**Final exam Date:27/5/2021 Time:2.30 hours**

**Instructor: Dr. E. Badran**

Q1:A thin uniform bar of mass m and length $\frac{3l}{2}$ is suspended by a string of length $l$and negligible mass.

1. Show that the kinetic and potential energy are for small oscillations are:

$$T=\frac{1}{2}ml^{2} \left(\frac{3}{4}\dot{θ}^{2}+\dot{φ}^{2}+\frac{3}{2}\dot{θ}\dot{φ} \right), and V=\frac{1}{2}mgl(φ^{2}+\frac{3}{4}θ^{2})$$

1. Write the secular equation.
2. Find the normal frequencies.
3. Find normal modes for small oscillations in a plane and discuss the physical meaning of the normal modes.

Q2: Orbits around a black hole be described in terms of the effective potential

$$V\_{eff}=-\frac{1}{r}+\frac{L^{2}}{2r^{2}}-\frac{L^{2}}{r^{3}}$$

Where *L* is the orbit angular momentum. With respect to be classical Keplerian case, the only modification is the last 1/r3 term (for simplicity we are setting GN =1 and the reduced mass µ =1.) The above potential should be interested and used in the standard way, I,e,. the radial equation of motion for a point particle orbiting a black hole is that associated with the Lagrangian

 $L= \frac{1}{2}\dot{r}^{2}-V\_{eff}(r)$

1. Show that for L2 < 12 there are no circular orbits, whereas for L2 >12 there are *two*.
2. Shetch a plot of *V*eff (r) for L2 < 12 and for L2 > 12.
3. Describe the possible orbits for L2 > 12 and L2 < 12.

Q3: : Show that the moment of inertia of a uniform rectangular sheet of sides ***a, 2a*** and mass ***m*** in the xy- plane and one corner at the origin along the diagonal is

I = $\left[\begin{matrix}\frac{4ma^{2}}{3}&\frac{-ma^{2}}{2}&0\\\frac{-ma^{2}}{2}&\frac{ma^{2}}{3}&0\\0&0&\frac{5ma^{2}}{3}\end{matrix}\right]$

1. Find the angular momentum and kinetic energy, when it is rotating with angular velocity $ω$ about the diagonal through the origin.
2. Find the angular momentum and kinetic energy, when it is rotating with angular velocity $ω$ about the z – axis.
3. Find the principal moment of inertia of the rectangular sheet about the corner.
4. Find the direction of the principal axis of the largest principal moment.

Q4: **A** merry-go-round (carousel) starts from rest and accelerates with a

constant **angular** acceleration of **0.02** revolution per second **per** second. **A**

woman sitting on a chair 6 meters from the axis of revolution holds a **2** kg

ball. Calculate the magnitude and direction of the force

she must exert to hold the ball ***5*** seconds after the merry-go-round begins

to rotate. Specify the direction with respect to the radius of the chair on

which she is sitting.