Images Formed by Plane Mirrors

Flat mirror

P'

P

A

B

d_o

d_i

Q'

Q

```
```
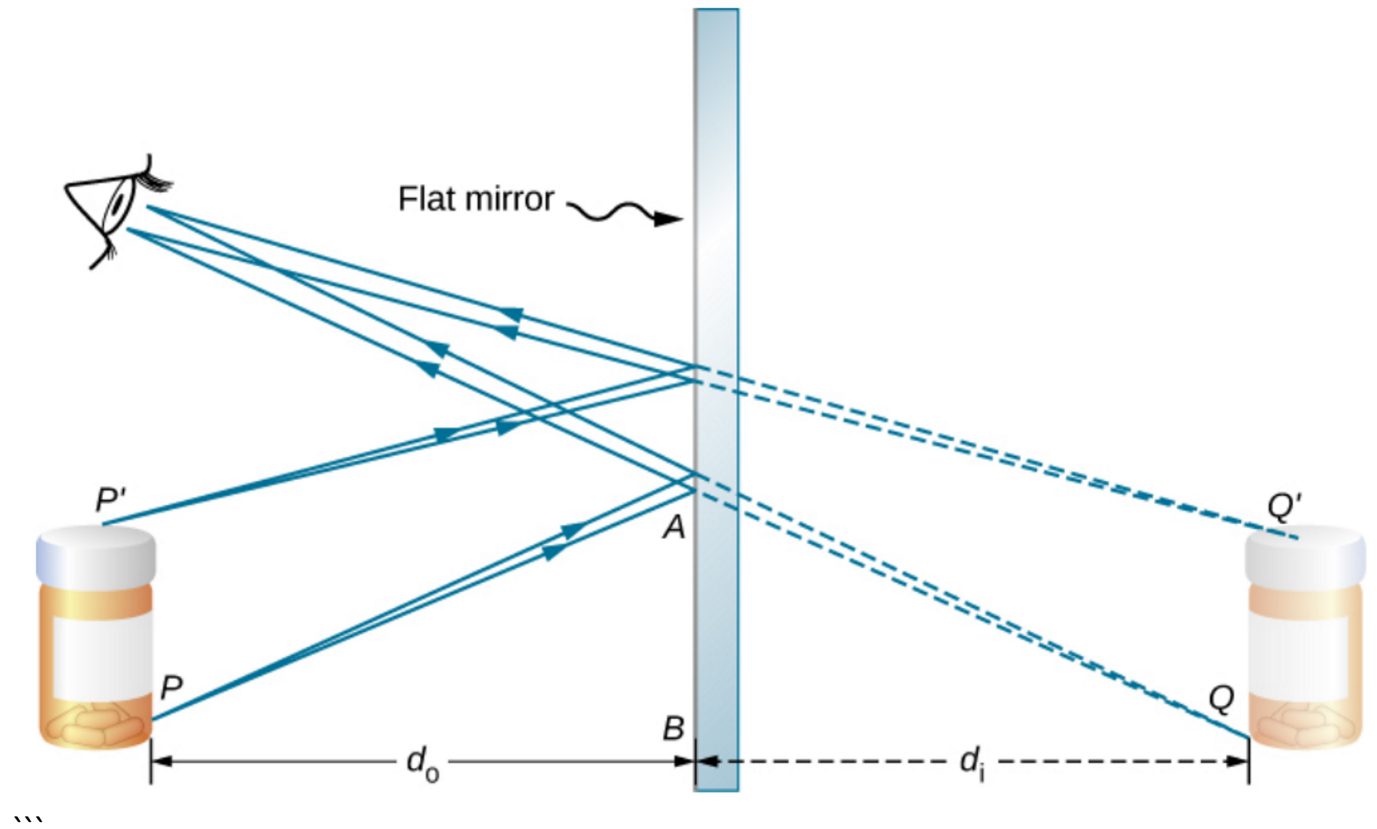
Two plane mirrors at right angle

mirror

object

detector

image
(virtual)

mirror

image
(virtual)

image
(virtual)

