Chapter 1 Nature of Light

1-1. a) $\lambda = \frac{h}{p} = \frac{h}{m\,v} = \frac{6.63 \times 10^{-34}\,\mathrm{J \cdot s}}{(0.05\,\mathrm{kg})\,(20\,\mathrm{m/s})} = 6.63 \times 10^{-34}\ \mathrm{m}$

b) $\lambda = \frac{h}{p} = \frac{h}{\sqrt{2\,m\,E}} = \frac{6.63 \times 10^{-34}\,\mathrm{J \cdot s}}{[(2 \cdot 9.11 \times 10^{-31}\,\mathrm{kg})\,(10 \cdot 1.602 \times 10^{-19}\,\mathrm{J})]^{1/2}} = 3.88 \times 10^{-10}\,\mathrm{m}$

1-2. $P = \frac{\text{Energy}}{\text{time}} = \frac{n\,h\,\nu}{t} = \frac{n\,h\,c}{t\lambda} = \frac{100\left(6.63 \times 10^{-34}\,\mathrm{J \cdot s}\right)\left(3 \times 10^{8}\mathrm{m/s}\right)}{(1\,\mathrm{s})\,(550 \times 10^{-9}\mathrm{m})} = 3.62 \times 10^{-17}\,\mathrm{W}$

1-3. The energy of a photon is given by $E = h\,\nu = h\,c/\lambda$

$$\text{At } \lambda = 380\,\mathrm{nm}: \quad E = \frac{\left(6.63 \times 10^{-34}\,\mathrm{J \cdot s}\right)\left(3 \times 10^{8}\mathrm{m/s}\right)}{380 \times 10^{-9}\,\mathrm{m}} = \left(5.23 \times 10^{-19}\mathrm{J}\right)\frac{1\,\mathrm{eV}}{1.60 \times 10^{-19}\mathrm{J}} = 3.27\,\mathrm{eV}$$

$$\text{At } \lambda = 770\,\mathrm{nm}: \quad E = \frac{\left(6.63 \times 10^{-34}\,\mathrm{J \cdot s}\right)\left(3 \times 10^{8}\mathrm{m/s}\right)}{770 \times 10^{-9}\,\mathrm{m}} = \left(2.58 \times 10^{-19}\mathrm{J}\right)\frac{1\,\mathrm{eV}}{1.60 \times 10^{-19}\mathrm{J}} = 1.61\,\mathrm{eV}$$

1-4. $p = E/c = m\,c^2/c = m\,c = 2.73 \times 10^{-22}\,\mathrm{kg \cdot m/s}, \quad \lambda = \frac{h}{p} = \frac{h\,c}{E} = \frac{h\,c}{m\,c^2} = \frac{h}{m\,c} = 2.43 \times 10^{-12}\,\mathrm{m}$

1-5. $E_{v=0} = m\,c^2 = \left(9.109 \times 10^{-31}\,\mathrm{kg}\right)\left(2.998 \times 10^{8}\,\mathrm{m/s}\right)^2 = \left(8.187 \times 10^{-14}\,\mathrm{J}\right)\frac{1\,\mathrm{MeV}}{1.602 \times 10^{-19}\,\mathrm{J}} = .511\,\mathrm{MeV}$

1-6. $c\,p = \sqrt{E^2 - m^2\,c^4}$, where $E = E_K + m\,c^2 = (1 + 0.511))\mathrm{MeV}$. So $c\,p = \sqrt{1.511^2 - 0.511^2}\,\mathrm{MeV}$

That is, $c\,p = 1.422\,\mathrm{MeV}$ and $p = 1.422\,\mathrm{MeV}/c$.

1-7. $\lambda = \frac{h\,c}{E} = \frac{\left(6.626 \times 10^{-34}\,\mathrm{J \cdot s}\right)\left(2.998 \times 10^{8}\,\mathrm{m/s}\right)}{E}\left(\frac{1\,\mathrm{eV}}{1.602 \times 10^{-19}\,\mathrm{J}}\right)\left(\frac{1\ \text{Å}}{10^{-10}\,\mathrm{m}}\right) = \frac{12,400}{E}\left(\text{Å} \cdot \mathrm{eV}\right)$

1-8. $E_K = m\,c^2\left(\frac{1}{\sqrt{1 - v^2/c^2}} - 1\right) = m\,c^2\left[\left(1 - v^2/c^2\right)^{-1/2} - 1\right] \simeq m\,c^2\left[\left(1 - (-1/2)\,v^2/c^2\right) - 1\right] = \frac{1}{2}m\,v^2$

1-9. The total energy of the proton is,

$$E = E_K + m_p c^2 = 2\times 10^9\left(\frac{1.60\times 10^{-19}\,\text{J}}{1\,\text{eV}}\right) + \left(1.67\times 10^{-27}\,\text{kg}\right)\left(3.00\times 10^8\,\text{m/s}\right)^2 = 4.71\times 10^{-10}\,\text{J}$$

a) $p = \dfrac{\sqrt{E^2 - m_p^2 c^4}}{c} = \dfrac{\left[\left(4.71\times 10^{-10}\,\text{J}\right)^2 - \left(1.67\times 10^{-27}\,\text{kg}\right)^2\left(3.00\times 10^8\,\text{m}\right)^2\right]^{-1/2}}{3.00\times 10^8\,\text{m/s}}$

$p = 1.49\times 10^{-18}\,\text{kg}\cdot\text{m/s}$

b) $\lambda = h/p = \left(6.63\times 10^{-34}\,\text{J}\cdot\text{s}\right)/\left(1.49\times 10^{-18}\,\text{kg}\cdot\text{m/s}\right) = 4.45\times 10^{-16}\,\text{m}$

c) $\lambda_{\text{photon}} = h\,c/E = \left(6.63\times 10^{-34}\,\text{J}\cdot\text{s}\right)\left(3.00\times 10^8\,\text{m/s}\right)/\left(4.71\times 10^{-10}\right) = 4.22\times 10^{-16}\,\text{m}$

1-10. $n_{\text{photons}} = \dfrac{\text{Energy}}{h\,\nu} = \dfrac{\text{Energy}}{h\,c/\lambda} = \dfrac{\left(1000\,\text{W/m}^2\right)\left(10^{-4}\ \text{m}^2\right)}{(6.63\times 10^{-34}\,\text{J})\,(3.00\times 10^8\,\text{m/s})/(550\times 10^{-9}\,\text{m})} = 2.77\times 10^{17}$

1-11. $\dfrac{n_1}{n_2} = \dfrac{E_e/h\,\nu_1}{E_e/h\,\nu_2} = \dfrac{E_e\,\lambda_1/h\,c}{E_e\,\lambda_2/h\,c} = \dfrac{\lambda_1}{\lambda_2}$

1-12. The wavelength range is 380 nm to 770 nm. The corresponding frequencies are

$$\nu_{770} = \frac{c}{\lambda} = \frac{3.00\times 10^8\,\text{m/s}}{770\times 10^{-9}\,\text{m}} = 3.89\times 10^{14}\,\text{Hz}\quad \nu_{380} = \frac{c}{\lambda} = \frac{3.00\times 10^8\,\text{m/s}}{380\times 10^{-9}\,\text{m}} = 7.89\times 10^{14}\text{Hz}$$

1-13. The wavelength of the radio waves is $\lambda = c/\nu = \left(3.00\times 10^8\ \text{m/s}\right)/\left(100\times 10^6\ \text{Hz}\right) = 3\ \text{m}$. The length of the half wave antenna is then $\lambda/2 = 1.5\,\text{m}$.

1-14. The wavelength is $\lambda = c/\nu = \left(3.0\times 10^8\ \text{m/s}\right)/\left(90\times 10^6\ \text{Hz}\right) = 3.33\,\text{m}$. The length of each of the rods is then $\lambda/4 = 0.83\,\text{m}$.

1-15. a) $t = D_l/c = (90\times 10^3/3.0\times 10^8)\,\text{s} = 3.0\times 10^{-4}\,\text{s}$. b) $D_s = v_s\,t = (340)\left(3.0\times 10^{-4}\right)\text{m} = 0.10\,\text{m}$

1-16. a) $I_e = \dfrac{\Phi_e}{\Delta\omega} = \dfrac{500\,\text{W}}{4\,\pi\,\text{sr}} = 39.8\,\text{W/sr}$ b) $M_e = \dfrac{\Phi_e}{A} = \dfrac{500\,\text{W}}{5\times 10^{-4}\,\text{m}^2} = 10^6\,\text{W/m}^2$

c) $E_e = \dfrac{\Phi_e}{A} = \dfrac{\Phi_e}{4\,\pi\,r^2} = \dfrac{500\,\text{W}}{4\,\pi(2\,\text{m})^2} = 9.95\,\text{W/m}^2$ e) $\Phi_e = E_e\,A = \left(9.95\,\text{W/m}^2\right)\pi\,(0.025\,\text{m})^2 = .0195\,\text{W}$

1-17. a) The half angle divergence $\theta_{1/2}$ can be found from the relation

$$\tan(\theta_{1/2}) \approx \theta_{1/2} = \frac{r_{\text{spot}}}{L_{\text{room}}} = \frac{0.0025\,\text{m}}{15\,\text{m}} = 1.67\times 10^{-4}\,\text{rad} = .0096^\circ$$

b) The solid angle is $\Delta\omega = \dfrac{A_{\text{spot}}}{L_{\text{room}}^2} = \dfrac{\pi\,r_{\text{spot}}^2}{L_{\text{room}}^2} = \dfrac{\pi\,(0.0025\,\text{m})^2}{(15\,\text{m})^2} = 8.73\times 10^{-8}\,\text{sr}$.

c) The irradiance on the wall is $E_e = \dfrac{\Phi_e}{A_{\text{spot}}} = \dfrac{\Phi_e}{\pi\,r_{\text{spot}}^2} = \dfrac{0.0015\,\text{W}}{\pi\,(0.0025\,\text{m})^2} = 76.4\,\text{W/m}^2$.

d) The radiance is (approximating differentials as increments)

$$L_e \approx \frac{\Phi_e}{\Delta\omega\,\Delta A_{\text{laser}}\cos\theta} = \frac{0.0015\,\text{W}}{(8.73\times 10^{-8}\,\text{sr})\left(\pi\,(0.00025\,\text{m})^2\right)\cos(0)} = 8.75\times 10^{10}\,\frac{\text{W}}{\text{m}^2\cdot\text{sr}}$$

Chapter 2 Geometrical Optics

2-1. $t = \frac{\sum d_{\text{op}}}{c} = \frac{\sum_i n_i x_i}{c}$

2-2. Referring to Figure 2 12 and with lengths in cm,

$$n_0 \left(x^2 + y^2\right)^{1/2} + n_i \left(y^2 + \left(s_o + s_i - x\right)^2\right)^{1/2} = n_o s_o + n_i s_i$$

$$(1) \left(x^2 + y^2\right)^{1/2} + 1.5 \left(y^2 + (30 - x)^2\right)^{1/2} = 20 + 1.5\,(10) = 35$$

$$2.25 \left(y^2 + (30 - x)^2\right) = \left(35 - \left(x^2 + y^2\right)^{1/2}\right)^2$$

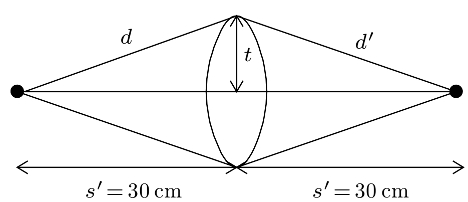
$$1.25 \left(x^2 + y^2\right) + 70 \left(x^2 + y^2\right)^{1/2} - 135\,x + 800 = 0$$

Using a calculator to guess and check or using a computer algebra system, (like the free program Maxima, for example) one can numerically solve this equation for x for given y values. Doing so results in,

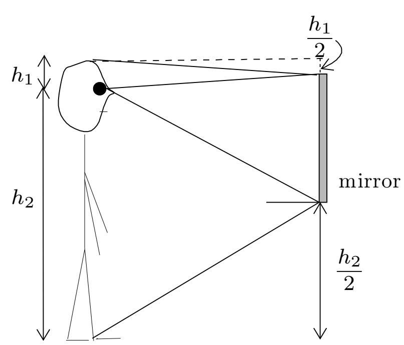
x (cm)	20	20.2	20.4	20.8	21.6	22.4	23.2	24.0	24.8	25.6	26.4	27.2
y (cm)	0	± 1.0	± 1.40	± 1.96	± 2.69	± 3.20	± 3.58	± 3.85	± 4.04	± 4.14	± 4.18	± 4.13

2-3. Refer to the figure for the relevant parameters.

$d = d' = \sqrt{30^2 + 2.5^2} = 30.104\,\text{cm}$
Fermat: $d + d' = s + s' - t + m\,t$
$d + d' = s + s' + t\,(m - 1)$
$2\,(30.10399) = 60 = t\,(1.52 - 1)$
$t = 4\,\text{mm}$
$n = 1.52$



2-4. See the figure below. Let the height of the person be $h = h_1 + h_2$.

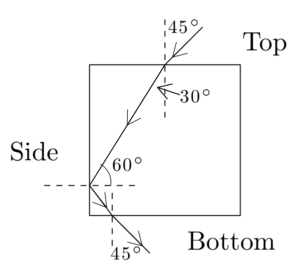


The person must be able to see the top of his head and the bottom of his feet. From the figure it is evident that the mirror height is:

$h_{\text{mirror}} = h - h_1/2 - h_2/2 = h/2$

The mirror must be half the height of the person. So for a person of height six ft person, the mirror must be 3 ft high.

2-5. Refer to the figure.

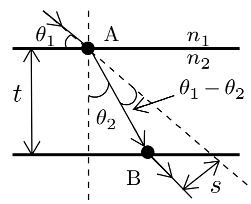


At Top: $(1) \sin 45 = \sqrt{2} \sin\theta' \Rightarrow \theta' = 30$
At Side: $\sqrt{2} \sin 60° = (1) \sin\theta', \sin\theta' = \sqrt{1.5} > 1$
Thus total internal reflection occurs.
At Bottom: reverse of Top: $\theta' = 45°$

2-6. The microscope first focuses on the scratch using direct rays. Then it focuses on the image I_2 formed in a two step process: (1) reflection from the bottom to produce an intermediate image I_1 and (2) refraction through the top surface to produce an image I_2. Thus, I_1 is at $2t$ from top surface, and I_2 is at the apparent depth for I_1, serving as the object: $s' = \dfrac{2t}{n}$ or $n = \dfrac{2t}{s'} = \dfrac{3}{1.87} = 1.60$

2-7. Refer to Figure 2 33 in the text. By geometry, $\tan\theta_c = \dfrac{7.60/4}{2.25}$ so $\theta_c = 40.18°$
Snell's law: $n\sin\theta_c = (1)\sin 90° \Rightarrow n = \dfrac{1}{\sin 40.18°} = 1.55$

2-8. Referring to the figure one can see that,
$s = \text{AB}\sin(\theta_1 - \theta_2)$ and $\text{AB} = \dfrac{t}{\cos\theta_2}$. Therefore,
$s = \dfrac{t\sin(\theta_1 - \theta_2)}{\cos\theta_2}$. For $t = 3\,\text{cm}$, $n_2 = 1.50$, $\theta_1 = 50°$,
Snell's law gives, $\sin\theta_2 = \dfrac{n_1}{n_2}\sin\theta_1 = \dfrac{1}{1.5}\sin 50°$.
Then, $\theta_2 = 30.71°$ and $s = \dfrac{3\sin(50° - 30.71°)}{\cos 30.71°} = 1.153\,\text{cm}$.



2-9. Image of near end: $s = 60\,\text{cm}$, $\dfrac{1}{60} = \dfrac{1}{s'} = \dfrac{1}{-40}$, $s' - 24\,\text{cm}$

Image of far end: $s = 60 + 100$ cm, $\dfrac{1}{160} + \dfrac{1}{s'} = \dfrac{1}{-40}$, $s' = -32\,\text{cm}$. So, $L' = \Delta s' = -24 - (-32) = 8\,\text{cm}$

2-10. (a) See Figure 2 34 in the text. Image due to rays directly from bubble through plane interface:
$\dfrac{n_1}{s} + \dfrac{n_2}{s'} = 0$ or $\dfrac{1.5}{s} + \dfrac{1}{s'} = 0 \Rightarrow s' = -3.33$ cm.

(b) Image due to rays first reflected in spherical mirror, then refracted through plane interface:
reflection: $\dfrac{1}{2} + \dfrac{1}{s_1'} = -\dfrac{2}{R}$ and $\dfrac{1}{2.5} + \dfrac{1}{s_1'} = -\dfrac{2}{-7.5}$ $s_1' = -7.5\,\text{cm}$
refraction: $\dfrac{n_1}{s} + \dfrac{n_2}{s_2'} = 0$ or $\dfrac{1.5}{15} + \dfrac{1}{s_2'} = 0$ $s_2' = -10\,\text{cm}$
Thus the images are at 3.33 cm and 10 cm behind the interface.

2-11. There are 5 unknowns: s_1 and s_1' in position (1), s_2 and s_2' in position (2), and the focal length f of the mirror. The five equations that, solved simultaneously, yield the results are:

(1) linear magnification: $s_1'/s_1 = 2$ (2) linear magnification: $s_2'/s_2 = 3$
(3) focal length from mirror equation: $f = \dfrac{s_1 s_1'}{s_1 + s_1'}$ (4) focal length from mirror equation: $f = \dfrac{s_2 s_2'}{s_2 + s_2'}$

(5) image distance relation: $s_2' = s_1' + 75$

One finds $s_1 = 112.5$ cm, $s_2 = 100$ cm, $s_1' = 225$ cm, $s_2' = 300$ cm, $f = 75\,\text{cm}$

2-12. The object distance from the front surface is the diameter of the sphere, 5 cm. Then,

$$\frac{n_1}{s} + \frac{n_2}{s'} = \frac{n_2 - n_1}{R} \Rightarrow \frac{1.5}{5} + \frac{1}{s'} = \frac{1 - 1.5}{-2.5} \Rightarrow s' = -10\,\text{cm} \quad \text{and, } m = -\frac{n_1 s'}{n_2 s} = -\frac{(1.5)(-10)}{(1)(5)} = +3.$$

2-13. Generally, $\dfrac{n_1}{s} + \dfrac{n_2}{s'} = \dfrac{n_2 - n_1}{R}$
(a) $\dfrac{n_1}{f} + \dfrac{n_2}{\infty} = \dfrac{n_2 - n_1}{R}$ or $f = \dfrac{n_1 R}{n_2 - n_1}$ (b) $n_2 > n_1$: then $R > 0$ (convex), $n_2 < n_1$: then $R < 0$ (concave)

2-14. (a) In this position the object distance is $s = 15\,\text{cm}$. Then, using, $\dfrac{n_1}{s} + \dfrac{n_2}{s'} = \dfrac{n_2 - n_1}{R}$ and $m = -\dfrac{n_1\, s'}{n_2\, s}$,

$$\frac{4}{3\,(15)} + \frac{1}{s'} = \frac{1 - 4/3}{-15} \Rightarrow s' = -15\,\text{cm (center)} \quad \text{and,}\ m = -\frac{(4/3)\,(-15)}{(1)\,(15)} \Rightarrow m = 4/3.$$

(b) Similarly, in this position $s = 7.5\,\text{cm}$ so that,

$$\frac{4}{3\,(15/2)} + \frac{1}{s'} = \frac{1 - 4/3}{-15} \Rightarrow s' = -6.4\,\text{cm},\ m = -\frac{n_1\, s'}{n_2\, s} = -\frac{(4/3)\,(-45/7)}{(1)\,(15/2)} = 8/7.$$

2-15. See Figure 2 35 in the text. Rays from the object are (a) refracted through the spherical window, (b) then reflected from the back plane mirror, (c) then refracted out again through the spherical window. Taking these in turn:

(a) $\dfrac{n_1}{s} + \dfrac{n_2}{s'} = \dfrac{n_2 - n_1}{R} \Rightarrow \dfrac{1}{30} + \dfrac{4}{3s'} = \dfrac{4/3 - 1}{5} \Rightarrow s' = 40\,\text{cm}.$ Then, $m = \dfrac{n_1\, s'}{n_2\, s} = \dfrac{(1)\,(40)}{(4/3)\,(30)} = -1$

(b) $s = 25 - 40 = -15\,\text{cm}$ (virtual object), $s' = -s = 15\,\text{cm}$, $m = -s'/s = 1$

(c) $\dfrac{4/3}{10} + \dfrac{1}{s'} = \dfrac{1 - 4/3}{-5} \Rightarrow s' = -15\text{cm}.$ Then, $m = \dfrac{-(4/3)\,(-15)}{(1)\,(10)} = +2.$

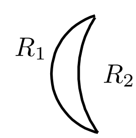
The overall magnification is $m = (-1)\,(+1)\,(+2) = -2$. Thus a virtual, inverted, double sized image appears 15 cm behind (right) the spherical window.

2-16. The plane side of the lens has $R_1 = \infty$. The radius of curvature R_2 of the convex side is then found from the lensmaker's equation:

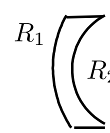
$$\frac{1}{f} = \frac{n_2 - n_1}{n_1}\left(\frac{1}{R_1} - \frac{1}{R_2}\right) \Rightarrow \frac{1}{25} = \frac{1.52 - 1}{1}\left(\frac{1}{\infty} - \frac{1}{R_2}\right) \Rightarrow R_2 = -13\,\text{cm}$$

2-17. In general the lensmaker's equation gives, $\dfrac{1}{f} = \dfrac{n_2 - n_1}{n_1}\left(\dfrac{1}{R_1} - \dfrac{1}{R_2}\right)$

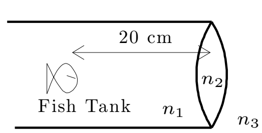
For the positive meniscus lens shown to the right, $R_1 = 5\,\text{cm}$ and $R_2 = 10\,\text{cm}$.
Then, $\dfrac{1}{f} = \dfrac{1.50 - 1}{1}\left(\dfrac{1}{5} - \dfrac{1}{10}\right) \Rightarrow f = +20\,\text{cm}$



For the negative meniscus lens shown to the right, $R_1 = 10\,\text{cm}$ and $R_2 = 5\,\text{cm}$.
For this case, $\dfrac{1}{f} = \dfrac{1.50 - 1}{1}\left(\dfrac{1}{10} - \dfrac{1}{5}\right) \Rightarrow f = -20\,\text{cm}$



2-18. The thin lens equation assumes identical, refractive indices on both sides. In this case we can modify the procedure, beginning with Eq. (2 23), to allow for three distinct media as shown.



$$\frac{n_1}{s_1} + \frac{n_2}{s_1'} = \frac{n_2 - n_1}{R_1}\ \text{, left lens surface}$$
$$\frac{n_2}{s_2} + \frac{n_3}{s_2'} = \frac{n_3 - n_2}{R_2}\ \text{, right lens surface}$$

For a thin lens, $s_2 \approx -s_1'$. Adding the equations, $\dfrac{n_1}{s_1} + \dfrac{n_3}{s_2'} = \dfrac{n_2 - n_1}{R_1} + \dfrac{n_3 - n_2}{R_2}$. Or, simply,

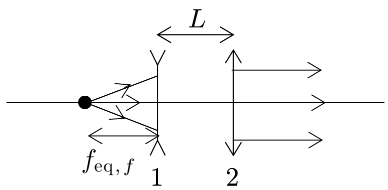
$$\frac{n_1}{s} + \frac{n_3}{s'} = \frac{n_2 - n_1}{R_1} + \frac{n_3 - n_2}{R_2} \Rightarrow \frac{4}{3\,(20)} + \frac{1}{s'} = \frac{3/2 - 4/3}{30} + \frac{1 - 3/2}{-30}\ \text{, so that}\ s' = -22.5\text{cm}.$$

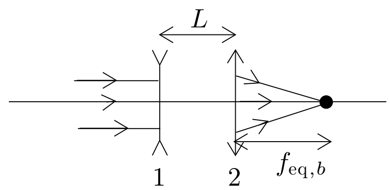
The total magnification is $m_T = m_1\, m_2 = \left(-\dfrac{n_1\, s_1'}{n_2\, s}\right)\left(-\dfrac{n_2\, s_2'}{n_3\, s_2}\right)$, where $s_2 = -s_1'$. So,

$$m_T = -\frac{n_1\, s'}{n_3\, s} = -\frac{(4/3)\,(-22.5)}{(1)\,(20)} = 1.50.$$

2-19. (a) Using $\dfrac{1}{f_{\rm eq}}=\dfrac{1}{f_1}+\dfrac{1}{f_2}$ as in Eq. (2 33), $\dfrac{1}{f_{\rm eq}}=\dfrac{1}{-5}+\dfrac{1}{20}$ or $f_{\rm eq}=6.67\,{\rm cm}$

(b) A pair of separated lenses has a front and a back focal length. The front focal length is the object posi tion from the first lens that leads to an image at infinity. The back focal length is the image position for an object at infinity. These cases are illustrated below. The drawings are generic and not to scale.





Working backwards, for the front focal length: Lens 2: $\dfrac{1}{s_2}+\dfrac{1}{\infty}=\dfrac{1}{f_2}$ or $s_2=f_2$, $s_2=L-s_1'$ or $s_1'=L-f_2$.

Lens 1: $\dfrac{1}{f_{\rm eq,\,f}}+\dfrac{1}{L-f_2}=\dfrac{1}{f_1}$ or $f_{\rm eq,\,f}=\dfrac{f_1\,(L-f_2)}{L-(f_1+f_2)}=\dfrac{(-5)\,(10-20)}{10-(-5+20)}\,{\rm cm}=-10\,{\rm cm}$

For the back focal length: for lens 1:

$\dfrac{1}{s_1}+\dfrac{1}{s_1'}=\dfrac{1}{f_1}\Rightarrow\dfrac{1}{\infty}+\dfrac{1}{s_1'}=\dfrac{1}{f_1}\Rightarrow s_1'=f_1$. Then $s_2=L-f_1$, so that, for lens 2:

$$\frac{1}{s_2}+\frac{1}{s_2'}=\frac{1}{f_2}\Rightarrow\frac{1}{L-f_1}+\frac{1}{f_{\rm eq,b}}=\frac{1}{f_2}\Rightarrow f_{\rm eq,b}=\frac{f_2\,(L-f_1)}{(L-f_1)-f_2}=\frac{(20)\,(10-(-5))}{(10-(-5))-20}\,{\rm cm}=-60\,{\rm cm}$$

2-20. See Figure 2 36 in the text. Consider the three media as a sequence of three thin lenses. Each has a focal length given by the lensmaker's equation, and the equivalent focal length is given Eq. (2 33) as,

$\dfrac{1}{f_{\rm eq}}=\dfrac{1}{f_1}+\dfrac{1}{f_2}+\dfrac{1}{f_3}$. Then, $\dfrac{1}{f_1}=(1.5-1)\left(\dfrac{1}{\infty}-\dfrac{1}{-15}\right)\Rightarrow f_1=30\,{\rm cm}$,

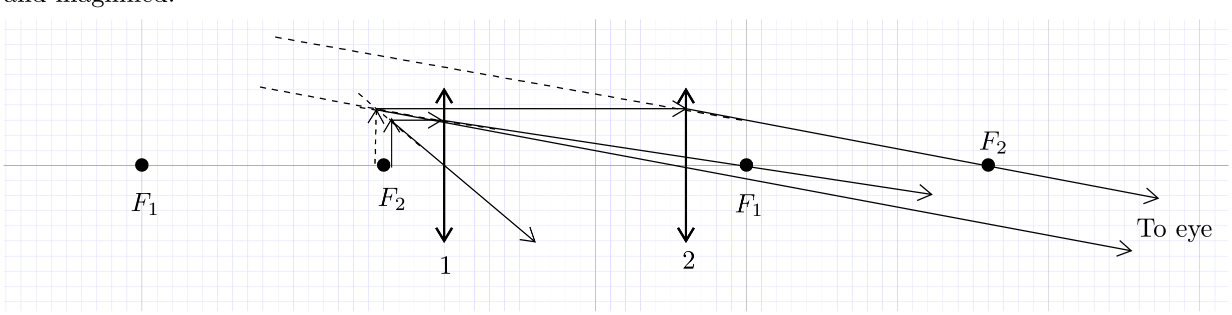
$\dfrac{1}{f_2}=(1.65-1)\left(\dfrac{1}{-15}-\dfrac{1}{15}\right)$ or $f_2=-\dfrac{150}{13}$ cm, and $\dfrac{1}{f_3}=$ same as for f_1 : $f_3=30$ cm. Then,

$\dfrac{1}{f_{\rm eq}}=\dfrac{1}{30}+\dfrac{-13}{150}+\dfrac{1}{30}$ and so $f_{\rm eq}=-50\,{\rm cm}$.

2-21. (a) One can use the formula derived in problem 2 19b, or do the calculation at first hand:

Second lens: $\dfrac{1}{s_2}+\dfrac{1}{\infty}=\dfrac{1}{20}$ or $s_2=20\,{\rm cm}$, First lens: $\dfrac{1}{s_1}+\dfrac{1}{-4}=\dfrac{1}{20}$ or $s_1=3.33\,{\rm cm}$. The object should be placed 3.33 cm before the first lens.

(b) In the figure below the dashed arrow is the intermediate image that acts as the object for the second lens. Since the image is "at infinity" it is described by an angular magnification. The image appears erect and magnified.



2-22. Refer to Figure 2 37 in the text.

