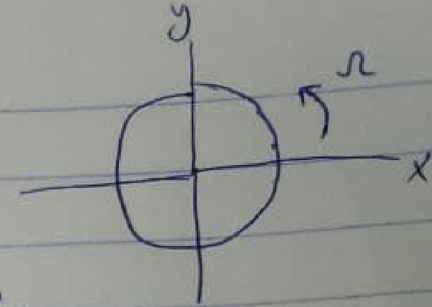
Q1) Initial conditions:

$(x_0, 0)$, (V_x, V_y)



$\omega = \Omega \hat{z}$

$F_{eff} = m\vec{a}_r = -m\vec{\omega}\times(\vec{\omega}\times\vec{r}) - 2m\vec{\omega}\times\vec{V}_r$

Let $\vec{r} = x\hat{x} + y\hat{y}$

$\vec{v} = \dot{x}\hat{x} + \dot{y}\hat{y}$

$\vec{\omega}\times\vec{r} = \Omega\hat{z}\times(x\hat{x} + y\hat{y})$

$= \Omega x\hat{y} - \Omega y\hat{x}$

$\square$ $w\times(w\times r) = \Omega\hat{z}\times(\Omega x\hat{y} - \Omega y\hat{x})$

$= -\Omega^2 x\hat{x} - \Omega^2 y\hat{y}$

$= -\Omega^2(x\hat{x} + y\hat{y}) = -\Omega^2\vec{r}$

$\square$ $\vec{\omega}\times\vec{V}_r = \Omega\hat{z}\times(\dot{x}\hat{x} + \dot{y}\hat{y})$

$= \Omega\dot{x}\hat{y} - \Omega\dot{y}\hat{x}$

$\Rightarrow$ $\ddot{x} = \Omega^2 x + 2w\dot{y}$

$\ddot{y} = \Omega^2 y - 2w\dot{x}$

Let $s = x + iy$

$\ddot{x} + i\ddot{y} = (x+iy)\Omega^2 - 2wi(\dot{x} + i\dot{y})$

$$\Rightarrow \begin{cases} \ddot{s} + 2wi\dot{s} - \Omega^2 s = 0 \\ \dot{s}(0) = V_x + iV_y \\ s(0) = x_0 \end{cases}$$

The solution of this equation is:

$\cancel{s(t) = -\frac{1}{2}isWt + C_1 e^{-2iwt} + C_2}$

$s\cancel{(0) = x_0} \Rightarrow \boxed{\cancel{C_1 + C_2 = x_0}}$

$\cancel{\dot{s}(t) = -\frac{1}{2}i}$

سؤال ناقص !

Q2) $\theta = 60^\circ$, $\psi = (3I - I_3)(MgL/3I\, I_3^2)^{1/2}$

$\phi = 2\sqrt{\frac{MgL}{3I}}$

A.) P_ϕ , P_ψ

$$P_\psi = I_3(\dot{\phi}\cos\theta + \dot{\psi})$$

$$P_\phi = (I_1 \sin^2\theta + I_3 \cos^2\theta)\dot{\phi} + I_3 \cos\theta\, \dot{\psi}$$

$$V_{eff}(\theta) = \frac{(P_\phi - P_\psi \cos\theta)^2}{2 I_1 \sin^2\theta} + Mgh\cos\theta$$

Q3) [illegible]

To simplify the solution:

$$\ddot{u} + u\left(1 + \frac{mk}{L^2}\right) = 0$$

or

$$\frac{\partial u^2(\phi)}{\partial \phi^2} + u\left(1 + \frac{mk}{L^2}\right) = 0$$

$$u(\phi) = u_0 \csc\left(\phi \sqrt{1 + \frac{km}{L^2}}\right)$$

$$\Rightarrow \boxed{r = r_0 \sec\left(\sqrt{1 + \frac{km}{L^2}}\, \phi\right)}$$

Q1) تكملة

The solution:

$$\boxed{s(t) = c_1 e^{-itw} + c_2 t e^{-iwt}}$$

$$s(0) = x_0 \Rightarrow \boxed{c_1 = x_0}$$

$$\dot{s}(0) = -iwc_1 + c_2 = V_x + V_y i$$

$$\boxed{V_x = c_2}, \boxed{V_y = -wx_0}$$

$$\Rightarrow \boxed{s(t) = x_0 e^{-iwt} + V_x t e^{-iwt}}$$

$$x(t) = Re\{s(t)\}$$

$$= Re\{x_0 \cos wt - x_0 i \sin(wt) + V_x t \cos wt - iV_x t \sin wt\}$$

$$y(t) = Im\{s(t)\}$$

$$\Rightarrow x(t) = x_0 \cos wt + V_x t \cos wt$$

$$y(t) = -x_0 \sin wt - V_x t \sin wt$$

Q3) $F(r) \propto \frac{1}{r^3} \Rightarrow F(r) = \frac{k}{r^3}$

(a) $U(r) = \frac{k}{2r^2}$

$$L = \frac{1}{2}\mu(\dot{r}^2 + r^2\dot{\theta}^2) + \frac{k}{2r^2}$$

$$\frac{\partial L}{\partial r} - \frac{d}{dt}\frac{\partial L}{\partial \dot{r}} = 0$$

$$\left(\mu r\dot{\theta}^2 - \frac{k}{r^3}\right) - \frac{d}{dt}(\mu\dot{r}) = 0$$

$$\boxed{\mu r\dot{\theta}^2 - \frac{k}{r^3} - \mu\ddot{r} = 0} \longrightarrow \text{The eq of motion}$$

$$\Rightarrow \boxed{\mu(\ddot{r} - r\dot{\theta}^2) = \frac{k}{r^3}}$$

(b) $\frac{\partial L}{\partial \theta} = 0 = \dot{P}_\theta = \frac{d}{dt}\frac{\partial L}{\partial \dot{\theta}}$

$$\Rightarrow \frac{\partial L}{\partial \dot{\theta}} = \text{constant} = \mu r^2\dot{\theta}$$

Let $\boxed{l \equiv \mu r^2\dot{\theta}}$ = constant of motion

$E = T + U$ = constant

$$= \frac{1}{2}\mu\dot{r}^2 + \frac{1}{2}\frac{l^2}{\mu r^2} - \frac{k}{2r^2} = \boxed{\text{constant} = E}$$

(c) $\frac{d^2u}{d\phi^2} + u = -\frac{m}{L^2u^2}\frac{k}{r^3}$ — orbit equation

Let $\frac{du(\phi)}{d\phi} = \dot{u}$

$$\Rightarrow \ddot{u} + u = -\frac{m}{L^2u^2}ku^3$$

$$\ddot{u} + u\left(1 + \frac{mk}{L^2}\right) = 0$$

$$u(\phi) = c_1\exp\left(i\frac{\sqrt{L^2+km}}{L}\phi\right) + c_2\exp\left(-i\frac{\sqrt{L^2+km}}{L}\phi\right)$$

$$u(\phi) = c_1\left[\cos\left(\frac{\sqrt{L^2+km}}{L}\phi\right) + i\sin\left(\frac{\sqrt{L^2+km}}{L}\phi\right)\right]$$

$$+ c_2\left[\cos\left(\frac{\sqrt{L^2+km}}{L}\phi\right) - i\sin\left(\frac{\sqrt{L^2+km}}{L}\phi\right)\right]$$