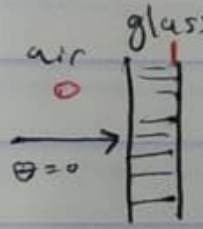
Homework 2: Hamza AlHasan - 1181636

P4.4 pg 111: $T_s^{tot} = \dfrac{T_s^{max}}{1 + F_s \sin^2\left(\frac{\phi}{2}\right)}$



(a)

$$T_s^{max} = \frac{T_s^{0\to1}\, T_s^{1-2}}{\left(1 - \sqrt{R_s^{0\leftarrow1}\, R_s^{1\to2}}\right)^2}$$

$$= \frac{\left[1 - \left(\frac{n-1}{n+1}\right)^2\right]^2}{\left[1 - \left(\frac{n-1}{n+1}\right)^2\right]^2} = 1$$

$$T_s^{0\to1} = 1 - R_s^{0\to1} = 1 - \left|r_s^{0\to1}\right|^2$$

$$= 1 - \left(-\frac{n-1}{n+1}\right)^2 = 1 - \left(\frac{n-1}{n+1}\right)^2$$

$$\Rightarrow T_s^{0\to1} = T_s^{1\to2}$$

where $r_s^{0\to1} = -\frac{n-1}{n+1}$ (at normal incident)

$$F_s = \frac{4\sqrt{R_s^{0\leftarrow1}\, R_s^{1\to2}}}{\left(1 - \sqrt{R_s^{0\leftarrow1}\, R_s^{1\to2}}\right)^2} = \frac{4R}{(1-R)^2} = \frac{4\left(\frac{n-1}{n+1}\right)^2}{\left[1 - \left(\frac{n-1}{n+1}\right)^2\right]^2}$$

$$= \left[\frac{2\,\frac{n-1}{n+1}}{1 - \left(\frac{n-1}{n+1}\right)^2}\right]^2 = \left[\frac{2}{\frac{n+1}{n-1} - \frac{n-1}{n+1}}\right]^2 = \left[\frac{2(n^2-1)}{(n+1)^2 - (n-1)^2}\right]^2$$

$$= \left[\frac{2(n^2-1)}{n^2+2n+1 - n^2+2n-1}\right]^2 = \left(\frac{2(n^2-1)}{4n}\right)^2 = \frac{(n^2-1)^2}{4n^2}$$

$\Phi_s = 2 k_1 d \underbrace{\cos\theta_1}_{1\ (\text{normal incident})} + \underbrace{\phi_{r_1}^{0\to1}}_{\text{zero}} + \underbrace{\phi_{r_2}^{1\to2}}_{\text{zero}}$

(we move from high n to low n)

$\Rightarrow \boxed{\Phi_s = \frac{4\pi n d}{\lambda_{vac}}}$; $k_1 = \frac{2\pi n_1}{\lambda_{vac}}$

we have $\Rightarrow$ $T^{tot} = \frac{1}{1 + \frac{(n^2-1)^2}{4n^2} \sin^2\left(\frac{2\pi n d}{\lambda_{vac}}\right)}$

(b) max transmittance :

$\boxed{T^{max} = 1}$

min transmittance :

$T^{tot}(\text{when } n=1.5) = \frac{1}{1 + 0.1736\,11 \sin^2(3\pi \frac{d}{\lambda_{vac}})}$

Let $0.1736\,11 = c$, $\frac{d}{\lambda_{vac}} = x$

$T = \frac{1}{1 + c \sin^2(3\pi x)}$

$\frac{dT}{dx} = \frac{-(2c \sin(3\pi x)\cos(3\pi x))\,3\pi}{[1 + c\sin^2(3\pi x)]^2} = 0$

$\Rightarrow \sin(3\pi x)\cos(3\pi x) = 0$

$3\pi x = m\pi$ or $3\pi x = m\frac{\pi}{2}$

$x = \frac{1}{3} m$ or $x = \frac{1}{6} m$

gives max $T = 1$; gives min T

$\Rightarrow$ at $x = \frac{d}{\lambda_{vac}} = \frac{1}{6} m$, T is minimum.

$\boxed{T^{min} = \frac{1}{1 + 0.1736\,11} = 0.85207}$

(c) $d = 150\ \mu m$, $n = 1.5$

As solved in part (b), $x = \frac{d}{\lambda_{vac}} = \frac{1}{3} m$ give T^{max}.

$\Rightarrow \frac{150\,\mu}{\lambda_{vac}} = \frac{1}{3} m$

$\Rightarrow \boxed{\lambda_{vac} = \frac{450\,\mu}{m}}$ gives maximum