Spherical Mirrors

Reflective surface

Concave mirror

Vertex

Vertex

Center of
curvature

Optical axis

Reflective surface

Convex mirror

Small spherical mirror

F

f

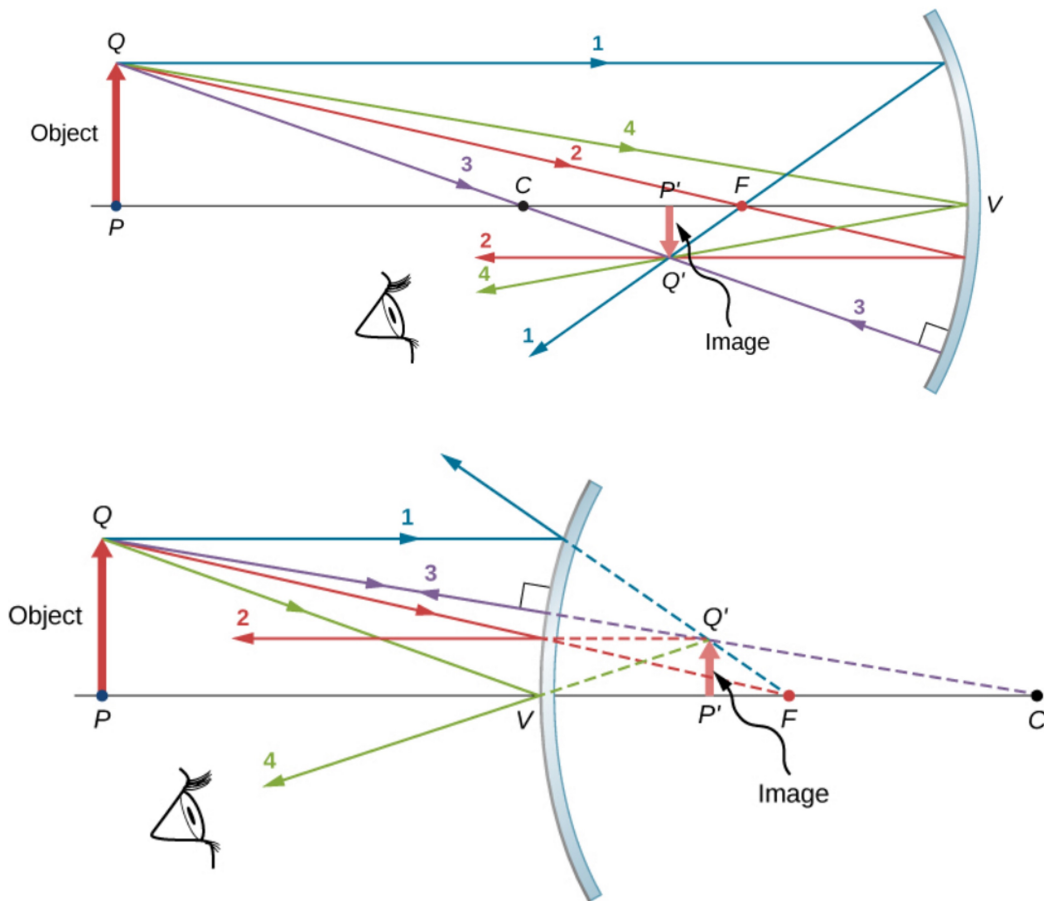Convex spherical mirror

F

f
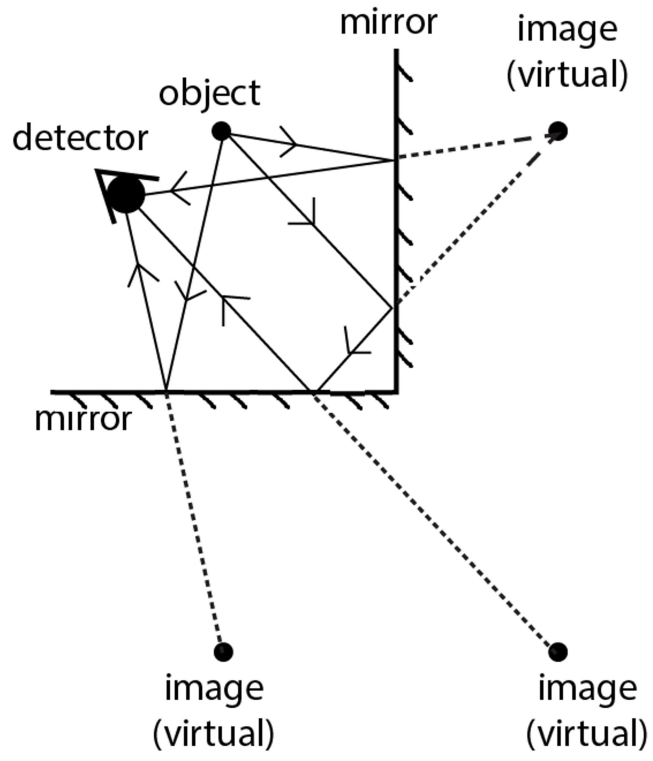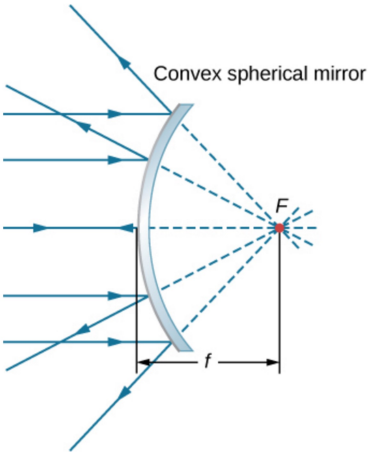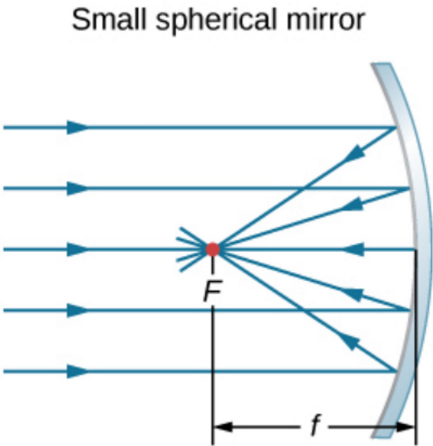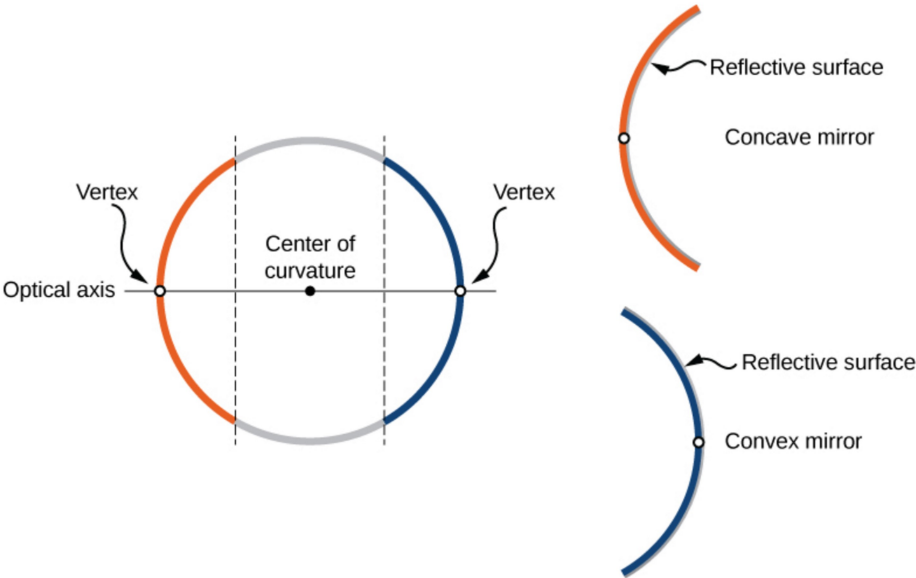
Convex mirror

Convex mirror

Concave mirror

Using Ray Tracing to Locate Images

Tuesday, April 5, 2022   11:03 PM

Ray-Tracing Rules

Ray tracing is very useful for mirrors. The rules for ray tracing are summarized here for reference:

- A ray traveling parallel to the optical axis of a spherical mirror is reflected along a line that goes through the focal point of the mirror (ray 1 in Figure 2.3.5).
