33

3. Three pendula of equal lengths, ℓ, and bob masses, m, swing in the vertical plane. Each is connected to the adjacent pendulum by identical massless springs of unstretched lengths equal to the separations of the hanging points. For small displacements the potential energy of the system is

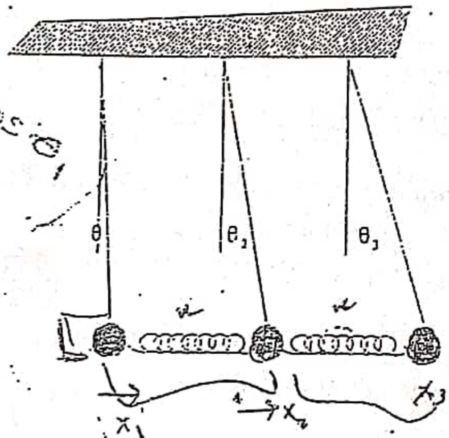
$$V = \frac{mg\ell}{2}\left(\theta_1^2 + \theta_2^2 + \theta_3^2\right) + \frac{1}{2}\varepsilon(\theta_1 - \theta_2)^2 + \frac{1}{2}\varepsilon(\theta_2 - \theta_3)^2$$

where ε is an energy parameter proportional to the spring constant.
Use $m = g = \ell = 1$ in this problem.

a) Find the normal mode frequencies.

b) Describe (qualitatively) the normal mode motions and assign the frequencies of part (a) to them. (If you prefer, you may use physical arguments rather than a full mathematical analysis.)

x_1, x_2, x_3



$\ell(1-\cos\theta_1)$

$L \approx \frac{1}{2}m(\dot{x}^2+\dot{y}^2)$

$\ell\theta$

$T = \frac{1}{2}m\left(\dot\theta_1^2 + \dot\theta_2^2 + \dot\theta_3^2\right)$

$\frac{1}{2}k\Delta x_1^2 +$

$V = V_{gravitational} + V_{elastic} = mg(\ell - \ell\cos\theta_1 + \ell - \ell\cos\theta_2 + \ell - \ell\cos\theta_3)$

$T = \frac{1}{2}m\ell^2\dot\theta_1^2 + \frac{1}{2}m\ell^2\dot\theta_2^2 + \frac{1}{2}m\ell^2\dot\theta_3^2$

$2\theta_1\theta_2$

$U = mgL(1 - \cos\theta_1 + 1 - \cos\theta_2 + 1 - \cos\theta_3)$

$+ \frac{1}{2}k(b^2)(\sin\theta_1 - \sin\theta_2)^2$

$+ kb^2(\sin\theta_2 - \sin\theta_3)^2$

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Problem 1: Show by any method that the following is a canonical transformation:

$$Q_1 = 2\sqrt{q_2 p_2}\ln q_2,$$
$$Q_2 = 2\sqrt{q_1 p_1}\ln q_1,$$
$$P_1 = \sqrt{q_2 p_2},$$
$$P_2 = \sqrt{q_1 p_1}.$$

Problem 2: Consider the Hamiltonian from the last problem set:

$$H = \frac{p^2}{2\alpha} - bqp\,e^{-at} + \frac{b\alpha q^2}{2}\,e^{-at}\,(a + b\,e^{-at}) + \frac{kq^2}{2},$$

where a, b, α and k are constants.

Consider the transformation:

$$Q = q; \qquad P = p - \alpha bq\,e^{-at}.$$

(a) Show that this is a canonical transformation by showing the symplectic condition for the Jacobian matrix is satisfied.
(b) Find the generating function (note there will be explicit time dependence).
(c) Find the new Hamiltonian function and show that it is a constant of the motion.

$\frac{dK}{dt} = H + \frac{\partial}{\partial t}$

Problem 3: Given the Hamiltonian $H = q_1 p_1 - q_2 p_2 + aq_1^2 - bq_2^2$, where a and b are constants:
(a) One obvious constant of motion is H itself; Show that $F = q_1 q_2$ and $G = (p_1 + aq_1)/q_2$ are constants of the motion.
(b) Find $q_1(t)$ and $q_2(t)$ using the initial conditions $q_1(t = 0) = \beta_1$ and $q_2(t = 0) = \beta_2$. Hence, find $p_1(t)$ and $p_2(t)$ without further integration using $G = \alpha_2$.
(c) Can there be any other constant of the motion which is explicitly independent of time t? Explain your answer.

Problem 4: Consider a particle moving in a potential $V(x, y, z)$ expressed in Cartesian coordinates.
(a) By direct evaluation in terms of canonical variables show that the Poisson bracket of the components of the angular momentum vector $\mathbf{L} = \mathbf{r} \times \mathbf{p}$ satisfies

$$[L_x, L_y] = L_z; \qquad [L_y, L_z] = L_x; \qquad \text{and} \quad [L_z, L_x] = L_y.$$

(b) If $V(x, y, z) = V(r^2)$ where $r^2 = x^2 + y^2 + z^2$, show that L_x and L_y are constants of motion using Poisson brackets.
(c) Given the results of (a) and (b) find another constant of motion (other than H).

$\frac{dK}{dt} = (K, K)$

$\frac{\partial K}{\partial t}$

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(c)The tent map is represented by

$$x_{n+1} = 2\alpha x_n \qquad \text{for } 0 < x < \tfrac{1}{2}$$

$$x_{n+1} = 2\alpha(1 - x_n) \qquad \text{for } \tfrac{1}{2} < x < 1$$

Show analytically that the Lyapunove exponent for the tent map is $\lambda = \ln(2\alpha)$

And discuss the dynamical behavior in term of α.

Q4: The moment of inertia of a uniform square sheet of side a and mass m in the xy plane and one corner at the origin along the diagonal is

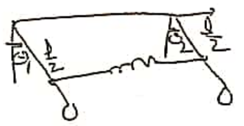
$$I = \begin{bmatrix} \frac{ma^2}{3} & \frac{-ma^2}{4} & 0 \\ \frac{-ma^2}{4} & \frac{ma^2}{3} & 0 \\ 0 & 0 & \frac{2ma^2}{3} \end{bmatrix}$$

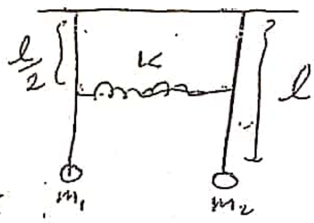
a) Find the principal moment of inertia of the square sheet about a corner.

b) Find a set of principal axis associated with the smallest principal moment of inertia.

Good Luck

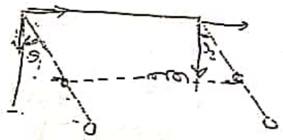
Q3: Determine the natural frequencies of small oscillations and the normal for two equal length b and two equal masses m connected to a spring of spring constant k as shown. The spring is un stretched in the equilibrium position. Consider: m = b = g =1.





l and massle and rigid

GOOD LUCK



$\frac{1}{4}(2\dot{x}_2)$.

$\frac{1}{2}(2\dot{x}_2$

$\frac{1}{4}(2\dot{x}_2)$

$\frac{1}{2}(2\dot{x}_1^2)$

1.

$\frac{1}{2}kx$

$(\frac{1}{4}\,2\dot{x}_1) = \frac{1}{2}(2\dot{x}_1)$

1

جامعة بيرزيت

BIRZEIT UNIVERSITY

Physics Department

Physics 334

Second Exam **Date:24/5/2018** **Time: 1.5 hours**

Instructor: Dr. E Badran

Q1: A bug crawls outward with constant speed $v1$ along the spoke of a wheel which is rotating with constant angular velocity ω about a vertical axis.

a) Find all the apparent forces acting on the bug.

b) Find how far the bug can crawl before it starts to slip given the (7 points)

Q2: The three principal axes of a tennis racket are (1) along the handle, (2) perpendicular to the handle in the plane of the string and (3) perpendicular to the handle and strings.The moments of inertia are in the following relation

$I1 < I2 < I3$,When a tennis racket is tossed in the air with a spin in the direction of either axis (1) or (3), the racket continues to spin uniformly about the initial axis and can be easily recaught. However, if the initial spin is around axis (2), the motion rapidly becomes irregular, and it is hard to catch the racket. Explain this behaviour by studying the stability.(7 points).

Q3: a) Show that the inertia tensor for a square plate of side a and mass m in coordinate system $Oxyz$ where O is at one corner of the plate and the x *and* y axes along the two edges. $I = \begin{pmatrix} ma^2/3 & -ma^2/4 & 0 \\ -ma^2/4 & ma^2/3 & 0 \\ 0 & 0 & 2ma^2/3 \end{pmatrix}$

b) Find the angular momentum and kinetic energy of the above plate if it is rotating about the diagonal.

c) Find the principal moments of inertia .

d) Find the principal axes associated with it.(8 points)

$(km + 2kM)^2$

$(k^2m^2 + 4k^2M^2 + 4k^2mM - 4k^2mM$

① تجاهل دوران الأرض

كثير من الوقت سوف تكون الصاروخ

في الهواء ... الآن ذلك

BIRZEIT UNIVERSITY

Physics Dept.

