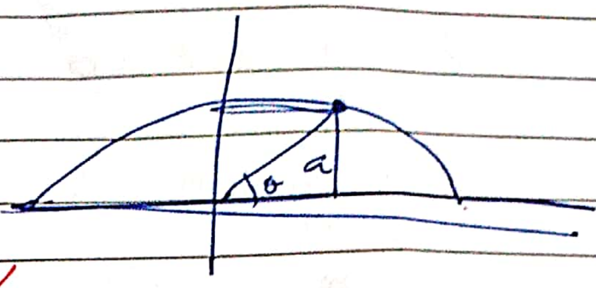
$\dot{P}_r = m\ddot{r} = -mr\dot{\phi}^2 - Ar^{\alpha-1} + mg\sin\phi$

$\ddot{r} + r\dot{\phi}^2 + \frac{A}{m}r^{\alpha-1} - g\sin\phi$ — (1)

$\dot{P}_\phi = mr^2\ddot{\phi} + 2m\dot{\phi}r\dot{r} = mgr\cos\phi$

$\ddot{\phi} + \frac{2\dot{r}}{r}\dot{\phi} - \frac{g}{r}\cos\phi = 0$ a

Q3 $f = r - a = 0$



~~$T = \frac{1}{2}m\left[\dot{r}^2 + r\sin\theta\right.$~~

$$T = \frac{1}{2}m\left[\dot{r}^2 + r^2\dot{\theta}^2\right]$$ ✓

$$u = mgy = -mgr\sin\theta$$

$$T - u = L = \frac{1}{2}m\dot{r}^2 + \frac{1}{2}mr^2\dot{\theta}^2 + mgr\sin\theta$$

~~$\frac{d}{dt}\frac{\partial L}{\partial \dot{r}} - \frac{\partial L}{\partial r} = 0$~~

$$\frac{d}{dt}\frac{\partial L}{\partial \dot{r}} - \frac{\partial L}{\partial r} + \lambda\frac{\partial f}{\partial r} = 0$$

$$\frac{\partial}{dt}\frac{\partial L}{\partial \dot{\theta}} - \frac{\partial L}{\partial \theta} + \lambda\frac{\partial f}{\partial \theta}$$ → what is f

$$\frac{d}{dt}\{m\dot{r}\} - mr\dot{\theta}^2 - mg\sin\theta + \lambda = 0$$

$$m\ddot{r} - mr\dot{\theta}^2 - mg\sin\theta + \lambda = 0 \quad \text{—— (1)}$$

$$\frac{d}{dt}\left[mr^2\dot{\theta}\right] - mgr\cos\theta + \theta = 0$$

$$mr^2\ddot{\theta} + 2mr\dot{r}\dot{\theta} - mgr\cos\theta = 0 \quad \text{—— (2)}$$

