$$\omega = 0 \quad \Rightarrow \quad R^2 \begin{bmatrix} 2k & -k & -k \\ -k & 2k & -k \\ -k & -k & 2k \end{bmatrix} \begin{pmatrix} A_1 \\ A_2 \\ A_3 \end{pmatrix} = 0$$

$$A_1 = A_2 = A_3$$

$$\frac{1}{\sqrt{3mR^2}} \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$$

$$\omega^2 = \frac{3k}{m} \quad \Rightarrow \quad \begin{pmatrix} -k & -k & -k \\ -k & -k & -k \\ -k & -k & -k \end{pmatrix} \begin{pmatrix} A_1 \\ A_2 \\ A_3 \end{pmatrix} = 0$$

$$A_1 + A_2 + A_3 = 0$$

The eigenvectors cannot be uniqutly fixed.

$$\frac{1}{\sqrt{6mR^2}} \begin{pmatrix} 2 \\ -1 \\ -1 \end{pmatrix}, \quad \frac{1}{\sqrt{2mR^2}} \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix}$$

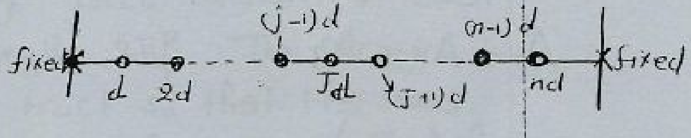
Spherical Double pendulum

$$x_1 = \sin(\theta_1)\cos(\varphi_1) \qquad x_2 = x_1 + \sin(\theta_2)\cos(\varphi_2)$$
$$y_1 = \sin(\theta_1)\sin(\varphi_1) \qquad y_2 = y_1 + \sin(\theta_2)\sin(\varphi_2)$$
$$z_1 = -\cos(\theta_1) \qquad z_2 = z_1 - \cos\theta_2$$

$$T = \frac{1}{2}(\dot{r}_1^2 + \dot{r}_2^2) \qquad U = g(z_1 + z_2)$$

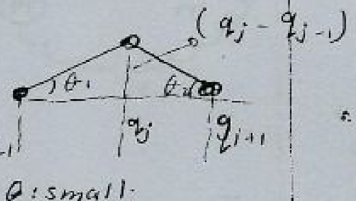
The ~~Lod~~ Loaded String

- Equal masses & equally spaced
- Fixed endpoints



- Considering only nearest neighbour interaction.

Now, Consider the j-th particle that is displaced in transverse fashion from its equilibrium



$$F_j = \tau \sin\theta_1 - \tau \sin\theta_2$$

$$= -\frac{\tau}{d}(q_j - q_{j-1}) - \frac{\tau}{d}(q_j - q_{j+1}) \quad \theta : \text{small}$$

$$m\ddot{q}_j = \frac{\tau}{d} q_{j-1} - \frac{2\tau}{d} q_j + \frac{\tau}{d} q_{j+1}$$

on the other hand if we want to solve in Lagrangian method.

$$U = \frac{\tau}{2d} \sum_{j=1}^{n+1} (q_{j-1} - q_j)^2$$

$$F_j = -\frac{\partial U}{\partial q_j} = \frac{\tau}{d}(q_{j-1} - 2q_j + q_{j+1}).$$

$$L = \frac{1}{2} \sum_{j=1}^{n+1} \left(m\dot{q}_j^2 - \frac{\tau}{d}(q_{j-1} - q_j)^2\right)$$

Equation of motion.

$$\frac{\partial}{\partial t}\left(\frac{\partial L}{\partial \dot{q}_j}\right) - \frac{\partial L}{\partial q_j} = 0$$

$$\frac{d}{dt}(m\dot{q}_j) - \frac{\tau}{d}(q_{j-1} - 2q_j + q_{j+1}) = 0$$

$$m\ddot{q} - \frac{\tau}{d}(q_{j-1} - 2q_j + q_{j+1}) = 0$$

بطلع عنا n من المعادلات

$$m\ddot{q}_{j-1} - \frac{\tau}{d}(q_{j-2} - 2q_{j-1} + q_j) = 0$$

$$m\ddot{q}_j - \frac{\tau}{d}(q_{j-1} - 2q_j + q_{j+1}) = 0$$

$$m\ddot{q}_{j+1} - \frac{\tau}{d}(q_j - 2q_{j+1} + q_{j+2}) = 0$$

$$\vdots$$

let $q_j = a_j e^{i\omega t}$

$$-m\omega^2 a_j - \frac{\tau}{d}(a_{j-1} - 2a_j + a_{j+1}) = 0$$

$$-\frac{\tau}{d} a_{j-1} + \left(\frac{2\tau}{d} - m\omega^2\right) a_j - \frac{\tau}{d}(a_{j+1}) = 0$$

عشان يطلع لنا nontrivial solution لازم تكون $\det(A) = $ صفر

A: coefficient matrix

لسى قبل هيك

For $j=1$ $\quad \left(\frac{2\tau}{d} - m\omega^2\right)a_1 - \frac{\tau}{d}a_2 = 0$

معادلة اول
جسيم وآخر
جسيم غير عن
الباقي

For $j=n$ $\quad -\frac{\tau}{d}a_{n-1} + \left(\frac{2\tau}{d} - m\omega^2\right)a_n = 0$

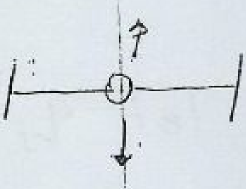
Now, let us construct the Matrix of the coefficients

first, let. $\frac{2\tau}{d} - m\omega^2 = \lambda$

$$\begin{vmatrix} \lambda & -\frac{\tau}{d} & 0 & 0 & 0 \\ -\frac{\tau}{d} & \lambda & -\tau/d & 0 & 0 \\ 0 & -\tau/d & \lambda & -\tau/d & 0 \\ \vdots & \vdots & \vdots & \lambda & \vdots \\ 0 & 0 & 0 & -\tau/d & \lambda \end{vmatrix}_{n\times n} = 0$$