Dr.Esma'el badran

$F = 8 + mg$

$= 0 \; mg$

Final Exam **Physics 334** **2nd semester 2014/2015**

ch 10

Q1: A rocket is fired from the ground toward East with initial velocity v_o, at an angle θ above the horizontal and at a latitude λ.

Assume that the height of the rocket trajectory is much smaller than the radius of the Earth. نفترض أن ارتفاع مسار الصاروخ هو أصغر بكثير من نصف القطر

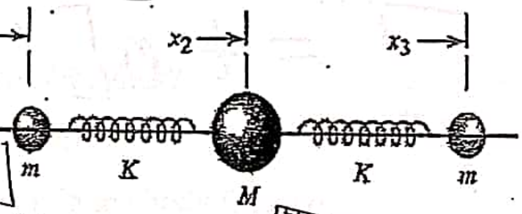
1) Ignoring earth's rotation, how long will the rocket be in the air, and how far does it land?

2) Now, taking into account Coriolis' force, answer the same questions as above.

3) In which direction and by how much is the rocket deviated from the east direction?

96

Q2: Linear motion of a tri atomic molecule. As an example CO_2 molecule can be modeled as shown. For simplicity consider only motion in one dimension, the x axis. The two end particles, each of mass m, are bound to the central particle, mass M, through a potential function that is equivalent to two springs of stiffness K, as shown. Use the coordinates x_1, x_2, and x_3.



87.8

a) Write the kinetic and potential energy in matrix form.

b) Find the natural frequencies and the normal modes (do not normalize them).

c) What is the physical meaning of the normal modes.

68.8 + 16 = 132

150

35

$k - m\left(\frac{k}{2m} + \frac{k}{2M}\sqrt{M^2 - 12mM}\right)$

$k - \frac{k}{2} - \frac{k}{2M}\sqrt{}$

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$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}}\right) - \frac{\partial L}{\partial x}$

جامعة بيرزيت
BIRZEIT UNIVERSITY
Physics Department

First Exam **Date:24/3/2015** **Time: 75 min.**

Instructor: Dr. E. Badran

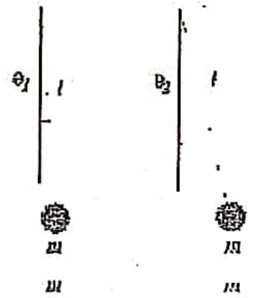
Q1: For a charged particle of mass m, and charge q is moving in the xy plane. The particle is subjected to an electromagnetic field, with vector potential $A = B_0(-y/2, x/2, 0)$ and scalar potential $\varphi(x) = -V_0 x$. Then, the Lagrangian can be written as $L =$

$$L = T - q\,\varphi + (q/c)\,A.v$$

Where v is the velocity in the xy plane and c is the speed of light.

A. Find the equations of motion.
B. Find the canonical momentum in each direction.
C. Find the Hamiltonian.

Q2: Two identical bobs of mass m are hanging from strings of length l. The two pendulums interact with each other through the another supporting string as shown and we assume that the mechanical energy of the system is conserved. The interaction between the two pendulums is modeled as



$$\boxed{V_{int} = -\lambda m g l \theta_1 \theta_2}$$

a) Write the kinetic and potential energy in matrix form.

b) Find the natural frequencies and normal modes of small oscillations

c) Discuss the physical meaning of the normal modes.

جامعة بيرزيت

BIRZEIT UNIVERSITY

Physics Department

First Exam

hours

Date:16/3/2010

Instructor: Dr. E. Badran

Q1: A spring of rest length L_o (no tension) is connected to a support at one end and has a mass M attached at the other. Neglect the mass of the spring, the dimension of the mass M, and assume that the motion is confined to a vertical plane. Also, assume that the spring only stretches without bending but it can swing in the plane.

1. Using the angular displacement of the mass from the vertical and the length that the string has stretched from its rest length (hanging with the mass m), find Lagrange's equations.
2. Solve these equations fro small stretching and angular displacements.
3. Find the Hamiltonian and set up Hamilton equations

Q2: a) Define chaotic system and how does your definition relate sensitivity to initial conditions to Liapunov exponents?

b) Consider a particle influenced by a force of the form

$$F(x) = -kx + \lambda x^2$$

For $\lambda > 0$, where the system could be considered hard or soft.

Draw a phase portrait for this motion. What is the maximum energy and we still have a pound motion. What is the period of small oscillations.

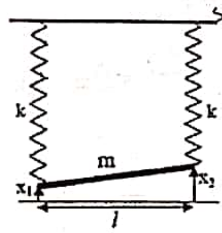
c) Show that the system $y_{n+1} = 1 - \gamma y_n^2$ with $-1<y<1$ and $0<\gamma<2$ can be transformed to the logistic map $x_{n+1} = \alpha x_n(1-x_n)$ by substituting $y = cx + d$. Find γ, c, d in terms of α. Find the first three bifurcations for the system.

second

Q3: Consider a frictionless puck on a horizontal turntable that rotates counterclockwise with a constant angular velocity Ω. Ignore the rotation of the Earth. In a coordinate system attached to the turntable with the origin on the rotation axis the puck has initial coordinate $(x_0, 0)$ and initial velocity (v_x, v_y). Determine the subsequent motion of the puck on the turntable given by $x(t)$, $y(t)$. Hint: It is convenient to consider the variable $s = x + iy$ (8 points)

first

Q4: : A rod of mass m and length l is suspended from two massless springs with a spring constant k as shown in the figure. (Note: *the gravity force does not need to be included.*)



A) Show that the kinetic and potential energy for the system in terms of displacements from equilibrium position of the two ends of the rod x_1 and x_2.

$$T = \frac{m}{6}(\dot{x}_1^2 + \dot{x}_1\dot{x}_2 + \dot{x}_2^2) \ and \ V(x_1, x_2) = \frac{1}{2}k(x_1^2 + x_2^2), where \ I = \frac{1}{12}ml^2$$

B) Find the natural frequencies.

C) Find the normal modes and give a physical explanation of them. (10 points)

first

Q5: Disscus the motion of a string when the initial conditions are $q(x,0) = \frac{4x(L-x)}{L^2}$, $\dot{q}(x,0) = 0$. Find the characteristic frequencies and calculate the amplitude nth mode. (8 points)

$$\mu_r = \frac{2}{l}\int_0^L q(x,0)\sin\frac{r\pi x}{L}\,dx$$

$$\nu_r = -\frac{2}{\omega_r L}\int_0^L \dot{q}(x,o)\sin\frac{r\pi x}{L}\,dx$$

Good luck

$x = (x_0 + v_x t)\cos\omega t + (v_y + x_0\omega)t\sin\omega t$

$y = (v_y + x_0\omega)t\cos\omega t - (x_0 + v_x t)\sin\omega t$

16 + 10 = 26

$$\Omega_1 = \sqrt{\frac{(I_1-I_2)(I-I_3)}{I_2 I_3}}$$

$$\Omega_2 = \sqrt{\frac{(I_2-I_3)(I_1-I_2)}{I_3 I_1}}$$

$$\Omega_3 = \sqrt{\frac{(I_3-I_1)(I_1 I_2)}{I_1 I_1}}$$

BIRZEIT UNIVERSITY

Physics Department

Final Exam | **Date:18/6/2014** | **Time: 150 m**

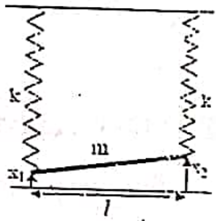
Instructor: Dr. E. Badran

(You have 45 points and the full mark is 40)

Q1) The three principal axes of a tennis racket are (1) along the handle, (2) perpendicular to the handle in the plane of the string and (3) perpendicular to the handle and strings. The moments of inertia are in the following relation $I_1 < I_2 < I_3$.

When a tennis racket is tossed in the air with a spin in the direction of either axis (1) or (3), the racket continues to spin uniformly about the initial axis and can be easily recaught. However, if the initial spin is around axis (2), the motion rapidly becomes irregular, and it is hard to catch the racket. Explain this behaviour by studying the stability. (10 points)

Q2: A rod of mass m and length l is suspended from two massless springs with a spring constant k as shown in the figure. (Note: *the gravity force does not need to be included.*)



A) Show that the kinetic and potential energy for the system in terms of displacements from equilibrium position of the two ends of the rod x_1 and x_2.

$$T = \frac{m}{6}(\dot{x}_1^2 + \dot{x}_1\dot{x}_2 + \dot{x}_2^2) \text{ and } V(x_1, x_2) = \frac{1}{2}k(x_1^2 + x_2^2), \text{ where } I = \frac{1}{12}ml^2$$

B) Find the natural frequencies.