- A ray traveling along a line that goes through the focal point of a spherical mirror is reflected along a line parallel to the optical axis of the mirror (ray 2 in Figure 2.3.5).
- A ray traveling along a line that goes through the center of curvature of a spherical mirror is reflected back along the same line (ray 3 in Figure 2.3.5).
- A ray that strikes the vertex of a spherical mirror is reflected symmetrically about the optical axis of the mirror (ray 4 in Figure 2.3.5).

Image Formation by Reflection—The Mirror Equation

$$\left.\begin{aligned}\tan\theta &= \frac{h_o}{d_o}\\[2mm]\tan\theta' &= -\tan\theta = \frac{h_i}{d_i}\end{aligned}\right\} = \frac{h_o}{d_o} = -\frac{h_i}{d_i}$$

$$-\frac{h_o}{h_i} = \frac{d_o}{d_i}.$$
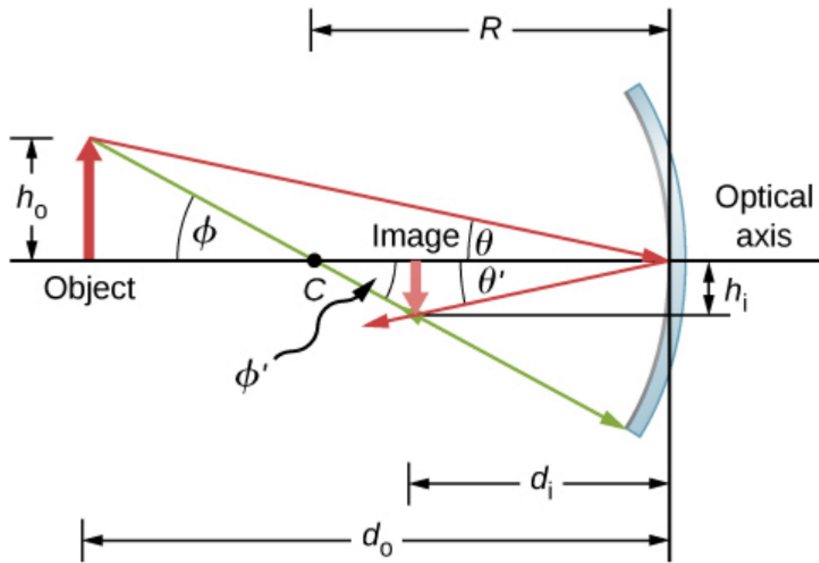
Figure 2.3.6: Image formed by a concave mirror.