(b) Lens heading towards mirror: $\frac{1}{3\,f/2}+\frac{1}{s'}=\frac{1}{-f}$ or $s'=-3\,f/5$. $m_1=-\frac{s'}{s}=-\frac{-3f/5}{3f/2}=2/5$

Mirror:

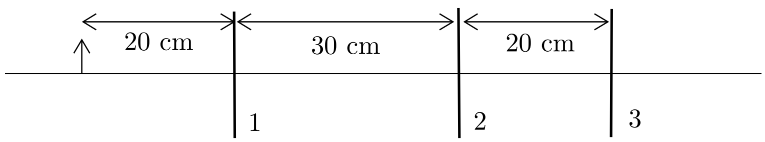
$s=3\,f+3\,f/5=18\,f/5\Rightarrow\frac{5}{18\,f}+\frac{1}{s'}=\frac{1}{f}\Rightarrow s'=18\,f/13,\ m_2=-\frac{s'}{s}=-(18\,f/13)/(18\,f/5)=-5/13$

Lens after reflection:

$s=3\,f-18\,f/13=21\,f/13\Rightarrow\frac{13}{21\,f}+\frac{1}{s'}=\frac{1}{-f}$ or $s'=21\,f/34, m_3=-s'/s=-(21\,f/34)/(21\,f/13)=\frac{13}{34}$

$m_T=\left(\frac{2}{5}\right)\left(-\frac{5}{13}\right)\left(\frac{13}{34}\right)=-17$. The image is inverted, (21/34) f behind (right of) lens, inverted, and 1/17 original size.

2-23. The arrangement of the object and lenses is shown below.



(a) $f_1=+10\,\text{cm}, f_2=+15\,\text{cm}, f_3=+20\text{cm}$

1st lens: $\frac{1}{20}+\frac{1}{s'}=\frac{1}{10}$ $\quad s'=20$ $\quad m_1=-20/20=-1$

2nd lens: $\frac{1}{10}+\frac{1}{s'}=\frac{1}{15}$ $\quad s'=-30$ $\quad m_2=-(-30)/10=+3$

3rd lens: $\frac{1}{50}+\frac{1}{s'}=\frac{1}{20}$ $\quad s'=100/3$ $\quad m_3=-100/3\,(50)=-2/3$

$m_T=m_1\,m_2\,m_3=+2$

(b) $f_1=+10\,\text{cm}, f_2=-15\,\text{cm}, f_3=+20\text{cm}$

1st lens: $\frac{1}{20}+\frac{1}{s'}=\frac{1}{10}$ $\quad s'=20$ $\quad m_1=-20/20=-1$

2nd lens: $\frac{1}{10}+\frac{1}{s'}=\frac{1}{-15}$ $\quad s'=-6$ $\quad m_2=-(-6)/10=+0.6$

3rd lens: $\frac{1}{26}+\frac{1}{s'}=\frac{1}{20}$ $\quad s'=520/6$ $\quad m_3=-520/(6\times 26)=-\frac{10}{3}$

$m_T=m_1\,m_2\,m_3=+2$

(c) $f_1=-10\,\text{cm}, f_2=+15\,\text{cm}, f_3=-20\text{cm}$

1st lens: $\frac{1}{20}+\frac{1}{s'}=\frac{1}{-10}$ $\quad s'=20/3$ $\quad m_1=-(-20)/3\,(20)=\frac{1}{3}$

2nd lens: $\frac{3}{110}+\frac{1}{s'}=\frac{1}{15}$ $\quad s'=330/13$ $\quad m_2=-\frac{(330)\,(3)}{(13)\,(110)}=-\frac{9}{13}$

3rd lens: $\frac{-13}{70}+\frac{1}{s'}=\frac{1}{-20}$ $\quad s'=140/19$ $\quad m_3=-\frac{(140)\,(13)}{(19)\,(-70)}=\frac{26}{19}$

$m_T=m_1\,m_2\,m_3=-6/19$

2-24. Using the lensmaker's formula, $\frac{1}{f}=\frac{n_2-n_1}{n_1}\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$ gives

in air: $\frac{1}{30}=\frac{1.50-1}{1}\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$

and in the liquid: $-\frac{1}{188}=\frac{1.50-n_L}{n_L}\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$.

Dividing the two equations gives, $\frac{-188}{30}=\frac{0.5n_L}{1.5-n_L}$ or $n_L=1.63$.

2-25. Use the lensmaker's formula to find the focal length of the lens,

$$\frac{1}{f}=\frac{n_2-n_1}{n_1}\left(\frac{1}{R_1}-\frac{1}{R_2}\right)=\frac{1.5-1}{1}\left(0+\frac{1}{60}\right)\Rightarrow f=120\,\text{cm}$$

The Newtonian equations are, $m=-\frac{f}{x}=-\frac{x'}{f}$. For $m=-4$, .

$$-4=-\frac{f}{x}=-\frac{120}{x}\ \text{ or } x=30\text{cm}$$
$$-4=-\frac{x'}{f}=-\frac{x'}{120}\ \text{ or } x'=480\text{cm}$$

Thus, $s=x+f=30+120=150\text{cm}$ and $s'=x'+f=480+120=600\text{cm}$.

Check: $\frac{1}{s}+\frac{1}{s'}=\frac{1}{150}+\frac{1}{600}=\frac{1}{120}=\frac{1}{f}$

2-26. (a) $f_1=10\,\text{cm}\Rightarrow P_1=\frac{1}{0.01}=+10\,D,\ f_2=20\,\text{cm}\Rightarrow P_2=\frac{1}{0.2}=+5\,D,\ f_3=-40\,\text{cm}\Rightarrow P_3=\frac{1}{-0.4}=-2.5\,D$

Then, $P=P_1+P_2+P_3=10+5-2.5=+12.5\,D$

(b) $\frac{1}{s}+\frac{1}{s'}=\frac{1}{f},\ V+V'=P$, where $V=\frac{1}{s}=\frac{1}{0.12}=+8.33\,D$,

$V'=4.167\,D$ or $s'=\frac{1}{V'}=\frac{1}{4.167}=0.24\,\text{m}=24\text{cm}$

2-27. See Figure 2 38 in the text. The applicable relations are:

Lens equations: $\frac{1}{s_1}+\frac{1}{s_1'}=\frac{1}{f}$ and $\frac{1}{s_2}+\frac{1}{s_2'}=\frac{1}{f}$,

Geometrical: $L=s_1+s_1'=s_2+s_2',\ D=s_2-s_1=s_1'-s_2'$
Thus,

$$f=\frac{s_1\,s_1'}{s_1+s_1'}=\frac{s_1\,s_1'}{L}=\frac{s_2\,s_2'}{s_2+s_2'}=\frac{s_2\,s_2'}{L}\tag{1}$$

Because the lens equation can be satisfied the second time by simply interchanging object and image dis tances,

$$s_2=s_1'\ \text{ and }\ s_2'=s_1\tag{2}$$

Adding and subtracting the equations $L=s_2+s_2$ and $D=-s_1+s_2$, we get,
$L-D=2s$ and $L+D=2s_2$. Their product is by Eq. (1), $L^2-D^2=4\,s_1\,s_2$, or by Eq. (2), $L^2-D^2=4\,f\,L$.
Thus, $f=\frac{L^2-D^2}{4\,L}$.

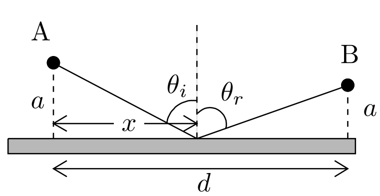
2-28. Lens equations: $\frac{1}{s_1}+\frac{1}{s_1'}=\frac{1}{f}$ and $\frac{1}{s_2}+\frac{1}{s_2'}=\frac{1}{f}$. Then calculate,

$$\frac{1}{M_1}-\frac{1}{M_2}=-\frac{s_1}{s_1'}+\frac{s_2}{s_2'}=-\frac{s_1}{s_1\,f/(s_1-f)}+\frac{s_2}{s_2\,f/(s_2-f)}=\frac{s_2-f}{f}-\frac{s_1-f}{f}=\frac{s_2-s_1}{f}$$

Thus,

$$f=\frac{s_2-s_1}{1/M_1-1/M_2}$$

2-29. Consider an arbitrary path from point A to point B by reflection from a mirror surface,

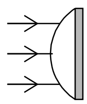


The path distance D from A to B is $D=\sqrt{a^2+x^2}+\sqrt{b^2+(d-x)^2}$

$$\frac{d\,D}{d\,x}=\frac{x}{\sqrt{a^2+x2}}+\frac{-(d-x)}{\sqrt{b^2+(d-x)^2}}=0$$

$\sin\theta_i-\sin\theta_r=0\Rightarrow\theta_i=\theta_r$

2-30. The two set ups are illustrated below,

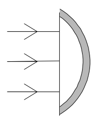


Refraction at curved side: $\frac{1}{\infty}+\frac{n}{s'}=\frac{n-1}{R}$; $s' - \frac{n\,R}{n-1}$

Reflection at plane side: $s'=-\,s=-\frac{n\,R}{n-1}$

Refraction at curved side: $\frac{n}{-\,n\,R/(n-1)}+\frac{1}{s'}=\frac{1-n}{-\,R}$ or $s'=R/2\,(n-1)$

Thus, $f_1=\frac{R}{2\,(n-1)}$



Reflection at curved face: $\frac{1}{\infty}+\frac{1}{s'}=-\frac{2}{R}$; $\quad s'=-\frac{R}{2}$

Refraction at plane face: $\frac{n}{-\,R/2}+\frac{1}{s'}=0$; $\quad s'=\frac{R}{s\,n}$

Refraction at plane face: $\frac{n}{-\,R/2}+\frac{1}{s'}=0$; $\quad s'=\frac{R}{s\,n}$

Thus, $f_2=R/2\,n$

Therefore the ratio of the focal lengths is $\frac{f_1}{f_2}=\frac{R/2\,(n-1)}{R/2\,n}=\frac{n}{n-1}$.

2-31. The distance between the object and the image is $D=s+s'=s+\frac{f\,s}{s-f}$. This is minimized when,

$\frac{d\,D}{d\,s}=1+\frac{(s-f)\,f-f\,s}{(s-f)^2}=0\Rightarrow s\,(s-2\,f)=0\Rightarrow s=\emptyset,\,2\,f$. The minimum distance D occurs when $s=2f$ and has the value $D=2f+\frac{f\,(2f)}{2f-f}=4f$. That is, in this configuration $s=s'=2f$.

2-32. Refer to Figure 2 39 in the text.

(a) Let the angle with the normal to the interface in each region of index of refraction n_i be θ_i. Then applying Snell's law sequentially at each interface leads to,

$$n_0\sin\theta_0=n_1\sin\theta_1=n_2\sin\theta_2...=n_i\sin\theta_i...=n_f\sin\theta_f$$

That is,

$$n_0\sin\theta_0=n_f\sin\theta_f$$

(b) In each medium the lateral displacement is $t_i\tan\theta_i$. The total lateral displacement y due to N media can be written as,

$$y=\sum_{i=1}^{n}t_i\tan\theta_i$$

where $\sin\theta_i=(n_0/n_i)\sin\theta_0$.

2-33. At each surface use the relation,

$$\frac{n_1}{s}+\frac{n_2}{s'}=\frac{n_2-n_1}{R}$$

For light incident first on the plane side:

1st surface (plane): no change, 2nd surface (curved): $\frac{1.5}{\infty}+\frac{1}{s'}=\frac{1-1.5}{-\,4}$ or $s'=8\,\text{cm}$.

For light incident first on the curved side:

1st surface (curved): $\frac{1}{\infty}+\frac{1.5}{s'}=\frac{1.5-1}{4}$ or $s'=12\text{cm}$.

2nd surface (plane): object distance $=4-12=-\,8$ cm (virtual), $\frac{1.5}{-\,8}+\frac{1}{s'}=\frac{1-1.5}{\infty}$ or $s'=5.33\,\text{cm}$

2-34. The focal length is the image position for incident parallel light rays (object at ∞). In all cases the fol lowing relation is to be used

$$\frac{n_1}{s}+\frac{n_2}{s'}=\frac{n_2-n_1}{R} \Rightarrow \frac{n_1}{\infty}+\frac{n_2}{f}=\frac{n_2-n_1}{R} \Rightarrow \frac{n_2}{f}=\frac{n_2-n_1}{R}$$

For the situation in which the center of curvature in medium with $n=4/3$:

For light incident from the medium of index 1: $\frac{4/3}{f}=\frac{4/3-1}{10}$ or $f=+40\,\mathrm{cm}$

For light incident from the medium of index 4/3: $\frac{1}{f}=\frac{1-4/3}{-10}$ or $f=+30\,\mathrm{cm}$

For the situation in which the center of curvature is in the medium with $n=1$,

For light incident from the medium of index 4/3: $\frac{4/3}{\infty}+\frac{1}{f}=\frac{1-4/3}{10}$ or $f=-30\,\mathrm{cm}$

For light incident from the medium of index 1: $\frac{1}{\infty}+\frac{4/3}{f}=\frac{4/3-1}{-10}$ or $f=-40\,\mathrm{cm}$

2-35. $|m|=\frac{s}{s'}=\frac{1}{50,000}=\frac{f}{s}=\frac{6\,\mathrm{in}}{s}$, since $s'=f$. So $s=50,000\times 6\,\mathrm{in}=25,000\,\mathrm{ft}$

2-36. Using, the lensmaker's equation the focal power of the cylindrical lens is,

$$\frac{1}{f}=\frac{n_2-n_1}{R}=\frac{1.60-1.0}{5\,\mathrm{cm}} \Rightarrow f=8.33\,\mathrm{cm}$$

The image distance is then found as,

$$\frac{1}{s'}=\frac{1}{f}-\frac{1}{s}=\frac{1}{8.33\,\mathrm{cm}}-\frac{1}{15\,\mathrm{cm}} \Rightarrow s'=18.73\,\mathrm{cm}$$

Then, Eq. (2 37) gives,

$$AB=\frac{s+s'}{s}CL=\frac{15+18.73}{15}7\,\mathrm{cm}=15.75\,\mathrm{cm}$$

The line image is real, 18.75 cm past the lens and 15.75 cm long.

2-37. Using, the lensmaker's equation the focal power of the cylindrical lens is,

$$\frac{1}{f}=\frac{n_2-n_1}{R}=\frac{1.52-1.0}{15\,\mathrm{cm}} \Rightarrow f=28.85\,\mathrm{cm}$$

The image distance is then found as, $\frac{1}{s'}=\frac{1}{f}-\frac{1}{s}=\frac{1}{28.85\,\mathrm{cm}}-\frac{1}{20\,\mathrm{cm}} \Rightarrow s'=-65.2\,\mathrm{cm}$

Then, Eq. (2 37) gives,

$$AB=\frac{s+s'}{s}CL=\frac{20-65.2}{20}2.5\,\mathrm{cm}=-5.65\,\mathrm{cm}$$

The line image is virtual, 65.2 cm from the lens on the object side of the lens and 5.65 cm long.

2-38. Using, the lensmaker's equation the focal power of the cylindrical lens is,

$$\frac{1}{f}=\frac{n_2-n_1}{R}=\frac{1.5-1.0}{-10\,\text{cm}}\Rightarrow f=-20\,\text{cm}$$

The image distance is then found as,

$$\frac{1}{s'}=\frac{1}{f}-\frac{1}{s}=\frac{1}{-20\,\text{cm}}-\frac{1}{25\text{cm}}\Rightarrow s'=-11.11\,\text{cm}$$

Then, Eq. (2 37) gives,

$$AB=\frac{s+s'}{s}CL=\frac{25-11.11}{25}5\,\text{cm}=2.78\,\text{cm}$$

The line image is virtual, 11.11 cm from the lens on the object side of the lens and 2.78 cm long.

2-39. Using, the lensmaker's equation the focal power of the cylindrical lens is,

$$\frac{1}{f}=\frac{n_2-n_1}{R}=\frac{1.5-1.0}{-20\,\text{cm}}\Rightarrow f=-40\,\text{cm}$$

The image distance is then found as,

$$\frac{1}{s'}=\frac{1}{f}-\frac{1}{s}=\frac{1}{-40\,\text{cm}}-\frac{1}{20\text{cm}}\Rightarrow s'=-13.33\,\text{cm}$$

Then, Eq. (2 37) gives,

$$AB=\frac{s+s'}{s}CL=\frac{20-13.33}{20}2\,\text{cm}=0.667\,\text{cm}$$

The line image is virtual, 13.33 cm from the lens on the object side of the lens and 0.67 cm long.

2-40. Using, the lensmaker's equation the focal power of the cylindrical lens is,

$$\frac{1}{f}=\frac{n_2-n_1}{R}=\frac{1.60-1.0}{5\,\text{cm}}\Rightarrow f=8.33\,\text{cm}$$

The image distance is then found as,

$$\frac{1}{s'}=\frac{1}{f}-\frac{1}{s}=\frac{1}{8.33\,\text{cm}}-\frac{1}{6\,\text{cm}}\Rightarrow s'=-21.45\,\text{cm}$$

Then, Eq. (2 37) gives,

$$AB=\frac{s+s'}{s}CL=\frac{6-21.45}{6}7\,\text{cm}=-18.0\,\text{cm}$$

The line image is virtual, 21.45 cm from the lens on the object side of the lens and 18.0 cm long.

Chapter 3 Optical Instrumentation

3-1. The *entrance pupil* is the aperture stop so no elements precede the aperture stop. The *exit pupil* is the image of the aperture stop formed by the lens. The position and size of the the exit pupil are found from the thin lens equation with the object being the aperture stop.

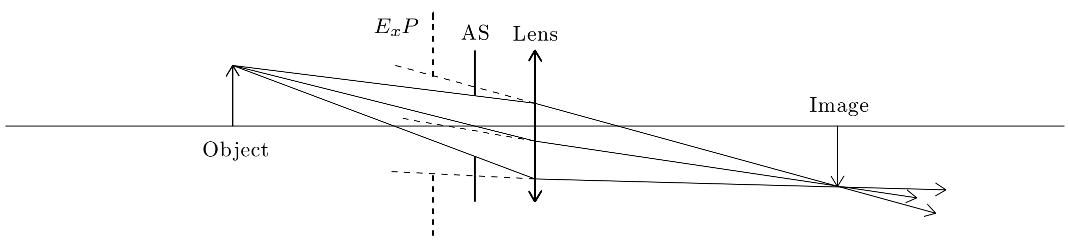
Exit Pupil and Size:

$$\frac{1}{s}+\frac{1}{s'}=\frac{1}{f}\Rightarrow\frac{1}{2\,\text{cm}}+\frac{1}{s'}=\frac{1}{5\,\text{cm}}\Rightarrow s'=-3.33\,\text{cm}$$
$$m=-\frac{s'}{s}=-\frac{3.33}{2}=\frac{5}{3}\Rightarrow\text{size}=\frac{5}{3}\times 2\,\text{cm}=3.33\,\text{cm}$$

Image Position and Size:

$$\frac{1}{10\,\text{cm}}+\frac{1}{s'}=\frac{1}{5\,\text{cm}}\Rightarrow s'=10\,\text{cm}$$
$$m=-s'/s=-10/10=-1\Rightarrow\text{size}=1\times 2\,\text{cm}=2\,\text{cm, inverted}$$

The system is drawn to scale below.



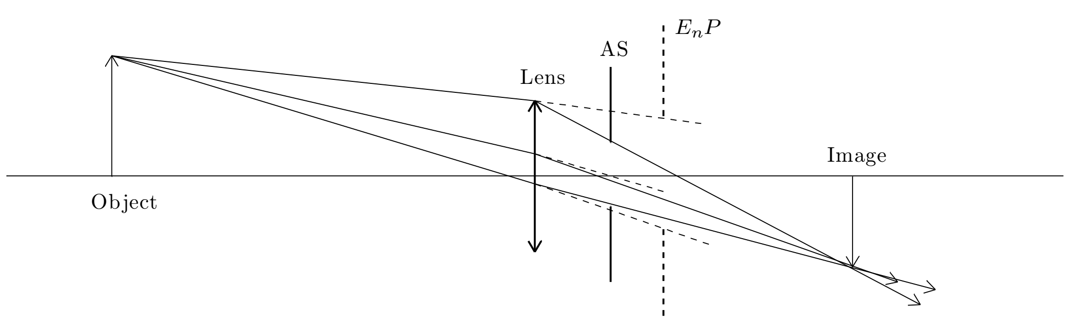
3-2. Exit Pupil: AS (no elements follow), **Entrance Pupil:** Image of AS formed by lens

Position of E_nP: $\frac{1}{2.5\,\text{cm}}+\frac{1}{s'}=\frac{1}{6\,\text{cm}}\Rightarrow s'=-4.29\,\text{cm}=-30/7\,\text{cm (right of lens)}$

Size of E_nP: $m=-s'/s=-\frac{-30}{7}\times\frac{2}{5}=\frac{12}{7}$ $\quad$ $\text{size}=\frac{12}{7}\times 2\,\text{cm}=3.43\,\text{cm}$

Image: $\frac{1}{14\,\text{cm}}+\frac{1}{s'}=\frac{1}{6\,\text{cm}}\Rightarrow s'=10.5\,\text{cm},\ m=-s'/s=-10.5/14=-0.75;\text{size}=\frac{3}{4}\times 4\,\text{cm}=3\,\text{cm, inverted}$

The system is drawn to scale below.

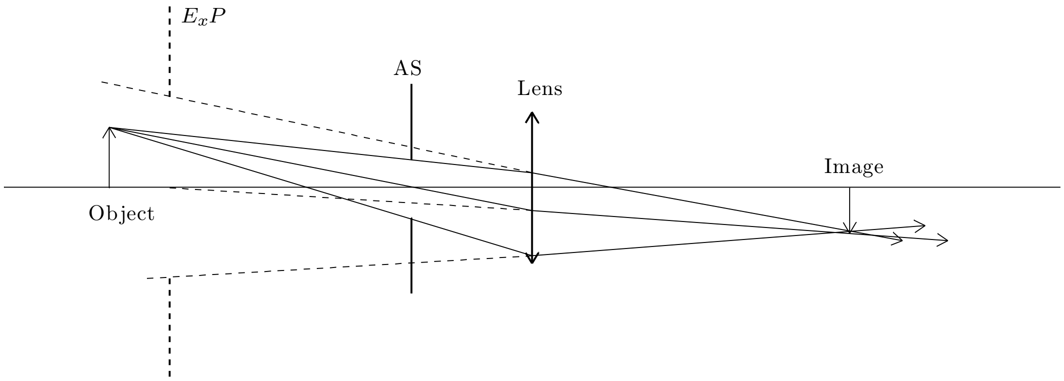


3-3. **Entrance Pupil:** AS (no preceding elements), **Exit Pupil**: Image of AS through lens

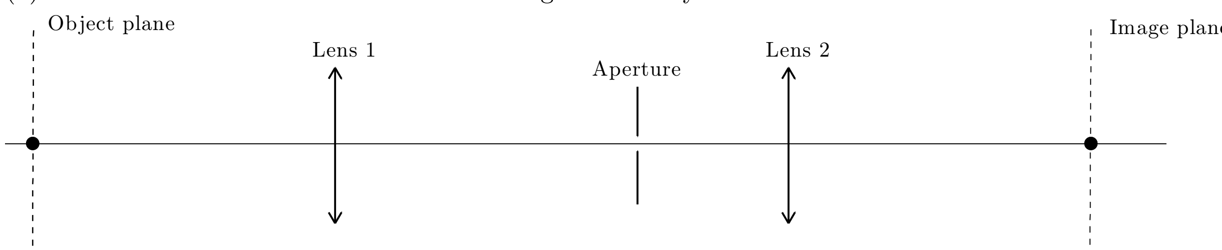
Exit Pupil: $\frac{1}{4\,\text{cm}} + \frac{1}{s'} = \frac{1}{6\,\text{cm}} \Rightarrow s' = -12\,\text{cm}$. $m = -s'/s = -\frac{-12}{4} = +3$, size $= 3 \times 2\,\text{cm} = 6\,\text{cm}$

Image: $\frac{1}{14\,\text{cm}} + \frac{1}{s'} = \frac{1}{6\,\text{cm}} \Rightarrow s' = 10.5\,\text{cm}$, $m = -s'/s = -10.5/14 = -0.75$, size $= 3/4 \times 2 = 1.5\,\text{cm}$

The system is drawn to scale below.



3-4. (a) The vertical and horizontal scales in the figure differ by a factor of 4.



(b) **Image Plane:**

$$s_1' = \frac{f_1 s_1}{s_1 - f_1} = \frac{(40/3)(40)}{40 - 40/3} = 20\,\text{cm (right of Lens 1)} \quad s_2' = \frac{f_2 s_2}{s_2 - f_2} = \frac{(20/3)(10)}{10 - 20/3} = 20\,\text{cm (right of Lens 2)}$$

(c) **Candidates for AS:** Lens1, Aperture, or Lens 2.

Lens subtends at an angle of $\theta_{L1} = \frac{2}{40} = 0.05\,\text{rad}$.

Aperture image in Lens 1: $s' = \frac{20\,(40/3)}{20 - 40/3} = 40\,\text{cm}$, or at object plane, then $\theta_{\text{Aperture}} = 0°$

Lens 2 image through Lens 1: $s' = \frac{30\,(40/3)}{30 - 40/3} = 24\,\text{cm}$ left of Lens 1 or 16 cm right of object.

$$m = -s'/s = -24/30 = -0.8 \text{ so size} = 0.8 \times 2 = 1.6\,\text{cm}, \; \theta_{L2} = \frac{1.6}{16} = 0.1 > 0.05$$

Thus Lens 1 behaves as the AS. It is also the E_nP, being the first in line.

(d) **Exit Pupil $\mathbf{E_xP}$:** Image of AS (i.e., Lens 1) in Lens 2:

$$s' = \frac{s\,f2}{s - f2} = \frac{30\,(20/3)}{30 - 20/3} = 8.57\,\text{cm, right of Lens 2}, \; m = -\frac{s'}{s} = -\frac{8.57}{30} = 0.2857 = 2/7 \text{ so } D_{E_xp} = \frac{2}{7} \times 2 = \frac{4}{7}\,\text{cm}$$

(e) **Field Stop:** Either Aperture or Lens 2, whichever subtends the smaller angle at center of E_nP=Lens 1:

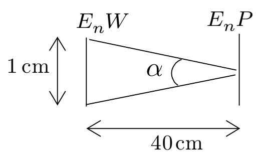
$$\theta_{\text{Aperture}} = \frac{0.5}{20} = 0.025; \theta_{\text{Lens 2}} = \frac{2}{30} = 0.067$$

Thus the Aperture Stop behaves as the field stop (FS). *Continued on next page*

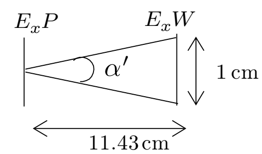
3-4. (e) *continued.*
Exit Window: Image of FS in following elements lens Lens 2.

$$s' = \frac{(10)\,(20/3)}{10-20/3} = 20\,\text{cm, right of Lens 2, in image plane,}$$
$$m = -\,s'/s = 20/10 = -\,2 \text{ so } D_{E_x\omega} = 2\times 0.5 = 1.0\,\text{cm}$$

(f) **Angular field of view:** Drawn not to scale,



$$\alpha = \frac{1}{40} = 0.0125\,\text{rad}$$
$$\alpha = 1.43^\circ$$



$$\alpha' = \frac{1}{10-8.43} = 0.0875\,\text{rad}$$
$$\alpha' = 5.01^\circ$$

3-5. Refer to Figure 3 5 in the text.
(a) First determine whether A, L_1, or L_2 (its image though L_1) subtends the smallest angle at he center of E_nP:
Rim of A at E_nP: 90°. Rim of L_1 at E_nP: $\tan^{-1}(3/3) = 45°$.
Rim of the image of L_2 in L_1:
This image is labeled L_2' in Figure 3 5 in the text and is shown there to be virtual, erect, located 15 cm from E_nP with half size 9 cm. Thus L_2' subtends an angle of $\tan^{-1}(9/15) = 31°$ at E_nP. So L_2' subtends the smallest angle at the entrance window making L_2 the field stop, of half size 3 cm.

(b) The entrance window is the image of the field stop in all optics to its left. Since the field stop is L_2, L_2' is the entrance window. As shown in Figure 3 5 in the text it is located 12 cm to the right of L_1 with half size 9 cm.

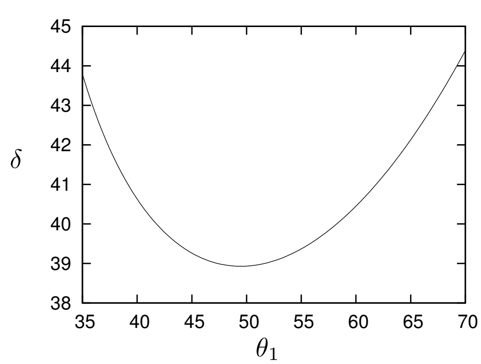
The exit window is the image of the field stop in all optics to its right. Since, in this case there are no optics to the right of the field stop (L_2), the exit window is L_2 of half size 3cm.

(c) The two marginal rays can be sketched onto Figure 3 5 in the text as follows. The marginal rays start from point P and skim the edges the entrance pupil A. Upon encountering L_1 the rays are head towards P'. Before they reach P' they are redirected towards the tip of the final image at P''. The backward exten sion of the marginal rays heading toward P'' skim the edges of the exit pupil.

3-6. The deviation δ is given as a function of the input angle θ_1 by Eqs. (3 7), (3 8), (3 10), and (3 11) as,

$$\delta = \theta_1 + \theta_2 - \theta_1' - \theta_2' = \theta_1 + \theta_2 - A$$
$$\delta = \theta_1 + \sin^{-1}\left\{ n \sin\left[A - \sin^{-1}\left(\frac{\sin(\theta_1)}{n} \right) \right] \right\} - A$$

For $n = 1.52$ and $A = 60°$, I obtain the plot below.



