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Optics - Final Exam

$\boxed{1}$ (a) $\vec{\nabla} R \cdot \vec{\nabla} R = [n(\vec{r})]^2$

$\vec{\nabla} R(r) = n(\vec{r})\, \hat{s}(\vec{r})$

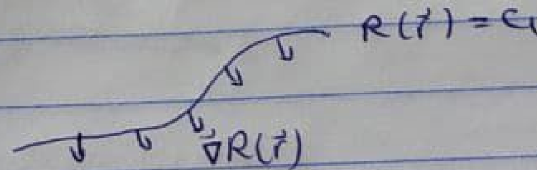
$\hat{s}(\vec{r})$: unit vector in the direction of $\vec{\nabla} R$

- $n(\vec{r})$: index as a function of $\vec{r}$
- $R(\vec{r})$ is in $\vec{E}(\vec{r},t)$ as: $\vec{E}(\vec{r},t) = \vec{E}_0(\vec{r})\, e^{i[K_{vac} \underbrace{R(\vec{r})}_{\text{here}} - \omega t]}$

$R(\vec{r})$ is a real scalar function

- $R(\vec{r})$ = constant represents a surface of wave

✓ and the wavefront travels in direction for which $R(\vec{r})$ change the fastest.



(b) single frequency

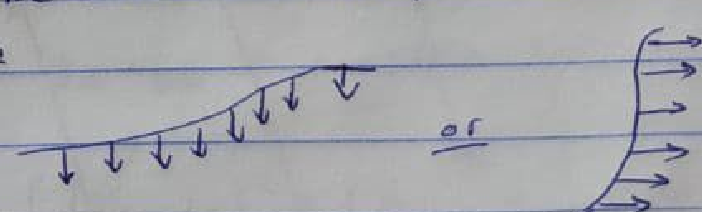
$n(\vec{r}) = \text{constant} = n$

✓ Eikonal equation: $\vec{\nabla} R(\vec{r}) = n\, \hat{s}$

✓ This means that all the points on the surface of the wave

✓ travels at the same direction.

For example:



2 (a) Fermat's Principle:

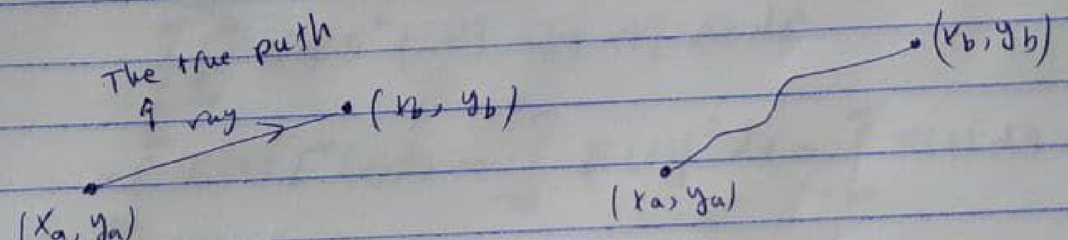
- The path taken by a light ray between two given points takes least time
- Optical path length $\equiv$ OPL

$$OPL\Big|_A^B = \min\left\{\int_A^B n\,dl\right\}$$

$$\Delta t\Big|_A^B = \int_A^B \frac{dl}{v(\vec{r})} = \int_A^B \frac{dl}{c/n(\vec{r})} = \frac{OPL|_A^B}{c}$$

To find the correct path of light, we minimize the OPL.

(b)



$$\text{distance} = d = \sqrt{(x_a - x_b)^2 + (y_a - y_b)^2}$$

$$OPL = nd = n\sqrt{(x_a - x_b)^2 + (y_a - y_b)^2}$$

x_a, x_b = fixed

$y_{tot} = y_a + y_b$ = constant

$$\Rightarrow OPL^2 = n^2\left((x_a - x_b)^2 + (y_a - y_{tot} + y_a)^2\right)$$

$$= n^2\left[(x_a - x_b)^2 + (2y_a - y_{tot})^2\right]$$

$$\frac{d\,OPL}{d y_a} = 0 \Rightarrow 2n^2(2) =$$

In Variational Calculus, the path that takes minimum time is the straight line between the two points

3 $\int_{-\infty}^{\infty} I(\vec{r},t)\,dt = \int_{-\infty}^{\infty} I(\vec{r},\omega)\,d\omega$

in general: $\int_{-\infty}^{\infty} |f(t)|^2\,dt = \int_{-\infty}^{\infty} |f(\omega)|^2\,d\omega$

and we know that $I \propto E.E^*$

special case: $I \propto |E|^2$

$$\int_{-\infty}^{\infty} I(\omega)\,d\omega = \int_{-\infty}^{\infty} |E(\omega)|^2\,d\omega = \int_{-\infty}^{\infty} E(\omega)\,E^*(\omega)\,d\omega$$

$$= \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}}\left(\int_{-\infty}^{\infty} E(t)\,e^{i\omega t}\,dt\right)\left(\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} E^*(t')\,e^{-i\omega t'}\right) d\omega$$

$$= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} E(t)\,E^*(-t')\left(\frac{1}{2\pi}\int_{-\infty}^{\infty} e^{i\omega(t'+t)}\,d\omega\right) dt\,dt'$$

$$= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} E(t)\,E^*(-t')\,\delta(t+t)\,dt\,dt'$$

$$= \int_{-\infty}^{\infty} E(t)\,E^*(t)\,dt = \int_{-\infty}^{\infty} |E(t)|^2\,dt = \int_{-\infty}^{\infty} I(t)\,dt$$

since $\delta(t'-t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} e^{i\omega(t'-t)}\,d\omega$