$$\left.\begin{aligned}\tan\phi &= \frac{h_o}{d_o - R}\\[2mm]\tan\phi' &= -\tan\phi = \frac{h_i}{R - d_i}\end{aligned}\right\} = \frac{h_o}{d_o - R} = -\frac{h_i}{R - d_i}$$

$$-\frac{h_o}{h_i} = \frac{d_o - R}{R - d_i}.$$

$$\frac{d_o}{d_i} = \frac{d_o - R}{R - d_i}.$$

$$\frac{1}{d_o} + \frac{1}{d_i} = \frac{2}{R}.$$

$$\underbrace{\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f}}_{\text{mirror equation}}.$$

Sign convention for spherical mirrors

Using a consistent sign convention is very important in geometric optics. It assigns positive or negative values for the quantities that characterize an optical system. Understanding the sign convention allows you to describe an image without constructing a ray diagram. This text uses the following sign convention:

1. The focal length f is positive for concave mirrors and negative for convex mirrors.
2. The image distance d_i is positive for real images and negative for virtual images.
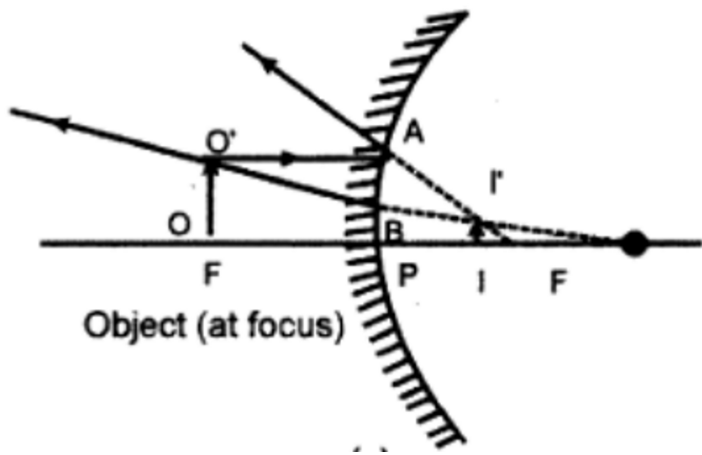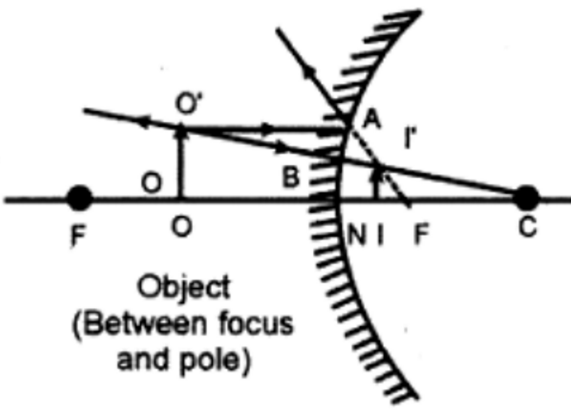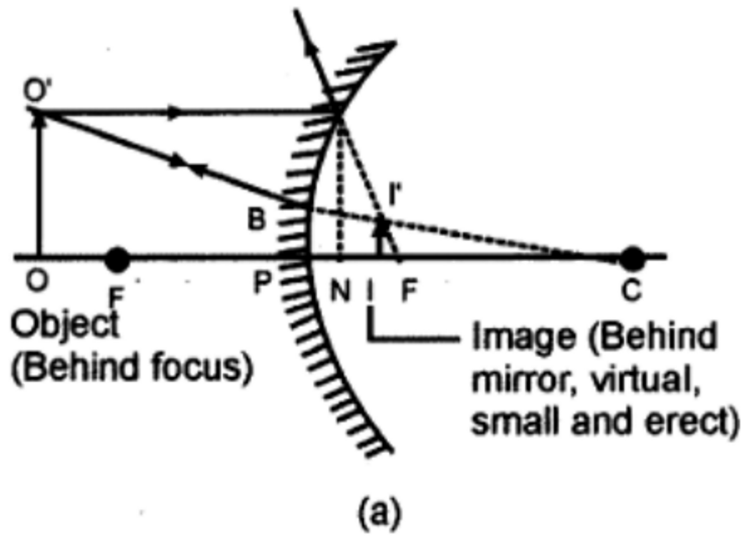
Image Magnification

$$-\frac{h_o}{h_i} = \frac{d_o}{d_i}.$$

$$m = \underbrace{\frac{h_i}{h_o}}_{\text{linear magnification}}.$$

$$m = \frac{h_i}{h_o} = -\frac{d_i}{d_o}.$$

O'

