wave equation

$$\frac{\partial^2\psi}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2\psi}{\partial t^2}$$

$$\psi = \sin(kx \pm \omega t)$$

$k = \frac{2\pi}{\lambda}$	$\omega = \frac{2\pi}{T}$	$\frac{\omega}{k} = v$

Phasors and addition of waves

$y_1 = A_1 e^{i\phi_1}$	$y_2 = A_2 e^{i\phi_2}$	$\delta = \phi_2 - \phi_1$
$y_r = y_1 + y_2$	$y_r = \lvert y_r\rvert e^{i\phi}$	

$\lvert y_r\rvert = A_1^2 + A_2^2 + 2A_1A_2\cos\delta$	$\tan\phi = \frac{A_1\sin\phi_1 + A_2\sin\phi_2}{A_1\cos\phi_1 + A_2\cos\phi_2}$

Spherical waves	Cylindrical waves
$\vec{E}(x,y,z,t) = \frac{\overrightarrow{E_0}}{r}e^{(\vec{k}.\vec{r}-\omega t)}$	$\vec{E}(x,y,z,t) = \frac{\overrightarrow{E_0}}{\sqrt{r}}e^{(\vec{k}.\vec{r}-\omega t)}$

Electromagnetic theory

$\vec{\nabla}.\vec{D} = \rho$	$\vec{\nabla}\times\vec{E} = -\frac{\partial\vec{B}}{\partial t}$
$\vec{\nabla}.B = 0$	$\vec{\nabla}\times\vec{H} = \vec{J} + \frac{\partial\vec{D}}{\partial t}$

$\vec{D} = \epsilon_0\vec{E} + \vec{P} = \epsilon_0(1+x)\vec{E} = \epsilon\vec{E} = \epsilon_0 n^2\vec{E}$	$\vec{B} = \mu_0\vec{H} + \vec{M}$
$n = \sqrt{\frac{\epsilon}{\epsilon_0}}$	$\vec{J} = \sigma\vec{E}$

Electromagnetic wave equation

$$\nabla^2\vec{E} = \mu\epsilon\frac{\partial^2\vec{E}}{\partial t^2}$$

$v = \frac{1}{\sqrt{\mu\epsilon}} = \frac{c}{n}$	$\vec{k}.\vec{E} = 0$	$\vec{K}\times\vec{E} = w\vec{B}$	$\lvert\vec{B}\rvert = \frac{n}{c}\lvert\vec{E}\rvert$	$\frac{\lvert\vec{E}\rvert}{\lvert\vec{H}\rvert} = \sqrt{\frac{\mu}{\epsilon}} = Z$

Energy and momentum

$\vec{S} = \vec{E}\times\vec{H}$ pointing vector

$I = \lvert\langle\vec{S}\rangle\rvert = \frac{1}{2}E_0H_0 = \frac{1}{2}c\epsilon_0E_0^2 = c\langle u\rangle$ I: irradiance, $\langle u\rangle$: average energy density

$\langle\vec{F}\rangle = \langle\frac{d\vec{P}}{dt}\rangle = \hat{k}q\langle v_xB\rangle$	$\langle\frac{dU}{dt}\rangle = cq\langle v_xB\rangle$	$\langle\frac{d\vec{P}}{dt}\rangle = \frac{1}{c}\langle\frac{dU}{dt}\rangle$

Damped oscillator model

$$m(\ddot{x} + \gamma\dot{x} + \omega_0^2x) = qE_0e^{i\omega t}$$

$$x_0 = \frac{qE_0}{m(\omega_0^2-\omega^2+i\gamma\omega)}$$

$$p = qx = \frac{Nq^2E_0e^{i\omega t}}{m(w_0^2-w^2+i\gamma\omega)}$$

dipole moment for N oscillator per unit volume (macroscopic polarization)

$$n = 1 + \frac{Nq^2}{2m\epsilon_0(\omega_0^2-\omega^2+i\gamma\omega)} = n' + in''$$

$n' = 1 + \frac{Nq^2}{2m\epsilon_0}\frac{\omega_0^2-\omega^2}{(\omega_0^2-\omega^2)^2+\gamma^2\omega^2}$	$n'' = \frac{Nq^2}{2m\epsilon_0}\frac{\gamma\omega}{(\omega_0^2-\omega^2)^2+\gamma^2\omega^2}$

For $\omega_0 \ll \omega$:

$n' = 1 + \frac{Nq^2}{2m\epsilon_0\omega_0}\frac{\omega_0-\omega}{(\omega_0-\omega)^2+(\frac{\gamma}{2})^2}$	$n'' = \frac{Nq^2}{8m\epsilon_0\omega_0}\frac{\gamma}{(\omega_0-\omega)^2+(\frac{\gamma}{2})^2}$

Refraction

$r_\perp = \frac{E_{0r}}{E_{0i}} = \frac{n_i\cos\theta_i - n_t\cos\theta_t}{n_i\cos\theta_i + n_t\cos\theta_t}$	$t_\perp = \frac{E_{0t}}{E_{0i}} = \frac{2n_i\cos\theta_i}{n_i\cos\theta_i + n_t\cos\theta_t}$
$r_\parallel = \frac{E_{0r}}{E_{0i}} = \frac{n_i\cos\theta_t - n_t\cos\theta_i}{n_i\cos\theta_t + n_t\cos\theta_i}$	$t_\parallel = \frac{E_{0t}}{E_{0i}} = \frac{2n_i\cos\theta_i}{n_i\cos\theta_t + n_t\cos\theta_i}$