C) Find the normal modes and give a physical explanation of them. (12 points)

Q3) Consider a frictionless puck on a horizontal turntable that rotates counterclockwise with a constant angular velocity Ω. Ignore the rotation of the Earth. In a coordinate system attached to the turntable with the origin on the rotation axis the puck has initial coordinate $(x_0, 0)$ and initial velocity (v_x, v_y). Determine the subsequent motion of the puck on the turntable given by $x(t)$, $y(t)$. Hint: It is convenient to consider the variable $s = x+iy$ (10 points)

$(\dot{x}_1 + \dot{x}_2)^2$

$\frac{1}{2}m\left(\frac{\dot{x}_1^2}{4} - \dot{x}_1^2\right) \quad \frac{1}{12}ml^2(\dot{x}_1^2 + 2\dot{x}_1\dot{x}_2 + \dot{x}_2^2)$

$\frac{3}{4} \quad \frac{1}{6} = \frac{1}{2} + x \quad \frac{6}{6l^2} - \frac{1}{12}$

$x = \frac{1}{6} - \frac{3}{12} = -\frac{2}{6} = -\frac{1}{3}$

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mechanics 2

جامعة بيرزيت

BIRZEIT UNIVERSITY

Physics Dept.

Physics 334

First Hour Exam | **Spring 2005/2006**

Time:1:15 Hours | **Date: 4/4/2006**

1) a) A geodesic is a line that represents the shortest path between any two points when the path is restricted to a particular surface. Find the geodesic of the cylinder. $\rho^2 d\theta + dz^2$. $\dot{z}^2 = \rho^2 C$

$H = \sum \dot{q} \frac{\partial L}{\partial \dot{q}} - L$

b) Show that the time invariance in the Lagrangian is associated with the conservation of Hamilton – Jaccobi function. How is this conservation is associated with energy conservation. $H = \sum p\dot{q} - L$.

$\dot{p} = -\frac{\partial H}{\partial q}$

2) A double pendulum of equal lengths, but not equal masses as shown, considering small oscillations only:

$\dot{q} = \frac{\partial H}{\partial p}$

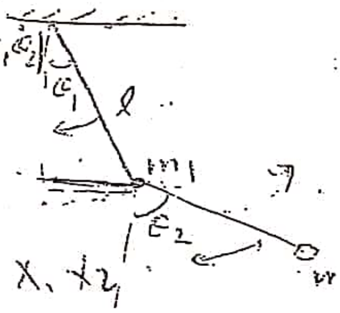
a) Show that $T = \frac{1}{2} m_1 l^2 \dot{\theta}_1^2 + \frac{1}{2} m_2 l^2 (\dot{\theta}_1^2 + \dot{\theta}_2^2 + 2\dot{\theta}_1\dot{\theta}_2)$

$V = m_1 g l \frac{\theta_1^2}{2} + m_2 g l \left(\frac{\theta_1^2 + \theta_2^2}{2}\right)$

b) Find the natural frequencies of the system.

c) Find the normal modes.

d) Give physical sketch of the normal modes.



$l - \cos\theta_1$ $(1 - \cos\theta)$ $(l\cos\theta_1) +$ x, x_2

~~l cos θ~~ $1 - \frac{\theta_1}{2}$ (θ_1)

GOOD LUCK

$mgl \frac{\theta_1^2}{2}$

$l \cos\theta_1$

$l\cos\theta_1 + l\cos\theta_2$

$y_1 = l(\cos\theta_1)$

جامعة بيرزيت
BIRZEIT UNIVERSITY
Physics Department

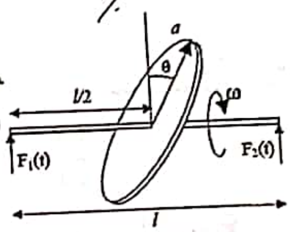
Final Exam | **Date:31/5/2018** | **Time: 150 min**

2.5 hours

Instructor: Dr. E. Badran

(The exam has 44 points and the full mark is 40)

Q1: A disk of mass m and radius is a mounted in the middle of massless rod of length l. The plane of the disk is tilted by an angle θ away from the normal to the rod. Initially the disk is at rest tilted up, as shown in the figure. At $t = 0$ it begins to rotate around the axis of the rod with an angular velocity ω in the direction shown. The rod is supported at the two ends and forces $F_1(t)$ and $F_2(t)$ are such that it rotates without wobble with a constant angular velocity .



10

A) What are direction of the principal axes of inertia of the disk?

B) Calculate the principal moments of inertia of the disk?

C) What are the components of the angular velocity ω in the body frame of reference?

D) Determine the torques needed to keep the disk spinning without wobble?

E) (10 points)

Q2: Consider a cube placed on a flat table with one face directly in front of you.

A) Find the matrix of rotation which represent the following displacement of the cube

matrix

1) The cube is rotated by $\pi/2$ about the y.

2) Then the cube is rotated by $\pi/2$ about the new x axis.

B) Find the axis and rotation angle that will accomplish the same displacement in a single rotation about a single axis. (8 points)

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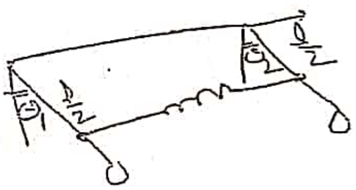
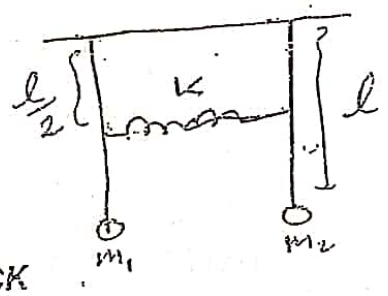
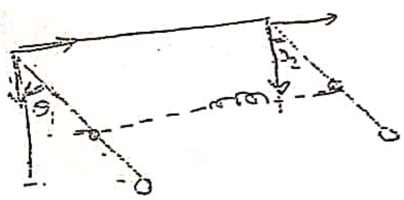
Q3: Determine the natural frequencies of small oscillations and the normal for two equal length b and two equal masses m connected to a spring of spring constant k as shown. The spring is un stretched in the equilibrium position. Consider: m = b = g =1.





and massless and rigid

GOOD LUCK



$\frac{1}{4}(2\dot{x}_2)$.

$\frac{1}{4}(2\dot{x}_2)$

$\frac{1}{2}(2\dot{x}_2$

$\frac{1}{2}(2\dot{x}_1^2)$

1

$(\frac{1}{2}\, 2\dot{x}_1) = \frac{1}{2}(2\dot{x}_1)$

1

جامعة بيرزيت

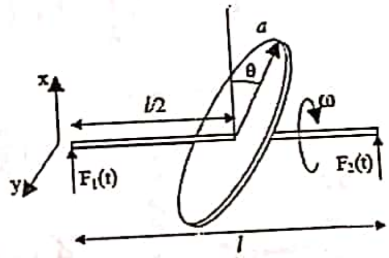
BIRZEIT UNIVERSITY

Physics Department

Final Exam | **Date:31/5/2018** | **Time: 150 min**

Instructor: Dr. E. Badran

(The exam has 44 points and the full mark is 40)



Q1: A disk of mass m and radius is a mounted in the middle of massless rod of length l. The plane of the disk is tilted by an angle θ away from the normal to the rod. Initially the disk is at rest tilted up, as shown in the figure. At $t = 0$ it begins to rotate around the axis of the rod with an angular velocity ω in the direction shown. The rod is supported at the two ends and forces $F_1(t)$ and $F_2(t)$ are such that it rotates without wobble with a constant angular velocity .

A) What are direction of the principal axes of inertia of the disk?

B) Calculate the principal moments of inertia of the disk?

C) What are the components of the angular velocity ω in the body frame of reference?

D) Determine the torques needed to keep the disk spinning without wobble?

E) (10 points)

Q2: Consider a cube placed on a flat table with one face directly in front of you.

A) Find the matrix of rotation which represent the following displacement of the cube

1) The cube is rotated by $\pi/2$ about the y.

2) Then the cube is rotated by $\pi/2$ about the new x axis.

B) Find the axis and rotation angle that will accomplish the same displacement in a single rotation about a single axis. (8 points)

$\omega = 0$

$\omega_2 = (k/m)^2$

$\omega_3 =$

جامعة بيرزيت

BIRZEIT UNIVERSITY

$[(k/m)(1+2m\ M)]^{\frac{1}{2}}$

Physics Dept.