B

O
F
Object
(Behind focus)

P   N I F

C

Image (Behind
mirror, virtual,
small and erect)

(a)

O'

A I'

O

B

F   O

N I F   C

Object
(Between focus
and pole)

O'

A

O

I'

B

F

P   I   F

Object (at focus)

Derivation of spherical mirror formula
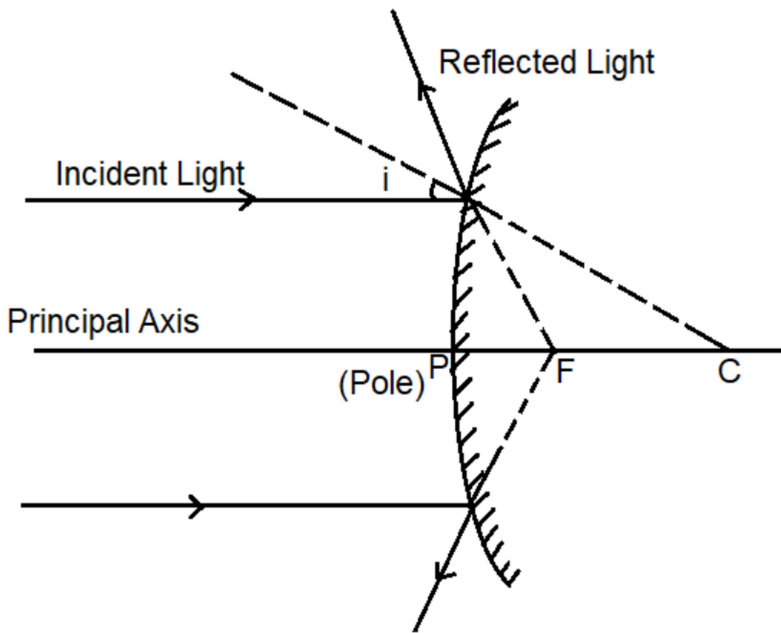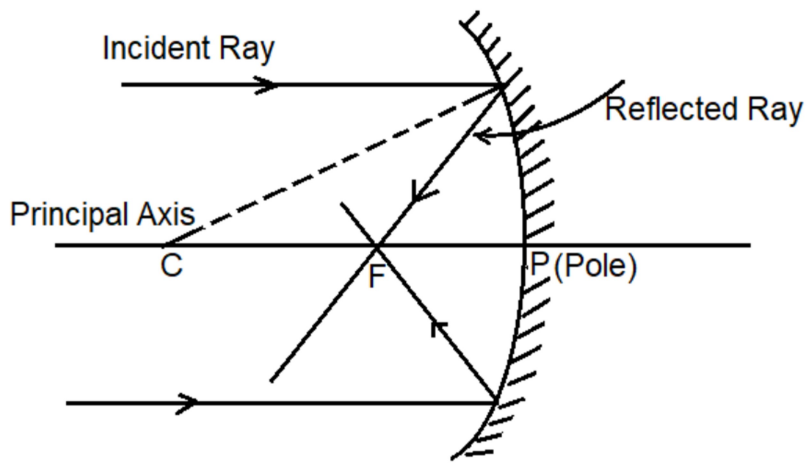
Cartesian Sign Convention

Incident Ray +ve -ve Mirror / Lens
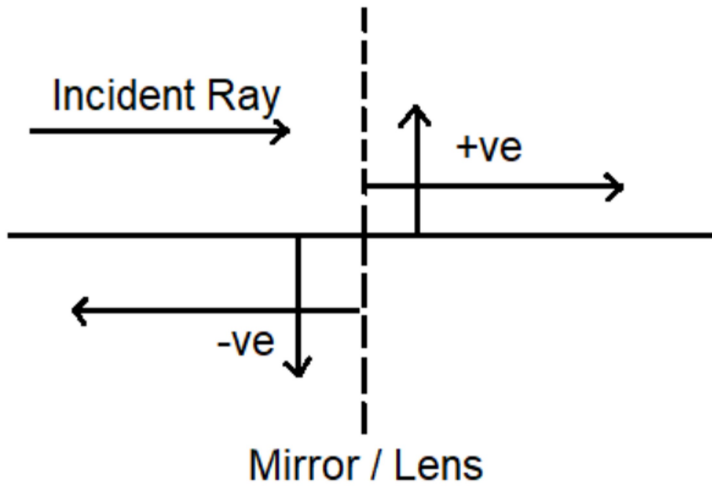
(i) All distances are measured from the pole of the mirror / lens / refracting surface which should be placed at the origin.