$r = a$, $\dot{r} = 0$, $\ddot{r} = 0$ →

(1) → $ma\dot{\theta}^2 + mg\sin\theta - \lambda = 0$

(2) → $ma^2\ddot{\theta} - mga\cos\theta = 0$

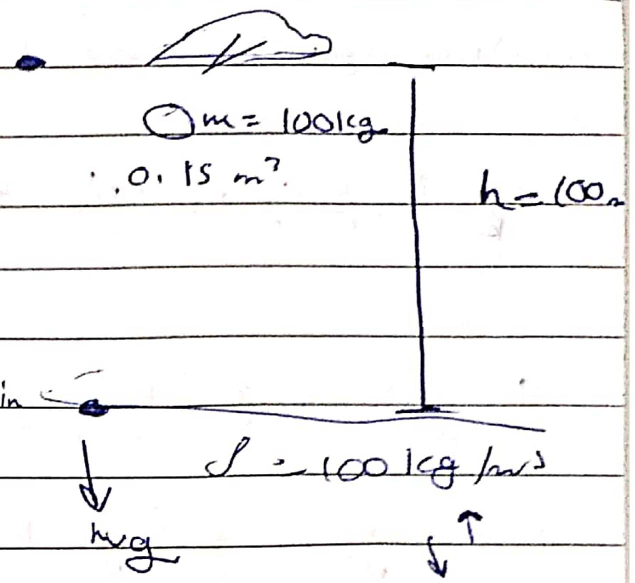
✓

⟹

$$\ddot{\theta} - \frac{g}{a}\cos\theta = 0$$

$$\dot{\theta}^2 + \frac{g}{a}\sin\theta = \lambda$$

Q1



$F_R = -kv \qquad k = 10\ \frac{N\,sec}{m}$

$ma = -mg - kv$ after going in water ↓ mg

when is in sky $\frac{1}{2}mv^2 = mgh$

$v = \sqrt{2gh}$

$v = \sqrt{~~2\times10\times100~~} = \sqrt{20\times100} = 20\sqrt{5}\ \frac{m}{s}$

$ma = -mg - \frac{k}{m}v$

$\frac{dv}{dt} = -g - \frac{k}{m}v$

Consider Force buoyancy or not ride.

$-\frac{k}{m}\int \frac{dv}{\left(g - \frac{k}{m}v\right)} = t$

g −

$v = \sqrt{2gh}$

$\frac{m}{k}\ln\left(g - \frac{k}{m}v\right) = t + c \qquad$ ~~v = 20√5 t = 0~~ $\quad t = 0$

~~c = m/k ln g~~

$\frac{m}{k}\ln\left(g - \frac{k}{m}v\right) = t + \frac{m}{k}\ln g$

$g - \frac{k}{m}v = e^{\ln g + \frac{k}{m}t} = g\,e^{\frac{k}{m}t}$

$v = g\frac{m}{v}\left(1 - e^{\frac{k}{m}t}\right)$

$\int dx = g\frac{m}{v}\int\left(1 - e^{\frac{k}{m}t}\right)dt = g\frac{m}{v}t - \frac{m}{k}e^{\frac{k}{m}t} + c$

$x = g\frac{m}{v}t -$

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$$V = \sqrt{gh2} \quad \text{at } t = 0 \qquad C = \frac{m}{k}\ln\left(g - \frac{k}{m}\sqrt{gh2}\right)$$

$$\frac{m}{k}\ln\left(g - \frac{k}{m}v\right) = \frac{k}{m}t + \frac{m}{k}\ln\left(g - \frac{k}{m}\sqrt{2gh}\right)$$

$$g - \frac{k}{m}v = \left(g - \frac{k}{m}\sqrt{2gh}\right) e^{\frac{k}{m}t}$$

$$v = \frac{m}{k}g - \frac{m}{k}\left(g - \frac{k}{m}\sqrt{2gh}\right) e^{\frac{k}{m}t}$$

$$\int dx = \int \frac{m}{k}\left(g - \left(g - \frac{k}{m}\sqrt{2gh}\, e^{\frac{k}{m}t}\right)\right) dt$$

2/6

$$x = \frac{m}{k}gt - \frac{m^2}{k^2}\left(g - \frac{k}{m}\sqrt{2gh}\right) e^{\frac{k}{m}t} + C$$

$$C = \frac{m^2}{k^2}\left(g - \frac{k}{m}\sqrt{2gh}\right) \quad \text{at } x = 0$$

$$x = \frac{m}{k}gt - \frac{m^2}{k^2}\left(g - \frac{k}{m}\sqrt{2gh}\right)\left(e^{\frac{k}{m}t} + 1\right)$$

λ

$V = 0$ max depth

$$\sqrt{2gh} = e^{\frac{k}{m}t}$$

$$t = \frac{m}{k}\ln\sqrt{2gh}$$

$$x = \frac{m^2}{k^2}g\ln\sqrt{2gh} - \frac{m^2}{k^2}\left(g - \frac{k}{m}\sqrt{2gh}\right)\left(\sqrt{2gh} + 1\right)$$

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Q2 $\nabla . F = \frac{1}{r}\frac{dF}{\partial r} = \frac{1}{r} 2n\left(r^{2n-1}\right) = 2nr^{2n}$