3-7. The index of refraction is given by Eq. (3 15)

$$n = \frac{\sin\left(\frac{A+\delta_m}{2}\right)}{\sin(A/2)} = \frac{\sin(30-\delta_m/2)}{1/2}$$

For $n_{\text{red}} = 1.525$ $\qquad \sin(30+\delta_m/2) = \frac{1}{2}(1.525)$ or $\delta_{m,\text{red}} = 39.37°$

For $n_{\text{blue}} = 1.535$ $\qquad \sin(30+\delta_m/2) = \frac{1}{2}(1.535)$ or $\delta_{m,\text{blue}} = 40.26°$

$$\delta_{m,\text{blue}} - \delta_{m,\text{red}} = 0.89° = 53'$$

3-8. The necessary data is

	n (crown)	n (flint)
F (481.6 nm)	1.5286	1.7328
C (656.3 nm)	1.5205	1.7076

(a) The Cauchy relations for λ_F and λ_C are: $n_F = A + B/\lambda_F^2, n_C = A + B/\lambda_C^2$. Solving these simultaneously,

$$A = n_F - \frac{B}{\lambda_F^2} \quad \text{and} \quad B = \frac{n_F - n_C}{(1/\lambda_F^2 - 1/\lambda_C^2)}$$

For crown glass: $A = 1.5286 - \frac{4240}{466.1^2} = 1.511 \qquad B = \frac{1.5286 - 1.5205}{\frac{1}{(486.1)^2} - \frac{1}{(656.3)^2}} = 4240\,\text{nm}^2$

For flint glass: $A = 1.7328 - \frac{1.319\times 10^4}{486.1^2} = 1.677;\ B = \frac{1.7328 - 1.7076}{\frac{1}{(486.1)^2} - \frac{1}{(656.3)^2}} = 1.319\times 10^4\,\text{nm}^2$

Then for a third wavelength $\lambda_D = 589.2\,\text{nm}$,

$$\text{Crown: } n_D = A + \frac{B}{\lambda_D^2} = 1.511 + \frac{4240}{(589.2)^2} = 1.523\,, \text{(table: 1.5230)}$$
$$\text{Flint: } n_D = 1.677 = \frac{1.319\times 10^4}{(589.2)^2} = 1.715\,, \text{(table: 1.7205)}$$

(b) The dispersion is $\mathfrak{D} = \frac{dn}{d\lambda} = -\frac{2B}{\lambda^3}$ so,

$$\mathfrak{D}_C = \frac{-2(4240\,\text{nm}^2)}{(589.2\,\text{nm})^3} = -4.146\times 10^{-5}\,\text{nm}^{-1} \quad \mathfrak{D}_F = \frac{-2\,(13190\,\text{nm}^2)}{(589.2\,\text{nm})^3} = 1.290\times 10^4\,\text{nm}^{-1}$$

(c) The resolving powers are,

$$\mathfrak{R}_C = b\,\mathfrak{D}_C = (7.5\times 10^7\,\text{nm})\,(4.146\times 10^{-5}\text{nm}^{-1}) = 3110$$
$$\mathfrak{R}_F = b\,\mathfrak{D}_F = (7.5\times 10^7\,\text{nm})\,(1.290\times 10^{-4}\,\text{nm}^{-1}) = 9675$$

The minimum resolvable wavelengths are,

$$\text{crown:}\quad (\Delta\lambda)_{\min} = \lambda/\mathfrak{R}_C = 5892\,\text{Å}/3110 = 1.9\text{Å}$$
$$\text{flint:}\quad (\Delta\lambda)_{\min} = \lambda/\mathfrak{R}_F = 5892\,\text{Å}/9675 = 0.61\,\text{Å}$$

3-9. (a) $1.6381 = \dfrac{\sin(30+\delta/2)}{\sin 30}$ or $\delta = 50°$

(b) $\Delta = \dfrac{n_F - n_C}{n_D - 1} = \dfrac{1.63461 - 1.64511}{1.6381 - 1} = -\dfrac{1}{55.5}$

(c) $D = -2B/\lambda^3$ where $B = \dfrac{n_1 - n_2}{\frac{1}{\lambda_1^2} - \frac{1}{\lambda_2^2}}$, as in problem 6 8a. For the long wavelength region, take $\lambda_1 = 587.6$ nm and $\lambda_2 = 656.3$ nm. Then $B = \dfrac{1.63810 - 1.63461}{1/587.6^2 - 1/656.3^2} = 6073.7\,\text{nm}^2$ and $A = n_1 - \dfrac{B}{\lambda^2} = 1.63810 - \dfrac{6073.7}{587.6^2} = 1.6205$.

Finally, $\mathfrak{D}\,(\text{at}\ \lambda = 656.3\,\text{nm}) = \dfrac{-2\,(6073.7)}{656.3^3} = 4.297 \times 10^{-5}\,\text{nm}^{-1}$

(d) The resolving power is $\mathfrak{R} = \dfrac{\lambda_{\text{AV}}}{\Delta\lambda} = b\,\mathfrak{D}$. Now $\Delta\lambda = 656.2852 - 656.2716 = 0.0136\,\text{nm}$, so that

$b = \dfrac{\lambda_{\text{AV}}}{\mathfrak{D}\,\Delta\lambda} = \dfrac{656.28\,\text{nm}}{(4.297 \times 10^{-5}\,\text{nm}^{-1})\,(0.0136\,\text{nm})} = 1.12 \times 10\,\text{nm} = 1.12\,\text{m}!$

3-10. The indices of refraction of the lines are,

$n_C - \dfrac{\sin[(60 + 38°20')/2]}{\sin(60/2)} = 1.51323$. Similarly, $n_D = 1.51570$ and $n_F = 1.52308$

Then, $\Delta = \dfrac{n_F - n_C}{n_D} = 0.01909$

3-11. Let the flint prism be prism (1) and the crown be prism (2). Then, dispersion of the flint prism is $\delta_{D1} = (1.635 - 1)\,(5°) = 3.175^{\circ}$. This dispersion must be matched by the crown prism, $\delta_{D2} = 3.175^{\circ} = (1.53 - 1)\,A_2$, or $A_2 = 5.99°$. For the C and F lines with this double prism,

$\delta_{F2} = (1.536 - 1)\,(5.99) = 3.211°$ $\delta_{F1} = (1.648 - 1)\,(5) = 3.240°$ Net: $\delta_{F2} - \delta_{F1} = 0.029°$
$\delta_{C2} = (1.527 - 1)\,(5.99) = 3.157°$ $\delta_{C1} = (1.6301 - 1)\,(5) = 3.150°$ Net: $\delta_{C2} - \delta_{C1} = -0.007°$

Dispersion $= \delta_{F12} - \delta_{C12} = 0.029 - (-0.007) = 0.036° = 2.16'$

3-12. Let the crown prism be prism 1 and the flint prism be prism 2. Then,

$$\delta_{C1} = (n_{C1} - 1)\,A_1 = (1.5205 - 1)\,15 = 7.8075°$$
$$\delta_{F1} = (n_{F1} - 1)\,A_1 = (1.5286 - 1)\,15 = 7.929°$$

The relative dispersion is $\delta_{F1} - \lambda_{C1} = 0.1215°$. This must be reversed by the flint prism:

$$\delta_{F2} - \delta_{C2} = 0.1215° = (n_{F2} - n_{C2})\,A_2 = (1.7328 - 1.7076)\,A_2 \Rightarrow A_2 = 4.82°$$

The deviation of line D is then, $\delta_{D1} - \delta_{D2} = (n_{D1} - 1)\,A_2 = (.523)\,(15) - (0.7205)\,(4.821) = 4.37°$

3-13. (a) The radiant exitance is given by $M_e = \dfrac{\Phi_{\text{tot}}}{A} = \dfrac{25\,\text{W}}{(0.05\,\text{m})^2} = 10^4\ \text{W/m}^2$.

Since the source is perfectly diffuse (i.e. is Lambertian) the radiant intensity can be written in the form, $I_e = I_e(0)\cos\theta$. Here $\theta = 0$ is the direction normal to the surface of the square source. Take the $\theta = 0$ direc tion to be along the positive z axis. The total power emitted in the forward direction is related to the intensity via the relation,

$$\Phi_{\text{tot}} = \int I_e d\omega = \int_0^{2\pi} d\varphi \int_0^{\pi/2} I_e(0)\cos(\theta)\sin(\theta)d\theta = 2\pi I(0)\frac{1}{2} = \pi I_e(0)$$

Thus,

$$I_e(0) = \frac{\Phi_{\text{tot}}}{\pi\,\text{sr}} = \frac{25\,\text{W}}{\pi\,\text{sr}} = 7.96\,\text{W/sr}$$

This is the radiant intensity in the direction of the surface normal to the source. Since the source is Lam bertian the radiance L_e is independent of the viewing angle θ and given by,

$$L_e = \frac{I(0)\cos\theta}{A\cos\theta} = \frac{I(0)}{A} = \frac{7.96\,\text{W/sr}}{(0.05\,\text{m})^2} = 3180\,\frac{\text{W}}{\text{m}^2\cdot\text{sr}}$$

(b) Assume that the camera aperture is parallel to the source surface and centered along the direction of the surface normal. Now, $f/D = 8 = (4\text{ cm})/D$. Thus, $D = 0.5$ cm. The flux Φ_{cam} entering the camera is well approximated by multiplying the intensity in the forward direction by the solid angle intercepted by the camera lens. Noting that the source to lens distance is $r = 1\,\text{m}$, this is, approximately,

$$\Phi_{\text{cam}} = I(0)\,\Delta\omega_{\text{lens}} = I(0)(A_{\text{lens}})/r^2 = (7.96\,\text{W/sr})\left(\frac{\pi\,(0.25\,\text{cm})^2}{(100\,\text{cm})^2}\right) = 1.56\times 10^{-4}\,\text{W}$$

(c) The irradiance at the film is then, $E_e = \dfrac{\Phi_{\text{cam}}}{A_{\text{image}}} = \dfrac{\Phi_{\text{cam}}}{|m|^2 A}$

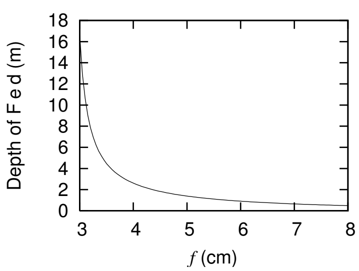
Here, we have noted that each image dimension is decreased from the source dimension by the magnifica tion,

$$|m| = \frac{s'}{s} = \frac{f}{s-f} = \frac{4}{100-4} = 0.0417$$

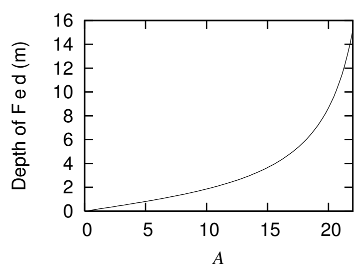
So,

$$E_e = \frac{1.56\times 10^{-4}\,\text{W}}{(0.0417)^2(0.05\,\text{m})^2} = 35.9\,\text{W/m}^2$$

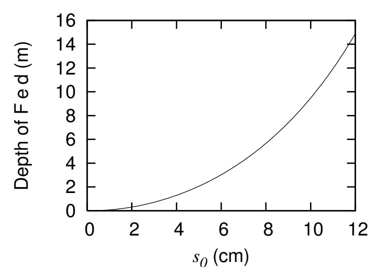
3-14. Plots as a function of the different parameters are shown below.



$A = 8$, $s_0 = 2\,\text{m}$, $d = 0.05\,\text{mm}$



$f = 5\,\text{cm}$, $s_0 = 2\,\text{m}$, $d = 0.05\,\text{mm}$



$A = 2$, $f = 5\,\text{cm}$, $d = 0.05\,\text{mm}$

3-15. From Eqs. (3 30) and (3 31),

$$s_0 - s_1 = \frac{A\,s_0\,d\,(s_0 - f)}{f^2 + A\,s_0\,d} = \frac{(4)\,(6)\,(10^{-6})\,(6-0.05)}{(.05)^2 + (4)\,(6)\,(10^{-6})}\,\text{m} = 0.057\,\text{m} = 5.7\text{cm}$$

$$s_2 - s_0 = \frac{A\,s_0\,d\,(s_0 - f)}{f^2 - A\,s_0\,d} \approx s_0 - s_1 = 5.7\text{cm}$$

Thus, points at 5.7 cm nearer or farther than the middle row create blur circles larger than the silver grain dimension. Under these demanding conditions, the picture could not be taken successfully.

3-16. Using Eq. (3 35),

$$\frac{1}{f}=\frac{1}{f_1}+\frac{1}{f_2}-\frac{L}{f_1\,f_2}=\frac{1}{20}+\frac{1}{-8}-\frac{15}{(20)\,(-8)}\Rightarrow f=53.3\,\text{cm}$$

If the film plane is in the image position for an object at infinity, its position can be found as follows. The image formed by the first lens of the object at infinity falls at the focal point of the first lens which is 5 cm past the second lens. This intermediate image serves as the object for the second lens. That is $s_2=-5$ cm. The final image position relative to the second lens can be found using the thin lens formula,

$$\frac{1}{s_2}+\frac{1}{s_2'}=\frac{1}{f_2}\Rightarrow\frac{1}{-5}+\frac{1}{s_2}=\frac{1}{-8}\Rightarrow s_2'=13.33\,\text{cm}$$

That is, the film should be placed 13.33 cm past the negative lens of the combination. The image size h' of a distant object subtending an angle of 2° at the camera is found using the effective focal length of the lens.

$$\tan 2^\circ=\frac{|h'|}{f}=\frac{h'}{53.33\,\text{cm}}\Rightarrow 1.86\,\text{cm}$$

Alternately one may find the image height as,

$$h'=m_1\,m_2\,h=-\frac{s_1'}{s_1}\left(-\frac{s_2'}{s_2}\right)=\frac{20\,\text{cm}}{s_1}\,\frac{13.33}{-5}\,h=(53.33\,\text{cm})\,\frac{h}{s_1}=(53.33\,\text{cm})\tan 2^\circ=1.86\,\text{cm}$$

3-17. Using the parameters $f=5\text{ cm}=0.1640\,\text{ft}$, $s_0=6\,\text{ft}$, $A=f/4=4$, and $d=0.05\text{ mm}=1.640\times 10^{-4}$ ft in Eqs, (3 30) and (3 31) gives,

$$s_1=\frac{s_0 f\,(f+A\,d)}{f^2-A\,d\,s_0}=5.26\,\text{ft}$$
$$s_2=\frac{s_0\,f\,(f-A\,d)}{f^2-A\,d\,S_0}=7.00\,\text{ft}$$

3-18. See Figure 3 38 in the text. The diameter of the image of the sun is about

$D_I=f_L\,\theta=(50\text{ cm})\,(0.5^\circ)\left(\frac{\pi\,\text{rad}}{180^\circ}\right)=0.44$ cm. Since all of the power intercepted by the lens, falls on the image, the irradiance of the image is

$$I_{\text{image}}=I_{\text{sun}}\,\frac{A_{\text{image}}}{A_{\text{lens}}}=I_{\text{sun}}\,\frac{D_L^2}{D_I^2}=\left(1000\,\text{W/cm}^2\right)\left(\frac{5}{0.44}\right)^2=1.29\times 10^5\,\text{W/cm}^2$$

3-19. (a) Using $f=15\,\text{cm}, s=100\,\text{ft}=3048\,\text{cm}$, and $h=6\,\text{ft}$, one finds,

$$m=-\frac{s'}{s}=\frac{-f}{s}=\frac{-15}{3048}$$
$$h'=|m|\,h=\frac{15}{3048}\,6\,\text{ft}=0.0295\text{ ft}=0.9\,\text{cm}$$

(b) Distant objects form an image in the first lens at $f_1=12\,\text{cm}$. Thus, the object distance for lens 2 is $s_2=(8-12)\text{ cm}=-4\text{ cm}$. Then the image distance from lens 2 is $s_2'=15$ cm so the focal length of the second lens must be,

$$\frac{1}{f_2}=\frac{1}{s_2}+\frac{1}{s_2'}=\frac{1}{-4\,\text{cm}}+\frac{1}{15\,\text{cm}}\Rightarrow f_2=-5.45\,\text{cm}$$

Then for the person at 100 feet,

$$m=m_1\,m_2=\left(-\frac{s_1'}{s_1}\right)\left(-\frac{s_2'}{s_2}\right)=\left(-\frac{-12}{3048}\right)\left(\frac{-15}{-4}\right)=-0.0148$$

The ratio of the magnifications is

$$\frac{|m|}{|m\,(\text{telephoto})|}=\frac{15/3048}{180/4\,(3048)}=\frac{1}{3}$$

3-20. (a) $D=\dfrac{f}{1.8}=\dfrac{50\,\text{mm}}{1.8}=27.8\,\text{mm}$

(b) Factor of $\sqrt{3}$: $f/1.8\rightarrow f/3.1\rightarrow f/5.4\rightarrow f/9.4$

(c) $D=50/3.1=16.0$ mm; $D=50/5.4=9.26$ mm; $D=50/9.4=5.35$ mm

(d) $f/1.8$ at $\dfrac{1}{100}$ is equivalent in exposure to $f/3.1$ at $3/100$ s, $f/5.4$ at $9/100$ s, $f/9.4$ at $27/100$ s

3-21. $M=\dfrac{25}{F_{\text{eq}}}$ where $\dfrac{1}{f_{\text{eq}}}=\dfrac{1}{f_1}+\dfrac{1}{f_2}-\dfrac{L}{f_1 f_2}$ and L is determined by the condition for elimination of chromatic aberration: $L=\dfrac{1}{2}(f_1+f_2)$. Then,

$$\frac{1}{f_{\text{eq}}}=\frac{1}{f_1}+\frac{1}{f_2}-\frac{f_1+f_2}{2f_1 f_2}=\frac{1}{2}\left(\frac{1}{f_1}+\frac{1}{f_2}\right)\text{ and }M=\frac{25}{\text{feq}}=12.5\left(\frac{1}{f_1}+\frac{1}{f_2}\right)$$

3-22. The focal lengths of the two lenses are, $f_1=f_2=3$ cm and they are separated by $L=2.8$ cm.

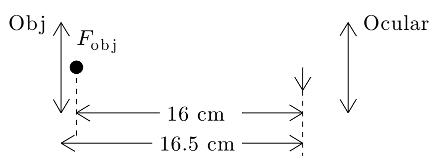
(a) The equivalent focal length is,

$$\frac{1}{f_{\text{eq}}}=\frac{1}{f_1}+\frac{1}{f_2}-\frac{L}{f_1 f_2}=\frac{1}{3}+\frac{1}{3}-\frac{2.8}{9}\qquad f_{\text{eq}}=2.8125\,\text{cm}$$

(b) The magnification is,

$$M=\frac{25}{f_{\text{eq}}}+1=\frac{25}{2.8125}+1=9.9\approx 10\times$$

3-23. (a) The system is sketched below.



$$M=m_{\text{obj}}\times M_{\text{oc}}=\frac{s'}{s}\times 10\text{ where }s'=f+16=16.5\,\text{cm}$$

$$s=\frac{s' f}{s'-f}=\frac{(16.5)\,(0.5)}{16.5-0.5}=\frac{8.25}{16}$$

$$|m|=\frac{s'}{s}=\frac{(16.5)\,(16)}{8.25}=32,\ M=(32)\,(16)=320\times$$

(b) $s=\dfrac{8.25}{16}=0.516$ cm

3-24. (a) $M=m_{\text{obj}}\times M_{\text{oc}}=\dfrac{s'_{\text{obj}}}{s_{\text{obj}}}\left(\dfrac{25}{f_{\text{oc}}}+1\right)$ where, $s'_{\text{obj}}=\dfrac{s_0 f_{\text{obj}}}{s_0-f_{\text{obj}}}=\dfrac{(1.20)\,(1)}{1.20-1}=6$ cm. So,

$M=\left(\dfrac{6}{1.20}\right)\left(\dfrac{25}{3}+1\right)=46.7\times$

(b) $L=s'_{\text{obj}}+s_{\text{oc}}=6+2.68=8.68$ cm where $s_{\text{oc}}=\dfrac{s'_{\text{oc}} f_{\text{oc}}}{s'_{\text{oc}}-f_{\text{oc}}}=\dfrac{(-25)\,(3)}{-25-3}=2.68$ cm

3-25. $M=-\left(\dfrac{25}{f_e}\right)\left(\dfrac{L}{f_o}\right)$ and $L=d-f_o-f_e\Rightarrow -20=-\left(\dfrac{25}{4}\right)\left(\dfrac{25-4-f_o}{f_o}\right)\Rightarrow f_o=5$ cm

3-26. $\dfrac{1}{30}+\dfrac{1}{s'}=\dfrac{1}{0.2}\Rightarrow s'=0.2013$ m, $m=\dfrac{s'}{s}=\dfrac{0.2013}{30}=0.006711$, $h'=m\,h\Rightarrow h=\dfrac{h'}{m}=\dfrac{0.001}{0.006711}=0.149$ m $=14.9$ cm

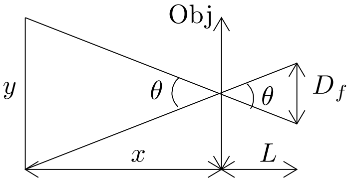
3-27. (a) $M = 7\times$ (b) $M = \dfrac{f_{\text{obj}}}{f_{\text{oc}}}$ or $f_{\text{oc}} = \dfrac{14}{7} = 2\,\text{cm}$ (c) $M = \dfrac{D_{\text{obj}}}{D_{\text{ex}}}$ or $D_{\text{ex}} = \dfrac{35}{7} = 5\,\text{mm}$

(d) The exit pupil position is the image of L_1 in L_2, or $s' = \dfrac{s\,f}{s-f} = \dfrac{L\,\text{foc}}{L - \text{foc}} = \dfrac{(14+2)\,(2)}{(14+2)-(2)} = \dfrac{16}{7} = 2.3\,\text{cm}$

(e) Let y be the linear dimension of the field of view at a distance of $x = 1000\,\text{yd}$. Then the angular field of view can be written as,

$$\theta = \frac{D_f}{L} = \frac{1.8}{14+2} = 0.1125 \text{ rad} = \frac{y}{x} \Rightarrow y = x\,\theta = (1000\,\text{yd})(0.1125 \text{ rad}) = 337\,\text{ft}.$$

The field of view is 337 ft at 1000 yd. The geometry is sketched (not to scale) below.

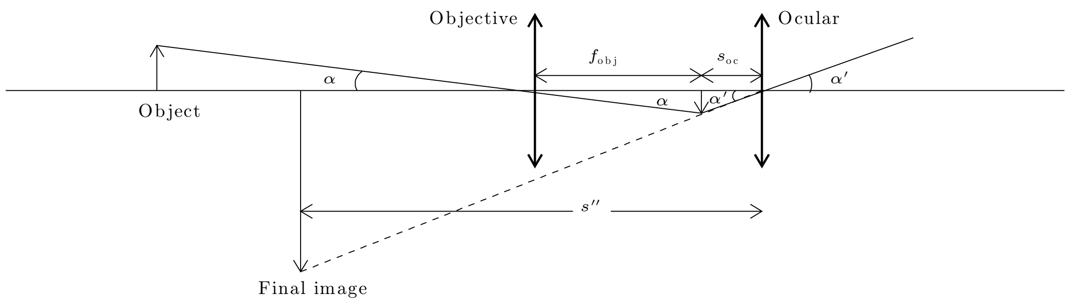


3-28. (a) Consider the right triangles in the figure below that share the image y as a common size. Form these two triangles one can write,

$$y = f_{\text{obj}} \tan\alpha = s_{\text{oc}} \tan\alpha' \Rightarrow \frac{\tan\alpha'}{\tan\alpha} = \frac{f_{\text{obj}}}{s_{\text{oc}}}$$

The angular magnification is then, $M = \dfrac{\alpha'}{\alpha} \approx \dfrac{\tan\alpha'}{\tan\alpha} = \dfrac{f_{\text{obj}}}{s_{\text{oc}}}$
Since $m_{\text{oc}} = -\,s''/s_{\text{oc}}$, the required result follows,

$$M = -\frac{f_{\text{obj}} m_{\text{oc}}}{s''}$$



(b) When viewed at ∞, $M = f_o/f_e = 30/4 = 7.5\times$. When viewed at 25 cm, $s'' = -\,25\,\text{cm}$ and so,

$$m_{\text{oc}} = -\frac{s''}{s_{\text{oc}}} = -\frac{s''}{s''\,f_{\text{oc}}/(s''-f_{\text{oc}})} = -\frac{s''-f_{\text{oc}}}{f_{\text{oc}}} = -\frac{(-25)-4}{4} = \frac{29}{4} = 7.25\times$$

Them by the formula from part (a),

$$M = -\frac{(30)\,(7.25)}{(-25)} = 8.70\times$$

3-29. The image size on the moon is determined by the angle α' shown in the figure associated with the solution to problem 3 28 above. In that problem solution it is shown that $M = \alpha'/\alpha$. Here $\alpha = 0.5° = \pi/360$. The diameter of the image of the moon is given by $D_{\text{Moon}} = s''\ \alpha' = (25\text{ cm})\ \alpha'$. To find α' use the relation obtained in the solution to problem 3 28), $M = f_{\text{obj}}/s_{\text{oc}}$. Here, from a rearrangement of the thin lens for mula,

$$s_{\text{oc}} = \frac{s''f_{\text{oc}}}{s''-f_{\text{oc}}} = \frac{(-25)\,(5)}{-25-5}\text{ cm} = \frac{25}{6}\text{ cm}$$

so that,

$$M = \frac{20}{(25/6)} = \frac{24}{5}\times$$

Then,

$$\alpha' = M\,\alpha = \frac{24}{5}\frac{\pi}{360} = \frac{\pi}{75} \text{ and } D_{\text{Moon}} = s''\,\alpha' = 25\,\frac{\pi}{75}\text{ cm} = \frac{\pi}{3}\text{ cm} = 1.05\text{ cm}$$

3-30. (a) $L = f_{\rm obj} + f_{\rm oc} = 12\,{\rm cm} - 4\,{\rm cm} = 8\,{\rm cm}$. $M = f_{\rm obj}/f_{\rm oc} = (12/4) = 3\times$.
(b) For the image at $s'' = 30\,{\rm cm}$:

$$s_{\rm oc} = \frac{s''\,f_{\rm oc}}{s'' - f_{\rm oc}} = \frac{-30\,(-4)}{-30+4}\,{\rm cm} = -\frac{60}{13}\,{\rm cm}$$
$$M = \frac{f_{\rm obj}}{s_{\rm oc}} = \frac{12}{60/13} = 2.6\times$$
$$L = f_{\rm obj} + s_{\rm oc} = 12\,{\rm cm} - (60/13)\,{\rm cm} = 7.38\,{\rm cm}$$

3-31. See Figure 3 39 in the text. In the normal position, $s_{\rm oc} = f_{\rm oc} = 5\,{\rm cm}$. But when $s'' = 25\,{\rm cm}$,

$$s_{\rm oc} = \frac{s''\,f_{\rm oc}}{s'' - f_{\rm oc}} = \frac{25\cdot 5}{25-5}\,{\rm cm} = 6.25\,{\rm cm}$$

Thus, the ocular must be moved further from the objective by an amount $6.25\,{\rm cm} - 5\,{\rm cm} = 1.25\,{\rm cm}$ in order to produce a real image on the screen 25 cm away.

3-32. Using Eq. (3 35) and the given information,

$$\frac{1}{f_{\rm eq}} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{L}{f_1\,f_2} = \frac{1}{2\,{\rm cm}} + \frac{1}{2\,{\rm cm}} - \frac{2}{4\,{\rm cm}} \Rightarrow f_{\rm eq} = 2\,{\rm cm}$$

(a) $M_\infty = \frac{25}{f_{\rm eq}} = \frac{25}{2} = 12.5\times$ (b) $M_{\rm tel} = \frac{f_{\rm obj}}{f_{\rm oc}} = \frac{30}{2} = 15\times$

(c) The exit pupil is the image of the AS or objective lens, formed by the ocular. Since the ocular consists of 2 lenses, one should argue as follows: To produce parallel rays *leaving* the ocular (image at ∞), the field lens must be at the focal point of the objective, 30 cm from the objective. Then the image there is 2 cm from the eye lens ($1/2 + 1/s' = 1/2$, $s' \to \infty$). Given this separation, the image of the objective in the *field* lens is

$$s_1' = \frac{30(2)}{30-2}\,{\rm cm} = \frac{15}{7}\,{\rm cm}$$

that is 1/7 cm *beyond* the eye lens. The image formed by the eye lens is then at,

$$s_2' = \frac{(-1/7)\,(2)}{-1/7-2}\,{\rm cm} = 0.133\,{\rm cm}$$

Thus the exit pupil falls 0.133 cm from the eye lens.
Using $M = D_{\rm obj}/D_{E_xP}$ we also have $D_{E_xP} = \frac{4.5}{15}\,{\rm cm} = 3\,{\rm mm}$.

(d) $\theta = D_{\rm FL}/L = 2/30 = (1/15)\,{\rm rad} = 3.8°$.