$\ddot{y} = -2\omega \dot{z} \cos\lambda$

$v_z = v_0 - \frac{1}{2} g t^2$

Second Exam **Physics 334** **2^{nd} semester 2014/2015**

ch 11

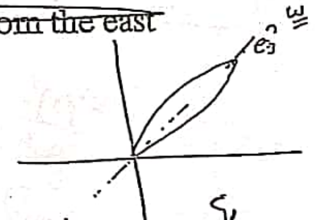
Q1: A quarterback in American football throws the ball in such a way that it appears to "wobble" تتأرجح rather than spinning smoothly as it flies. For this problem assume that the football can be treated as an axis symmetric top having $I_1 = I_2 = 2I_3$, where the $\hat{e}_3$ axis is the long axis of the football. Assume further that the football is released with a rapid spin, i.e., $\omega_3 \neq 0$, where ω_3 is the component of the football's angular velocity along the $\hat{e}_3$ body axis. $\omega = \omega_3 e_3$ $(0, 0, \omega_3)$

a) What does the wobbling motion indicate? In particular what does it imply about the other ($\hat{e}_1$ and $\hat{e}_2$) components of ω (No calculation is required here.)

b) Expressing vectors with respect to the body fixed axes, $\hat{e}_1$; $\hat{e}_2$; $\hat{e}_3$, derive three equations that relate the components of ω to one another and to the moments of inertia I_1; I_2; I_3.

c) If the football is spinning rapidly, then one can assume that $\omega_3 >> \omega_{1,2}$. Use this assumption to derive an approximate expression for ω as a function of time. Find the period of the wobble in terms of $I_1 = I_2$, I_3, and ω_3.

~~In which direction and by how much is the rocket deviated from the east direction?~~



ch 6 Q2: If a particle is projected vertically upward to a height h above the ground a point on earth's surface at a northern latitude λ, show that it strikes the ground at a point $(4/3)\omega(8h^3/g)^{1/2}\cos\lambda$ to the west. Note: neglect air resistance, and consider only small vertical heights.

west

East

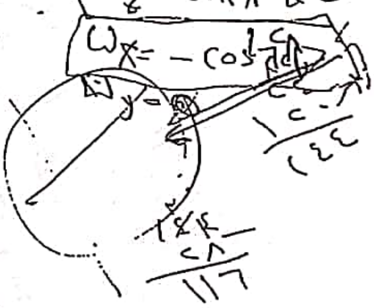
$\omega_z = \sin\lambda$

$\omega_x = -\cos\lambda$

$\ddot{y} =$

$I_1 = I_2 = 2I_3$

$I_1 > I_3$ $I_2 > I_3$



$v_z = v_0 - gt$

$v_z^2 = v_0^2 - 2hg$

$v_0 = \sqrt{2hg}$

$\frac{4}{49} - \frac{28}{144}$

$\frac{144-28}{144} = \frac{116}{144}$

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Q3: *Part one*: The tent map is represented by the following interaction:

$$x_{n+1} = 2\alpha x_n \quad for\ 0 < x < \frac{1}{2}$$

$$x_{n+1} = 2\alpha(1 - x_n) \quad for\ \frac{1}{2} < x < 1$$

Where $0 < \alpha < 1$.

a) Plot the bifurcation diagram for the tent map starting
$x_1 = 0.3$ *and* $\alpha = 0.3, 0.4, \cancel{5.2}, 0.6, 0.5$

b) Find analytically the Lyapunov exponent. And how it is related to your bifurcation diagram.

Part two:

a)Define chaotic system and how does your definition relate sensitivity to initial conditions to Poincare surface?

b) Consider a particle influenced by a force of the form

$$F(x) = -kx + \varepsilon x^4$$

For $k < 0$, where the system could be considered hard or soft.

Good Luck

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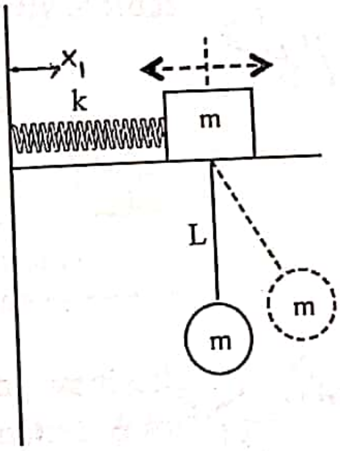
Physics Dept.

Second Exam **Physics 334** **2^{nd} semester 2016/2017**



Q1: A pendulum of length *L* and mass *m* is connected to a block also of mass *m* that is free to move horizontally on a frictionless surface. The block is connected to a wall with a spring of spring constant *k*. For the special case where

$$\sqrt{\frac{g}{l}}=\sqrt{\frac{k}{m}}=\omega_0 \text{ , determine:}$$

(a) The frequencies of the normal modes of this system for small oscillations aroundthe equilibrium positions.
(b) The motion of each of the normal modes.

Q2: Consider a frictionless puck on a horizontal turntable that rotates counterclockwise with a constant angular velocity Ω. Ignore the rotation of the Earth. In a coordinate system attached to the turntable with the origin on the rotation axis the puck has initial coordinate $(x_0,0)$ and initial velocity (v_x,v_y). Determine the subsequent motion of the puck on the turntable given by x(t), y(t). Hint: It is convenient to consider the variable s = x+iy

Q3: : ***Part one***: The tent map is represented by the following interaction:

$$x_{n+1}=2\alpha x_n \;\; for\; 0<x<\frac{1}{2}$$

$$x_{n+1}=2\alpha(1-x_n)\;\; for\; \frac{1}{2}<x<1$$

Where $0<\alpha<1$.

a) Plot the bifurcation diagram for the tent map starting

$$x_1=0.3 \; and \; \propto = \; 0.3\,, 0.4, 5.2, 0.6, 0.5$$

$\omega_1 = 0$

$L = \frac{1}{2} m(\dot{x}+\dot{y} - q(\quad -\frac{B_0 y \dot{x}}{2} \quad + q\left(\frac{B_0}{2} x\dot{y}\right)$

$\frac{1}{2} m(\dot{x}^2 + \dot{y}^2 + qV(r) \quad + \quad \frac{1}{2} B_0 y \dot{x} - \frac{1}{2} B_0 \cdot x \dot{y}$

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Physics Department

$K \quad - q_i V(r)$

$q_i V(r)$

Physics 334

First Exam **Date:11/4/2018** **Time: 1.5 hours**

Instructor: Dr. E Badran

plane x, y

$B \rightarrow \hat{z}$

Q1: A particle of mass ***m*** and electric charge ***q*** moves in a plane under the influence of a central potential ***V(r)*** and a constant uniform magnetic field B, perpendicular to the plane, generated by a vector potential

$$\vec{A} = 1/2\, \vec{B} \times \vec{r}$$

$\boxed{B = \nabla \times A}$

$L = T - V + \left(\frac{q}{c}\right) \vec{A} \cdot \vec{v}$

$L = T - \boxed{qV} + \frac{q}{c} A \cdot \vec{v}$

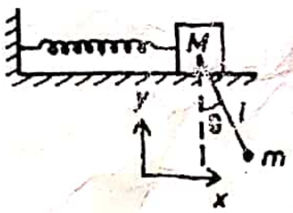
A) Write the Lagrangian of the system.

B) Find the Hamiltonian.

3 C) Find the equations of motion and solve them.

Q2: A simple pendulum consisting of a *mass* m and weightless string of length *l* is mounted on a support of mass *M* which is attached to a horizontal spring with force constant *k as* shown in Fig..

a) Set up the Kinetic and potential energy in matrix form.
b) Find the frequencies for small oscillations.
c) Find the normal modes
d) What is the physical meaning of the normal modes



Q3) Consider a coupled torsional pendulum of spring and a mass as shown. A reasonable mathematical model for the system potential energy of the form

$$V = \tfrac{1}{2}kx^2 + \tfrac{1}{2}\beta\theta^2 + \varepsilon x\theta^2$$

Where x is the downward displacement of the mass from its equilibrium hanging position, θ is the angular rotation of the mass from its angle when hanging in its equilibrium position, and the coupling ε is assumed to be small.

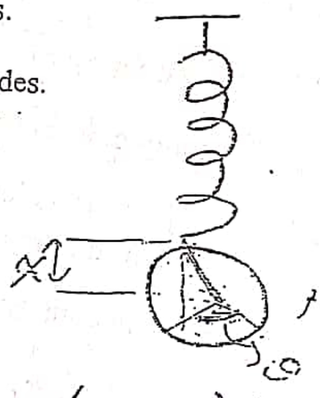
a) Give a brief physical justification for the coupling between x and θ used in this model.

b) Find the natural frequencies and the normal modes.

c) Normalize the normal modes.

d) Give a brief physical description of the normal modes.