When applying the mirror formula, it is necessary to observe the following points:

- That all distances are measured from the mirror as the origin.
- All real distances are positive while all virtual distances are negative.
- A concave mirror has a positive focal length while a convex mirror has a negative focal length.

Example

1. Determine the position, size and nature of the image of an object 4cm tall placed on the principal axis of a concave mirror of focal length 15cm at a distance 30cm from the mirror.

solution

u=30cm, f= 15cm, h_o=4cm

1/v=1/f-1/u

= 1/15 − 1/30 =1/30

v =30cm

Also, m=v/u =h_i/h_o

Thus, h_i=(30cm x 4cm)/30cm =4cm.

Thus the image formed is real and same size as the object.

If an object is 30cm in front of a convex mirror that had a focal length of 60cm, how far behind the mirror will the image appear to an observer? How tall will the image be?
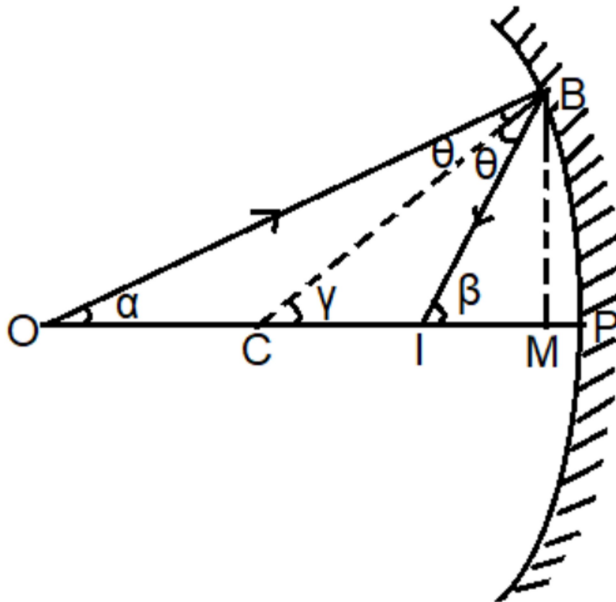
If an object is 30cm in front of a convex mirror that had a focal length of 60cm, how far behind the mirror will the image appear to an observer? How tall will the image be?

d_i= -20cm, h_i= (2/3)h_o

$\alpha + \theta = \gamma$, $\gamma + \theta = \beta$

$\Rightarrow \alpha + \beta = 2\gamma$

For small aperture of the mirror, α , β , $\gamma \rightarrow 0$

$\Rightarrow \alpha \approx \tan \alpha$, $\beta \approx \tan \beta$, $\gamma \approx \tan \gamma$ & M $\rightarrow$ P (for paraxial rays)

$\Rightarrow \tan \alpha + \tan \beta = 2 \tan \gamma$

$$\frac{BM}{MO} + \frac{BM}{MI} = 2\frac{BM}{MC}$$

$$\frac{1}{MO} + \frac{1}{MI} = \frac{2}{MC}$$

$$\frac{1}{PO} + \frac{1}{PI} = \frac{2}{PC}$$

Applying Sign convention,

$$-\frac{1}{u} + \frac{1}{-v} = -\frac{2}{R}$$

$$\frac{1}{u} + \frac{1}{v} = \frac{2}{R}$$
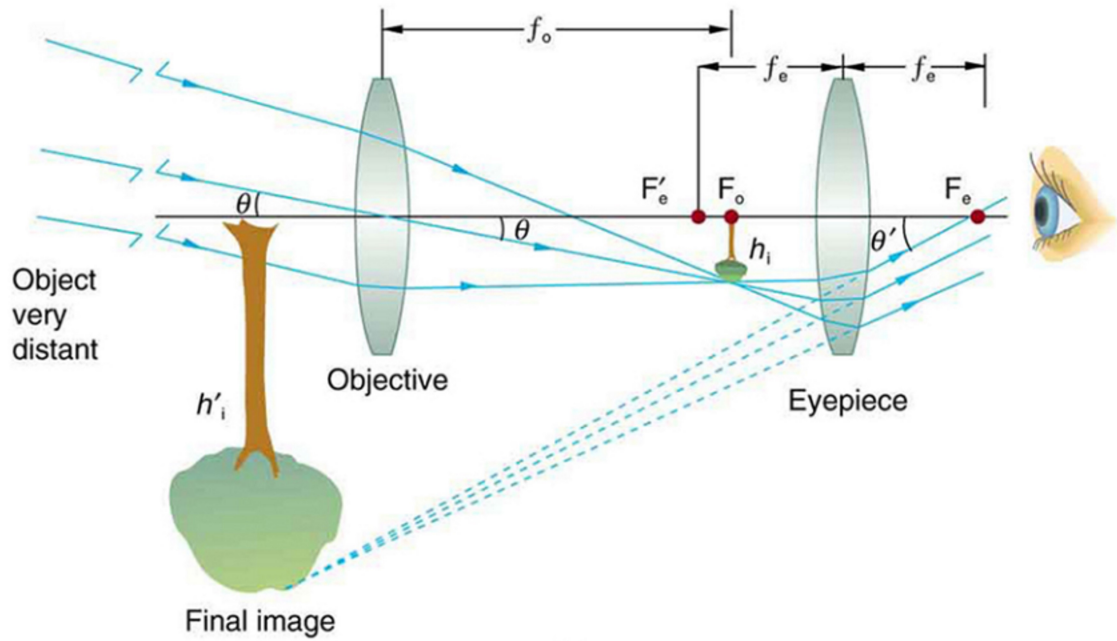
u = ∞