3-33. "Unfolding" the right angle in the optical axis of Figure 3 34a in the text, the equivalent optics shown in Figure 3 40 associated with the problem in the text is seen. From the diagram in Figure 3 40,

$$|M| = \frac{\alpha_m}{\alpha_o} = \frac{h/f_e}{h/f_o} = \frac{f_o}{f_e}$$

3-34. Refer to figure 3 41 in the text. For M_2:

$$\frac{1}{s} + \frac{1}{s'} = \frac{1}{f} \Rightarrow f = \frac{s\,s'}{s+s'} = \frac{(-2)(10)}{-2+10}\,{\rm ft} = -2.5\,{\rm ft}$$

Without the intermediate convex mirror, the optical system is the same as a Newtonian telescope, whose magnification is given (see problem 3 33) by

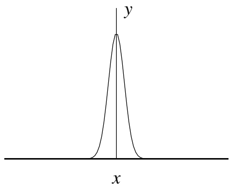
$$M = -\frac{f_{\rm obj}}{f_{\rm oc}} = -\frac{12}{-4/12} = -36\times$$

The magnification is increased by the convex mirror by the factor, $m = -s'/s = -(-10)/(-2) = +5$

The overall magnification is then, $M_{\rm overall} = m\,M = (5)(-36) = -180\times$

Chapter 4 Wave Equations

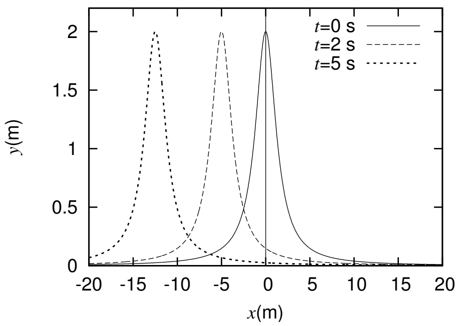
4-1. The waveform is sketched below. As a function of time $y = A\,e^{-b(x+10\,t)^2}$



4-2. (a) Replace x by $x + v\,t$:

$$y(x,t) = \frac{4\,\mathrm{m}^3}{(x + (2.5\,\mathrm{m/s}))^2 + 2\,\mathrm{m}^3}$$

(b)



4-3. (a), (b)

Wave form 1 is a traveling wave since it is a function of $u = z - v\,t$ with $v = -1$ m/s. This means that it travels in the negative z direction with a speed of 1 m/s.

Wave form 2 is a traveling wave since it is a function of $u = x - v\,t$, with $v = 1\,\mathrm{m/s}$. This is a wave traveling in the positive x direction with a speed of 1 m/s.

Wave form 3 is not a traveling wave since it is not a function of simply $u = x - v\,t$.

With more labor one can show that waveforms 1 and 2 do and waveform 3 does not satisfy the differential wave equation (Eq. (4 2)).

4-4. The waveform can be rewritten as,

$$y = \frac{(100\,\mathrm{m}^2)}{x - (10\,\mathrm{m/s})\,t}\,e^{(x-(10\,\mathrm{m/s})t)^2/\mathrm{m}^2}$$

Thus $y = y(x - (10\,\mathrm{m/s})\,t)$ and so the waveform is a traveling wave with $v_x = 10\,\mathrm{m/s}$.

4-5. From the given information, $k = 2\,\pi/\lambda = (2\,\pi/5)\,\mathrm{m}^{-1}$, $|v| = f\,\lambda = \lambda/T = (5/3)\,\mathrm{m/s}, v_z = -5/3\,\mathrm{m/s}$

(a) $y = A\sin 2\,\pi(z/\lambda \pm t/T) = 2\sin 2\,\pi\left[\frac{z}{5\,\mathrm{m}} + \frac{t}{3\,\mathrm{s}}\right]$

(b) $y = A\sin k(z \pm v\,t) = 2\sin\left\{\frac{2\,\pi}{5\,\mathrm{m}}\left[z + \left(\frac{5}{3}\,\mathrm{m/s}\right)t\right]\right\}$

(c) $\tilde{y} = A\,e^{2\pi i(z/\lambda \pm t/T)} = 2\,e^{2\pi i[z/(5\mathrm{m}) + t/(3\mathrm{s})]}$, $y = \mathrm{Im}(\tilde{y})$

4-6. (a) $y = A\sin 2\,\pi(x/\lambda \pm t/T) = (5\,\mathrm{m})\sin[2\,\pi\,x/(50\,\mathrm{m})]$

(b) $y = A\sin k\,(x \pm v_x\,t) = (5\,\mathrm{m})\sin\left[\frac{2\,p}{50\,\mathrm{m}}(x + (2\,\mathrm{m/s})\,(4\,\mathrm{s})\right] = (5\,\mathrm{m})\sin\left[\frac{\pi}{25\,\mathrm{m}}(x + 8\,\mathrm{m})\right]$

4-7. Compare the general form, $y = \sin 2\pi\left(\frac{x}{\lambda} - \frac{t}{T}\right)$ to the given form, $y = (10\,\text{cm})\sin\left(\frac{628.3\,x}{\text{cm}} - \frac{6283}{\text{s}}t\right)$

(a) $2\pi/\lambda = 628.3/\text{cm} \Rightarrow \lambda = 0.01\,\text{cm}$
(b) $\nu = 1/T$; $2\pi/T = 6283/\text{s} \Rightarrow T = 0.001\,\text{s}, \nu = 1000\,\text{Hz}$
(c) $k = 2\pi/\lambda = 628.3/\text{cm}$ (d) $\omega = 2\pi\nu = 2\pi(1000)/\text{s} = 6283/\text{s}$ (e) $T = 0.001\,\text{s}$
(f) $v = \nu\lambda = (1000)\,(0.01)\,\text{cm/s} = 10\,\text{cm/s}$ (g) $A = 10\,\text{cm}$

4-8. (a) $\varphi = y - Bt = \text{constant}, \frac{d\varphi}{dt} = 0 = \frac{dy}{dt} - B \Rightarrow v_y = \frac{dy}{dt} = B$. ($B$ has units of m/s ✓)

(b) $\varphi = Bx + Ct + D = \text{constant}, \frac{d\varphi}{dt} = 0 = B\frac{dx}{dt} + C \Rightarrow v_x = \frac{dx}{dt} = -C/B$.
(Bx/Ct is unitless so C/B in m/s ✓)

(c) $\varphi^2 = Bz^2 + BC^2t^2 - 2BCzt = B(z - Ct)^2$, $\varphi = \sqrt{B}\,(z - Ct)$=constant, $\frac{d\varphi}{dt} = 0 = \sqrt{B}\left(\frac{dz}{dt} - C\right)$.
Therefore, $v_z = \frac{dz}{dt} = C$. Apparently Ct and the same dimensions as z so C has dimensions of m/s ✓.

4-9. Taking the general form at $t = 0$ as $y = A\sin[2\pi(x/\lambda) + \varphi]$, the given conditions are,

$$y(x = 0) = 13 = A\sin\varphi \quad (1)$$
$$y(x = 3\lambda/4) = -7.5 = A\sin(1.5\pi + \varphi) = -A\cos\varphi \quad (2)$$

Dividing (1) by (2) gives,

$$-13/7.5 = -\tan\varphi \Rightarrow \varphi = \tan^{-1}(13/7.5) = 1.047 \approx \pi/3$$

Then, from (1)

$$A = 13/\sin\varphi = 13/\sin(\pi/3) = 15.0$$

So that,

$$y = 15\sin(2\pi x/\lambda + \pi/3)$$

4-10. (a) Take the waveform at $t = 0$ to be $y = A\sin(2\pi x/\lambda + \varphi_0)$. We are given that $y = A$ at $x = x_0$. That is,

$$y(x_0) = A\sin(2\pi x_0/\lambda + \varphi_0) = A$$
$$\sin(2\pi x_0/\lambda + \varphi_0) = 1$$
$$2\pi x_0/\lambda + \varphi_0 = \pi/2$$
$$\varphi_0 = \frac{\pi}{2} - \frac{2\pi}{\lambda}x_0$$

(b) For $\lambda = 10\,\text{cm}$,

$$x_0 = 0,\ \varphi_0 = \pi/2$$
$$x_0 = 5/6\,\text{cm},\ \varphi_0 = \frac{\pi}{3}$$
$$x_0 = 5/2\,\text{cm},\ \varphi_0 = 0$$
$$x_0 = 5\,\text{cm},\ \varphi_0 = -\frac{\pi}{2}$$
$$x_0 = -1/2\,\text{cm},\ \varphi_0 = \frac{3\pi}{5}$$

(c) Generally $y = A\cos(2\pi x/\lambda + \varphi_c)$ the same argument given in (a) requires that

$$\cos(2\pi x_0/\lambda + \varphi_c) = 1$$
$$2\pi x_0/\lambda + \varphi_c = 0$$
$$\varphi_c = -2\pi x_0/\lambda = \varphi_0 - \pi/2$$

Thus we should subtract $\pi/2$ from the answers in part (b)

4-11. Generally, $\boldsymbol{k}\cdot\boldsymbol{r}=k_x x+k_y y+k_z z$.

(a) For propagation along the z axis, $k_x=k_y=0$, So $\boldsymbol{k}\cdot\boldsymbol{r}=k_z z$, with $k_z=2\pi/\lambda$. The waveform can then be written as,

$$\psi=A\sin(\boldsymbol{k}\cdot\boldsymbol{r}-\omega t)=A\sin(k_z z-\omega t)=A\sin\left[\frac{2\pi}{\lambda}(z-v t)\right]=A\sin 2\pi(z/\lambda-v t)$$

(b) In this case, $k_z=0$ and $k_x=k_y=|k|/\sqrt{2}=\frac{1}{\sqrt{2}}\frac{2\pi}{\lambda}$. The general form of the wave is then,

$$\psi=A\sin(\boldsymbol{k}\cdot\boldsymbol{r}\pm\omega t)=A\sin(k_x x+k_y y\pm\omega t)=A\sin\left[\frac{2\pi}{\sqrt{2}\lambda}(x+y\pm v t)\right]=A\sin 2\pi\left(\frac{x}{\sqrt{2}\lambda}+\frac{y}{\sqrt{2}\lambda}\pm v t\right).$$

If one is interested in the wave displacement only on the line $x=y$,

$$\psi=A\sin 2\pi\left(\frac{2x}{\sqrt{2}\lambda}\pm v t\right).$$

(c) In this case $\boldsymbol{k}=\frac{k}{\sqrt{3}}(\hat{\boldsymbol{x}}+\hat{\boldsymbol{y}}+\hat{\boldsymbol{z}})$ and $\boldsymbol{k}\cdot\boldsymbol{r}=\frac{k}{\sqrt{3}}(x+y+z)$, with $k=2\pi/\lambda$. The waveform is then,

$$\psi=A\sin(\boldsymbol{k}\cdot\boldsymbol{r}\pm\omega t)=A\sin\left[\frac{k}{\sqrt{3}}(x+y+z)\pm\omega t\right]=A\sin\left[\frac{2\pi}{\sqrt{3}\lambda}(x+y+z\pm v t)\right]$$

4-12. Let $\tilde{z}=a+i b$ where a and b are real.

(a) $(\tilde{z}+\tilde{z}^*)/2=(a+i b+a-i b)/2=a=\mathrm{Re}(\tilde{z})$

(b) $(\tilde{z}-\tilde{z}^*)/2i=(a+i b-a+i b)/2i=b=\mathrm{Im}(\tilde{z})$

(c) Let $\tilde{z}=e^{i\theta}=\cos\theta+i\sin\theta$ and apply the result from (a): $\cos\theta=\left(e^{i\theta}+e^{-i\theta}\right)/2$

(d) Let $\tilde{z}=e^{i\theta}=\cos\theta+i\sin\theta$ and apply the result from (b): $\sin\theta=\left(e^{i\theta}-e^{-i\theta}\right)/2i$

4-13. (a) Note that $e^{i\pi/2}=\cos(\pi/2)+i\sin(\pi/2)=i$ so that

$$i\,A\,e^{i(\boldsymbol{k}\cdot\boldsymbol{r}-\omega t+\varphi_0)}=e^{i\pi/2}\,A\,e^{i(\boldsymbol{k}\cdot\boldsymbol{r}-\omega t+\varphi_0)}=A\,e^{i(\boldsymbol{k}\cdot\boldsymbol{r}-\omega t+\varphi_0+\pi/2)}$$

(b) Similarly, $e^{i\pi}=\cos(\pi)+i\sin(\pi)=-1$ so that

$$-A\,e^{i(\boldsymbol{k}\cdot\boldsymbol{r}-\omega t+\varphi_0)}=e^{i\pi}\,A\,e^{i(\boldsymbol{k}\cdot\boldsymbol{r}-\omega t+\varphi_0)}=A\,e^{i(\boldsymbol{k}\cdot\boldsymbol{r}-\omega t+\varphi_0+\pi)}$$

4-14. One can write $\psi=\mathrm{Im}(\tilde{\psi})$ with

$$\tilde{\psi}=A\,e^{i(ky+\omega t)}+A\,e^{i(ky-\omega t+\pi)}=A\,e^{i(ky+\omega t)}+A\,e^{i\pi}e^{i(ky-\omega t)}=A\,e^{i(ky+\omega t)}-A\,e^{i(ky-\omega t)}$$

where I have used the result from problem 4 13b. Regrouping gives,

$$\tilde{\psi}=A\,e^{iky}\left(e^{i\omega t}-e^{-i\omega t}\right)=2\,i\,A\,e^{iky}\sin(\omega t)=2\,A\sin(\omega t)\,[i\cos(k\,y)-\sin(k\,y)]$$

Here I have used the result from problem 4 12d. Reforming the real wavefunction gives the desired relation:

$$\psi=\mathrm{Im}(\tilde{\psi})=2\,A\cos(k\,y)\sin(\omega t)$$

4-15. The irradiance is related to the electric field amplitude by the relation $E_e=(\varepsilon_0\,c/2)E_0^2$ so that,

$$E_0=\sqrt{\frac{2\,E_e}{\varepsilon_0 c}}=\sqrt{\frac{2000}{8.85\times10^{-12}\times3\times10^{8}}}\,\mathrm{V/m}=870\,\mathrm{V/m}$$

Then,

$$B_0=E_0/c=870/(3\times10^{8})\,\mathrm{T}=2.90\times10^{-6}\,\mathrm{T}$$

4-16. (a) $B_0 = E_0/v = E_0\, n/c = (100)\,(1.5)/(3\times 10^8)\,\mathrm{T} = 5.0\times 10^{-7}$ T
(b) $\langle S\rangle = E_e = \left(\varepsilon_0\, c^2/2\right)E_0 B_0 = (8.85\times 10^{-12}\, 9.0\times 10^{16}/2)\,(100)\,(5.0\times 10^{-7})\mathrm{W/m^2} = 19.9\,\mathrm{W/m^2}$

4-17. (a) From the solution to problem 4 15,

$$E_0 = \sqrt{\frac{2\,E_e}{\varepsilon_0 c}} = \sqrt{\frac{2\cdot 1350}{8.85\times 10^{-12}\times 3\times 10^8}}\,\mathrm{V/m} = 1010\,\mathrm{V/m},\ B_0 = E_0/c = 3.37\times 10^{-6}\,\mathrm{T}$$

(b) $\text{Photon flux} = \dfrac{E_e}{h\,\nu} = \dfrac{E_e}{h\,c/\lambda} = \dfrac{1350}{(6.626\times 10^{-34})\,(3\times 10^8)/(7\times 10^{-7})}\,\mathrm{m^{-2}\,s^{-1}} = 4.76\times 10^{21}\mathrm{m^{-2}\,s^{-1}}$

(c) $k = 2\,\pi/\lambda = 2\,\pi/(7\times 10^{-7}\,\mathrm{m}) = 1.43\times 10^6/\mathrm{m}$, $\nu = c/\lambda = 4.28\times 10^{14}$ Hz. So,

$$E = (1010\,\mathrm{V/m})\sin 2\,\pi\Big(\big(1.43\times 10^6/\mathrm{m}\big)s \pm \big(4.28\times 10^{14}\,/\mathrm{s}\big)\,t\Big)$$

4-18. (a) $E_e = \Phi/A = \dfrac{220\cdot 0.05}{4\,\pi\,(10)^2}\,\mathrm{W/m^2} = 8.75\times 10^{-3}\,\mathrm{W/m^2}$,

$E_0 = \sqrt{\dfrac{2\,E_e}{\varepsilon_0 c}} = \sqrt{\dfrac{2\cdot 8.75\times 10^{-3}}{8.85\times 10^{-12}\times 3\times 10^8}}\,\mathrm{V/m} = 2.57\,\mathrm{V/m}$

(b) $E_e = \Phi/A = \dfrac{2000}{10^{-10}}\,\mathrm{W/m^2} = 2\times 10^{13}\,\mathrm{W/m^2}$

$E_0 = \sqrt{\dfrac{2\,E_e}{\varepsilon_0 c}} = \sqrt{\dfrac{2\cdot 2\times 10^{13}}{8.85\times 10^{-12}\times 3\times 10^8}}\,\mathrm{V/m} = 1.23\times 10^8\,\mathrm{V/m},\ B_0 = E_0/c = 0.409\,\mathrm{T}$

4-19. The energy flow in a cylindrical wave is in the (cylindrical) radial direction. The total flux Φ_e through a cylindrical surface of length L and radius r must be independent of the radial coordinate r. Now the total flux is related to the irradiance E_e by the relation,

$$\Phi_e = E_e\,A = E_e\,2\,\pi\,r\,L$$

For the flux to be independent of r, the irradiance E_e must be proportional to $1/r$. The irradiance is pro portional to the square of electric field amplitude E_0 so the electric field amplitude must be proportional to $1/\sqrt{r}$. That is,

$$E_e \propto \frac{1}{r} \propto E_0^2 \Rightarrow E_0 \propto \sqrt{1/r}$$

4-20. Eq. (4 44) gives,

$$\frac{\lambda'}{\lambda} = \frac{(1-v/c)^{1/2}}{(1+v/c)^{1/2}} = \sqrt{(1-v/c)\,(1+v/c)^{-1}}$$

A Taylor series expansion indicates that, for small v/c,

$$(1+v/c)^{-1} = 1 - v/c + O(v^2/c^2) \approx 1 - v/c$$

Using this in the expression for λ'/λ gives the desired result,

$$\frac{\lambda'}{\lambda} \approx \sqrt{(1-v/c)\,(1-v/c)} = 1 - v/c$$

4-21. $\dfrac{\lambda'}{\lambda} = \dfrac{540}{640} = \sqrt{\dfrac{1-v/c}{1+v/c}} \Rightarrow v = 0.168\,c$

4-22. $\dfrac{\lambda'}{\lambda} = 4.8 = \sqrt{\dfrac{1-v/c}{1+v/c}} \Rightarrow v = -\,0.917\,c$

4-23. From thermodynamics:

$$v_{\mathrm{rms}} = \sqrt{\frac{3\,R\,T}{M}} = \sqrt{\frac{3\,(8.31)\,(1000)}{0.004}} = 2497\,\mathrm{m/s}$$

Since the speed is small compared to c, we can use Eq. (4 45),

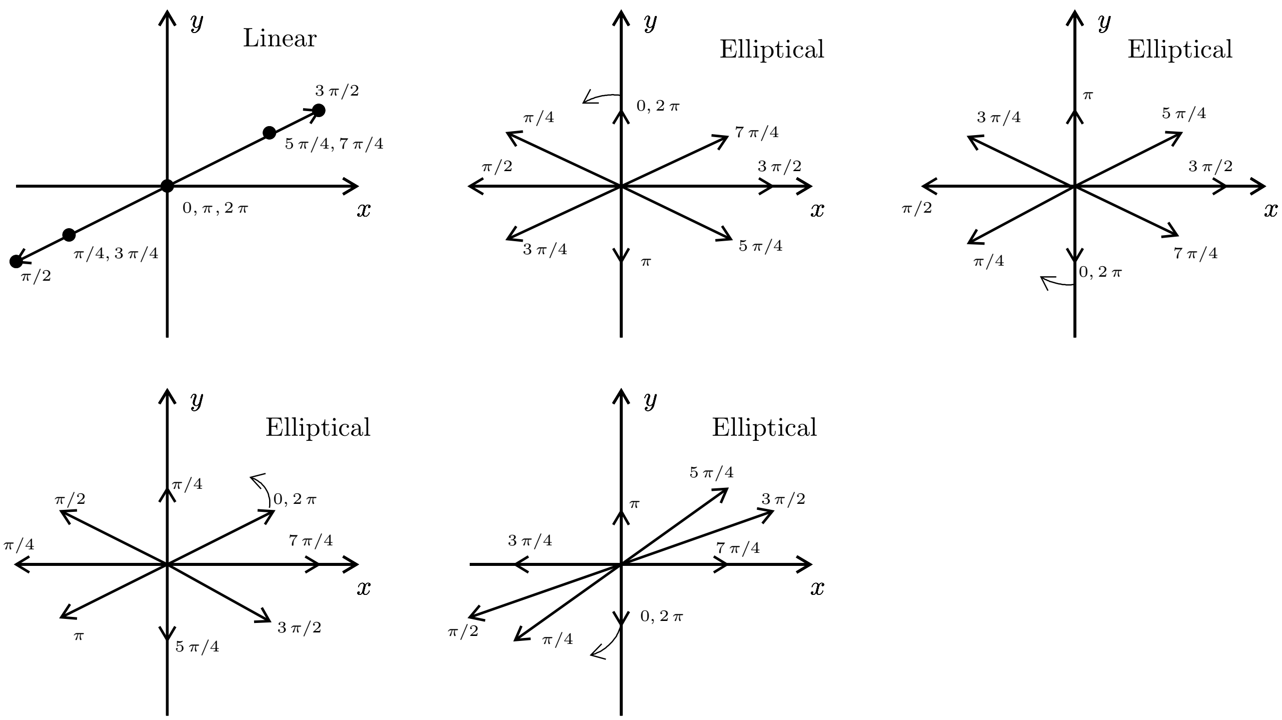
$$\frac{\lambda'}{\lambda} = 1 - v/c \Rightarrow (\lambda' - \lambda) = \Delta\lambda = (v/c)\,\lambda$$

$$\Delta\lambda = \frac{2497}{3\times 10^8}\left(706.52\times 10^{-9}\right)\mathrm{m} = 0.0059\,\mathrm{nm}$$

Now $\Delta\lambda$ is added to $\lambda_0 = \lambda(v=0)$ for receding atoms and subtracted from λ_0 for approaching atoms. Thus an estimate for the Doppler width is

$$\text{Doppler Width} \approx 2\,\Delta\lambda = 0.012\,\mathrm{nm}$$

4-24. For the stated conditions the electric fields are,
(a) $\boldsymbol{E} = -\,E_y\,[2\sin(\omega\,t)\,\hat{\boldsymbol{x}} + \sin(\omega\,t)\,\hat{\boldsymbol{y}}]$ (b) $\boldsymbol{E} = -\,E_y\,[2\sin(\omega\,t)\,\hat{\boldsymbol{x}} + \sin(\omega\,t - \pi/2)\,\hat{\boldsymbol{y}}]$
(c) $\boldsymbol{E} = -\,E_y\,[2\sin(\omega\,t)\,\hat{\boldsymbol{x}} + \sin(\omega\,t + \pi/2)\,\hat{y}]$ (d) $\boldsymbol{E} = -\,E_y\,[2\sin(\omega\,t - \pi/4)\,\hat{\boldsymbol{x}} + \sin(\omega\,t + \pi/4)\,\hat{\boldsymbol{y}}]$
(e) $\boldsymbol{E} = -\,E_y\,[2\sin(\omega\,t)\,\hat{\boldsymbol{x}} + \sin(\omega\,t + \pi/4)\,\hat{\boldsymbol{y}}]$.
I'll place all the field vectors for each case on the same diagram, showing the evolution over one complete cycle. For all the curves, the labels indicate the values of $\omega\,t$. For the linear polarization, the dots mark the end point of $\boldsymbol{E}$ at the indicated values of $\omega\,t$



Chapter 5 Superposition of Waves

5-1. (a) E_1 is a function of $u_1 = k\,x - \omega\,t = (3/\mathrm{m})\,x - (4/\mathrm{s})\,t$. The velocity component of the wave can be deter mined by setting $u_1 = \text{constant}$ so that,

$$\frac{du_1}{dt} = 0 = k\,\frac{dx}{dt} - \omega \Rightarrow v_x = \frac{dx}{dt} = \omega/k = (4/3)\ \mathrm{m/s}$$

Similarly, E_2 is a function of $u_2 = k\,x + \omega\,t = (3/\mathrm{m})\,x + (4/\mathrm{s})\,t$ and so,

$$\frac{du_2}{dt} = 0 = k\,\frac{dx}{dt} + \omega \Rightarrow v_x = \frac{dx}{dt} = -\,\omega/k = -\,(4/3)\ \mathrm{m/s}$$

The waveforms have the same speed and are oppositely directed.
(b),(c) For notational convenience I will write $k = 3/\mathrm{m}$ and $\omega = 3/\mathrm{s}$. The requirement is,

$$E_1 + E_2 = 0 = \frac{5E_0}{(k\,x - \omega\,t)^2 + 2} - \frac{5E_0}{(k\,x + \omega\,t - 6)^2 + 2}$$
$$(k\,x + \omega\,t - 6)^2 + 2 - (k\,x - \omega\,t)^2 - 2 = 0$$
$$k\,x + \omega\,t - 6)^2 = (k\,x - \omega\,t)^2$$
$$k\,x + \omega\,t - 6 = \pm\,(k\,x - \omega\,t)$$

Choosing the +sign:

$$k\,x + \omega\,t - 6 = k\,x - \omega\,t \Rightarrow 2\,\omega\,t = 6 \Rightarrow t = 3/\omega = 3/(4/\mathrm{s}) = 0.75\ \mathrm{s}$$

This is the answer to (b). Using the solution with the minus sign,

$$k\,x + \omega\,t - 6 = -\,k\,x + \omega\,t \Rightarrow 2\,k\,x = 6 \Rightarrow x = 3/k = 3(3/\mathrm{m}) = 1\ \mathrm{m}$$

This is the answer to (c).

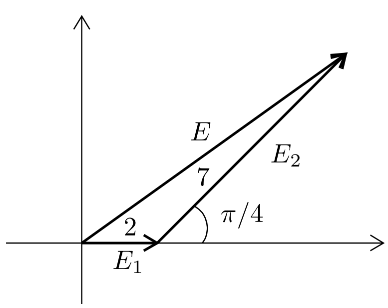
5-2. (a) First put the functions in a common form. Since the cosine function is even, I can write,

$$E_1 = 2\cos(-\,\omega\,t) \text{ and } E_2 = 7\cos(\pi/4 - \omega\,t)$$

The associated complex functions are then,

$$\tilde{E}_1 = 2e^{-i\omega t},\ E_2 = 7\,e^{i\pi/4}e^{-i\omega t}$$

Note that $E_1 = \mathrm{Re}(\tilde{E}_1)$ and $E_2 = \mathrm{Re}(\tilde{E}_2)$. A phasor diagram displaying the complex amplitudes is shown below at the left,



(b) Using Eq. (5 9):
$E_0 = \sqrt{E_{01}^2 + E_{02}^2 + 2\,E_{01}\,E_{02}\cos(\alpha_2 - \alpha_1)}$
$E_0 = \sqrt{2^2 + 7^2 + 2\cdot 2\cdot 7\cos(\pi/4)} = 8.53$
Using Eq. (5 10):
$\tan(\alpha) = \dfrac{E_{01}\sin(\alpha_1) + E_{02}\sin(\alpha_2)}{E_{01}\cos(\alpha_1) + E_{02}\cos(\alpha_2)} = \dfrac{0 + 7/\sqrt{2}}{2 + 7/\sqrt{2}} \Rightarrow \alpha = 0.619\text{in} \approx 0.2\,\pi$
The resultant is then,
$E = E_1 + E_2 = E_0\cos(\alpha - \omega\,t) = 8.53\cos(0.2\,\pi - \omega\,t)$

5-3. The resultant is, $E = 3\cos(\pi/6 - \omega\,t) + 4\cos(\pi/2 - \omega\,t)$. The frequency is $\omega = 2\,\pi/T = 2\,\pi/\mathrm{s}$. Using Eqs. (5 9) and (5 10),

$$E_0 = \sqrt{E_{01}^2 + E_{02}^2 + 2\,E_{01}\,E_{02}\cos(\alpha_2 - \alpha_1)} = \sqrt{3^2 + 4^2 + 2\cdot 3\cdot 4\cos(\pi/2 - \pi/6)} = 6.08$$
$$\tan\alpha = \frac{E_{01}\sin(\alpha_1) + E_{02}\sin(\alpha_2)}{E_{01}\cos(\alpha_1) + E_{02}\cos(\alpha_2)} = \frac{3\sin(\pi/6) + 4\sin(\pi/2)}{3\cos(\pi/6) + 4\cos(\pi/2)} \Rightarrow \alpha = 0.36\,\pi$$
$$E = 6.08\cos(0.36\,\pi - (2\,\pi/\mathrm{s})\,t)$$

5-4. One could proceed directly by mimicking the development for the cosine waves leading to Eqs. (5 9) and (5 10). I choose to first convert the given fields to the cosine form and then using those equations. That is,

$$y_1 = 5\sin(\omega t + \pi/2) = 5\cos(\omega t) = 5\cos(0 - \omega t)$$
$$y_2 = 7\sin(\omega t + \pi/3) = 7\cos(\omega t + \pi/3 - \pi/2) = 7\cos(\omega t - \pi/6) = 7\cos(\pi/6 - \omega t)$$

Them using Eqs. (5 9) and (5 10):

$$y_0 = \sqrt{5^2 + 7^2 + 2\cdot 5\cdot 7\cos(\pi/6)} = 11.6$$
$$\tan\alpha = \frac{0 + 7\sin(\pi/6)}{5 + 7\cos(\pi/6)} = \alpha = 0.098\,\pi$$
$$y = 11.6\cos(0.098\,\pi - \omega t) = 11.6\cos(\omega t - 0.098\,\pi) = 11.6\sin(\omega t - 0.098\,\pi + \pi/2)$$
$$y = 11.6\sin(\omega t + 0.402\,\pi)$$

5-5. The frequency of all of the waves is $\omega = 2\pi/T = \pi/\mathrm{s}$. Putting all of the waves into cosine form gives,

$$E_1 = \cos(\pi/18 - \pi/2 - \omega t) = \cos(-4\pi/9 - \omega t)$$
$$E_2 = 3\cos(5\pi/9 - \omega t)$$
$$E_3 = 2\cos(\pi/6 - \pi/2 - \omega t) = 2\cos(-\pi/3 - \omega t)$$

Using Eq. (5 13) the amplitude of the resultant wave can be found as

$$E_0^2 = [\sin(4\pi/9) + 3\sin(5\pi/9) + 2\sin(-\pi/3)]^2 + [\cos(4\pi/9) + 3\cos(5\pi/9) + 2\cos(-\pi/3)]^2$$
$$E_0 = 0.695$$

Using Eq. (5 12),

$$\tan\alpha = \frac{\sin(-4\pi/9) + 3\sin(5\pi/9) + 2\sin(-\pi/3)}{\cos(-4\pi/9) + 3\cos(5\pi/9) + 2\cos(-\pi/3)} \Rightarrow \alpha = 0.349$$

So the resultant wave has the form, $E = 0.649\cos(0.349 - (\pi/\mathrm{s})\,t)$

5-6. (a) If they are in phase the amplitudes add, $E_R = (100)\,(0.02\,\mathrm{V/m}) = 2\,\mathrm{V/m}$
(b) If the sources have random phase differences, the irradiances add. Since the irradiances add, the square of the amplitudes add. Then

$$E_R^2 = (100)\,E_{01}^2 \Rightarrow E_R = 10\,(0.02\,\mathrm{V/m}) = 0.2\,\mathrm{V/m}$$

5-7. At the indicated position,

$$\psi_1 = A_1\cos(8\pi/3 - \omega t),\quad A_1 = 4\,\mathrm{cm}, \omega = 20/\mathrm{s}$$
$$\psi_2 = A_2\cos(3\pi/2 - \omega t),\quad A_2 = 2\,\mathrm{cm}, \omega = 20/\mathrm{s}$$

Using Eqs (5 9) and (5 10),

$$A = \sqrt{A_1^2 + A_2^2 + 2\,A_1\,A_2\cos(\alpha_2 - \alpha_1)} = \sqrt{20 + 16\cos(3\pi/2 - 8\pi/3)}\ \mathrm{cm} = 2.48\,\mathrm{cm}$$
$$\tan\alpha = \frac{4\sin(8\pi/3) + 2\sin(3\pi/2)}{4\cos(8\pi/3) + 2\sin(3\pi/2)} \Rightarrow \alpha = 2.51$$
$$\psi_R = (2.48\,\mathrm{cm})\cos(2.51 - (20/\mathrm{s})\,t)$$

5-8. Now, $v_g = v_p - \lambda\,(dv_p/d\lambda)$, with $v_p = c/n$.
(a) $\lambda = c/\nu = \frac{2\pi c}{\omega}$. Further, $\frac{dv_p}{d\lambda} = \frac{d(c/n)}{d(2\pi c/\omega)} = \frac{-c\,dn/n^2}{-2\pi c\,d\omega/\omega^2} = \frac{\omega^2}{2\pi n^2}\frac{dn}{d\omega}$. Therefore,

$$v_g = v_p - \frac{2\pi c}{\omega}\frac{\omega^2}{2\pi n^2}\frac{dn}{d\omega} = v_p\left(1 - \frac{\omega}{n}\frac{dn}{d\omega}\right)$$

(b) For normal dispersion, $dn/d\lambda < 0$ so that $dn/d\omega > 0$ and $v_g > v_p$.