(7 points)



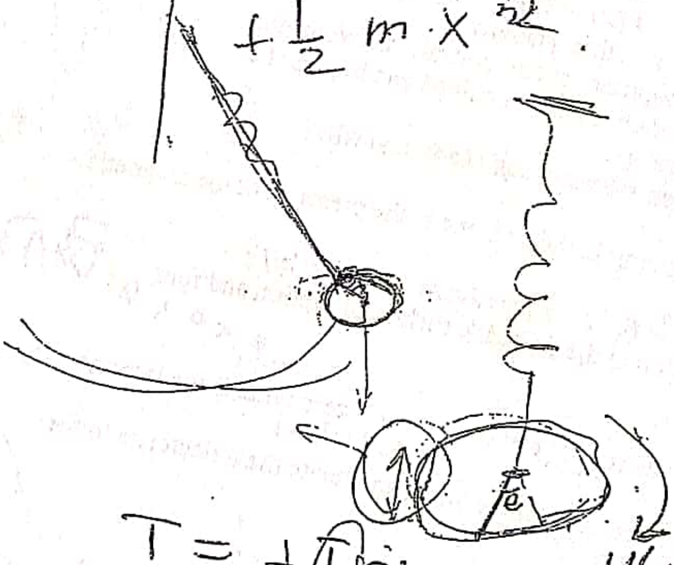
Good Luck

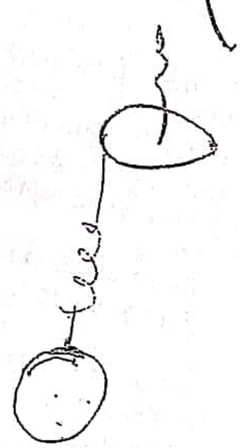
$u = \frac{1}{2}kx^2$

$\frac{1}{2}RM^2\dot{\theta}^2$

$+\frac{1}{2}m\dot{x}^2$

$(a_{11}\ a_{12})\,(T)\begin{pmatrix} a_{11} \\ a_{12} \end{pmatrix} =$





$\sqrt{\frac{k}{m}}$

$T = \frac{1}{2}I\dot{\theta}^2$

$T = \frac{1}{2}MR^2$

$\frac{1}{2}m\dot{x}^2$

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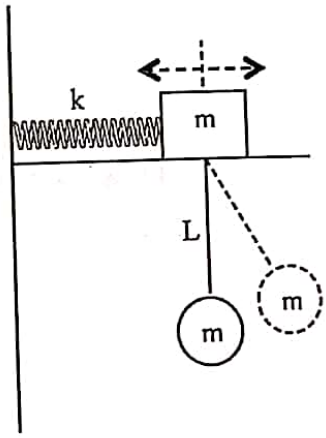
Physics Dept.

Second Exam **Physics 334** **2^{nd} semester 2016/2017**



Q1: A pendulum of length L and mass m is connected to a block also of mass m that is free to move horizontally on a frictionless surface. The block is connected to a wall with a spring of spring constant k. For the special case where

$$\sqrt{\frac{g}{l}} = \sqrt{\frac{k}{m}} = \omega_0 \text{ , determine:}$$

(a) The frequencies of the normal modes of this system for small oscillations aroundthe equilibrium positions. .

(b) The motion of each of the normal modes.

Q2: Consider a frictionless puck on a horizontal turntable that rotates counterclockwise with a constant angular velocity Ω. Ignore the rotation of the Earth. In a coordinate system attached to the turntable with the origin on the rotation axis the puck has initial coordinate $(x_0,0)$ and initial velocity (v_x,v_y). Determine the subsequent motion of the puck on the turntable given by x(t), y(t). Hint: It is convenient to consider the variable s = x+iy

Q3: : ***Part one***: The tent map is represented by the following interaction:

$$x_{n+1} = 2\alpha x_n \;\; for\; 0 < x < \frac{1}{2}$$

$$x_{n+1} = 2\alpha(1 - x_n) \;\; for\; \frac{1}{2} < x < 1$$

Where $0 < \alpha < 1$.

a) Plot the bifurcation diagram for the tent map starting

$x_1 = 0.3$ *and* $\propto = 0.3, 0.4, 5.2, 0.6, 0.5$

0.52

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BIRZEIT UNIVERSITY

Physics Department

Physics 334

First Exam | Date:17/3/2011 | Time: 1.5 hours

Instructor: Dr. E Badran

{Note: Full mark is 20 the exam has 24 points }

Q1) Consider the Lagrangian of the form

$$L = e^{\gamma t}(\tfrac{1}{2}m\dot{q}^2 - \tfrac{1}{2}kq^2)$$

a) Find the equation of motion and what does it represent.

b) Is there any constant of motion

c) Use the point transformation $s = e^{\gamma t/2}q$ and find the new Lagrangian in term of s.

d) Is there any constant of motion in the new formulation

(7 Points)

Q2: *Part one* :A particle of mass m moves along the x-axis with potential energy

$$V(x) = \tfrac{1}{2}kx^2 - \tfrac{1}{4}\varepsilon x^4$$

a) Find the fixed points and their stability.

b) Sketch the velocity-phase space portrait showing the possible types of motion. Point out elliptic and hyperbolic points and the separatrix.

c) This system represents *hard* or *soft* chaos, and why?

Part two:

For an electron moving in the xy plane in the presence of the magnetic field of the form

$$\vec{B} = B_o\,[1 + r\,(\cos 2\pi x/a + \cos 2\pi y/a)]\,\hat{z}$$

$\nabla \times A = B$.

In the last page shown the Poincar'e surface of section and four trajectories.

Questions:

1) If one of the λ's is ± 1.2 and two others is zero what is the last one and why

conditions

2) Discuss the sensitivity of the initial and relate the trajectories to the surface of section.

(10 points)

$L = T + q(v \cdot A)$

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Physics Dept.

Second Exam **Physics 334** **2^{nd} semester 2016/2017**

Q1: A bug crawls outward with constant speed v along the spoke of a wheel which is rotating with constant angular velocity ω about a vertical axis.

A) Find all the apparent forces acting on the bug?

B) Find how far the bug can crawl before it start to slip, given the coefficient of static friction μ_s. (6 points)

Q2: Show that the moment of inertia of a uniform rectangular sheet of sides a, $2a$ and mass m in the xy- plane and one corner at the origin along the diagonal is

$$I = \begin{bmatrix} \frac{4ma^2}{3} & \frac{-ma^2}{2} & 0 \\ \frac{-ma^2}{2} & \frac{ma^2}{3} & 0 \\ 0 & 0 & \frac{5ma^2}{3} \end{bmatrix}$$

$\frac{18}{3}$ $\frac{15}{3}$

$\frac{14}{3}$

42

A) Find the angular momentum and kinetic energy, when it is rotating with angular velocity ω about the diagonal through the origin.

B) Find the angular momentum and kinetic energy, when it is rotating with angular velocity ω about the z – axis.

C) Find the principal moment of inertia of the rectangular sheet about the corner.

D) Find the direction of the principal axis of the largest principal moment. (10 points)

1 14 4 6

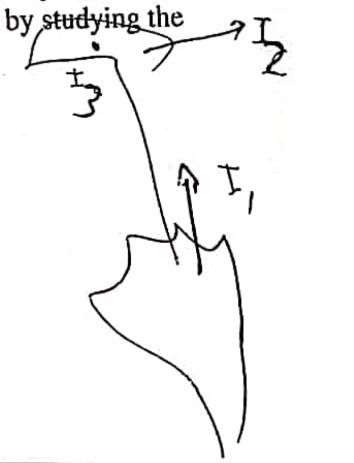
$\cancel{56}\sqrt{3}$ 56 6

$56\sqrt{3}$ 14 49

Q3: The three principal axes of a tennis racket are (1) along the handle, (2) perpendicular to the handle in the plane of the string and (3) perpendicular to the handle and strings. The moments of inertia are in the following relation

$$I_1 < I_2 < I_3$$

When a tennis racket is tossed in the air with a spin in the direction of either axis (1) or (3), the racket continues to spin uniformly about the initial axis and can be easily recaught. However, if the initial spin is around axis (2), the motion rapidly becomes irregular, and it is hard to catch the racket. Explain this behaviour by ~~studying~~ the stability. (6 points)



$$\Omega_1 = \omega_1 \sqrt{\frac{(I_1 - I_3)(I_1 - I_2)}{I_2 I_3}}$$

Good Luck

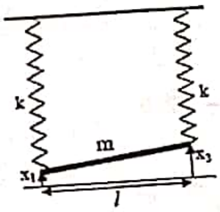
$$\Omega_2 = \omega_2 \sqrt{\frac{(I_2 - I_1)(I_2 - I_3)}{I_1 I_3}}$$

34 $\times 5$ 70

$$\Omega_3 = \omega_3 \sqrt{\frac{(I_3 - I_1)(I_3 - I_2)}{I_2 I_1}}$$

Q3: Consider a frictionless puck on a horizontal turntable that rotates counterclockwise with a constant angular velocity Ω. Ignore the rotation of the Earth. In a coordinate system attached to the turntable with the origin on the rotation axis the puck has initial coordinate $(x_0,0)$ and initial velocity (v_x, v_y). Determine the subsequent motion of the puck on the turntable given by $x(t)$, $y(t)$.
Hint: It is convenient to consider the variable $s = x+iy$ (8 points)

Q4: : A rod of mass m and length l is suspended from two massless springs with a spring constant k as shown in the figure. (Note: *the gravity force does not need to be included.*)



A) Show that the kinetic and potential energy for the system in terms of displacements from equilibrium position of the two ends of the rod x_1 and x_2.

$$T = \frac{m}{6}(\dot{x}_1^2 + \dot{x}_1\dot{x}_2 + \dot{x}_2^2) \ and\ V(x_1, x_2) = \frac{1}{2}k(x_1^2 + x_2^2), where\ I = \frac{1}{12}ml^2$$

B) Find the natural frequencies.