$$\frac{1}{v} = \frac{2}{R}$$

If u = ∞ , v = f

Hence , f = R/2 and

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$$

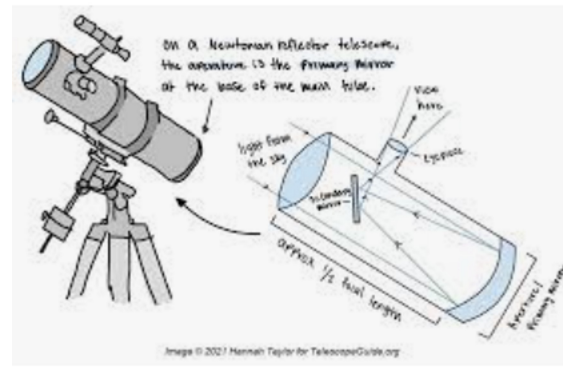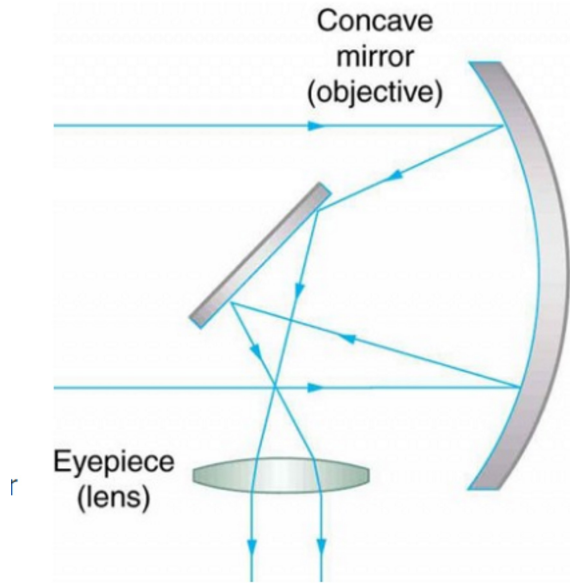# Simple telescope (Refracting and reflecting)

Object very distant

Objective

Eyepiece

Final image

$$M = \frac{\theta'}{\theta} = -\frac{f_o}{f_e}$$

Reflecting Telescopes

Concave
mirror
(objective)

Eyepiece
(lens)

On a Newtonian reflector telescope,
the aperture is the primary mirror
at the base of the main tube.