5-9. It is given that $\Delta \equiv \dfrac{n_D - n_C}{n_0 - 1} = \dfrac{1}{30}$. Then,

$$v_g = v_p[1 + (\lambda/n)\, dn/d\lambda] \approx v_p\left[1 + \frac{\lambda_D}{n_D}\frac{\Delta n}{\Delta\lambda}\right]$$

Here, $-\Delta n = n_C - n_F = \Delta\,(n_0 - 1) = \dfrac{1}{30}(1.50 - 1) = 1/60$ and $\Delta\lambda = \lambda_C - \lambda_F = (656.3 - 486.1)\text{nm} = 170.2\,\text{nm}$ Then,

$$v_g = \frac{c}{n_D}\left[1 + \frac{\lambda_D}{n_D}\frac{\Delta n}{\Delta\lambda}\right] = \frac{c}{1.50}\left[1 - \frac{589}{1.5}\frac{1}{60 \cdot 170.2}\right] = c/1.56$$

5-10. Now, $n = A + B/\lambda^2$ and $v_g = v_p\,[1 + (\lambda/n)\, dn/d\lambda]$. Therefore,

$$\frac{dn}{d\lambda} = -2\,B/\lambda^3 = -\frac{2\,(2.5 \times 10^6)}{(5000)^3}\,\text{Å}^{-1} = -4 \times 10^{-5}\,\text{Å}^{-1}$$

Then,

$$v_p = \frac{c}{n} = \frac{c}{1.50} \text{ and } v_g = \frac{c}{1.50}\left[1 + \frac{5000}{1.5}\left(-4 \times 10^{-5}\right)\right] = \frac{c}{1.73}$$

5-11. (a) We know that $K = n^2$ and $v_g = v_p(1 + (\lambda/n)\, dn/d\lambda)$. So, $v_p = c/n = c/\sqrt{K}$. Then

$$\frac{dn}{n} = \frac{d(\sqrt{K})}{\sqrt{K}} = \frac{1}{2}\frac{K^{-1/2}}{K^{1/2}} = \frac{dK}{2\,K}$$

Also, $v = \nu\,\lambda$ so that $dv = \lambda\, d\nu + \nu d\lambda$. At a particular velocity, $\lambda\, d\nu = -\,\nu\, d\lambda$, so that

$$\frac{\lambda}{d\lambda} = -\frac{\nu}{d\nu} = -\frac{\omega}{d\omega}$$

Thus,

$$v_g = \frac{c}{\sqrt{K}}\left[1 + \frac{dK}{\sqrt{K}}\left(-\frac{\omega}{d\omega}\right)\right] = \frac{c}{\sqrt{K}}\left(1 - \frac{\omega}{2K}\frac{dK}{d\omega}\right)$$

(b) Given $K = 1 + A/\left(\omega_0^2 - \omega^2\right)$ where the second term is small compared to the first. Then,

$$\frac{dK}{d\omega} = \frac{-A\,(-2\,\omega)}{(\omega_0^2 - \omega^2)^2} = \frac{2\,A\,\omega}{(\omega_0^2 - \omega^2)^2}$$

Using the result of (a),

$$v_g = \frac{c}{\sqrt{K}}\left(1 - \frac{\omega}{2\,K}\frac{2\,A\,\omega}{(\omega_0^2 - \omega^2)^2}\right) = \frac{c}{\sqrt{K}}\left(1 - \frac{A\,\omega^2}{(\omega_0^2 - \omega^2)^2}\right)$$

Now in the second term in the parentheses the difference of K from 1 can be ignored to lowest order. How ever this difference can not be neglected outright in the pref actor $c/\sqrt{K}$. In fact, a Taylor series expansion gives,

$$K^{-1/2} \approx 1 + \frac{1}{2}\frac{A}{\left(\omega_0^2 - \omega^2\right)}$$

so that keeping only the lowest order corrections,

$$v_g = c\left(1 + \frac{1}{2}\frac{A}{\left(\omega_0^2 - \omega^2\right)} - \frac{A\,\omega^2}{(\omega_0^2 - \omega^2)^2}\right)$$

Finally as is typically the case of interest, (and if K is close to 1), $\omega^2 \gg |\omega_0^2 - \omega^2|$ so that the middle term can be dropped giving,

$$v_g = c\left(1 - \frac{A\,\omega^2}{(\omega_0^2 - \omega^2)^2}\right)$$

5-12. (a) According to Eq. 5 38, $v_g = \dfrac{d\omega}{dk} = v_p + k\,\dfrac{dv_p}{dk}$. Then with $k = 2\,\pi/\lambda$ and $dk = -\,(2\,\pi/\lambda^2)d\lambda$. Thus,

$$v_g = v_p + \frac{2\,\pi}{\lambda}\,\frac{d\,v_p}{(-2\,\pi)}\,\frac{\lambda^2}{d\lambda} = v_p - \lambda\frac{dv_p}{d\lambda}$$

(b) $v_g = v_p - \lambda\,\dfrac{dv_p}{d\lambda} = (A + B\,\lambda) - \lambda\,B = A = \text{constant}$. Since the group velocity is independent of wavelength, any waveform constructed from harmonic waves of different wavelengths is transmitted by the medium without a change in shape.

5-13. As in problem 5 12a, $v_g = v_p - \lambda\dfrac{dv_p}{d\lambda}$. Then,

Long λ:

$$\frac{dv_p}{d\lambda} = \frac{d}{d\lambda}\sqrt{\frac{g}{2\,\pi\,\lambda}} = \frac{1}{2}\sqrt{\frac{g}{2\,\pi\,\lambda}} = \frac{1}{2}\,v_p$$

Short λ:

$$\frac{dv_p}{d\lambda} = \frac{d}{d\lambda}\left(\frac{2\,\pi\,T}{\lambda\,\rho}\right)^{1/2} = -\,\frac{1}{2\lambda}\left(\frac{2\,\pi\,T}{\lambda\,\rho}\right)^{1/2}$$

So,

$$v_g = \left(\frac{2\,\pi\,T}{\lambda\,\rho}\right)^{1/2} + \frac{1}{2}\left(\frac{2\,\pi\,T}{\lambda\,\rho}\right)^{1/2} = \frac{3}{2}\left(\frac{2\,\pi\,T}{\lambda\,\rho}\right)^{1/2} = \frac{3}{2}\,v_p$$

5-14. The frequency received by the mirror is $\nu_1' = \nu_0/(1 + v/c)$. Since $\lambda'/\lambda = \nu/\nu' = 1 - v/c$ and $v < 0$ due to recession. The reflected beam is also due to a receding source so the reflected frequency is

$$\nu_2' = \frac{\nu_1'}{1 + v/c}$$

Thus,

$$\nu_2' = \frac{\nu_0}{1 + v/c}\,\frac{1}{1 + v/c} = \frac{\nu_0}{(1 + v/c)^2} = \nu_0(1 + v/c)^{-2}$$

For $v/c \ll 1$, a Taylor series expansion is appropriate, so that $\nu_2' \approx \nu_0\,(1 - 2v/c)$ and the beat frequency is

$$\nu_2' - \nu_0 = 2(v/c)\,\nu_0$$

5-15. If the wave is inverted upon reflection, the standing wave will be of the form given in Eq. (5 23),

$$y_S = 2\,y_0 \sin(k\,x)\cos(\omega\,t)$$

From the given traveling wave information, $k = 4/\text{cm}$, $\omega = 2\,\pi/T$, and $y_0 = 7$ cm. So, the standing wave has the form,

$$y = (14\,\text{cm})\sin[(4/\text{m})\,x]\cos[(2\,\pi/T)\,t]$$

This wave has an amplitude of 14 cm, a wavelength of $\lambda = 2\,\pi/k = 1.57\,\text{cm}$, in the sense that it is a standing wave, the net speed of energy transmission is zero. The length of one loop is $\lambda/2 = 0.785$ cm. The period is T.

5-16. The standing wave has the form,

$$y = 2\,A\sin(k\,x)\cos(\omega\,t) = (3\,\text{cm})\sin\!\left(\frac{\pi\,x}{10\,\text{cm}}\right)\cos\!\left(\frac{50\,\pi}{\text{s}}\,t\right)$$

(a) The amplitude of each of the oppositely directed traveling waves that add to the standing wave is $A = 1.5\,\text{cm}$. One of these component waves is traveling in the $+x$ direction and the other is traveling in the $-x$ direction. The wave speed of the component waves is $v = \omega/k = (50\,\pi/(\pi/10))$ cm/s $= 500$ cm/s. The wavelength is $\lambda = 2\,\pi/k = (2\,\pi)/(\pi/10)\text{cm} = 20\,\text{m}$. The frequency is $\nu = \omega/2\,\pi = 25\,\text{Hz}$.
(b) The internodal distance is $\lambda/2 = 10\,\text{cm}$
(c) The displacement, velocity component, and acceleration component at $x = 5\,\text{cm}$ and $t = 0.22\,\text{s}$ are,

$$y(x = 5\,\text{cm}, t = 0.22\,\text{s}) = -\,3\,\text{cm}$$
$$v_y = \frac{dy}{dt} = -\,2\,A\,\omega\sin(k\,x)\sin(\omega\,t)$$
$$v_y(x = 5\,\text{cm}, t = 0.22\,\text{s}) = 0$$
$$a_y = \frac{dv_y}{dt} = -\,\omega^2\,2\,A\sin(k\,x)\cos(\omega\,t) = -\,\omega^2\,y$$
$$a_y(x = 5\,\text{cm}, t = 0.22\,\text{s}) = 7.40\times 10^4\,\text{cm/s}^2$$

5-17. In the complex representation,

$$E_1 = E_0\sin(\omega\,t + k\,x) = \text{Im}\!\left[E_0\,e^{i(\omega t + kx)}\right]$$
$$E_2 = E_0\sin(\omega\,t - k\,x - \varphi_R) = \text{Im}\!\left[E_0 e^{i(\omega t - kx - \varphi_R)}\right]$$
$$E_R = E_1 + E_2 = E_0\,\text{Im}\!\left[e^{i(\omega t + kx)} + e^{i(\omega t - kx - \varphi_R)}\right]$$
$$E_R = E_0\,\text{Im}\!\left[e^{i(\omega t - \varphi_R/2)}e^{i(kx + \varphi_R/2)} + e^{i(\omega t - \varphi_R/2)}e^{-i(kx + \varphi_R/2)}\right]$$
$$E_R = E_0\,\text{Im}\!\left\{e^{i(\omega t - \varphi_R/2)}\left[e^{i(kx + \varphi_R/2)} + e^{-i(kx + \varphi_R/2)}\right]\right\}$$
$$E_R = E_0\,\text{Im}\!\left[e^{i(\omega t - \varphi_R/2)}\,2\cos(k\,x + \varphi_R/2)\right]$$
$$E_R = 2\,E_0\cos(k\,x + \varphi_R/2)\,\text{Im}\!\left[e^{i(\omega t - \varphi_R/2)}\right]$$
$$E_R = 2\,E_0\cos(k\,x + \varphi_R/2)\;\sin(\omega\,t - \varphi_R/2)$$

5-18. The separation between cavity modes is $\Delta\nu = \nu_{m+1} - \nu_m = (m+1)\,c/2d - m\,c/2d = c/2d$. That is,

$$\Delta\nu = \left(3\times 10^8\,\text{m/s}\right)/(2\cdot 1\,\text{m}) = 0.15\,\text{GHz}$$

Then the number of lasing modes would be the ration of the frequency range for lasing to the separation of cavity modes:

$$\#\text{of lasing modes} = \frac{6\,\text{GHz}}{0.15\,\text{GHz}} = 40$$

Chapter 6 Properties of Lasers

6-1. The allowed energies of the electron in the hydrogen atom are $E_n = -(13.6\text{ eV})/n^2$, $n = 1, 2, 3, \ldots$ The energies required for this problem are:

$$E_1 = -13.6\,\text{eV} \quad E_2 = -3.40\,\text{eV} \quad E_3 = -1.51\,\text{eV} \quad E_4 = -0.850\,\text{eV} \quad E_5 = -0.544\,\text{eV}$$

The wavelength associated with a transition is

$$\lambda_{mn} = \frac{h\,c}{E_m - E_n} = \frac{1240\text{ eV}\cdot\text{nm}}{E_m - E_n}$$

(a) Lyman Series:

$$\lambda_{21} = \frac{1240}{-3.4-(-13.6)}\text{nm} = 122\,\text{nm} \qquad \lambda_{31} = \frac{1240}{-1.51-(-13.6)}\text{nm} = 103\,\text{nm} \qquad \lambda_{41} = 97.3\,\text{nm}$$

These are in the ultraviolet (shorter than visible).

(b) Balmer Series:

$$\lambda_{32} = \frac{1240}{-1.51-(-3.4)}\text{nm} = 656\,\text{nm} \qquad \lambda_{42} = \frac{1240}{-0.85-(-3.4)}\text{nm} = 486\,\text{nm} \qquad \lambda_{52} = 434\,\text{nm}$$

These are in the visible range.

6.2. (a) From the solution to problem 6 1 it is evident that an energy of 5 eV does not correspond to the energy difference between the ground state of hydrogen and an excited state of hydrogen. Therefore, since the photon must give up its energy in one lump, the photon will not be absorbed by the hydrogen atom. (Neglecting very unlikely nonlinear processes, of course.)

(b) An electron in hydrogen is ionized (leaves the proton) when its energy exceeds zero. Thus the energy of the photon must exceed $E_{\text{final}} - E_{\text{initial}} = 0 - (-13.6\,\text{eV}) = 13.6$ eV. Any energy greater than this is possible since the free electron can have any energy. The corresponding wavelength range is,

$$\lambda < \frac{h\,c}{13.6\,\text{eV}} = \frac{1240\,\text{eV}\cdot\text{nm}}{13.6\,\text{eV}} = 91.2\,\text{nm}$$

(c). In this case the photon energy must exceed $0 - E_2 = 0 - (-3.4\,\text{eV}) = 3.4\,\text{eV}$. The wavelength range is then,

$$\lambda < \frac{h\,c}{3.4\text{eV}} = \frac{1240\,\text{eV}\cdot\text{nm}}{3.4\,\text{eV}} = 365\,\text{nm}$$

6-3. (a) The rotational inertia is

$$I = 2\,m_H r^2 = 2(1.67\times10^{-27})\left(0.074\times10^{-9}/2)^2\,\text{kg}\cdot\text{m}^2 = 4.6\times10^{-48}\,\text{kg}\cdot\text{m}^2$$

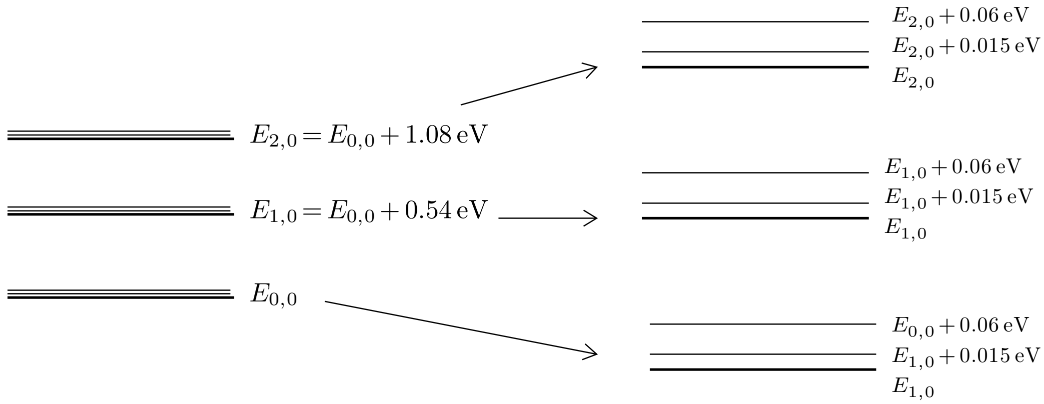
(b) $\Delta E = E_1^{\text{rot}} - E_0^{\text{rot}} = \hbar^2/I = (6.626\times10^{-34}/2\,\pi)^2/(4.6\times10^{-48})\,\text{J} = 2.4\times10^{-21}\,\text{J} = 0.015\,\text{eV}$

(c) $P_1/P_0 = e^{-(\Delta E)/k_B T} = e^{-0.015/(8.62\times10^{-5}\cdot293)} = 0.55$

6-4. (a) $\Delta E = E_1^{\text{vib}} - E_0^{\text{vib}} = h\,f = (6.626\times10^{-34})\left(1.3\times10^{14}\right)\text{J} = 8.6\times10^{-20}\,\text{J} = 0.54\,\text{eV}$

(b) $P_1/P_0 = P_1/P_0 = e^{-(\Delta E)/k_B T} = e^{-0.54/(8.62\times10^{-5}\cdot293)} = e^{-21.4} \approx 5\times10^{-10}$

6-5. The first few states of the energy level diagram in the electronic ground state is shown below. The energy labels are of the form $E_{k,l}$. Associated with each vibrational state there are a number of rotational sub levels. An expanded view of each vibrational state with associated sublevels is shown at the right.



6-6. The energy separation of the states is $\Delta E = h\,c/\lambda = (1240\,\mathrm{eV}\cdot\mathrm{nm})/(632.8\,\mathrm{nm}) = 01.96\,\mathrm{eV}$

Neglecting degeneracies:$N_2/N_1 = P_2/P_1 = e^{-(E_2-E_1)/k_BT} = e^{-1.96/(8.62\times10^{-5}\cdot 300)} \approx e^{-76}$.

6-7. The differential equation can be integrated as shown below,

$$\frac{dN_2}{dt} = -A_{21}\,N_2 \Rightarrow \frac{dN_2}{N_2} = -A_{21}\,dt$$

$$\int_{N_{20}}^{N_2(t)} \frac{dN_2}{N_2} = -A_{21}\int_0^t dt' \Rightarrow \ln(N_2(t)/N_{20}) = -A_{21}(t-0) = -A_{21}\,t$$

$$\frac{N_2(t)}{N_{20}} = e^{-A_{21}\,t} \Rightarrow N_2(t) = N_{20}\,e^{-A_{21}\,t}$$

At time $t = 1/A_{21}$, $N_2(t) = N_{20}\,e^{-A_{21}(1/A_{21})} = N_{20}\,e^{-1} = N_{20}/e$

6-8. The black body spectral radiance is $M_\lambda = \dfrac{2\,\pi\,h\,c^2}{\lambda^5}\dfrac{1}{e^{hc/\lambda k_BT}-1}$.

This function is maximized when it's denominator is minimized:

$$\frac{d}{d\lambda}\Big[\lambda^5\,(e^{hc/\lambda k_BT}-1)\Big] = 0 \Rightarrow \lambda^5\Big[e^{hc/\lambda k_BT}\,\frac{h\,c}{k_B\,T}\Big(-\frac{1}{\lambda^2}\Big) + \Big(e^{hc/\lambda k_BT}-1\Big)\,5\,\lambda^4\Big] = 0$$

Let $x = h\,c/(\lambda\,k_B\,T)$: $\lambda^4\,[-x\,e^x + 5\,(e^x-1)] = 0 \Rightarrow x\,e^x = 5\,e^x - 1$. This is a transcendental equation in x with solution, $x = 4.999986$. Then,

$$x_{\max} = 4.999986 = \frac{h\,c}{\lambda_{\max}\,k_B\,T} \Rightarrow \lambda_{\max}T = \frac{h\,c}{4.999986\,k_B} = 2.898\times10^{-3}\,\mathrm{m}\cdot\mathrm{K} = 2898\,\mu\mathrm{m}\cdot\mathrm{K}$$

6-9. Let $x = h\,c/(\lambda\,k_B\,T)$. Then $dx = (h\,c/k_B\,T)\,\big(-d\lambda/\lambda^2\big)$ so that, $d\lambda = -\lambda^2\,(k_B\,T/h\,c)\,dx$. Then,

$$M = \int_0^\infty M_\lambda\,d\lambda = \int_0^\infty \frac{2\,\pi\,h\,c^2}{\lambda^5}\frac{1}{e^{hc/\lambda k_BT}-1}\,d\lambda = \frac{2\,\pi\,(k_B\,T)^4}{h^3\,c^2}\int_0^\infty \frac{x^3}{e^x-1} = \frac{2\,\pi\,(k_B\,T)^4}{h^3\,c^2}\frac{\pi^4}{15} = \left(\frac{2\,k_B^4\,\pi^5}{15\,h^3\,c^2}\right)T^4$$

$$M = \left(\frac{2\,k_B^4\,\pi^5}{15\,h^3\,c^2}\right)T^4 \equiv \sigma\,T^4 = \big(5.668\times10^{-8}\big)\,\mathrm{W/m^2\;K^4}$$

6-10. As in the solution to problem (6 9),

$$M_\lambda = \frac{2\pi c^2}{\lambda^5}\frac{1}{e^x - 1}, \text{with } x = \frac{h\,c}{\lambda\,k_B T}$$

Then,

$$M = \int_0^\infty M_\lambda\, d\lambda$$

The change of variable is accomplished by noting that for $\lambda = 0, x = \infty$ and for $\lambda = \infty, x = 0$. Further,

$$dx = \frac{h\,c}{k_B T}\left(-\frac{d\lambda}{\lambda^2}\right) \Rightarrow d\lambda = -\frac{\lambda^2 k_B T}{hc} dx$$

So,

$$M = 2\pi h c^2 \int_\infty^0 \frac{1}{\lambda^5}\frac{1}{e^x - 1}\frac{-\lambda^2 k_B T}{h\,c} dx = 2\pi\; c\,k_B T \int_0^\infty \frac{1}{\lambda^3}\frac{dx}{e^x - 1} = 2\pi\; c\,k_B T \int_0^\infty \left(\frac{x\,k_B T}{h\,c}\right)^3 \frac{dx}{e^x - 1}$$

$$M = \frac{2\pi\,(k_B T)^4}{h^3 c^2}\int_0^\infty \frac{x^3}{e^x - 1}\, dx$$

The definite integral has the value:

$$\int_0^\infty \frac{x^3}{e^x - 1}\, dx = \frac{\pi^4}{15}$$

so that,

$$M = \left(\frac{2k_B^4 \pi^5}{15\,h^3 c^2}\right)^4 T^4 \equiv \sigma T^4, \sigma = 5.668 \times 10^{-8}\frac{\mathrm{W}}{\mathrm{m^2\,K^4}}$$

6-11. The original temperature is determined from the Wien displacement law,

$$\lambda_{\max} T_1 = 2898\;\mu\mathrm{m}\cdot\mathrm{K} \Rightarrow T_1 = \frac{2898\;\mu\mathrm{m}\cdot\mathrm{K}}{\lambda_{\max}} = \frac{2898\;\mu\mathrm{m}\cdot\mathrm{K}}{0.55\;\mu\mathrm{m}} = 5269\,\mathrm{K}$$

For the total radiant exitance to double,

$$M_2 = 2\,M_1 \Rightarrow \sigma (T_2)^4 = 2\sigma T_1^4 \Rightarrow T_2 = 2^{1/4} T_1 = 6266\,\mathrm{K}$$

The new maximum wavelength is,

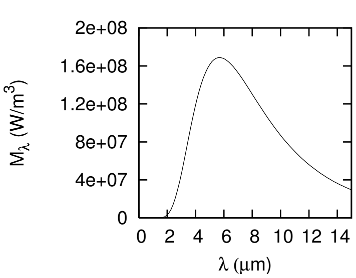
$$\lambda_{\max} = \frac{2898\;\mu\mathrm{m}\cdot\mathrm{K}}{T_2} = \frac{2898\;\mu\mathrm{m}\cdot\mathrm{K}}{6266\,\mathrm{K}} = 0.462\;\mu\mathrm{m} = 462\,\mathrm{nm}$$

6-12. The gray body at T_2 must match the the black body at the lower temperature, T_1. That is,

$$(M_{\mathrm{GB}})_{T_2} = (M_{\mathrm{BB}})_{T_1} \Rightarrow (\varepsilon M_{\mathrm{BB}}\Big)_{T_2} = (M_{\mathrm{BB}})_{T_1}$$

$$\varepsilon\,\sigma T_2^4 = \sigma T_1^4 \Rightarrow T_2^4 = T_1^4/\varepsilon = (5000\,\mathrm{K})^4/0.45 \Rightarrow T_2 = 6105\,\mathrm{K}$$

6-13. On the absolute temperature scale $451°\mathrm{F} = 506\,\mathrm{K}$. Then $M_\lambda = \varepsilon\frac{2\pi h c^2}{\lambda^5}\frac{1}{e^{hc/\lambda k_B T} - 1}$. The plot is as shown.



6-14. Equation (6 12) indicates that A_{21}/B_{21} is inversely proportional to the cube of the wavelength. This ratio is the ratio of spontaneous emission to stimulated emission. Stimulated emission contributes to the laser gain while spontaneous emission (essentially) does not. Thus, all else being equal, the gain on a transition resonant with short wavelength radiation is less than that of a transition resonant with long wavelength radiation.