C) Find the normal modes and give a physical explanation of them. (10 points)

Q5: Disscus the motion of a string when the initial conditions are

$q(x,0) = \frac{4x(L-x)}{L^2}$, $\dot{q}(x,0) = 0$. Find the characteristic frequencies and calculate the amplitude nth mode. (8 points)

$$\mu_r = \frac{2}{l}\int_0^L q(x,0)\sin\frac{r\pi x}{L}dx$$

$$\nu_r = -\frac{2}{\omega_r L}\int_0^L \dot{q}(x,o)\,sin\frac{r\pi x}{L}dx$$

Good luck

$$-\frac{8x+l}{l^2}\left(-\frac{l}{n\pi}\cos\frac{n\pi x}{l}\right) - \frac{8}{n^2\pi^2}\sin\frac{n\pi x}{l}$$

2006 – 2011

first

2 ميكانيك

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BIRZEIT UNIVERSITY

Physics Dept.

Physics 334

First Hour Exam
Time: 1:15 Hours

Spring 2005/2006
Date: 4/4/2006

1) a) A geodesic is a line that represents the shortest path between any two points when the path is restricted to a particular surface. Find the geodesic of the cylinder. $\rho^2 d\theta + dz^2$. $\dot{z}^2 = \rho^2 c$

b) Show that the time invariance in the Lagrangian is associated with the conservation of Hamilton – Jaccobi function. How is this conservation is associated with energy conservation.

$= \sum \dot{q} \frac{\partial L}{\partial \dot{q}} - L$ $\quad h = \sum p\dot{q} - L$

A double pendulum of equal lengths, but not equal masses as shown, considering small oscillations only:

$\dot{p} = -\frac{\partial H}{\partial q}$ $\quad \dot{q} = \frac{\partial H}{\partial p}$

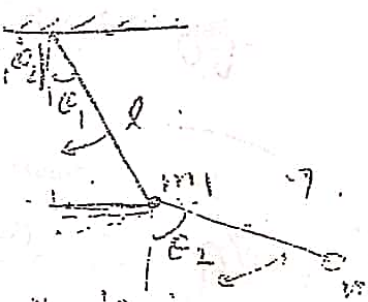
a) Show that $T = \frac{1}{2} m_1 l^2 \dot{\theta}_1^2 + \frac{1}{2} m_2 l^2 (\dot{\theta}_1^2 + \dot{\theta}_2^2 + 2\dot{\theta}_1\dot{\theta}_2)$

$V = m_1 g l \frac{\theta_1^2}{2} + m_2 g l \left(\frac{\theta_1^2 + \theta_2^2}{2}\right)$

b) Find the natural frequencies of the system.

c) Find the normal modes.

d) Give physical sketch of the normal modes.



$l - \cos\theta_1 \quad (1 - \cos\theta) \quad (l\cos\theta_1) +$

GOOD LUCK

$\cancel{l - l\cos\theta} \quad 1 - \frac{\theta_1}{2} \quad (\theta_1) = k \quad mgl\frac{\theta_1^2}{2}$

$l\cos\theta_1$

$l\cos\theta_1 + l\cos\theta_2$

$y_1 = l(\cos\theta_1)$

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(c)The tent map is represented by

$$x_{n+1} = 2\alpha x_n \qquad \text{for } 0 < x < ½$$

$$x_{n+1} = 2\alpha(1 - x_n) \qquad \text{for } ½ < x < 1$$

Show analytically that the Lyapunove exponent for the tent map is $\lambda = \ln(2\alpha)$

And discuss the dynamical behavior in term of α.

Q4: The moment of inertia of a uniform square sheet of side a and mass m in the xy plane and one corner at the origin along the diagonal is

$$I = \begin{bmatrix} \frac{ma^2}{3} & \frac{-ma^2}{4} & 0 \\ \frac{-ma^2}{4} & \frac{ma^2}{3} & 0 \\ 0 & 0 & \frac{2ma^2}{3} \end{bmatrix}$$

a) Find the principal moment of inertia of the square sheet about a corner.
b) Find a set of principal axis associated with the smallest principal moment of inertia.

Good Luck

Problem 1: Show by any method that the following is a canonical transformation:

$$Q_1 = 2\sqrt{q_2 p_2}\ln q_2,$$
$$Q_2 = 2\sqrt{q_1 p_1}\ln q_1,$$
$$P_1 = \sqrt{q_2 p_2},$$
$$P_2 = \sqrt{q_1 p_1}.$$

Problem 2: Consider the Hamiltonian from the last problem set:

$$H = \frac{p^2}{2\alpha} - bqp\,e^{-at} + \frac{b\alpha q^2}{2}\,e^{-at}\,(a + b\,e^{-at}) + \frac{kq^2}{2},$$

where a, b, α and k are constants.

Consider the transformation:

$$Q = q; \qquad P = p - \alpha bq\,e^{-at}.$$

(a) Show that this is a canonical transformation by showing the symplectic condition for the Jacobian matrix is satisfied.
(b) Find the generating function (note there will be explicit time dependence).
(c) Find the new Hamiltonian function and show that it is a constant of the motion.

$\frac{dK}{dt} = \dot{H} + \frac{\partial}{\partial t}$

Problem 3: Given the Hamiltonian $H = q_1p_1 - q_2p_2 + aq_1^2 - bq_2^2$, where a and b are constants:
(a) One obvious constant of motion is H itself; Show that $F = q_1q_2$ and $G = (p_1 + aq_1)/q_2$ are constants of the motion.
(b) Find $q_1(t)$ and $q_2(t)$ using the initial conditions $q_1(t=0) = \beta_1$ and $q_2(t=0) = \beta_2$. Hence, find $p_1(t)$ and $p_2(t)$ without further integration using $G = \alpha_2$.
(c) Can there be any other constant of the motion which is explicitly independent of time t? Explain your answer.

Problem 4: Consider a particle moving in a potential $V(x, y, z)$ expressed in Cartesian coordinates.
(a) By direct evaluation in terms of canonical variables show that the Poisson bracket of the components of the angular momentum vector $\mathbf{L} = \mathbf{r} \times \mathbf{p}$ satisfies

$$[L_x, L_y] = L_z; \qquad [L_y, L_z] = L_x; \qquad \text{and} \quad [L_z, L_x] = L_y.$$

(b) If $V(x,y,z) = V(r^2)$ where $r^2 = x^2 + y^2 + z^2$, show that L_x and L_y are constants of motion using Poisson brackets.
(c) Given the results of (a) and (b) find another constant of motion (other than H).

$\frac{dK}{dt} = (K, K)$

$\frac{\partial K}{\partial t}$

33

3. Three pendula of equal lengths, ℓ, and bob masses, m, swing in the vertical plane. Each is connected to the adjacent pendulum by identical massless springs of unstretched lengths equal to the separations of the hanging points. For small displacements the potential energy of the system is

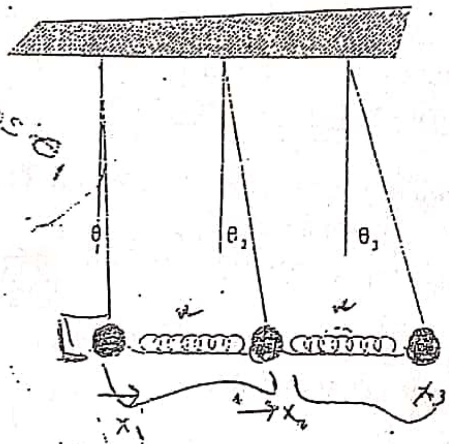
$$V = \frac{mg\,\ell}{2}\left(\theta_1^2 + \theta_2^2 + \theta_3^2\right) + \frac{1}{2}\,\varepsilon(\theta_1 - \theta_2)^2 + \frac{1}{2}\,\varepsilon(\theta_2 - \theta_3)^2$$

where ε is an energy parameter proportional to the spring constant. Use $m = g = \ell = 1$ in this problem.

a) Find the normal mode frequencies.

b) Describe (qualitatively) the normal mode motions and assign the frequencies of part (a) to them. (If you prefer, you may use physical arguments rather than a full mathematical analysis.)

x_1, x_2, x_3



$\ell(1-\cos\theta_1$

$L = \frac{1}{2}m(\dot{x}^2+\dot{y}^2)$

$\ell\theta$

$T = \frac{1}{2}m\left(\dot{\theta}_1^2 + \dot{\theta}_2^2 + \dot{\theta}_3^2\right)$

$\frac{1}{2}k\Delta x_1^2 +$

$V = V_{gravitational} + V_{elastic} = mg(\ell - \ell\cos\theta_1 + \ell - \ell\cos\theta_2 + \ell - \ell\cos\theta_3$

$T = \frac{1}{2}m\ell^2\dot{\theta}_1^2 + \frac{1}{2}m b^2\dot{\theta}_2^2 + \frac{1}{2}m\ell^2\,\theta_2^2$

$2\,\theta_1\theta_2$

$U = mgl(1-\cos\theta_1 + 1-\cos\theta_2 + 1-\cos\theta_3)$

$+\frac{1}{2}k\left(b^2\right)\left(\sin\theta_1 - \sin\theta_2\right)^2$

$+ k b^2 (\sin\theta_2 - \sin\theta_3)^2$

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$(km+2kM)^2$

$(k^2m^2+4k^2M^2+4k^2mM - 4k^2mM$

① تجاهل دوران الأرض

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BIRZEIT UNIVERSITY

Physics Dept.

