Q1: A Y- shaped massless wire with two arms at 45° is free to rotate around a vertical axis. A small bead of mass *m* is free to slide on one arm of the wire.

1. Write down the Lagrangian for the system.
2. Find two constants of motion.
3. Find Lagrange equations of motion.
4. Find the equilibrium position of the bead in term of .

Q2: Consider a Lagrangian of the form

Where the particle of mass m moves in one dimension. Assume all constants are positive.

1. Find the Hamiltonain of the system.
2. Find Hamilton equations of motion.
3. Determine whether H is a constant of motion and how is it related to the total energy

Q3: The Fermat principle states that light always propagates along a path that takes the minimum amount of time. Consider a medium with an index of refraction given by . Recall that the speed of light in a medium with index is given by . Find the function that describes the path of light in this medium. Determine a specific equation for the path of a laser beam that initially starts at the origin propagating in the direction, using the second form’ of the Euler-Lagrange equation.