أما المجال المغناطيسي مهمل لأنه أصلاً لازم الذرات تتحرك، فبيكون ضعيف جداً

Wave equation form here, is:

$$\nabla^2 \vec{E} - \epsilon_0 \mu_0 \frac{\partial^2 \vec{E}}{\partial t^2} = \mu_0 \frac{\partial^2 \vec{P}}{\partial t^2} \quad \text{— } (2.13)$$

Solutions:

$$\left.\begin{aligned} \vec{E} &= \vec{E}_0\, e^{i(\vec{k}\cdot\vec{r} - \omega t)} \\ \vec{P} &= \vec{P}_0\, e^{i(\vec{k}\cdot\vec{r} - \omega t)} \end{aligned}\right\} \rightarrow \text{same frequency} \quad (2.14)$$

Substitute 2.14 into 2.13:

$$\nabla^2 \vec{E} = \nabla^2 \vec{E}_0\, e^{i(k\cdot r - \omega t)}$$

$$= \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}\right) E_0\, e^{i(k_x x + k_y y + k_z z - \omega t)}$$

$$= -\vec{E}_0 \left(k_x^2 + k_y^2 + k_z^2\right) e^{i(\vec{k}\cdot\vec{r} - \omega t)}$$

$$= -k^2 \vec{E}_0\, e^{i(\vec{k}\cdot\vec{r} - \omega t)}$$

$$-\epsilon_0 \mu_0 \frac{\partial^2 \vec{E}}{\partial t^2} = -\epsilon_0 \mu_0 \vec{E}_0 \frac{\partial^2}{\partial t^2}\left(e^{i(\vec{k}\cdot\vec{r} - \omega t)}\right)$$

$$= \epsilon_0 \mu_0 \omega^2 \vec{E}_0\, e^{i(\vec{k}\cdot\vec{r} - \omega t)}$$

$$\mu_0 \frac{\partial^2 \vec{P}}{\partial t^2} = \mu_0 \vec{P}_0 \frac{\partial^2}{\partial t^2}\left(e^{i(\vec{k}\cdot\vec{r} - \omega t)}\right)$$

$$= -\mu_0 \omega^2 \vec{P}_0\, e^{i(\vec{k}\cdot\vec{r} - \omega t)}$$

$\Rightarrow$ eq (2.13) becomes;

$$-k^2 \vec{E}_0\, e^{i(\vec{k}\cdot\vec{r} - \omega t)} + \epsilon_0 \mu_0 \omega^2 \vec{E}_0\, e^{i(\vec{k}\cdot\vec{r} - \omega t)} = -\mu_0 \omega^2 \vec{P}_0\, e^{i(\vec{k}\cdot\vec{r} - \omega t)}$$

$\rightarrow$ eq (2.15)

* In a linear medium:

eq (2.16) — $\vec{P}_0(\omega) = \epsilon_0 \chi(\omega) \vec{E}_0(\omega)$ (This is known as constitutive relation)

$\chi(\omega)$: dimensionless susceptibility.

↳ can be complex

Sub (2.16) into (2.15) :

First, divide both sides in (2.15) by $e^{i(\vec{k}\cdot\vec{r} - \omega t)}$,

$$-k^2 \vec{E}_0 + \epsilon_0 \mu_0 \omega^2 \vec{E}_0 = -\mu_0 \omega^2 \epsilon_0 \chi(\omega) \vec{E}_0$$

$$-k^2 + \epsilon_0 \mu_0 \omega^2 = -\mu_0 \omega^2 \epsilon_0 \chi(\omega)$$

$$k^2 = \epsilon_0 \mu_0 \omega^2 + \mu_0 \epsilon_0 \omega^2 \chi(\omega)$$

$$k^2 = \epsilon_0 \mu_0 [1 + \chi(\omega)] \omega^2$$

$$\Rightarrow k = \frac{\omega}{c} \sqrt{1 + \chi(\omega)} \quad , \quad c = \frac{1}{\sqrt{\epsilon_0 \mu_0}} = 3 \times 10^8 \text{ m/s}$$