Dr.Esma'el badran

$F = 8 + mg$
$= 0\ mg$

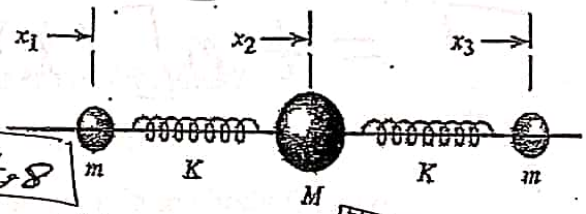
Final Exam **Physics 334** **2nd semester 2014/2015**

ch 10

Q1: A rocket is fired from the ground toward East with initial velocity v_o, at an angle θ above the horizontal and at a latitude λ.

Assume that the height of the rocket trajectory is much smaller than the radius of the Earth. نفترض أن ارتفاع مدار الصاروخ هو أصغر بكثير من نصف القطر

1) Ignoring earth's rotation, how long will the rocket be in the air, and how far does it land?
2) Now, taking into account Coriolis' force, answer the same questions as above.
3) In which direction and by how much is the rocket deviated from the east direction?



Q2: Linear motion of a tri atomic molecule. As an example CO_2 molecule can be modeled as shown. For simplicity consider only motion in one dimension, the x axis. The two end particles, each of mass m, are bound to the central particle, mass M, through a potential function that is equivalent to two springs of stiffness K, as shown. Use the coordinates x_1, x_2, and x_3.

a) Write the kinetic and potential energy in matrix form.

b) Find the natural frequencies and the normal modes (do not normalize them).

c) What is the physical meaning of the normal modes.

$k - m\left(\frac{k}{2m} + \frac{k}{2M}\sqrt{M^2 - 12mM}\right)$

$k - \frac{k}{2} - \frac{k}{2M}\sqrt{}$

جامعة بيرزيت

BIRZEIT UNIVERSITY

Physics Department

Physics 334

Second Exam **Date:24/5/2018** **Time: 1.5 hours**

Instructor: Dr. E Badran

Q1: A bug crawls outward with constant speed $v1$ along the spoke of a wheel which is rotating with constant angular velocity ω about a vertical axis.

a) Find all the apparent forces acting on the bug.

b) Find how far the bug can crawl before it starts to slip given the (7 points)

Q2: The three principal axes of a tennis racket are (1) along the handle, (2) perpendicular to the handle in the plane of the string and (3) perpendicular to the handle and strings.The moments of inertia are in the following relation

$I1 < I2 < I3$,When a tennis racket is tossed in the air with a spin in the direction of either axis (1) or (3), the racket continues to spin uniformly about the initial axis and can be easily recaught. However, if the initial spin is around axis (2), the motion rapidly becomes irregular, and it is hard to catch the racket. Explain this behaviour by studying the stability.(7 points).

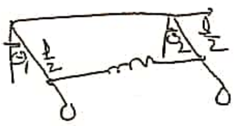
Q3: a) Show that the inertia tensor for a square plate of side a and mass m in coordinate system $Oxyz$ where O is at one corner of the plate and the x *and* y axes along the two edges. $I = \begin{pmatrix} ma^2/3 & -ma^2/4 & 0 \\ -ma^2/4 & ma^2/3 & 0 \\ 0 & 0 & 2ma^2/3 \end{pmatrix}$

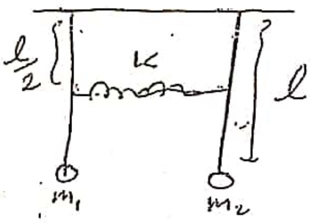
b) Find the angular momentum and kinetic energy of the above plate if it is rotating about the diagonal.

c) Find the principal moments of inertia .

d) Find the principal axes associated with it.(8 points)

Q3: Determine the natural frequencies of small oscillations and the normal for two equal length b and two equal masses m connected to a spring of spring constant k as shown. The spring is un stretched in the equilibrium position. Consider: m = b = g =1.





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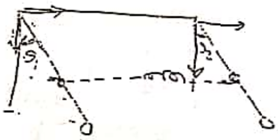
and rigid

GOOD LUCK



$\frac{1}{4}(2\dot{x}_2).$

$\frac{1}{4}(2\dot{x}_2)$

$\frac{1}{2}(2\dot{x}_2$

$\frac{1}{2}(2\dot{x}_1^2)$

$\frac{1}{2}kx$

1.

$\left(\frac{1}{4}\ 2\dot{x}_1\right) = \frac{1}{2}(2\dot{x}_1)$

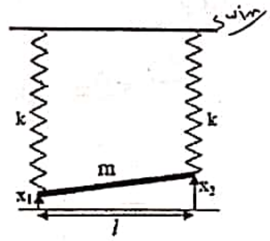
1

second

Q3: Consider a frictionless puck on a horizontal turntable that rotates counterclockwise with a constant angular velocity Ω. Ignore the rotation of the Earth. In a coordinate system attached to the turntable with the origin on the rotation axis the puck has initial coordinate $(x_0,0)$ and initial velocity (v_x, v_y). Determine the subsequent motion of the puck on the turntable given by $x(t)$, $y(t)$. Hint: It is convenient to consider the variable $s = x+iy$ (8 points)

first

Q4: : A rod of mass m and length l is suspended from two massless springs with a spring constant k as shown in the figure. (Note: *the gravity force does not need to be included.*)



A) Show that the kinetic and potential energy for the system in terms of displacements from equilibrium position of the two ends of the rod x_1 *and* x_2.

$$T = \frac{m}{6}(\dot{x}_1^2 + \dot{x}_1\dot{x}_2 + \dot{x}_2^2) \ \ and\ V(x_1,x_2) = \frac{1}{2}k(x_1^2 + x_2^2),\ where\ I = \frac{1}{12}ml^2$$

B) Find the natural frequencies.

C) Find the normal modes and give a physical explanation of them. (10 points)

first

Q5: Disscus the motion of a string when the initial conditions are

$q(x,0) = \frac{4x(L-x)}{L^2}$, $\dot{q}(x,0) = 0$. Find the characteristic frequencies and calculate the amplitude nth mode. (8 points)

first

$$\mu_r = \frac{2}{l}\int_0^L q(x,0)\sin\frac{r\pi x}{L}\,dx$$

$$v_r = -\frac{2}{\omega_r L}\int_0^L \dot{q}(x,o)\sin\frac{r\pi x}{L}\,dx$$

Good luck

$x = (x_0 + v_x t)\cos\omega t + (v_y + x_0\omega)t \sin\omega t$

$y = (v_y + x_0\omega)t\cos\omega t - (x_0 + v_x t)\sin\omega t$

16 + 10 = 26

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جامعة بيرزيت

BIRZEIT UNIVERSITY

Physics Department

First Exam **Date:16/3/2010**

hours

Instructor: Dr. E. Badran

Q1: A spring of rest length L_o (no tension) is connected to a support at one end and has a mass M attached at the other. Neglect the mass of the spring, the dimension of the mass M, and assume that the motion is confined to a vertical plane. Also, assume that the spring only stretches without bending but it can swing in the plane.

1. Using the angular displacement of the mass from the vertical and the length that the string has stretched from its rest length (hanging with the mass m), find Lagrange's equations.

2. Solve these equations fro small stretching and angular displacements.

3. Find the Hamiltonian and set up Hamilton equations

Q2: a) Define chaotic system and how does your definition relate sensitivity to initial conditions to Liapunov exponents?

b) Consider a particle influenced by a force of the form

$$F(x) = -kx + \lambda x^2$$

For $\lambda > 0$, where the system could be considered hard or soft.

Draw a phase portrait for this motion. What is the maximum energy and we still have a pound motion. What is the period of small oscillations.

c) Show that the system $y_{n+1} = 1 - \gamma y_n^2$ with $-1<y<1$ and $0<\gamma<2$ can be transformed to the logistic map $x_{n+1} = \alpha x_n(1-x_n)$ by substituting $y = cx + d$. Find γ, c, d in terms of α. Find the first three bifurcations for the system.