$\theta_{Broster} = \tan^{-1}\frac{n_t}{n_i}$	$\theta_{Critical} = \sin^{-1}\frac{n_t}{n_i}$

Reflected power	Transmittance power	
$R = \frac{I_rA_r}{I_iA_i} = r^2$	$T = \frac{I_tA_t}{I_iA_i} = \frac{n_t\cos\theta_t}{n_i\cos\theta_i}t^2$	$I = \frac{n\epsilon_0c_0}{2}\lvert E_0\rvert^2$

Lenses

Refracting prisms	Spherical	Convex lens
$\frac{n_{prism}}{n_0} = \sin\frac{\frac{1}{2}(A+D)}{\sin\frac{1}{2}A}$	$\frac{n_1}{s_{01}} + \frac{n_2}{s_{i1}} = \frac{n_2 - n_1}{R_1}$	$\frac{1}{s_{01}} + \frac{1}{s_{i1}} = \left(\frac{n_l}{n_m} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) = \frac{1}{f}$

Mirror

$-\frac{h_0}{h_i} = \frac{d_0}{d_i}$	$\frac{1}{d_0} + \frac{1}{d_i} = \frac{2}{R} = \frac{1}{f}$	$magnification = \frac{h_i}{h_0}$

Simple telescope

$M = \frac{\theta\prime}{\theta} = \frac{f_0}{f_e}$

Compound lens

$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{d}{f_1 f_2}$

Ray tracing

Translation	Refraction
$\begin{matrix} x_2 \\ \gamma_2 \end{matrix} = \begin{bmatrix} 1 & d \\ 0 & 1 \end{bmatrix} \begin{matrix} x_1 \\ \gamma_1 \end{matrix}$	$\begin{matrix} x_2 \\ \gamma_2 \end{matrix} = \begin{bmatrix} 1 & d \\ 0 & \frac{n_2}{n_1} \end{bmatrix} \begin{matrix} x_1 \\ \gamma_1 \end{matrix}$

Curved lens

n_1 to n_2	n_2 to n_3
$\begin{matrix} x_2 \\ \gamma_2 \end{matrix} = \begin{bmatrix} 1 & 0 \\ \frac{n_1 - n_2}{n_2 R_1} & \frac{n_1}{n_2} \end{bmatrix} \begin{matrix} x_1 \\ \gamma_1 \end{matrix}$	$\begin{matrix} x_3 \\ \gamma_3 \end{matrix} = \begin{bmatrix} 1 & 0 \\ \frac{n_2 - n_3}{n_3 R_2} & \frac{n_2}{n_3} \end{bmatrix} \begin{matrix} x_2 \\ \gamma_2 \end{matrix}$

Thin lens

$$\begin{matrix} x_3 \\ \gamma_3 \end{matrix} = \begin{bmatrix} 1 & 0 \\ \frac{n_2 - n_3}{n_3 R_2} + \frac{n_1 - n_2}{n_3 R_1} & \frac{n_1}{n_3} \end{bmatrix} \begin{matrix} x_1 \\ \gamma_1 \end{matrix}$$

For $n_1 = n_3 = 1$ and $R_1 = -R_2$:

$$\begin{matrix} x_3 \\ \gamma_3 \end{matrix} = \begin{bmatrix} 1 & 0 \\ -\frac{1}{f} & 1 \end{bmatrix} \begin{matrix} x_1 \\ \gamma_1 \end{matrix}$$

$$f = \frac{R}{2(n-1)}$$

Imaging by thin lens

$$\begin{matrix} x_i \\ \gamma_i \end{matrix} = \begin{bmatrix} 1 - \frac{s_i}{f} & s_0 + s_i - \frac{s_0 s_i}{f} \\ -\frac{1}{f} & 1 - \frac{s_0}{f} \end{bmatrix} \begin{matrix} x_0 \\ \gamma_0 \end{matrix}$$

$$s_0 + s_i - \frac{s_0 s_i}{f} = 0 \rightarrow \frac{1}{s_0} + \frac{1}{s_i} = \frac{1}{f}$$

$magnification$	$angular\ magnification$
$\beta_l = 1 - \frac{s_i}{f}$	$\beta_\gamma = 1 - \frac{s_0}{f}$

$\beta_l \beta_\gamma = 1$	$x_i \gamma_i = x_0 \gamma_0$

Thick lens and lenses system

$$M = \begin{bmatrix} 1 & 0 \\ \frac{n_2 - n_3}{n_3 R_2} & \frac{n_2}{n_3} \end{bmatrix} \begin{bmatrix} 1 & d \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ \frac{n_1 - n_2}{n_2 R_1} & \frac{n_1}{n_2} \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix}$$

$$A = 1 + \frac{n_1 - n_2}{n_2 R_1} d$$

$$B = \frac{n_1}{n_2} d$$

$$C = \frac{1}{n_3}\left(\frac{n_2 - n_3}{R_2} + \frac{n_1 - n_2}{R_1} + \frac{n_1 - n_2}{n_2 R_1}\frac{n_2 - n_3}{R_2} d\right)$$

$$D = \frac{n_1}{n_3}\left(1 + \frac{n_2 - n_3}{n_2 R_2} d\right)$$

Thin lens

Take: $n_1 = n_3$ and $d = 0$

$A = 1$	$B = 0$	$C = \frac{1}{n_1}\left(\frac{n_2 - n_1}{R_2} + \frac{n_1 - n_2}{R_1}\right)$	$D = 1$

At $-R_1 = R_2 = -R$:

$$C = -\frac{2}{n_1}\left(\frac{n_2 - n_1}{R}\right)$$

At $n_1 = 1$: $C = -\frac{1}{f}$