For $\boxed{n=1}$

$$\frac{2\tau}{d} - m\omega^2 = 0 \quad \Rightarrow \omega = \sqrt{\frac{2\tau}{md}}$$



For $\boxed{n=2}$

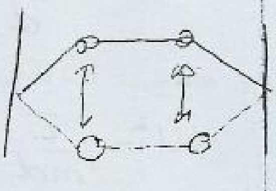
$$\begin{vmatrix} \lambda & -\tau/d \\ -\tau/d & \lambda \end{vmatrix} = 0 \quad \Rightarrow \frac{2\tau}{d} - m\omega^2 = \mp\frac{\tau}{d}$$

$$\omega_{12}^2 = \frac{\tau}{md}(2 \mp 1).$$

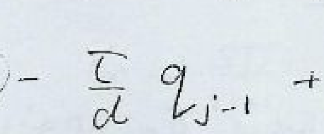
$$\boxed{\lambda = \frac{\tau}{d}} \qquad \begin{bmatrix} \tau/d & -\tau/d \\ -\tau/d & \tau/d \end{bmatrix} \begin{bmatrix} a_{11} \\ a_{12} \end{bmatrix} = 0$$

$$\frac{\tau}{d} a_{11} - \frac{\tau}{d} a_{12} = 0$$

$$\boxed{a_{11} = a_{12}}$$



$$\lambda = -\frac{\tau}{d}$$



$$a_{12} = -a_{22}$$

For Large Number of Particles.

هذا الحد الوحيد المشتق بالنسبة لـ t

$$m\ddot{q}_j - \frac{\tau}{d} q_{j-1} + \frac{2\tau}{d} q_j - \frac{\tau}{d} q_{j+1} = 0$$

$$q_j = a_j e^{i\omega t} = \underbrace{a e^{i(j\gamma - \delta)}}_{a_i} e^{i\omega t}$$

phase

$$q_{j-1} = a e^{i((j-1)\gamma - \delta)} e^{i\omega t}$$

$$q_{j+1} = a e^{i((j+1)\gamma - \delta)} e^{i\omega t}$$

$$m\ddot{q}_j - \frac{\tau}{d} q_{j-1} + \frac{2\tau}{d} q_j - \frac{\tau}{d} q_{j+1} = 0$$

$$-m\omega^2 a\, e^{i((j-1)\gamma)} + \frac{2\tau}{d} e^{ij\gamma} - \tau/d\, e^{i(j+1)\gamma} = 0.$$

$$-m\omega^2 - \frac{\tau}{d} e^{-i\gamma} + \frac{2\tau}{d} - \frac{\tau}{d} e^{i\gamma} = 0$$

$$-m\omega^2 + -\frac{\tau}{d}\left(e^{i\gamma} + e^{-i\gamma}\right) + \frac{2\tau}{d} = 0$$

$$\omega^2 = \frac{\tau}{md}\left(2 - e^{i\gamma} + e^{-i\gamma}\right).$$

$$\omega^2 = \frac{2\tau}{md}\left(1 - \cos\gamma\right).$$

$$= \frac{4\tau}{md} \sin^2\left(\frac{\gamma_r}{2}\right) \qquad r = 1, 2, \ldots$$

$$\omega = 2\sqrt{\frac{\tau}{md}} \sin\left(\frac{\gamma_r}{2}\right).$$

عشان نوجد γ_r نطبق BC'S وهون BC's انه عل الأطراف fixed.

From last lecture

$$\omega = 2\sqrt{\frac{T}{md}}\ \sin\left(\frac{\gamma_r}{2}\right)$$

let $a_{jr} = a_r\, e^{i(j\gamma_r - \delta_r)}$

If we consider the real part

$$a_{jr} = a_r \cos(j\gamma_r - \delta_r)$$

حسب BC's نطلع δ_r ، γ_r

الأطراف fixed $\quad a_{0r} = a_{(n+1)r} = 0$

$$0 = a_r \cos\left(j\gamma_r - \frac{\pi}{2}\right) = a_r \sin(j\gamma_r).$$

$$a_r \sin((n+1)\gamma_r) = 0$$

$$(n+1)\gamma_r = s\pi$$

$$\gamma_r = \frac{s\pi}{n+1}, \quad s = 1, 2, 3, \ldots$$

$$\gamma_r = \frac{r\pi}{n+1}$$

$$a_{jr} = a_r \sin j\left(\frac{r\pi}{n+1}\right)$$

The General Solution

$$q_j = \sum \beta_r' a_r \sin j\left(\frac{r\pi}{n+1}\right) e^{i\omega_r t}$$

$$\omega_r = 2\sqrt{\frac{I}{md}} \sin\left(\frac{\theta_r}{2}\right)$$

$$= 2\sqrt{\frac{I}{md}} \sin\left(\frac{r\pi}{2(n+1)}\right)$$

$$\eta_r(t) = \beta_r e^{i\omega_r t} \quad \& \quad \beta_r = \beta_r' a_r$$

$$q_j(t) = \sum_r \eta_r(t) \sin j\left(\frac{r\pi}{n+1}\right)$$

Real part $Re[q(t)]$

$$q_j = \sum \sin j\left(\frac{r\pi}{n+1}\right)\left(\mu \cos\omega_r t - \nu \sin\omega_r t\right)$$

$\beta_r = \mu + i\nu$

HW 13 14 16 18 24 21