6-15. The frequency bandwidth $\Delta\nu = \nu_2 - \nu_1$ is related to the wavelength range $\Delta\lambda = \lambda_2 - \lambda_1$ by,

$$\Delta\nu = \frac{c}{\lambda_2} - \frac{c}{\lambda_1} = c\,\frac{\lambda_1 - \lambda_2}{\lambda_2\,\lambda_1} \approx -\,c\frac{\Delta\lambda}{\lambda^2} \Rightarrow |\Delta\lambda| = \frac{\lambda^2|\Delta\nu|}{c}$$

From Table 6 1, $\lambda = 1.06 \times 10^{-6}\,\text{m}$, so, $|\Delta\lambda| = \dfrac{\left(1.06 \times 10^{-6}\right)^2\left(1.2 \cdot 10^{11}\right)}{\left(3 \cdot 10^{8}\right)}\,\text{m} = 4.5 \times 10^{-10}\,\text{m} = 0.45\,\text{nm}$

6-16. (a) $t_c \approx 1/\Delta\nu = 1/(10^5\,\text{Hz}) = 10^{-5}\,\text{s} \quad L_c = c\,t_c = \left(3 \cdot 10^8\,\text{m/s}\right)\left(10^{-5}\,\text{s}\right) = 3000\ \text{m}$

(b) $t_c \approx 1/\Delta\nu = 1/(2 \cdot 10^9\,\text{Hz}) = 5 \cdot 10^{-10}\,\text{s} \quad L_c = c\,t_c = \left(3 \cdot 10^8\,\text{m/s}\right)\left(10^{-5}\,\text{s}\right) = 0.15\,\text{m} = 15\,\text{cm}$

6-17. The separation between cavity modes is $\Delta\nu = \nu_{m+1} - \nu_m = (m+1)\,c/2d - m\,c/2d = c/2d$. That is,

$$\Delta\nu = \left(3 \times 10^8\,\text{m/s}\right)/(2 \cdot 0.5\,\text{m}) = 0.3\,\text{GHz}$$

Then the number of lasing modes would be the ration of the frequency range for lasing to the separation of cavity modes:

$$\#\text{of lasing modes} = \frac{2\,\text{GHz}}{0.3\,\text{GHz}} = 6.67 \Rightarrow 6$$

6-18. (a) The half angle beam spread is,

$$\theta = \frac{\lambda}{\pi\,w_0} = \frac{6.33 \cdot 10^{-7}\,\text{m}}{\pi \cdot 5 \cdot 10^{-4}\,\text{m}} = 4.0 \times 10^{-4}\,\text{rad} = 0.023°$$

(b) The diameter is about,

$$D = 2\,d\,\theta = 2\,(1000\,\text{m})\left(4.0 \times 10^{-4}\right) = 0.8\,\text{m} = 80\,\text{cm}$$

6-19. Assuming that that irradiance is constant over the spot:

$$I_{\text{laser}} = \frac{P_{\text{laser}}}{A_{\text{laser}}} = \frac{P_{\text{laser}}}{\pi D^2/4} = I_{\text{sun}}$$

$$D = \sqrt{\frac{4\,P_{\text{laser}}}{\pi\,I_{\text{sun}}}} = \sqrt{\frac{4 \cdot 0.01}{\pi\,1000}}\,\text{m} = 0.0036\,\text{m} = 3.6\text{mm}$$

6-20. (a) The energy difference ΔE between the levels must have near the same energy as the pump photons. That is,

$$\Delta E = h\,\nu = \frac{h\,c}{\lambda} \Rightarrow \lambda = \frac{h\,c}{\Delta E} = \frac{1240\,\mathrm{eV\cdot nm}}{\Delta E}$$

Then, $\lambda_{1.53} = (1240/1.53)\,\mathrm{nm} = 810$ nm. Similarly, $\lambda_{1.653} = 750$ nm, $\lambda_{2.119} = 585\,\mathrm{nm}$, and $\lambda_{2.361} = 525\,\mathrm{nm}$.
(b) The quantum efficiency η_Q is,

$$\eta_Q = \frac{\text{Energy of an output photon}}{\text{Energy of a pump event}} = \frac{h\,c/\lambda_{\mathrm{out}}}{h\,c/\lambda_p} = \frac{\lambda_p}{\lambda_{\mathrm{out}}}$$

So using the results from part (a) for the various transitions:

$$\eta_{Q,1.53} = \frac{810}{1064} = 0.76,\ \eta_{Q,1.653} = \frac{750}{1064} = 0.70,\ \eta_{Q,2.119} = \frac{585}{1064} = 0.55, \text{ and } \eta_{Q,2.361} = \frac{525}{1064} = 0.49.$$

6-21. (a) The laser output beam would have a power of $P_{\mathrm{output}} = 0.8 \times 0.3 \times 0.7 \times 0.15 \times 0.5 \times 2500\,\mathrm{W} = 31.5\,\mathrm{W}$.

(b) The overall efficiency is $\eta = \dfrac{P_{\mathrm{output}}}{P_{\mathrm{pump}}} = \dfrac{31.5}{2500} = 0.0126$.

6-22. The diode laser beam is generated in a very small region. The radius w_0 of the beam waist of the diode laser beam must therefore by very small (compared to other types of lasers). As indicated by Equation (6 16), the divergence angle is inversely proportional proportional to w_0 and so will be larger for diode lasers than for other types of lasers.

6-23. As the irradiance in the laser cavity increases, the population inversion decreases. Given that there is a positive population inversion, the population densities of the upper and lower lasing levels satisfy the inequality $N_{\mathrm{upper}} > N_{\mathrm{lower}}$. The magnitude of the net rate of depletion of the population inversion due to stimulated processes is proportional to $I\,(N_{\mathrm{upper}} - N_{\mathrm{lower}})$. Thus increased irradiance causes a larger rate of depletion of the population inversion, leading to a lessened inversion.

6-24. The diode laser frequency can be tuned to match one of the pump transitions (see problem 6 20) of the Nd:YAG gain medium. Thus nearly all of the diode laser pump energy can be used to pump the laser. Much of the arc lamp energy (see Figures (6 8) and (6 9)) is not resonant with a pump transition and so is wasted. This wasted pump energy lowers the overall efficiency of the laser system.

Chapter 7 Interference of Light

7-1. (a) $I_1 = (\varepsilon_0\, c/2)\, E_{01}^2 = \left(8.85\times 10^{-12}\cdot 3\times 10^8/2\right)\left(3000\,\right)^2\,\mathrm{W/m^2} = (0.0013275)\left(3000\right)^2 = 11,950\,\mathrm{W/m^2}$
$I_2 = (\varepsilon_0\, c/2)\, E_{02}^2 = (0.0013275)\left(4000\right)^2 = 21,240\,\mathrm{W/m^2}$

(b) $I_{12} = 2\sqrt{I_1\, I_2}\cos\delta = 2\sqrt{I_1\, I_2}\cos\left[\pi/3 - (\pi/5 - \pi/6)\right] = 18,720\,\mathrm{W/m^2}$

(c) $I = I_1 + I_2 + I_{12} = 51,900\,\mathrm{W/m^2}$

(d) The visibility is,

$$\text{visibility} = \frac{I_{\max} - I_{\min}}{I_{\max} + I_{\min}} = \frac{I_1 + I_2 + 2\sqrt{I_1\, I_2} - (I_1 + I_2 - 2\sqrt{I_1\, I_2})}{I_1 + I_2 + 2\sqrt{I_1\, I_2} + I_1 + I_2 - 2\sqrt{I_1\, I_2})} = \frac{4\sqrt{I_1\, I_2}}{2\,(I_1 + I_2)} = \frac{2\sqrt{I_1\, I_2}}{I_1 + I_2} = 0.96$$

7-2. As in the solution to problem 7 1,

$$\text{visibility} = \frac{2\sqrt{I_1\, I_2}}{I_1 + I_2}$$

Since irradiance is proportional to the square of the field amplitude, for parallel fields, this relation can be rewritten as,

$$\text{visibility} = \frac{2\, E_{01}\, E_{01}}{E_{01}^2 + E_{02}^2} = \frac{2\,(1.6)\,(2.8)}{1.6^2 + 1.8^2} = 0.86$$

For perpendicular fields, $I_{12} \propto \boldsymbol{E}_1\cdot\boldsymbol{E}_2 = 0$ so the visibility is zero.

7-3. As in the solution to problem 7 2,

$$\text{visibility} \equiv \mathcal{V} = \frac{2\sqrt{I_1\, I_2}}{I_1 + I_2} = \frac{2\, E_{01}\, E_{02}}{E_{01}^2 + E_{02}^2}$$

Then,

$$\mathcal{V} = \frac{2\, E_{01}\, E_{02}}{E_{01}^2 + E_{02}^2}\frac{1/E_{02}^2}{1/E_{02}^2} = \frac{2\,R}{R^2 + 1},\ \text{with}\ R = E_{01}/E_{02}$$

For $R = 2$,

$$\mathcal{V} = 0.8$$

If $\mathcal{V} = 0.5$,

$$\mathcal{V}\,(1 + R^2) = 2\,R \Rightarrow 0.5 + 0.5\,R^2 = 2\,R$$

$$R^2 - 4\,R + 1 = 0 \Rightarrow R = \frac{4\pm\sqrt{16 - 4}}{2} = 2\pm\sqrt{3} = 3.732, 1/3.732$$

7-4. (a) $\text{visibility} = \dfrac{I_{\max} - I_{\min}}{I_{\max} + I_{\min}} = \dfrac{I_1 + I_2 + 2\sqrt{I_1\, I_2} - (I_1 + I_2 - 2\sqrt{I_1\, I_2})}{I_1 + I_2 + 2\sqrt{I_1\, I_2} + I_1 + I_2 - 2\sqrt{I_1\, I_2})} = \dfrac{4\sqrt{I_1\, I_2}}{2\,(I_1 + I_2)} = \dfrac{2\sqrt{I_1\, I_2}}{I_1 + I_2}$

So if $I_1 = N\, I_2$,

$$\text{visibility} \equiv \mathcal{V} = \frac{2\sqrt{N}\; I_2}{(N + 1)\, I_2} = \frac{2\sqrt{N}}{N + 1}$$

(b) Solving the above relation for N,

$$N = \left[\frac{\sqrt{1 - \mathcal{V}^2} + 1}{\mathcal{V}}\right]$$

For $\mathcal{V} = 0.96, N = 1.78$. For $\mathcal{V} = 0.90, N = 2.55$. For $\mathcal{V} = 0.8, N = 4$. For $\mathcal{V} = 0.5, N = 13.9$

7-5. See Figure 7 24 in the test. Using $m\,\lambda = a\sin\theta = a\,y/L$ leads to $\Delta y \equiv y_{m+1} - y_m = \dfrac{\lambda\,L}{m}$. So the fringe sepa ration is,

$$\Delta y = \frac{546.1\times10^{-7}\cdot 100}{0.2}\,\text{cm} = 0.0273\,\text{cm}$$

The irradiance on the screen relative to $y=0$ at a maximum is given by,

$$I = 4\,I_0\cos^2\!\left[\frac{\pi\,a\,y}{\lambda\,L}\right] = 4\,I_0\cos^2\!\left[\frac{\pi\,0.2}{546.1\times10^{-7}\cdot100}y\right] = 4\,I_0\cos^2[(115/\text{cm})\,y]$$

To take into account phase changes on reflection, and using $y=0$ to be the plane of the mirror surface where $I=0$, one can use the sin function instead,

$$I = 4\,I_0\sin^2[(115/\text{cm})\,y]$$

7-6. Let $\lambda_1 = 436\,\text{nm}$ and the unknown wavelength be λ_2. At the position of overlap,

$$(3+1/2)\,\lambda_1 = a\sin\theta = 3\,\lambda_2 \Rightarrow \lambda_2 = (3.5/3)\,\lambda_1 = (7/6)\,(436\,\text{nm}) = 508.7\ \text{nm}$$

7-7. See Figure 7 25 in the text. The position of the m^{th} minima relative to the central maximum is given by,

$$(m-1/2)\,\lambda = a\sin\theta = a\,y_m/L \Rightarrow y_m = L\,(m-1/2)\lambda/a$$

The distance on the screen between the fifth minima on either side of the central maximum is

$$\Delta y \equiv y_5 - y_{-5} = [4.5-(-4.5)]\,\text{sL}\lambda/a = 9\ L\,\lambda/a$$

$$\lambda = \frac{\Delta y\,a}{9\,L} = \frac{(3.473)\,(0.02\,)}{9\,(150)}\,\text{cm} = 5.15\times10^{-5}\,\text{cm} = 514\,\text{nm}$$

7-8. The set up is like that of Figure 7 3 in the text with $L=7$ m and $a=1$ mm. The single slit is used to render the the quasi monochromatic light source more coherent. If a laser is used, the single silt is not needed. The wavelength of the light can be determined using Eq. (7 23),

$$y_{m+1} - y_m \equiv \Delta y = \lambda\,L/a = \lambda = \frac{\Delta y\,a}{L} = \frac{(0.0056)\,(0.001)}{7}\,\text{m} = 8.0\times10^{-7}\,\text{m} = 800\,\text{nm}$$

7-9. The fringe separation is,

$$y_{m+1} - y_m \equiv \Delta y = \lambda\,L/a$$

(a) So the slit to screen distance should be,

$$L = \frac{\Delta y\,a}{\lambda} = \frac{(0.001)\,(0.0005)}{6\times10^{-7}}\,\text{m} = 0.833\,\text{m}$$

(b) The optical path difference can be written in terms of wavelengths as $\Delta = m\,\lambda$. The with and without the plate of thickness t,

$$\Delta_2 - \Delta_1 = \Delta m\,\lambda \Rightarrow \Delta m = \frac{n\,t - t}{\lambda} = (n-1)\frac{t}{\lambda} = (1.5-1)\frac{10^{-4}}{6\times10^{-7}} = 83.3\,\text{fringes}$$

(c) $I = 4\,I_0\cos^2\!\left(\dfrac{\pi\,\Delta}{\lambda}\right)$ where $\Delta = a\,y/L$. At $\Delta = 0, I = I_{\max} = 4\,I_0$. Then for $I = 2\,I_0 = I_{\max}/2$:

$$2\,I_0 = I = 4\,I_0\cos^2\!\left(\frac{\pi\,\Delta}{\lambda}\right) \Rightarrow \Delta = \lambda/4 = 150\,\text{nm}$$

7-10. At a pinhole, destructive interference occurs for wavelengths given by $(m+1/2)\,\lambda = a\sin\theta = a\,y/L$. So,

$$\lambda = \frac{y}{(m+1/2)L} = \frac{(1.25)\,(3)}{(m+1/2)\,1500}\,\text{mm} = \frac{2500}{m+1/2}\,\text{nm}$$

In the visible range, $m=4$ and $m=5$ give the wavelengths 556 nm and 455 nm.

7-11. See Figure 7 26 in the text. Constructive interference occurs at screen locations,

$$y = \frac{m\,\lambda(d+L)}{a} = \frac{m\,\lambda(d+2\,d)}{a} = \frac{m\,\lambda(3\,d)}{2\,d\,\alpha\,(n-1)}$$
$$\alpha = \frac{3}{2}\frac{\lambda}{n-1}\frac{\Delta m}{\Delta\lambda} = \frac{3}{2}\frac{589.3\times10^{-9}}{1.5-1}\frac{1}{3\times10^{-4}} = 0.005893\,\text{rad} = 0.3376^\circ = 20.3'$$

7-12. See Figure 7 27 that accompanies the statement of the problem in the text. Also refer to the discussion related to Figure 7 8 in the body of the text.

$$y_m = \frac{m\,\lambda\,s}{2\,\theta\,d} = \frac{m\,\lambda\,(d+D)}{2\,\theta\,d}$$

So,

$$y_{m+1} - y_m \equiv \Delta y = \frac{\lambda\,(d+D)}{2\,\theta\,d}$$
$$\theta = \frac{\lambda\,(d+D)}{2\,d\Delta y} = \frac{(589.3\times10^{-7})\,(50+100)}{2\cdot50\cdot0.05} = 0.001768\,\text{rad} = 6'5''$$

7-13. Refer to Figure 7 9 for the setup. Equation (7 25) gives, $a = 2\,d\,\alpha\,(n-1)$. Using this in Eq. (7 22) gives,

$$\Delta y = \frac{\lambda\,L}{a} = \frac{\lambda\,L}{2\,d\,\alpha\,(n-1)} = \frac{\lambda\,(d+s)}{2\,d\,\alpha\,(n-1)}$$

Rearrangement leads to,

$$\alpha = \frac{\lambda\,(d+s)}{2\,n\,(n-1)\,\Delta y}$$

It is given that $\Delta y = 0.5\,\text{cm}/19$. Then,

$$\alpha = \frac{\left(546.1\times10^{-7}\right)(d+4\,d)}{2\,d\,(1.5-1)\,0.5/19} = 0.010376\,\text{rad} = 35'40''$$

7-14. The condition for a minimum in the reflecting light is

$$\Delta_p + \Delta_r == 2\,n\,t = (m+1/2)\,\lambda$$

For the two wavelengths then

$$2\,n\,t = (m_1+1/2)\,\lambda_1 = (m_2+1/2)\,\lambda_2$$
$$\frac{m_1+1/2}{m_2+1/2} = \frac{\lambda_2}{\lambda_1} = \frac{675}{525} = 1.2875$$

By trial and error, this relation is satisfied with $m_1 = 4$ and $m_2 = 3$. Then,

$$t = \frac{(m_1+1/2)\,\lambda_1}{2\,n} = \frac{(4.5)\,(525\,\text{nm})}{2\,(1.30)} = 908.65\,\text{nm}$$

7-15. At normal incidence, $(m+1/2)\,\lambda = 2\,n\,t$. At 45°, $(m+1/2)\,\lambda' = 2\,n\,t\cos\theta_t$.
Here θ_t is the angle the ray makes with the normal in the film which can be found from Snell's law,

$$\sin(45^\circ) = n\sin\theta_t \Rightarrow \sin\theta_t = \frac{1}{\sqrt{2}\,n} = \frac{1}{\sqrt{2}\,(1.38)} = 0.5124 \Rightarrow \theta_t = 30.825^\circ = \cos\theta_t = 0.859$$

Then,

$$\lambda' = \frac{2\,n\,t\cos\theta_t}{m+1/2} = \frac{(m+1/2)\,\lambda\cos\theta_t}{m+1/2} = \lambda\cos\theta_1 = (580\,\text{nm})\,(0.859) = 498\,\text{nm}$$

7-16. Using Eq. (7 28),the index of refraction of the film is $n_f = \sqrt{n_0\,n_s} = \sqrt{(1)\,(1.78)} = 1.33$.
The required thickness is $t = \lambda_f/4 = \lambda/(4\,n_f) = (550\,\text{nm})/(4\cdot1.33) = 103\,\text{nm}$

7-17. (a) $r = \dfrac{1-n}{1+n}$ where $n = n_2/n_1$. So $r = \dfrac{1-1.4}{1+1.4} = -\,1/6$ and $R = r^2 = 0.0278$. That is 2.78% of the light is reflected.

(b) The thickness should be $t = \lambda_{n_f}/4 = (500\,\text{nm})/(4\cdot 1.4) = 89.3\,\text{nm}$

(c) The reflection from the bottom of the film layer is

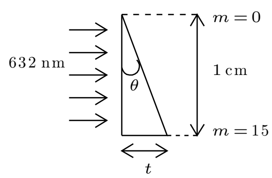
$$r_{\text{bottom}} = \frac{1-1.6/1.4}{1+1.6/1.4} = -\,1/15$$

An estimate gives

$$r^2_{\text{effective}} = (|r_{\text{top}}| - |r_{\text{bottom}}|)^2 = (1/6 - 1/15)^2 = 0.01 \Rightarrow 1\%\ \text{energy reflected}$$

The reflection coefficients from the top and bottom are subtracted because the film thickness is chosen for destructive interference so the emerging reflected beams are π out of phase. This estimate ignores the inte rior reflection at the top surface as the beam reflected from the bottom surface emerges from the film. The light lost at this interior reflection is a small part of the total.

7-18. The soap film forms a wedge like structure since the soapy water collects at the bottom of the wire frame. This structure is shown in the diagram below.



We assume a wedge angle θ. Interference occurs by reflection of the incident 632.8 nm light. Consider dark fringes: $m\,\lambda = 2\,n\,t$. At $m = 0$, $\Delta = \lambda/2$ due to the relative phase shift on reflection. At the film bottom,where $m = 15$,

$$(15)\,\lambda = 2\,(1.33)\,t = 2.66\,(1\,\text{cm})\,\theta \Rightarrow \theta \approx 15\,\lambda/2.66 = 3.57\times 10^{-4}\,\text{rad} = 1'14''$$

7-19. See Figure 7 28 that accompanies this problem in the text. The dark lines are wavelengths for which destructive interference occurs on reflection. These satisfy.

$$m\,\lambda = 2\,n\,t\cos\theta_t$$

Here the film is the air layer. The angle in the air film is the same as the incident angle of 45° since the angle that the ray emerges from the top glass slide into the air film is the same as the angle at which the ray entered the glass slide from the top ambient air. Then,

$$\lambda_m = \frac{2\,n\,t\cos(45°)}{m} = \frac{2\cdot 1\cdot(10^4\,\text{nm})\,0.707}{m} = \frac{14,142\,\text{nm}}{m}$$

There are 15 orders in the visible with m ranging from 21 to 35. The dark lines occur at:
$\lambda_{21} = 673.4\,\text{nm}$, $\lambda_{22} = 642.8\,\text{nm}$..., $\lambda_{35} = 404.1\,\text{nm}$.

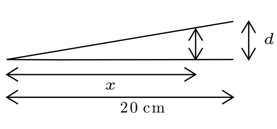
7-20. Refer to Figure 7 15b in the body of the text. Constructive interference will occur for

$$(m + 1/2)\,\lambda = 2\,t$$

Here, t is the thickness of the air wedge at a given horizontal position. The 40$^{\text{th}}$ bright fringe corresponds to $m = 39$ since the first bright fringe occurs for $m = 0$. Then, for the 40$^{\text{th}}$ fringe,

$$39.5\,\lambda = 2\,t \Rightarrow t = \frac{(39.5)(589\times 10^{-7})}{2}\ \text{cm} = 1.16\times 10^{-3}\ \text{cm}$$

7-21. Refer to the figure below. The maximum thickness of the air wedge is $d = 0.05$ mm and the length of the wedge is 20 cm. Let the horizontal distance from the point of contact of the glass plates to a given position in the air wedge be x. For dark fringes, $2\,t + \lambda/2 = (m + 1/2)\,\lambda \Rightarrow t = m\,\lambda/2$



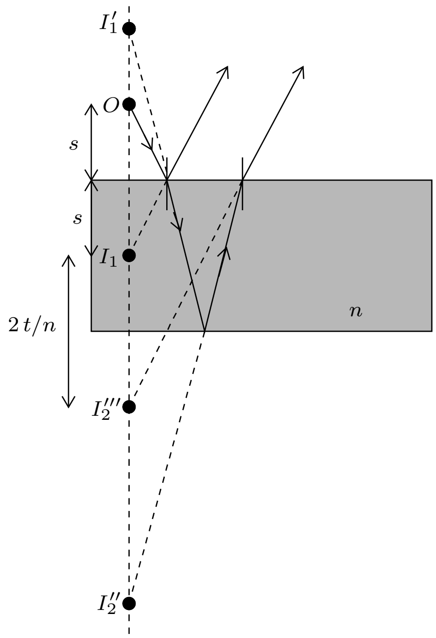
From the geometry: $x/t = (0.2\,\text{m})/d$ so $x = (t/d)\,(0.2\,\text{m}) = \dfrac{m\,\lambda}{2}\,(0.2\,\text{m})$.

For $\Delta m = 1$, $\Delta x = (0.1\,\text{m})\ \lambda/d = \dfrac{(0.1\,\text{m})\left(546.1\times 10^{-9}\,\text{m}\right)}{5\times 10^{-5}\,\text{m}} = 1.09\,\text{mm}$

Also, solving for $m = \dfrac{x\,d}{(0.1\,\text{m})\,\lambda} = \dfrac{(0.2\,\text{m})\,(5\times 10^{-5}\,\text{m})}{(0.1\,\text{m})\,(546.1\times 10^{-9}\,\text{m})} = 181.3$

Counting the $m = 0$ dark fringe, 184 fringes would appear.

7-22. The physical arrangement is sketched below.



The two image positions viewed by the reflected light are I_1 and I_2'''. These serve as the virtual sources. There position can be determined in a step by step fashion. The point source is O. The virtual source I_1 due to the top surface reflection is located a distance s below the top surface. The position of the virtual source I_2''' is found as:

Reflection at top surface forming I_2':

$$\frac{1}{s}+\frac{1}{s_2'}=0\Rightarrow s_2'=-\,n\,s$$

Reflection at bottom forming I_2'':

$$s_2=t-s_2'=t+n\,s\Rightarrow s_2''=-\,s_2=-\,(t+n\,s)$$

Refraction out of top surface forming I_2''':

$$s_2=t-s_2''=2\,t+n\,s$$

$$\frac{n}{s_2}+\frac{1}{s_2'''}=0\Rightarrow s_2'''=-\frac{s_2}{n}=-\left(s+\frac{2\,t}{n}\right)$$

That is I_2''' is formed a distance of $s+2\,t/n$ below the top surface.

Thus the separation of virtual sources is $2\,t/n$.

7-23. Refer to Figure 7 17 in the text. Using Eq. (7 38),

$$2\,t_m+\Delta_r=2\,t_m+\lambda/2=m\,\lambda\Rightarrow t_m=(m-1/2)\,\lambda/2$$

The 10$^{\text{th}}$ bright fringe occurs for $m=10$, so that

$$t_{10}=9.5\,\lambda/2=(9.5\,/2)\,(546.1\times10^{-6}\,\text{mm})=2.59\times10^{-3}\,\text{mm}$$

Using Eq. (7 39), the radius of curvature R of the lens surface can be found:

$$R=\frac{r_{10}^2+t_{10}^2}{2\,t_{10}}=\frac{(7.89/2)^2+(2.59\times10^{-3})^2}{2\,(2.59\times10^{-3})}=3000\,\text{mm}=3\text{m}$$

7-24. Using Eq. (7 39) with $t_m\ll r_m$,

$$r_{m,\text{air}}^2\approx2\,R\,t_{m,\text{air}}=2\,R\,(m-1/2)\,\lambda/2$$
$$r_{m,\text{liq}}^2\approx2\,R\,t_{m,\text{liq}}=2\,R\,(m-1/2)\,\lambda/2n_{\text{liq}}$$
$$\frac{r_{m,\text{air}}^2}{r_{m,\text{liq}}^2}=n_{\text{liq}}\Rightarrow\frac{r_{m,\text{air}}}{r_{m,\text{liq}}}=\sqrt{n_{\text{liq}}}$$

7-25. Using Eq. (7 39) with $t_m\ll r_m$,

$$r_{11,1}^2\approx2\,R\,t_{11,1}=2\,R\,(11-1/2)\,\lambda_1/2=r_{10,2}^2\approx2\,R\,t_{10,1}=2\,R\,(10-1/2)\,\lambda_2/2$$
$$\lambda_2=\lambda_1\,(10.5/9.5)=(546\,\text{nm})(10.5/9.5)=603.5\,\text{nm}$$
$$r_{11}=\sqrt{2\,R\,(11-1/2)\,\lambda_1/2}=\sqrt{2\,(1)\,(10.5)\,(546\times10^{-9})/2}\,\text{m}=0.002398\,\text{m}=2.39\,\text{mm}$$
$$t_{11}=10.5\,\lambda_1/2=2.87\times10^{-4}\,\text{cm}$$

7-26. Refer to Figure 7 20 and the surrounding discussion in the body of the text.

$$\Delta m=\Delta x/x==3.4/1=3.4\Rightarrow d=\Delta m\,\lambda/2=(3.4)\left(\frac{546.1}{2}\,\text{nm}\right)=928.4\,\text{nm}$$

7-27. Refer to Figure 7 30 that accompanies the statement of the problem in the text. The cross sections of the emergent reflected and transmitted beams are the same as that of the incident beam. Inside the film the cross section is modified due to refraction.

(a) $E_e = \dfrac{\Phi}{A} = \dfrac{\Phi}{\pi r^2} = \dfrac{10^{-3}\,\text{W}}{\pi\left(5\times 10^{-4}\right)^2\,\text{m}^2} = 1273\,\text{W/m}^2 = (\varepsilon_0\,c/2)\,E_0^2 \Rightarrow E_0 = 980\,\text{V/m}.$

(b) Snell's law gives: $(1)\sin(45°) = (1.414)\sin\theta_f \Rightarrow \theta_f = 30°$

(c) Using the Stokes relations,

$$|r'| = |r| = 0.28$$
$$t\,t' = 1 - r^2 = 0.9216$$

(d) Reflected beams:

$$E_1 = r\,E_0 = (0.28)\left(980\,\text{V/m}\right) = 274\,\text{V/m},\ (E_1/E_0)^2 = 0.078 = 7.8\%$$
$$E_2 = r'\,t\,t'\,E_0 = (0.28)\,(0.9216)\,(980\,\text{V/m}) = 253\ \text{V/m},\ (E_2/E_0)^2 = 0.067 = 6.7\%$$
$$E_3 = (r')^3 t\,t'\,E_0 = \left(0.28\right)^3(0.9216)\,(980\,\text{V/m}) = 19.8\,\text{V/m},\ (E_3/E_0)^2 = 0.00041 = 0.041\%$$

(e) Transmitted beams:

$$E_1 = t\,t'\,E_0 = (0.9216)\,(980\,\text{V/m}) = 274\,\text{V/m},\ (E_1/E_0)^2 = 0.85 = 85\%$$
$$E_2 = (r')^2\,t\,t'\,E_0 = (0.28)^2(0.9216)\,(980\,\text{V/m}) = 70.8\,\text{V/m},\ (E_2/E_0)^2 = 0.0052 = 0.52\%$$

(f) $m\,\lambda = 2\,n_f\,t\cos\theta_f \Rightarrow t = \dfrac{\lambda}{2\,n_f\cos\theta_f} = \dfrac{632.8\,\text{nm}}{2\,(1.414)\cos(30°)} = 258\,\text{nm}$

7-28. From Eq. (7 27) and the Stokes relations,

$$|r| = |r'| = \left|\frac{1-n}{1+n}\right| = \left|\frac{1-1.52/1}{1+1.52/1}\right| = 0.206$$

$$t\,t' = 1 - r^2 = 0.95742$$

(a) Reflected beams:

$$E_1 = r\,E_0 = (0.20635)\,(1\,\text{V/m}) = 0.206\,\text{V/m}$$
$$E_2 = r'\,t\,t'\,E_0 = (0.20635)\,(0.95742)\,(1\,\text{V/m}) = 0.198\,\text{V/m}$$
$$E_3 = (r')^3\,t\,t'\,E_0 = (0.20635)^3\,(0.95742)\left(1\,\text{V/m}\right) = 0.084\,\text{V/m}$$

Transmitted beams:

$$E_1 = t\,t'\,E_0 = (0.95742)\,(1\,\text{V/m}) = 0.957\,\text{V/m}$$
$$E_2 = (r')^2 t\,t'\,E_0 = (0.20635)^2\,(0.95742)\left(1\,\text{V/m}\right) = 0.041\,\text{V/m}$$
$$E_3 = (r')^4 t\,t'\,E_0 = (0.20635)^4\,(0.95742)\left(1\,\text{V/m}\right) = 0.0017\,\text{V/m}$$

(b) As shown in problem 7 3 the visibility is $\mathcal{V} = \dfrac{2\,R}{R^2+1}$, where R is the ratio of the amplitudes.