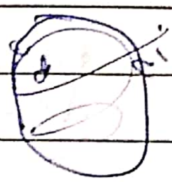
$$\nabla \times F = \begin{vmatrix} \vec{r} & \vec{\varphi} & \vec{z} \\ \frac{1}{r}\frac{\partial}{\partial r} & \frac{1}{r}\frac{\partial}{\partial \varphi} & \frac{\partial}{\partial z} \\ 0 & 0 & r^{2n} \end{vmatrix} = r\left(\frac{1}{r}\left(\frac{\partial r^{2n}}{\partial \varphi}\right)\right) = 0$$

$\nabla \times F = 0$

$\tau = \frac{d(\beta)}{dt}$

$I \omega$

part (2) $F_B = q\, v \times B = q \begin{vmatrix} i & j & k \\ v_{ox} & v_{oy} & 0 \\ 0 & 0 & B_0 \end{vmatrix}$



$F = qE$

$= qB_0 [\, v_{oy}\, i - v_{ox}\, j \,]$

$= qB_0 (v_{oy}\, i - v_{ox}\, j)$

$F = qE = qE_0 x\, i$

in x axis $F_x = qE_0 x + qB_0 v_{oy}$

in y axis $F_y = qB_0 v_{ox}$

3/3

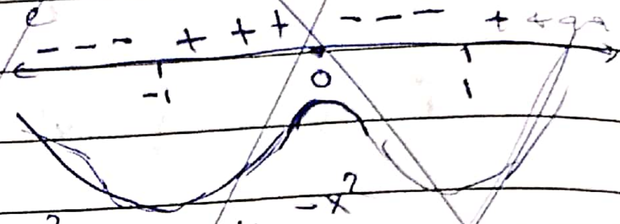
dx

$2xe^{-x^2} = 2x^3 e^{-x^2}$

$x = 0$

$x^2 = 1 \Rightarrow x = \pm 1$

$\frac{d^2u}{dx} = -2V_0 e^{-x^2} + 4x^2 V_0 e^{-x^2} + V_0 6x^2 e^{-x^2} - 4V_0 x^4 e^{-x^2}$



$\frac{d^2u}{dx} = -2V_0 e^{-x^2} + 4x^2 V_0 e^{-x^2} + V_0 6x^2 e^{-x^2} - 4V_0 x^4 e^{-x^2}$

$= (-2V_0 + 10V_0 x^2 - 4V_0 x^4) e^{-x^2}$

$\left.\frac{du}{dx}\right|_{x=0} = -2V_0$

$u|_{x=1} = -V_0 e^{-1}$ ← max Energy at both $x = -1, 1$

$u|_{x=-1} = -V_0 = e^{-1}$

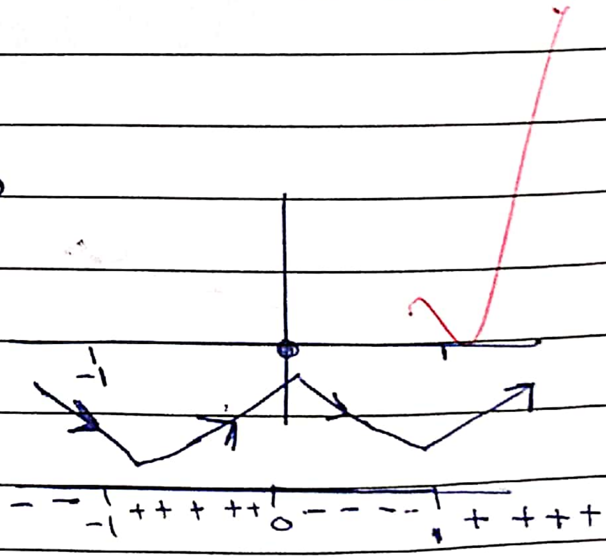
$m\ddot{x} = -kx$ $\quad -k = \left.\frac{dF}{dx}\right|_x$