light from
the sky

Secondary
mirror

Eyepiece

approx ½ focal length

Image © 2021 Hannah Taylor for TelescopeGuide.org
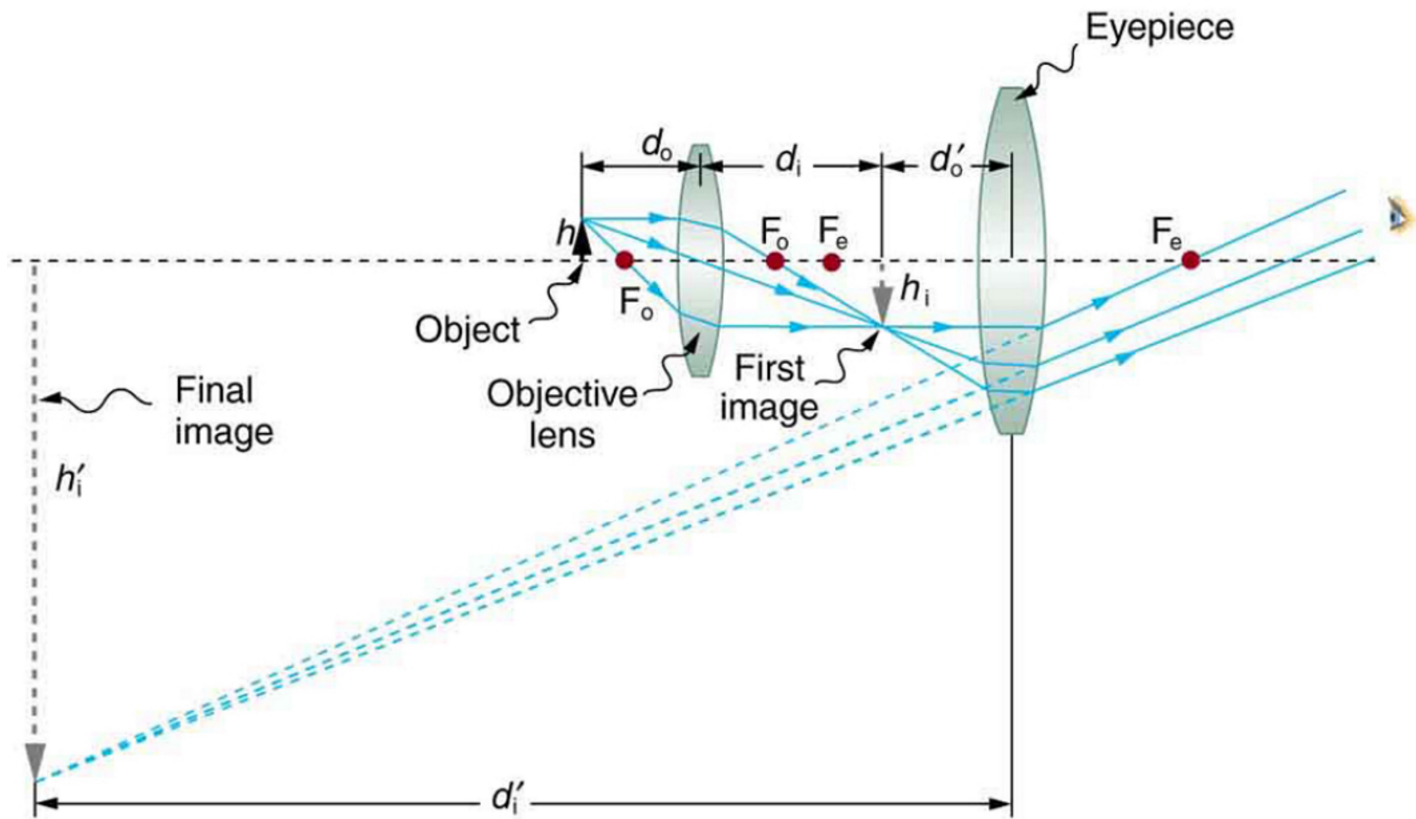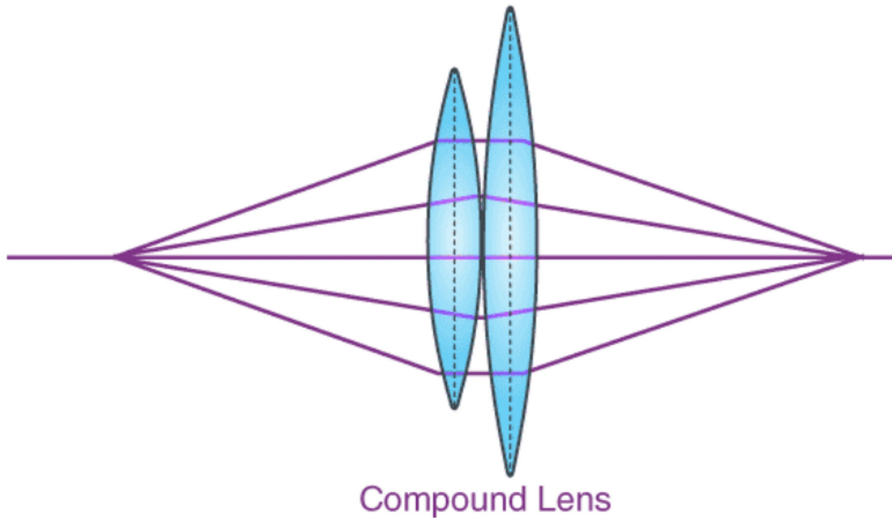
Simple Microscope

Figure 2. A compound microscope composed of two lenses, an objective and an eyepiece. The objective forms a case 1 image that is larger than the object. This first image is the object for the eyepiece. The eyepiece forms a case 2 final image that is further magnified.

$$m_o = -\frac{d_i}{d_o}$$

Compound lenses

Compound Lens

$$\frac{1}{f} = \frac{1}{f1} + \frac{1}{f2}$$

$$\frac{1}{f} = \frac{1}{f1} + \frac{1}{f2} - \frac{d}{f1.f2}$$

Application of Compound Lenses

We find compounded lenses used in telescopes and microscopes where two or more lenses are combined in order to:

- Reduce defects caused by using a single lens.
- Get an erect image of an object.
- Increase the magnification of the image.