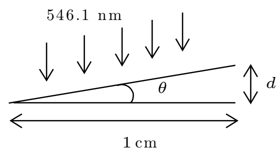
Reflected: $R = E_1/E_2 = 0.206/0.198 = 1.044.\ \mathcal{V} = \dfrac{2\,(1.044)}{1+1.044^2} = 0.999$

Transmitted: $R = E_1/E_2 = 0.957/0.041 = 23.485.\ \mathcal{V} = \dfrac{2\,(23.485)}{1+23.485^2} = 0.085$

Chapter 8 Optical Interferometry

8-1. $\lambda = (2\,\Delta d/\Delta m) = (2\cdot 0.014\,\text{cm})/523 = 4.6\times 10^{-5}\,\text{cm} = 436\,\text{nm}$

8-2. Straight fringes are due to a wedge between one mirror and the image of the other ($M2$ and $M1'$ in Figure 8 1). Interference then occurs as from reflection by an air wedge.



There are 12 fringes/cm so there are 11 fringe spaces/cm.

$m\,\lambda = 2\,d \Rightarrow d = m\,\lambda/2 = (11/2)\,\lambda = 5.5\,\lambda.$

$\theta = t/(1\,\text{cm}) = 5.5\left(5.461\times 10^{-6}\right)/1 = 3.00\times 10^{-4}\,\text{cm} = 0.0172° = 1'2''$

8-3. The optical path difference due to the insertion of the thin sheet of width t and index n is

$$\Delta = m\,\lambda = 2\,(n\,t - t) = 2\,t\,(n-1)$$
$$t = \frac{m\,\lambda}{2\,(n-1)} = \frac{35\,(589\times 10^{-9}\,\text{m})}{2\,(1.434-1)} = 23.75\times 10^{-6}\,\text{m} = 23.75\,\mu\text{m}$$

8-4. (a) Using Eq. (8 5), $m_{\max} = 2\,d/\lambda = (2\cdot 2\,\text{cm})/(500\times 10^{-7}\,\text{cm}) = 80,000$. (b) $m = m_{\max} - 6 = 79,994$

8-5. (a) The optical path length due to the presence of the cell of length L and index of refraction n is

$$\Delta = N\,\lambda = 2\,n\,L - 2\,L = 2\,L(n-1) \Rightarrow n = 1 + N\,\lambda/(2\,L)$$

(b) Rearranging the result from (a), $N = 2\,L(n-1)/\lambda = \dfrac{2\,(0.1\,\text{m})(1.00045-1)}{589\times 10^{-9}\,\text{m}} = 153$

8-6. At $\theta = 0$, $m = 2\,d/\lambda = (20\,\mu\text{m})/(0.6238\,\mu\text{m}) = 31.6$. The smallest diameter dark ring corresponds to $m = 31$. Then,

$$m\,\lambda = 2\,d\cos\theta_m \Rightarrow \cos\theta_{31} = \frac{31\,\lambda}{2\,d} = \frac{31\,(0.6328)}{20} = 0.98084 \Rightarrow \theta_{31} = 11.23°$$

The 10^{th} dark ring is then of order $m = 22$:

$$\cos\theta_{22} = \frac{22\,\lambda}{2d} = \frac{22\,(0.6328)}{20} = 0.69608 \Rightarrow \theta_{22} = 45.89°$$

8-7. In general after a path length difference of d, $m\,\lambda = 2\,d$. A defect of depth Δd then satisfies $2\,\Delta d = \lambda\,\Delta m$. So, $\Delta d = \Delta m\,\lambda/2 = (1/4)\,(632.8\,\text{nm})/2 = 79.1\,\text{nm}$ or $\lambda/8$.

8-8. (a) The resolving power is,

$$\mathcal{R} = \frac{\lambda}{\Delta\lambda} = \frac{656.3}{1.360} = 48,257 \approx 48,260$$

(b) The plate thickness d is $d = m\,\lambda/2$. The mode number m can be found by comparing the coefficient of finesse, F and the resolving power. The coefficient of finesse is,

$$F = \frac{4\,r^2}{(1-r^2)^2} = \frac{4\,(0.99)^2}{(1-(0.99)^2)^2} = 9900$$

The resolving power is

$$\mathcal{R} = \frac{\pi}{2}\,m\sqrt{F} \Rightarrow m = \frac{2\,\mathcal{R}}{\pi\sqrt{F}} = \frac{(2)\,(48,260)}{\pi\,\sqrt{9900}} = 308.8 \approx 309$$

Then

$$d = m\,\frac{\lambda}{2} = (309)\,\frac{656.3\,\text{nm}}{2} = 0.01013\,\text{cm}$$

8-9. (a) The coefficient of finesse is,

$$F=\frac{4\,r^2}{(1-r^2)^2}=\frac{4\,(0.999)}{(1-0.999)^2}=3.996\times10^6$$

(b) The frequency separation between the laser modes is related to the wavelength separation,

$$|\Delta\nu|=\frac{c}{\lambda^2}\,\Delta\lambda\Rightarrow\Delta\lambda=\frac{\lambda^2}{c}|\Delta\nu|=\frac{(632.8\times10^{-9})\,(150\times10^6)}{3\times10^8}\,\mathrm{m}=2.00\times10^{-13}\,\mathrm{m}$$

The resolving power required is then,

$$\mathcal{R}=\frac{\lambda}{\Delta\lambda}=\frac{632.8\times10^{-9}}{2.00\times10^{-13}}=3.16\times10^6$$

(c) The mode number is,

$$m=\frac{2\,\mathcal{R}}{\pi\sqrt{F}}=\frac{2\,(3.16\times10^6)}{\pi\sqrt{3.996\times10^6}}=1006$$

The plate spacing is

$$d=\frac{m\,\lambda}{2}=\frac{(1006)\,(6.328\times10^{-5}\,\mathrm{cm})}{2}=0.0318\,\mathrm{cm}$$

(d) The free spectral range is,

$$\nu_{fsr}=\frac{c}{2\,d}=\frac{3\times10^8\,\mathrm{m/s}}{2\,(3.18\times10^{-4}\,\mathrm{m})}=4.72\times10^{11}\,\mathrm{Hz}$$

Expressed as a wavelength range,

$$\lambda_{fsr}=\frac{\lambda^2}{c}\,\nu_{fsr}=\frac{(632.8\times10^{-9})^2}{3\times10^8}\left(4.72\times10^{11}\right)\mathrm{m}=6.300\times10^{-10}\,\mathrm{m}=0.63\,\mathrm{nm}$$

(e) The minimum resolvable wavelength difference is

$$\Delta\lambda_{\min}=\frac{\lambda^2}{2\,d\,\mathcal{F}}=\frac{\lambda^2}{2\,d\left(\pi\sqrt{\mathcal{F}}/2\right)}=\frac{\lambda^2}{d\,\pi\sqrt{F}}=\frac{(632.8\times10^{-9})^2}{(3.18\times10^{-4}\,\mathrm{m})\pi\sqrt{3.996\times10^6}}=2.00\times10^{-13}\,\mathrm{m}$$

8-10. (a) The mode number is,

$$m=\frac{2\,n\,d}{\lambda}=\frac{(2)\,(4.5)(2\,\mathrm{cm})}{5.46\times10^{-5}\,\mathrm{cm}}=330,000$$

(b) From Eqs. (8 27) and (8 28),

$$F=\frac{T_{\max}-T_{\min}}{T_{\min}}=\frac{T_{\max}}{T_{\min}}-1\Rightarrow\frac{T_{\max}}{T_{\min}}=1+F=1+\frac{4\,r^2}{(1-r^2)^2}=1+\frac{4\,(0.9)}{(1-0.9)^2}=361$$

(c) The resolving power is

$$\mathcal{R}=\frac{2\,d\,\mathcal{F}}{\lambda}=m\,\mathcal{F}=m\,\pi\sqrt{F}/2=(330,000)\,\pi\,\sqrt{360}/2=9.84\times10^6$$

8-11. The spacing d at which overlap occurs is, $d=m\,\lambda_2/2=(m+1)\,\lambda_1/2$. Thus,

$$m\left(\lambda_2-\lambda_1\right)=\lambda_1$$

$$\frac{2\,d}{\lambda}\,\Delta\lambda=\lambda$$

$$d=\frac{\lambda^2}{2\,\Delta\lambda}=\frac{(490\times10^{-9})^2}{2\,(0.0055\times10^{-9})}\,\mathrm{m}=0.0218\,\mathrm{m}=2.18\,\mathrm{cm}$$

8-12. The bright bands due to white light satisfy, $m\,\lambda=2\,d$. Thus,

$$m_1\,\lambda_1=m_2\,\lambda_2=2\,d\quad\text{and}\quad m_1-m_2=149$$

These equations solved simultaneously gives $m_2=588.7\rightarrow588$. Then,

$$d=m_2\,\lambda_2/2=(588)\,(546.1\times10^{-9}\,\mathrm{nm})/2=0.161\,\mathrm{mm}$$

8-13. The irradiance as a function of phase has the form $I = 4I_0\cos^2(\delta/2)$.

(a) The fringe maximum occur for,

$$\cos\delta_m/2 = \pm 1, \delta_m/2 = m\,\pi \Rightarrow \delta_m = 2\,m\,\pi$$

Here m is an integer or zero. Thus the fringe maxima are separated by $\delta_{m+1} - \delta_m = 2\,\pi$.

(b) At the half maxima, $\delta_{1/2} = \delta_m \pm \delta_{\mathrm{FW}}/2$

$$\cos^2(\delta/2) = 1/2$$
$$\cos^2((\delta_m \pm \delta_{\mathrm{FW}})/2) = \cos^2((2\,m\,\pi \pm \delta_{\mathrm{FW}}/2)/2) = \cos^2(\delta_{\mathrm{FW}}/4) = 1/2$$
$$\cos(\delta_{\mathrm{FW}}/4) = \pm 1/\sqrt{2}$$
$$|\delta_{\mathrm{FW}}/4| = \pi/4 \Rightarrow \delta_{\mathrm{FW}} = \pi$$

(c) The finesse is then, $\mathcal{F} = \dfrac{\delta_{m+1} - \delta_m}{\delta_{\mathrm{FW}}} = \dfrac{2\,\pi}{\pi} = 2.$

8-14. Case 1: The irradiances of the emerging beams are

$$I_1 = (0.2)\,(0.8)\,I_0 = 0.16\,I_0 \text{ and } I_2 = (0.8)\,(0.2)\,I_0 = 0.16\,I_0,\ I_1 = I_2$$

The visibility is

$$\mathcal{V} = \frac{I_{\max} - I_{\min}}{I_{\max} + I_{\min}} = \frac{I_1 + I_2 + 2\sqrt{I_1 I_2} - \left(I_1 + I_2 - 2\sqrt{I_1 I_2}\right)}{I_1 + I_2 + 2\sqrt{I_1 I_2} + \left(I_1 + I_2 - 2\sqrt{I_1 I_2}\right)} = \frac{2\sqrt{I_1 I_2}}{I_1 + I_2} = \frac{2\,I_2}{2\,I_2} = 1$$

Case 2: In this case,

$$I_1 = (0.2)\,(0.2)\,I_0 = 0.04\,I_0 \text{ and } I_2 = (0.8)\,(0.8)\,I_0 = 0.64\,I_0,\ I_2 = 16\,I_1$$

The visibility is,

$$\mathcal{V} = \frac{2\sqrt{16\,I_1^2}}{I_1 + 16\,I_1} = \frac{8}{17} = 0.47$$

8-15. (a) Following the method of Section 8 5 and using Eq. (8 22):

$$E_{0R} = -r\,E_{0I} + r\,t\,e^{-i\delta}E_{01}^{+} = \left(-r + \frac{r\,t^2e^{-i\delta}}{1 - r^2e^{-i\delta}}\right)E_{0I} = \frac{-r + r\,(r^2 + t^2)\,e^{-i\delta}}{1 - r^2e^{-i\delta}}E_{0I}$$
$$E_{0R} = \frac{r(1 - e^{-i\delta})}{1 - r^2e^{-i\delta}}E_{0I} = \frac{2\,i\,r\,e^{-i\delta/2}\sin\delta/2}{1 - r^2e^{-i\delta}}$$
$$R = \frac{E_{0R}E_{0R}^*}{E_{0I}\,E_{0I}^*} = \frac{4r^2\sin^2\delta/2}{(1 - r^2)^2 + 4r^2\sin^2\delta/2}$$

Note that I have assumed lossless mirrors in setting $r^2 + t^2 = 1$.

(b) $R + T = \dfrac{4r^2\sin^2\delta/2 + (1 - r^2)^2}{(1 - r^2)^2 + 4r^2\sin^2\delta/2} = 1$

8-16. Using the relation between the finesse and the ratio of the maximum and minimum transmission,

$$F = \frac{T_{\max} - T_{\min}}{T_{\min}} = \frac{T_{\max}}{T_{\min}} - 1 \Rightarrow \frac{T_{\max}}{T_{\min}} = 1 + F = 1 + \frac{4\,r^2}{(1 - r^2)^2} = 1 + \frac{4\,(0.6)}{(1 - 0.6)^2} = 16$$

8-17. In this case, in steady state, Eq. (8 21) should be written as, $E_{01}^{+} = t_1 E_{0I} + E_{01}^{+}\sqrt{R_1 R_2}e^{-i\delta}$ where $\delta = 2kd + \varphi_{1R} + \varphi_{2R}$ and $r_{1,2} = \sqrt{R_{1,2}}e^{-i\varphi_{1,2R}}$. Rearranging,

$$E_{01}^{+} = \frac{t_1}{1 - \sqrt{R_1 R_2}e^{-i\delta}} E_{0I}$$

The complex amplitude of the transmitted field is then obtained in a manner that mirrors the development leading to Eq. (8 23),

$$E_{0T} = \frac{t_1 t_2 e^{-i\delta/2}}{1 - \sqrt{R_1 R_2}e^{-i\delta}} E_{0I}$$

Then,

$$T = \frac{I_T}{I_I} = \frac{E_{0T}E_{0T}^*}{E_{0I}E_{0I}^*} = \frac{|t_1|^2 |t_2|^2}{1 + R_1 R_2 - 2\sqrt{R_1 R_2}\cos\delta} = \frac{T_1 T_2}{(1 - \sqrt{R_1 R_2})^2 + 4\sqrt{R_1 R_2}\sin^2(\delta/2)}$$

Here $T_{1,2} = |t_{1,2}|^2$. If the mirrors are lossless, then $T_{1,2} = 1 - R_{1,2}$ and $R = 1 - T$.

8-18. (a) Estimating from Figure 8 15, $\mathfrak{F} = \frac{\text{FSR}}{\text{FWHM}} \approx 70$ (b) $\nu_{fsr} = c/2d = (3\times 10^8)/(2\cdot 0.1)\,\text{Hz} = 1.5\,\text{GHz}$

(c) $\text{FWHM} = \frac{\nu_{fsr}}{\mathfrak{F}} = \frac{1.5\,\text{GHz}}{70} = 21\,\text{MHz}$ (d) $Q = \frac{\nu}{\text{FWHM}} = \frac{4.53\times 10^{14}}{2.1\times 10^7} = 2.2\times 10^7$

(e) $\tau_p \approx Q/(2\pi\nu) = (2.2\times 10^7)/(2\pi\cdot 4.53\times 10^{14}) \sim 8\times 10^{-9}\,\text{s}$

8-19.

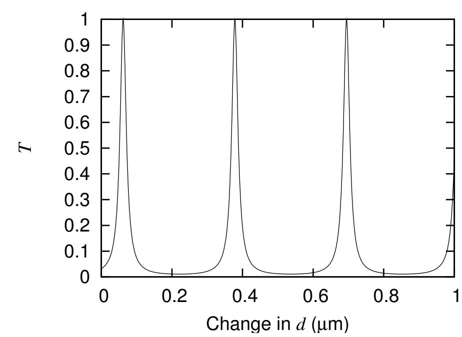
The transmittance function has the form,

$$T = \frac{1}{1 + \frac{4\mathcal{F}^2}{\pi^2}\sin^2(kd)}$$

For the case at hand,

$\mathcal{F} = 15$ and $k = 2\pi/\lambda = 2\pi/(632.8\times 10^{-9}\,\text{m}) = 9.93\times 10^6/\text{m}$.

The plot is shown to the right.



8-21. (a) The peak separation for a given wavelength is $d_{fsr} = \lambda/2 \Rightarrow \lambda = 2\,\Delta d = 2\cdot 2.95\,\mu\text{m} = 5.9\,\mu\text{m}$

(b) Using Eq. (8 37), $\Delta\lambda \approx \frac{\Delta d}{d}\lambda = \frac{0.01\times 10^{-6}}{0.05} 5.9\,\mu\text{m} = 1.2\times 10^{-6}\,\mu\text{m}$

(c) $\lambda_2 - \lambda_1 = \frac{2}{m_2}d_2 - \frac{2}{m_1}d_1 = \frac{2m_1 d_2 - 2m_2 d_1}{m_1 m_2} = \frac{2m_1(d_1 + \Delta d) - 2(m_1+1)d_1}{m_1(m_1+1)} \approx \frac{2m_1\Delta d - 2d_1}{m_1^2}$

Now, $m_1 = 2d_1/\lambda \approx 2d/\lambda$. So,

$$\lambda_2 - \lambda_1 = \lambda\frac{\Delta d}{d} - \frac{\lambda^2}{2d} = 1.2\times 10^{-6}\,\mu\text{m} - \left(\frac{5.9^2}{2\cdot 5\times 10^{-4}}\right)\mu\text{m} = -3.5\times 10^{-4}\,\mu\text{m}.$$

8-22. (a) From the plot, 28 divisions $\approx$ 11.6 GHz. The separation of the absorption dips is then,

$$\Delta\nu_{\text{dips}} \approx (25.5\,\text{divisions})\frac{11.6\,\text{GHz}}{28\,\text{divisions}} = 10.6\,\text{GHz}$$

(b) Similarly, the full widths at half depth are about 2 divisions and 1.5 divisions leading to widths of about 0.83 GHz and 0.62 GHz, respectively.

8-23. The FWHM of the transmittance peaks is determined as in the text. Picking up the development with Eq. (8 33),

$$\delta_{1/2} = \left(\frac{2\,\pi}{\lambda} - \frac{2\,\pi}{\lambda + \delta\lambda_{1/2}} \right) 2d = \pi / \mathcal{F} \Rightarrow$$

$$\frac{1}{\lambda} - \frac{1}{\lambda + \delta\lambda_{1/2}} = \frac{\lambda + \delta\lambda_{1/2} - \lambda}{\lambda\,(\lambda + \delta\lambda_{1/2})} = \frac{1}{4\,d\,\mathcal{F}}$$

$$\frac{\delta\lambda_{1/2}}{1 + \delta\lambda_{1/2}/\lambda} \approx \delta\lambda_{1/2} = \frac{\lambda}{4\,d}\frac{\lambda}{\mathcal{F}} = \frac{\lambda}{2\,m\,\mathcal{F}}$$

$$2\,\delta\lambda_{1/2} \approx \lambda / (m\mathcal{F})$$

Chapter 9 Coherence

9-1. The given function is

$$f(x)=\begin{cases}-1, & -L/2<x<0\\ +1, & 0<x<L/2\end{cases}\qquad \text{Odd function: } a_0=a_n=0$$

Its Fourier series representation is formed as,

$$f(x)=\sum_{m=1}^{\infty} b\,m\sin mkx,\ \text{where}\ b_m=\frac{2}{L}\int_{-L/2}^{+L/2} f(x)\sin mkx\,dx=$$

$$b_m=\frac{2}{L}\left\{\int_{-L/2}^{0}-\sin mkx\,dx+\int_0^{L/2}\sin mkx\,dx\right\}=\frac{2}{m\,k\,L}\left\{[\cos(m\,k\,x)]_{-L/2}^{0}-[\cos(m\,k\,x)]_0^{L/2}\right\}$$

$$b_m=\frac{2}{m\,k\,L}\left\{1-\cos\frac{m\,k\,L}{2}-\cos\frac{m\,k\,L}{2}+1\right\}=\frac{4}{m\,k\,L}\left[1-\cos\left(\frac{m\,k\,L}{2}\right)\right]\ \text{with}\ k=\frac{2\,\pi}{L},\ b_m=\frac{2}{m\,\pi}(1-\cos m\,\pi)$$

$$f(x)=\frac{2}{\pi}\sum_{m=1}^{\infty}\left(\frac{1-\cos m\,\pi}{m}\right)\sin\frac{2\,\pi\,m\,x}{L}=\frac{2}{\pi}\left(2\sin\left(\frac{2\,\pi x}{L}\right)+\frac{2}{3}\sin\left(\frac{6\,\pi\,x}{L}\right)+\ldots\right)$$

$$\text{or } f(x)=\frac{4}{\pi}\left(\frac{\sin(k\,x)}{1}+\frac{\sin(3\,k\,x)}{3}+\frac{\sin(5\,k\,x)}{5}+\ldots\right)$$

9-2. The function is,

$$f(t)=\begin{cases}E_0\cos\omega\,t, & \frac{3}{2}\pi,\omega\,t,\frac{5}{2}\pi\\ 0\,, & \frac{5}{2\,\pi}<\omega\,t<\frac{7}{2}\pi\end{cases}$$

The Fourier series representation is formed as:

$$f(t)=\frac{a_0}{2}+\sum_{m=1}^{\infty}a_m\cos m\omega t\quad \text{where}\ a_0=\frac{2}{T}\int_{t0}^{t_0+T}f(t)\,dt\ \text{ and }\ a_m=\frac{2}{T}\int_{t_0}^{t_0+T}f(t)\cos m\omega t\,dt$$

$$a_0=\frac{2}{T}\int_{t_0}^{t_0+T}f(t)\,d\,t=\frac{2}{\omega\,T}\int E_0\cos(\omega\,t)\,d(\omega\,t)=\frac{2E_0}{\omega\,T}\sin\omega\,t\Big]_{3\pi/2}^{5\pi/2}=\frac{2\,E_0}{\pi}$$

$$a_m=\frac{2\,E_0}{\omega\,T}\int_{3\pi/2}^{5\pi/2}\cos\omega t\cos m\omega t\,d(\omega t)$$

$$m=1\text{: } a_1=\frac{2\,E_0}{\omega\,T}\int_{3\,\pi/2}^{5\,\pi/2}\cos^2\omega t\,d(\omega\,t)=\frac{2\,E_0}{2\,\pi}\left[\frac{\omega\,t}{2}+\frac{2\,\omega\,t}{4}\right]_{3\pi/2}^{5\,\pi/2}=\frac{E_0}{2}$$

$$m>1\text{: } a_m=\frac{2\,E_0}{\omega\,T}\left[\frac{\sin[(m-1)\,\omega\,t]}{2(m-1)}+\frac{\sin[(m+1)\,\omega\,t]}{2(m+1)}\right]_{3\pi/2}^{5\pi/2},\quad m=2\text{: } a_2=\frac{2\,E_0}{2\,\pi}\left[\frac{\sin\omega\,t}{2}+\frac{\sin 3\,\omega\,t}{6}\right]_{3\pi/2}^{5\pi/2}=\frac{2\,E_0}{3\,\pi}$$

$$m=3\text{: } a_3=a_5=a_7=\ldots=0,\ m=4\text{: } a_4=\frac{E_0}{\pi}\left[\frac{\sin 3\omega t}{6}+\frac{\sin 5\omega t}{10}\right]_{3\pi/2}^{5\pi/2}=-\frac{2\,E_0}{15\,\pi}$$

All $b_m=0$ since function is even or, more tediously, by integration.

$$\text{Thus }\ f(t)=\frac{E_0}{\pi}+\frac{E_0}{2}\cos\omega t+\frac{2\,E_0}{3\,\pi}\cos 2\omega t-\frac{2\,E_0}{15\,\pi}\cos 4\omega t+\ldots$$

9-3. The Fourier transform is formed as,

$$g(\omega)=\frac{1}{2\,\pi}\int_{-\infty}^{\infty} f(t)e^{i\omega t}dt=\frac{1}{2\,\pi}\int_{-\infty}^{\infty} h\,e^{-t^2/2\sigma^2}e^{i\omega t}dt=\frac{h}{2\,\pi}\int_{-\infty}^{\infty} e^{-\left[\frac{t^2}{2\sigma^2}-i\omega t\right]}dt$$

Completing the square:

$$\frac{t^2}{2\,\sigma^2}-i\,\omega\,t=\left(\frac{t}{\sqrt{2}\sigma}-\frac{i\,\sigma\,\omega}{\sqrt{2}}\right)^2+\frac{\sigma^2\omega^2}{2}$$

Then,

$$g(\omega)=\frac{h}{2\,\pi}\int_{\infty}^{\infty} e^{-\left[\left(\frac{t}{\sqrt{2}\sigma}-\frac{i\sigma\omega}{\sqrt{2}}\right)^2+\frac{\sigma^2\omega^2}{2}\right]}dt=\frac{h}{2\pi}\,e^{\frac{\sigma^2\omega^2}{2}}\int_{-\infty}^{\infty} e^{-\left(\frac{t}{\sqrt{2}\sigma}-\frac{i\sigma\omega}{\sqrt{2})}\right)^2}dt$$

Let, $x=\dfrac{t}{\sqrt{2}\,\sigma}-\dfrac{i\,\sigma\,\omega}{\sqrt{2}}$ so $dx=\dfrac{dt}{\sqrt{2}\,\sigma}$, so $dx=\dfrac{dt}{\sqrt{2}\,\sigma}$. Then,

$$g(\omega)=\frac{h}{2\pi}\,e^{\frac{-\sigma^2\omega^2}{2}}\int_{-\infty}^{\infty} e^{-x^2}\sqrt{\ }\sigma\,dx=\frac{h}{2\,\pi}\,e^{\frac{-\sigma^2\omega^2}{2}}\sqrt{2}\ \sigma\int_{-\infty}^{\infty} e^{-x^2}dx=\frac{h}{2\,\pi}\,e^{\frac{-\sigma^2\omega^2}{2}}\sqrt{2}\ \sigma\sqrt{\pi}$$

$$g(\omega)=\frac{\sigma h}{\sqrt{2\pi}}e^{-\sigma^2\omega^2/2}$$

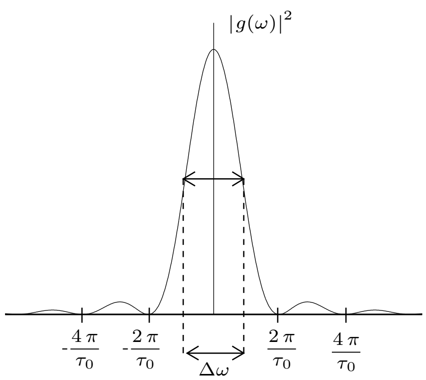
9-4. In general,

$$f(t)=\int_{-\infty}^{\infty} g(\omega)e^{i\omega t}\,d\omega\ \text{ where } g(\omega)=\frac{1}{2\,\pi}\int_{-\infty}^{\infty} f(t)e^{i\omega t}\,dt$$

For the given function,

$$g(\omega)=\frac{1}{2\,\pi}\int_{-\tau_0/2}^{\tau_0/2} A\,e^{i\omega t}\,dt=\frac{A}{2\,\pi}\left[\frac{e^{i\omega t}}{i\,\omega}\right]_{-\tau_0/2}^{\tau_0/2}=\frac{A}{\pi\,\omega}\left[\frac{e^{i\omega\tau_0/2}-e^{-i\omega\tau_0/2}}{2\,i}\right]=\frac{A}{\pi\,\omega}\sin\left(\frac{\omega\tau_0}{2}\right)=\frac{A\tau_0}{2\,\pi}\text{sinc}\,(\tau_0/2)$$

$$|g(\omega)|^2=\frac{A^2\tau_0^2}{4\,\pi^2}\text{sinc}^2(\omega\,\tau_0/2)$$



From the plot,

$$\Delta\omega\approx\frac{2\,\pi}{\tau_0}\Rightarrow 2\,\pi\,\Delta\nu\approx 2\,\pi/\tau_0\ \ \Delta\nu\approx\frac{1}{\tau_0}$$

9-5. Generally, $l_t=\lambda^2/\Delta\lambda$. Then, $(l_t)_1=\dfrac{(590)^2}{100}=3.48\times 10^{-4}\,\text{m}$ and $(l_t)_2=\dfrac{(590)^2}{10}=3.48\times 10^{-5}\,\text{m}$.
Use longer coherence − length light, thus shorter − bandwidth filter.

9-6. The coherence length is $l_t=c\tau_0=c(10^{-6}\,\text{s})=300\,\text{m}$. The bandwidth is, $\Delta\nu=1/\tau_0=10^6\,\text{Hz}$. The resultant linewidth is $\Delta\lambda=\lambda^2/l_t=(632.8\times 10^{-9}\text{m})^2/(300\,\text{m})=1.33\times 10^{-15}\,\text{m}$.