$dr\,\hat{r} + r\,d\theta\,\hat{\theta}$

$\dot{r}^2 + r^2\dot{\theta}^2$

$\sum q_k p$

$\frac{\partial H}{\partial q_k} = -\dot{p}_k$

$\frac{\partial H}{\partial p_k} = \dot{q}_k$

$V = \frac{1}{2}kx^2$

ميكانيك ٢

$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}}\right) - \frac{\partial L}{\partial x}$

جامعة بيرزيت

BIRZEIT UNIVERSITY

Physics Department

First Exam | **Date:24/3/2015** | **Time: 75 min.**

Instructor: Dr. E. Badran

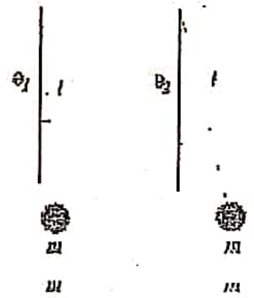
Q1: For a charged particle of mass m, and charge q is moving in the xy plane. The particle is subjected to an electromagnetic field, with vector potential $A = B_0(-y/2, x/2, 0)$ and scalar potential $\varphi(x) = -V_0 x$. Then, the Lagrangian can be written as $L =$

$$L = T - q\,\varphi + (q/c)\,A.v$$

Where v is the velocity in the xy plane and c is the speed of light.

A. Find the equations of motion.
B. Find the canonical momentum in each direction.
C. Find the Hamiltonian.

Q2: Two identical bobs of mass m are hanging from strings of length l. The two pendulums interact with each other through the another supporting string as shown and we assume that the mechanical energy of the system is conserved. The interaction between the two pendulums is modeled as



$$\left\{ V_{int} = -\lambda mgl\theta_1\theta_2 \right\}$$

a) Write the kinetic and potential energy in matrix form.

b) Find the natural frequencies and normal modes of small oscillations

c) Discuss the physical meaning of the normal modes.

جامعة بيرزيت

BIRZEIT UNIVERSITY

Physics Department

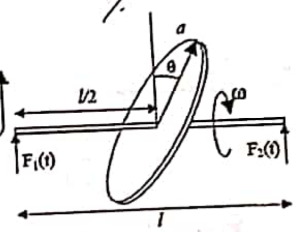
Final Exam | **Date:31/5/2018** | **Time: 150 min**

2.5 hours

Instructor: Dr. E. Badran

(The exam has 44 points and the full mark is 40)

Q1: A disk of mass m and radius is a mounted in the middle of massless rod of length l. The plane of the disk is tilted by an angle θ away from the normal to the rod. Initially the disk is at rest tilted up, as shown in the figure. At $t = 0$ it begins to rotate around the axis of the rod with an angular velocity ω in the direction shown. The rod is supported at the two ends and forces $F_1(t)$ and $F_2(t)$ are such that it rotates without wobble with a constant angular velocity .



10

A) What are direction of the principal axes of inertia of the disk?

B) Calculate the principal moments of inertia of the disk?

C) What are the components of the angular velocity ω in the body frame of reference?

D) Determine the torques needed to keep the disk spinning without wobble?

E) (10 points)

Q2: Consider a cube placed on a flat table with one face directly in front of you.

A) Find the matrix of rotation which represent the following displacement of the cube

matrix

1) The cube is rotated by $\pi/2$ about the y.

2) Then the cube is rotated by $\pi/2$ about the new x axis.

B) Find the axis and rotation angle that will accomplish the same displacement in a single rotation about a single axis. (8 points) mechanics 2

جامعة بيرزيت

BIRZEIT UNIVERSITY

Physics Dept.

Physics 334

First Hour Exam
Time: 1:15 Hours

Spring 2005/2006
Date: 4/4/2006

1) a) A geodesic is a line that represents the shortest path between any two points when the path is restricted to a particular surface. Find the geodesic of the cylinder. $\rho^2 d\theta + dz^2$. $\dot{z}^2 = \rho^2 C$

$h = \sum_i \dot{q}_i \frac{\partial L}{\partial \dot{q}} - L$

b) Show that the time invariance in the Lagrangian is associated with the conservation of Hamilton – Jaccobi function. How is this conservation is associated with energy conservation. $h = \sum p\dot{q} - L$.

$\dot{P} = -\frac{\partial H}{\partial q}$

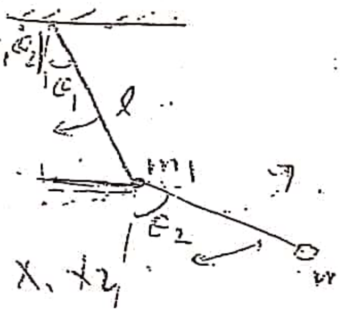
$\dot{q} = \frac{\partial H}{\partial P}$

A double pendulum of equal lengths, but not equal masses as shown, considering small oscillations only:

a) Show that $T = \frac{1}{2} m_1 l^2 \dot{\theta}_1^2 + \frac{1}{2} m_2 l^2 (\dot{\theta}_1^2 + \dot{\theta}_2^2 + 2\dot{\theta}_1\dot{\theta}_2)$

$V = m_1 g l \frac{\theta_1^2}{2} + m_2 g l \left(\frac{\theta_1^2 + \theta_2^2}{2}\right)$

b) Find the natural frequencies of the system.
c) Find the normal modes.
d) Give physical sketch of the normal modes.



$l - \cos\theta_1$ $(1 - \cos\theta)$ $(l\cos\theta_1) +$ x_1, x_2

~~l cos θ~~ $1 - \frac{\theta_1}{2}$ (θ_1)

GOOD LUCK

$mgl\frac{\theta_1^2}{2}$

$l\cos\theta_1$

$l\cos\theta_1 + l\cos\theta_2$

$y_1 = l(\cos\theta_1)$

$$\Omega_1 = \sqrt{\frac{(I_1 - I_2)(I - I_3)}{I_2 I_3}}$$

$$\Omega_2 = \sqrt{\frac{(I_2 - I_3)(I_1 - I_2)}{I_3 I_1}}$$

$$\Omega_3 = \sqrt{\frac{(I_3 - I_1)(I_1 - I_2)}{I_1 I_2}}$$

جامعة بيرزيت

BIRZEIT UNIVERSITY

Physics Department

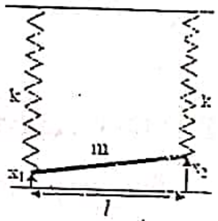
Final Exam **Date:18/6/2014** **Time: 150 m**

Instructor: Dr. E. Badran

(You have 45 points and the full mark is 40)

Q1) The three principal axes of a tennis racket are (1) along the handle, (2) perpendicular to the handle in the plane of the string and (3) perpendicular to the handle and strings. The moments of inertia are in the following relation $I_1 < I_2 < I_3$.
When a tennis racket is tossed in the air with a spin in the direction of either axis (1) or (3), the racket continues to spin uniformly about the initial axis and can be easily recaught. However, if the initial spin is around axis (2), the motion rapidly becomes irregular, and it is hard to catch the racket. Explain this behaviour by studying the stability. (10 points)

Q2: A rod of mass m and length l is suspended from two massless springs with a spring constant k as shown in the figure. (Note: *the gravity force does not need to be included.*)



A) Show that the kinetic and potential energy for the system in terms of displacements from equilibrium position of the two ends of the rod x_1 and x_2.

$$T = \frac{m}{6}(\dot{x}_1^2 + \dot{x}_1\dot{x}_2 + \dot{x}_2^2) \text{ and } V(x_1, x_2) = \frac{1}{2}k(x_1^2 + x_2^2), \text{ where } I = \frac{1}{12}ml^2$$

B) Find the natural frequencies.

C) Find the normal modes and give a physical explanation of them. (12 points)

Q3) Consider a frictionless puck on a horizontal turntable that rotates counterclockwise with a constant angular velocity Ω. Ignore the rotation of the Earth. In a coordinate system attached to the turntable with the origin on the rotation axis the puck has initial coordinate $(x_0, 0)$ and initial velocity (v_x, v_y). Determine the subsequent motion of the puck on the turntable given by $x(t)$, $y(t)$. Hint: It is convenient to consider the variable $s = x + iy$ (10 points)

$(\dot{x}_1 + \dot{x}_2)^2$

$\frac{1}{2}m\left(\frac{\dot{x}_1^2}{4} - \dot{x}_1^2\right) \quad \frac{1}{12}ml^2(\dot{x}_1^2 + 2\dot{x}_1\dot{x}_2 + \dot{x}_2^2)$

$\frac{3}{4} \quad \frac{1}{6} = \frac{1}{2} + x \quad \frac{6}{6 \cdot 12} - \frac{1}{12}$

$x = \frac{1}{6} - \frac{3}{12} = -\frac{2}{6} = -\frac{1}{3}$

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