$V(x) = -V_0 x^2 e^{-x^2}$

$\frac{dV}{dx} = -V_0 \left(2xe^{-x^2} + -2x^3 e^{-x^2}\right) = 0$

$x = 0$

$x = \pm 1$



$x = 0$ is a max

$x = \pm 1$ is a min

$E_{max} = U\big|_{x=0}$ $\frac{dU}{dx} = 0$

$E = 0$

$m\ddot{x} = -kx$ $k = -\frac{dF}{dt}\Big|_x$

$= -\frac{d^2U}{dx^2}\Big|_{x=1,-1}$

$\frac{dU}{dx} = -$ ~~$V_0 \cdot 2x^2$~~

$= -V_0\left[-2x^3e^{-x^2} + 2xe^{-x^2}\right]$

$\frac{d^2U}{dx} = -V_0 2\left[e^{-x^2}(3x^2+2) + (-x^3+x)2x\, e^{-x^2}\right]$

$\frac{d^2U}{dx}\Big|_{x=1} = 2V_0 e^{-1}$

$\frac{d^2U}{dx}\Big|_{x=-1} = 2V_0 e^{-1}$

6/6

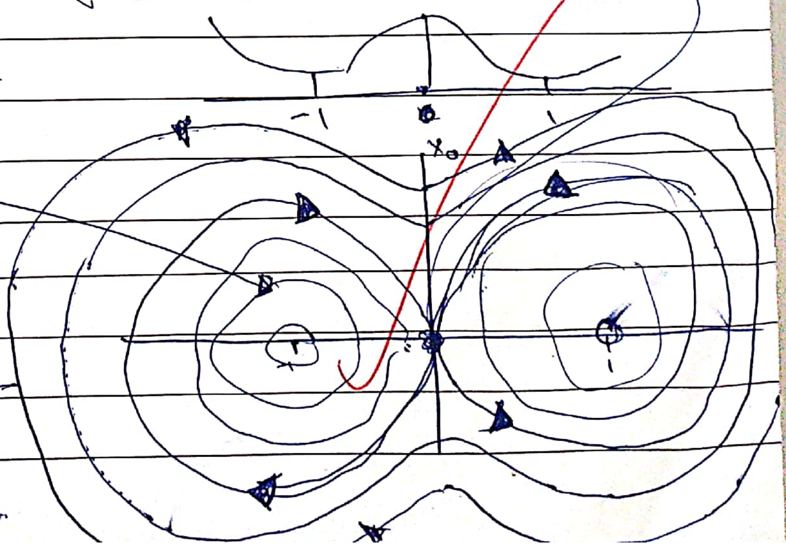
$\omega_0\big|_{x=-1} = \omega_0\big|_{x=1} = \sqrt{\frac{k}{m}} = \sqrt{\frac{2V_0}{em}}$

$T = \frac{1}{f} = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{em}{2V_0}}$

$x=0$ sparatix point

elliptic are circles

hyprbolic



Q4

$$r^2 + 2\beta r + \omega_0^2 = 0$$

$$r = \frac{-2\beta + \sqrt{4\beta^2 - 4\omega_0^2}}{2} = \frac{2\beta}{2} = \beta$$

$$x_c = (a + bt)e^{-\beta t}$$

$$x_p = A\cos(2\omega_0 t + \delta_1) + B\cos(3\omega_0 t + \delta_2)$$

A = ? B = ?

6

~~$-4\omega_0^2 A\cos 2\omega_0 t - 9\omega_0^2 B\cos 3\omega_0 t + \omega_0^2$~~

$$-4\omega_0^2 A\cos 2\omega_0 t - 9\omega_0^2 B\cos 3\omega_0 t - 4\beta\omega_0 A\sin 2\omega_0 t - 6\beta\omega_0 B\sin 3\omega_0 t + \omega_0^2 A\cos 2\omega_0 t + \ldots$$

$$= F_0\cos 2\omega_0 t + F_0\cos 3\omega_0 t$$

$$x_p = \frac{1}{m}\sum\left(\frac{\alpha_1}{\sqrt{(\omega_0^2 - (2\omega_0)^2)^2 + 8\omega_0^2\beta^2}}\right)\cos 2\omega_0 t - \varphi_0$$

3/6