9-7. The angular diameter of the sun viewed from the earth is $\theta = 0.5° = 0.008727$ rad. The spatial coherence width is then,

$$l_s < \frac{1.22\,\lambda}{\theta} = \frac{1.22(550\times 10^{-7}\text{cm})}{0.008727} = 0.00769\,\text{cm}$$

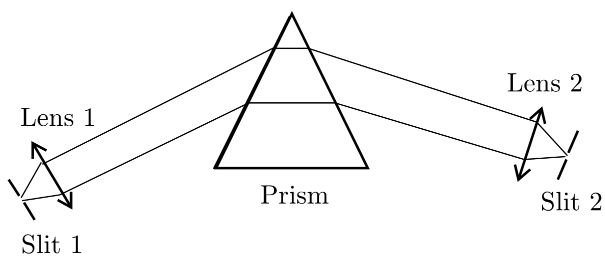
If l_s is the diameter of a circle of coherence, then "good" coherence exists over an area of 10% the area, or a diameter of $\sqrt{0.1}\,l_s < \sqrt{.1}(0.0769\,\text{mm}) = 0.0243\,\text{mm}$.

9-8. (a) $\Delta\lambda = \dfrac{\lambda^2}{l_t} = \dfrac{(643.8\,\text{nm})^2}{30\times 10^7\,\text{nm}} = 0.00138\,\text{nm}$ (b) $\tau_c = \dfrac{l_t}{c} = \dfrac{30\,\text{cm}}{c} = 1\,\text{ns}$

9-9. The coherence length of the transmitted light is $l_t = \dfrac{\lambda^2}{\Delta\lambda} = \dfrac{(5000\,\text{Å})^2}{1\,\text{Å}} = 0.25\,\text{cm} = 2.5\,\text{mm}$.

9-10. The linewidth is for the 0.2 mm exit slit is $\Delta\lambda = (20\,\text{Å/mm})\,(0.2\,\text{mm}) = 4\,\text{Å}$. The coherence length and time are then,

$$l_t = \frac{\lambda^2}{\Delta\lambda} = \frac{(5\times 10^{-5}\text{cm})^2}{4\times 10^{-8}\,\text{cm}} = 0.0625\,\text{cm}$$
$$\tau_0 = l_t/c = 2.08\times 10^{-12}\,\text{s}$$



9-11. See Figure 9 14 in the text for the arrangement. The slit separation a must be within the spatial coherence width of the light emerging from the pinhole. The distance from the pinhole to the slits is $r = 1$ m. If the diameter of the pinhole is $s = 5\times 10^{-4}$ m, then the angle subtended by the pinhole from the plane con taining the two slits is $\theta = s/r$. Since the pinhole is circular the spatial coherence width is given by Eq. (9 38). Therefore,

$$a < l_s = \frac{1.22\,\lambda}{\theta} = \frac{1.22\,r\,\lambda}{s} = \frac{1.22(100\,\text{cm})(5.89\times 10^{-5}\,\text{cm})}{0.05\,\text{cm}} = 0.144\,\text{cm}$$

9-12. (a) $\Delta\lambda = \dfrac{\lambda^2}{l_t} = \dfrac{(6328\times 10^{-10}\text{m})^2}{10^4\,\text{m}} = 4\times 10^{-7}\,\text{Å}$, $\Delta\nu = \dfrac{1}{\tau_0} = \dfrac{c}{l_t} = \dfrac{c}{10^4\text{m}} = 3\times 10^4$ Hz

9-13. (a) The linewidth is for the 0.2 mm exit slit is $\Delta\lambda = (20\,\text{Å/mm})\,(0.2\,\text{mm}) = 4\,\text{Å}$. The coherence length and time are then (as in problem 9 10),

$$l_t = \frac{\lambda^2}{\Delta\lambda} = \frac{(5\times 10^{-5}\text{cm})^2}{4\times 10^{-8}\,\text{cm}} = 0.0625\,\text{cm}$$
$$\tau_0 = l_t/c = 2.08\times 10^{-12}\,\text{s}$$

(b) The normalized correlation function is $|\gamma(\tau)| = 1 - \tau/\tau_0$. But, $\tau/\tau_0 = \Delta l/l_t = \dfrac{0.4\text{mm}}{0.625\text{mm}} = 0.64$. So, $V = |\gamma(\tau)| = 1 - \tau/\tau_0 = 0.36$.

(c) $V = \dfrac{I_{\max} - I_{\min}}{I_{\max} + I_{\min}} = 0.36 = \dfrac{100 - I_{\min}}{100 + I_{\min}} \Rightarrow I_{\min} = 47.06$.

9-14. The coherence length l_t and spatial coherence width l_s are the length and diameter of the cylindrical coher ence volume. Now,

$$l_t = \frac{\lambda^2}{\Delta\lambda} = \frac{550^2}{700-400}\,\text{nm} = 1.0083\times 10^{-4}\text{cm} = 1.83\,\lambda_{\text{ave}}$$
$$l_s = \frac{1.22\lambda}{\theta} = \frac{1.22(550)}{0.008727}\,\text{nm} = 76890\,\text{nm} = 7.689\times 10^{-3}\,\text{cm}$$
$$\text{“Good”}\;\; l_s = 0.25(7.689\times 10^{-3}) = 1.922\times 10^{-3}\text{cm} = 35\,\lambda_{\text{ave}}$$

The area of the base of the cylindrical coherence volume is

$$A_{\text{base}} = \frac{\pi\, l_s^2}{4} = \frac{\pi}{4}\,(1.922\times 10^{-3})^2 = 2.90\times 10^{-6}\,\text{cm}^2$$

9-15. (a) For unequal beams and partial coherence, $I = I_1 + I_2 + 2\sqrt{I_1 I_2}\,\text{Re}(\gamma)$ where,

$$\text{Re}(\gamma) = \text{Re}\big[(1-\tau/\tau_0)e^{i\omega\tau}\big] = (1-\tau/\tau_0)\cos(\omega\tau) = |\gamma(\tau)|\cos(\omega\tau)$$

So,

$$I_{\max} = I_1 + I_2 + 2\sqrt{I_1 I_2}\,|\gamma(\tau)|$$
$$I_{\min} = I_1 + I_2 - 2\sqrt{I_1 I_2}\,|\gamma(\tau)|$$
$$V = \frac{I_{\max}-I_{\min}}{I_{\max}+I_{\min}} = \frac{I_1+I_2+2\sqrt{I_1 I_2}\,|\gamma(\tau)| - \big(I_1+I_2-2\sqrt{I_1 I_2}\,|\gamma(\tau)|\big)}{I_1+I_2+2\sqrt{I_1 I_2}\,|\gamma(\tau)| + \big(I_1+I_2-2\sqrt{I_1 I_2}\,|\gamma(\tau)|\big)} = \frac{2\sqrt{I_1 I_2}|\gamma(\tau)|}{I_1+I_2}$$

(b) For beams of the same irradiance

$$V_{\text{eq}} = \frac{2\sqrt{I_1 I_1}|\gamma(\tau)|}{I_1+I_2} = |\gamma(\tau)|$$

When $V = 0.9\,V_{\text{eq}} = 0.9|\gamma(\tau)|$, we have,

$$\frac{2\sqrt{I_1 I_2}|\gamma(\tau)|}{I_1+I_2} = 0.9|\gamma(\tau)| \Rightarrow 4\,I_1 I_2 = 0.81\,(I_1+I_2)\quad\text{with } R = I_1/I_2,$$
$$0.81\,R^2 - 2.38\,R + 0.81 = 0 \Rightarrow R = 2.55$$

9-16. The optical path difference in m^{th} order is $\Delta = m\,\lambda$. Then, $\frac{\tau}{\tau_0} = \frac{\Delta}{l_t} = \frac{\Delta}{\lambda^2/\Delta\lambda} = \frac{m\,\lambda}{\lambda^2/\Delta\lambda} = \frac{m\,\Delta\lambda}{\lambda}$

As shown in problem (9 15b) for equal irradiance beams, $V = |\gamma(\tau)| = 1 - \tau/\tau_0 = 1 - \frac{m\,\Delta\lambda}{\lambda}$

9-17. (a) $l_t = \lambda^2/\Delta\lambda = (546.1)^2/0.05 = 5.965\times 10^6\,\text{nm}$

(b) With a filtered Hg lamp, $V = 1 - \frac{m\,\Delta\lambda}{\lambda} = 1 - \frac{20\,(0.05)}{546.1} = 0.998.$

With filtered white light, $\lambda_{\text{ave}} = 546\,\text{nm}$ and $\Delta\lambda = 10\text{nm}$, $V = 1 - \frac{20\,(10)}{546} = 0.63$

9-18. $\Delta = m\,\lambda = 2\times 1\,\text{cm} = 2\times 10^7\,\text{nm}$, $V = 1 - m\,\frac{\Delta\lambda}{\lambda} = 1 - \frac{(\Delta)\,(\Delta\lambda)}{\lambda^2} = 1 - \frac{(2\times 10^7)\,(.0013)}{(643.847)^2} = 0.937$

For a 5 cm movement, $\Delta = 2\times 5\,\text{cm} = 10^8\,\text{nm}$, and $V = 1 - \frac{(10^8)\,(0.0013)}{(643.847)^2} = 0.686$

$V \to 0$ when $1 - \frac{(2\,d)\,(\Delta\lambda)}{\lambda^2} \to 0$ or when $d = \lambda^2/2\,\Delta\lambda$, so $d = 0.0895\,\text{cm}$.

9-19. (a) $d = 1\text{cm}$: $V = 1 - \frac{(2\times 10^7)\,(0.025)}{(546.1)^2} < 0$ or $V = 0$ (See Prob. 9 16). $d = 5\,\text{cm}, V = 0$

$V = 0$ first when $d = \frac{\lambda^2}{2\,\Delta\lambda} = \frac{(546.1)^2}{2\,(0.025)} = 0.596\text{cm}$

(b) $V = 0.85 = 1 - 2\,d\,\frac{\Delta\lambda}{\lambda^2} \Rightarrow\; d = 0.15\left(\frac{\lambda^2}{2\,\Delta\lambda}\right) = \frac{(0.15)\,(546.1)^2}{2\,(0.025)} = 0.0895\,\text{cm}$

Chapter 10 Fiber Optics

10-1. The information rate is,

$$(4000\,\text{Hz})(2\times\text{sampling rate})\,(8\,\text{bits/sample}) = 64\text{kbps}$$

$$\text{Number of channels} = \frac{44.7\,\text{Mbps}}{64\,\text{kbps}} - 26(\text{assigned}) = 672$$

10-2. Each TV channel requires a bandwidth of 6 MHz.

(a) $\#\text{of TV stations} = \dfrac{\nu}{6\times10^6\,\text{Hz}} = \dfrac{c/\lambda}{6\times10^6\,\text{Hz}} = \dfrac{(3\times10^8/1.55\times10^{-6})}{6\times10^6} = 32\,\text{million}$

(b) $\#\text{of TV stations} = \dfrac{4\times10^{12}}{6\times10^6\,\text{Hz}} = 0.67\,\text{million}$

10-3. (a) In general, $L_s = d\sqrt{\left(\dfrac{n_1}{\sin\theta}\right)^2 - 1}$. When $\theta = \theta_m, \sin\theta = \sin\theta_m = \sqrt{n_1^2 - n_2^2}$. Then,

$$L_s = \sqrt{\left(\frac{n_1}{n_1^2 - n_2^2}\right)^2 - 1} = \frac{n_2\,d}{\sqrt{n_1^2 - n_2^2}}$$

(b) In one meter of fiber, there are $1/L_s$ reflections, so the number of reflections per meter N_s is,

$$N_s = \frac{\sqrt{n_1^2 - n_2^2}}{n_2\,d} = \frac{(1.46^2 - 1.457^2)^{1/2}}{1.457\,(50\times10^{-6}\,\text{m})} = 1284/\text{m}$$

10-4. (a) $1.52\sin\varphi_c = 1.41 \Rightarrow \varphi_c = 68.1°$ (b) $\text{N.A.} = n_0\sin\theta_m = n_1\cos\varphi_c = 1.52\cos(68.1°) = 0.567$
(c) $\sin\theta_m = (\text{N.A.})/n_0 = (0.567/1) = 0.567 \Rightarrow \theta_m = 34.5°$

10-5. (a) $\text{N.A.} = \sqrt{n_1^2 - n_2^2} = \sqrt{1.53^2 - 1.39^2} = 0.64$
(b) Since $n_0\sin\theta_m = \text{N.A.}$, $\theta_m = \sin^{-1}(0.6394) = 39.7°$. The cone angle is $2\,\theta_m = 79.5°$.
(c) The number of reflections per inch is $N_s = 1/L_s$, so,

$$N_s = \frac{1}{L_s} = \frac{1}{d\sqrt{\left(\frac{n_1}{n_0\sin\theta}\right)^2 - 1}} = \frac{1}{\left(0.0025\,\text{in}\right)\sqrt{\left(\frac{1.53}{\sin(39.75°)}\right)^2 - 1}} = 184/\text{inch}$$

That is, there will be 6624 reflections in 3 ft. At $\theta = \theta_m/2 = 19.87°$, $1/L_s = 91.14/\text{inch}$ so that there would be 3281 reflections in 3 ft.

10-6. Refer to Figure 10 3b in the body of the text. In that figure, x_s is the hypotenuse of the triangle formed by θ', L_s, and d. Combining a relation that follows from examination of that triangle with Snell's law gives,

$$x_s = \frac{d}{\sin\theta'} = \frac{n_1\,d}{n_0\sin\theta} = \frac{n_1\,d}{\sin\theta}$$

(b) The total distance is $x_t = x_s\,N_s$ where N_s is the total number of skips. That is,

$$x_t = x_s\,\frac{L}{L_s} = \frac{n_1 d}{\sin\theta}\left[\frac{L}{d\sqrt{(n_1/\sin\theta)^2 - 1}}\right] = \frac{n_1\,L}{\left(n_1^2 - \sin^2\theta\right)^{1/2}}$$

(c) For the given parameters,

$$x_s = \frac{n_1\,d}{\sin\theta} = \frac{(1.5)\,(50\,\mu\text{m})}{\sin(10°)} = 432\,\mu\text{m}$$

$$L_s = d\,\sqrt{(n_1/\sin\theta)^2 - 1} = (50\,\mu\text{m})\,\sqrt{(1.5/\sin(10°))^2 - 1} = 429\,\mu\text{m}$$

$$x_t = \frac{n_1\,L}{\left(n_1^2 - \sin^2\theta\right)^{1/2}} = \frac{1.5\,(10\,\text{m})}{\sqrt{1.5^2 - \sin^2(10°)}} = 10.07\,\text{m}$$

10-7. The number of modes is,

$$m_{\max} = \frac{1}{2}\left(\frac{\pi d}{\lambda}\right)^2 ((\text{N.A.})^2 = \frac{1}{2}\left(\frac{\pi d}{\lambda}\right)^2 \left(n_1^2 - n_2^2\right) = \frac{1}{2}(\pi\, 40/0.85)^2 \left(1.461^2 - 1.456^2\right) = 159$$

10-8. The necessary condition is,

$$\frac{d}{\lambda} < \frac{2.4}{\pi\,(\text{N.A.})} \Rightarrow d = \frac{2.4\,\lambda}{\pi\,(\text{N.A.})} = \frac{2.4\,(1.25\,\mu\text{m})}{\pi\sqrt{1.46^2 - 1.457^2}} = 10.2\,\mu\text{m}$$

10-9. In general the number of modes is given by $N = 1 + (d/\lambda)\sqrt{n_1^2 - n_2^2}$. Then,
For $d = 5\,\lambda$: $N = 1 + 2\,(5)\sqrt{3.6^2 - 3.55^2} = 6$ (or 12 for two polarizations).
For $d = 50\lambda$: $N = 1 + 2\,(5)\sqrt{3.6^2 - 3.55^2} = 60$ (or 120 for two polarizations).

10-10. The loss in db/(100 m) is

$$\alpha_{\text{db}} = 10\,\text{db}/(100\,\text{m})\log_{10}(P_1/P_2) = (0.1\,\text{db/m})\log_{10}(5) = 0.070\,\text{db/m} = 70\,\text{db/km}$$

10-11. The entire cable has a loss of $3\times 5\,\text{db} + 2\,\text{db} = 17\,\text{db}$ in 3 km. So $\alpha_{\text{db}} = 17\text{db}/3\text{km}$. Then, using Eq. (10 13):

$$\alpha_{\text{db}} = (10\,\text{db/km})\log_{10}(P_1/P_2) \Rightarrow P_2 = P_1\left(10^{-\alpha_{\text{db}} L/10(\text{db/km})}\right)$$

$$P_2 = (4\text{ mW}\big)\,10^{-[(17/3)3/10]} = (4\text{ mW}\big)\,10^{-17/10} = 0.080\,\text{mW}$$

10-12. The loss in db that can be tolerated in a length L of fiber is,

$$\alpha_{\text{db}}\,L = (10\,\text{db})\log_{10}(0.01/10^{-6}) = 40\,\text{db}$$

So for the coaxial cable: $L = (40\,\text{db}/12\,\text{db/km}) = 3.33\,\text{km}$. For the fiber: $L = (40\,\text{db}/4\,\text{db/km}) = 10\,\text{km}$.

10-13. Rayleigh scattering has a $1/\lambda^4$ dependence so,

$$\alpha_{1.55} = \alpha_{0.90}\ (0.9/1.55)^4 = (1.2\,\text{db/km})(0.9/1.55)^4 = 0.136\,\text{db/km}$$

10-14. (a) The fractional power loss is

$$f = \frac{P_1 - P_2}{P_1} = 1 - P_2/P_1 \Rightarrow P_2/P_1 = 1 - f$$

Then,

$$\alpha_{\text{db}} = (10\,\text{db/km})\log_{10}\left(\frac{1}{1-f}\right) = -\,(10\,\text{db/km})\log_{10}(1-f)$$

(b) Using the relation from part (a),For $f = 0.25, \alpha_{\text{db}} = 1.25\,\text{db/km}$, for $f = 0.75, \alpha_{\text{db}} = 6.02\,\text{db/km}$, for $f = 0.90, \alpha_{\text{db}} = 10\text{db/km}$ and for $f = 0.99, \alpha_{\text{db}} = 20\,\text{db/km}$

10-15. (a) The ratio of the longest and shortest path lengths is,

$$\frac{L_{\text{short}}}{L_{\text{long}}} = \frac{n_2}{n_1} = \frac{1.45}{1.46}$$

so,

$$L_{\text{long}} = L_{\text{short}}\,(1.46/1.45) = (1\,\text{km})(1.46/1.45) = 1.0069\,\text{km}$$

(b) The transit times are ,

$$t_{\text{short}} = \frac{L_{\text{short}}}{v} = \frac{n_1}{c}L_{\text{short}} = \frac{1.46}{3\times 10^5\,\text{km/s}}(1\,\text{km}) = 4.867\times 10^{-6}\,\text{s}$$

$$t_{\text{short}} = \frac{L_{\text{short}}}{v} = \frac{n_1}{c}L_{\text{short}} = \frac{1.46}{3\times 10^5\,\text{km/s}}(1.0069\,\text{km}) = 4.900\times 10^{-6}\,\text{s}$$

10-16. First find the index of refraction of the cladding:

$$\sin\theta_m = \sqrt{n_1^2 - n_2^2} \Rightarrow n_2 = \sqrt{n_1^2 - \sin^2\theta_m} = \sqrt{1.446^2 - \sin^2(35°)} = 1.3274$$

Then,

$$\delta\left(\frac{\tau}{L}\right) = \frac{n_1}{c}\left(\frac{n_1 - n_2}{n_2}\right) = \frac{1.446}{3\times 10^{-4}\,\text{km/ns}}\left(\frac{1.446 - 1.3274}{1.3274}\right) = 431\,\text{ns/km}$$

For non overlapping pulses,

$$\delta(\tau) > T = 1/\nu \Rightarrow \nu_{\max} = 1/\delta(\tau) = 1/(431\,\text{ns}) = 2.32\,\text{MHz}$$

10-17. The entrance angle $\theta = 15°$ is not necessarily θ_m so Eq. (10 14),

$$\delta(\tau) = \frac{\Delta L}{v} = \frac{n_1}{c}\Delta L = \frac{n_1}{c}(x_t - L) = \frac{n_1}{c}L\left(\frac{n_1}{\sqrt{n_1^2 - \sin^2\theta}} - 1\right) = \frac{(1.48)\,(1\,\text{km})}{3\times 10^{-4}\,\text{km/ns}}\left[\frac{1.48}{\sqrt{1.48^2 - \sin^2(15°)}} - 1\right]$$

$$\delta(\tau) = 77.2\,\text{ns}$$

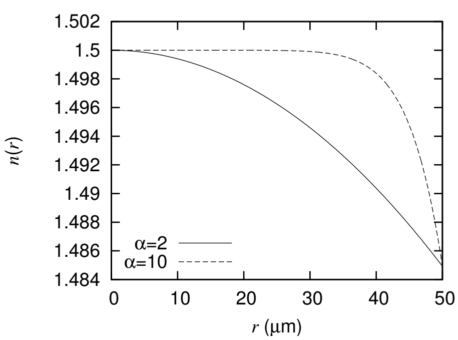
Here the expression for x_t is given by problem 10 6b.

10-18. The group delay is

$$\delta(\tau/L) = \frac{n_1}{c}\left(\frac{n_1 - n_2}{n_2}\right) = \frac{1.46}{3\times 10^{-4}\,\text{km/ns}}(0.003) = 14.6\,\text{ns/km}$$

10-19. Using Eq. (10 15) the refractive index profiles is seen to be,

$$n(r) = n_1\sqrt{1 - 2\Delta(r/a)^\alpha} = 1.5\sqrt{1 - (0.02)\,(r/50\ \mu\text{m})^\alpha}$$



10-20. The relative index difference is,

$$\Delta = \frac{n_1 - n_2}{n_2} = \frac{1.46 - 1.44}{1.46} = 0.0137$$

Using Eq. (10 16) and the relation that follows,

$$\delta(\tau)_G = \frac{n_1 L}{2c}\Delta^2 = \frac{1.46\,(1\,\text{km})}{2\,(3\times 10^{-4}\,\text{km/ns})}(0.0137)^2 = 457\,\text{ps}$$

$$\delta(\tau)_{SI} = \frac{2}{\Delta}\delta(\tau)_G = \frac{2}{0.0137}\delta(\tau)_G = 146\times 457\,\text{ps} = 66.7\,\text{ns}$$

10-21. $\nu_{\max} L = \dfrac{0.5}{\delta(\tau/L)} \Rightarrow \nu_{\max} = \dfrac{0.5}{L\,\delta(\tau/L)} = \dfrac{0.5}{(1\,\text{km})\,(20\,\text{ns/km})} = 25\,\text{MHz}$

10-22. At $\lambda = 820\,\text{nm}$, Figure 10 13 gives $M \approx 100\,\text{ps/nm km}$. Then $\delta(\tau/L) = -M\,\Delta\lambda$. So,

For the LED: $\delta(\tau) = -M\,\Delta\lambda\,L = (100)\,(40)\,(1)\,\text{ps} = 4000\,\text{ps} = 4\,\text{ns}$.
For the LD: $\delta(\tau) = -M\,\Delta\lambda\,L = (100)\,(4)\,(1)\,\text{ps} = 400\,\text{ps} = 0.4\,\text{ns}$.

10-23. The total time delay is found as

$$\delta\tau = \sqrt{\delta\tau_{\text{mod}}^2 + \delta\tau_{\text{mat}}^2}$$

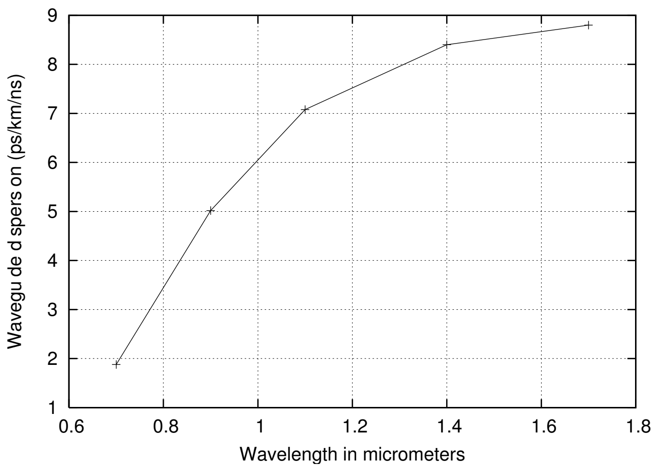
where,

$$\delta\tau_{\text{mod}} = \frac{L\,n_1}{c}\,\Delta = \frac{(1\,\text{km})\,(1.46)}{3\times10^{-4}\,\text{km/ns}}\,(0.01) = 48.7\,\text{ns}$$
$$\delta\tau_{\text{mat}} = M\,L\,\Delta\lambda = (100)\,(1)\,(40)\,\text{ps} = 4\,\text{ns}$$

So,

$$\delta\tau = \sqrt{48.7^2 + 4^2}\,\text{ns} = 48.9\,\text{ns}$$

10-24. (a)The plot is shown below,



(b) From the plot:
$M' \approx 3.9\,\text{ps/km nm at } 1.27\,\mu\text{m}$
$M' \approx 4.3\,\text{ps/km nm at } 1.55\,\mu\text{m}$
Then,
At $1.27\,\mu$m:
$\Delta(\tau/L) = M'\,\Delta\lambda = (3.9)\,(1)\frac{\text{ps}}{\text{km}} = 3.9\,\frac{\text{ps}}{\text{km}}$
At $1.55\,\mu$m:
$\Delta(\tau/L) = M'\,\Delta\lambda = (4.3)(1)\frac{\text{ps}}{\text{km}} = 4.3\,\frac{\text{ps}}{\text{km}}$

10-25. (a) Modal:

$$\delta\tau_{\text{mod}} = \frac{n_1\,L}{c}\left(\frac{n_1 - n_2}{n_2}\right) = \frac{(1.47)(1)}{3\times10^{-4}}\left(\frac{1.47 - 1.455}{1.455}\right)\text{ns} = 50.5\,\text{ns}$$

Material:

$$\delta\tau_{\text{mat}} = L\,M\,\Delta\lambda = (1)\,(43)\,(25)\,\text{ps} = 1.075\,\text{ns}$$

Waveguide:

$$\delta\tau_{\text{WG}} = L\,M'\,\Delta\lambda = (1)\,(3)\,(25)\,\text{ps} = 0.075\,\text{ns}$$

(b) The total is then,

$$\delta\tau = \sqrt{\delta\tau_{\text{mod}}^2 + \delta\tau_{\text{mat}}^2 + \delta\tau_{\text{WG}}^2} \approx 50.5\,\text{ns}$$

10-26. Consider Figure 10 16a in the body of the text. Let the phase difference due to the differing lengths of the paths of the two beams be $k\,\Delta L$. Then the phase difference for the light exiting through Output 1 is $\Delta\varphi_1 = k\,\Delta L$ and that for the light exiting through Output 2 is $\Delta\varphi_2 = k\,\Delta L + \pi$, where the addition of π is due to the fact that $r_1' = -1/\sqrt{2} = e^{i\pi}\,1/\sqrt{2} = e^{i\pi}\,r_2$. Now the transmission coefficients at each beam splitter must be $t_1 = t_2 = 1/\sqrt{2}$ so that the amplitudes each of the four individual beams exiting through an output port is

$$E_{0,\text{out}} = (1/\sqrt{2})(1/\sqrt{2})E_{0,\text{in}} = \frac{1}{2}\,E_{\text{in}}$$

The irradiance of each of the four individual output beams is then, $I_{\text{out}} = I_{\text{in}}/4$. The total irradiance at each output port is then given by the two beam, equal irradiance, interference relation,

$$I_{T,\,\text{port 1}} = 2\,I_{\text{out}}(1 + \cos(\Delta\varphi_1)) = 2\frac{I_{\text{in}}}{4}[1 + \cos(k\,\Delta L)] = \frac{I_{\text{in}}}{2}[1 + \cos(k\,\Delta L)]$$
$$I_{T,\,\text{port 2}} = 2\,I_{\text{out}}(1 + \cos(\Delta\varphi_2)) = 2\frac{I_{\text{in}}}{4}[1 + \cos(k\,\Delta L + \pi)] = \frac{I_{\text{in}}}{2}[1 - \cos(k\,\Delta L)]$$

Thus the total irradiance through the two outputs is, $I_T = I_{T,\,\text{port 1}} + I_{T,\,\text{port 2}} = I_{\text{in}}$.

10-27. (a) The frequency difference is $\Delta\nu = \dfrac{\Delta\lambda}{\lambda}\,\nu = \dfrac{0.8}{1550}\,\dfrac{3\times 10^{8}}{1550\times 10^{-9}}\,\text{Hz} = 10\,0.0\ \text{GHz}$

(b) $40\times 10\,\text{GHz} = 4\,\text{THz}$.

10-28. (a) The conditions to be satisfied are,

$$\Delta L = m\,\lambda_{1n} = (m+1/2)\,\lambda_{2n}$$

Here $\lambda_{1,2n} = \lambda_{1,2}/n = \lambda_{1,2}/1.5$. No ΔL satisfies this relation with exactly integer m but a reasonably good solution is

$$\Delta L = 950.01\,\lambda_{1n} = (950.01)\,(1550.8\,\text{nm})/1.5 = 950.500\,\lambda_{2n}$$

Many other approximate solutions exist.

(b) Since,

$$\frac{\Delta L}{\lambda_{3n}} = \frac{950.01\,\lambda_{1n}}{\lambda_{3n}} = 950.01\left(\frac{1550.8}{1551.6}\right) = 949.5201778808971$$

is close to an integer $+1/2$. This light will exit primarily through Output 2.

(c) The total irradiances are given by the expressions in the solution to problem 10 26:

$$R = \frac{I_{T,2}}{I_{T,1}} = \frac{1-\cos(2\,\pi\Delta L/\lambda_n)}{1+\cos(2\,\pi\,\Delta L/\lambda_n)}$$

Here,

$$2\,\pi\,\Delta L/\lambda_n = 2\,\pi(950.01)\,\lambda_{1n}/\lambda_{2n} = (2\,\pi)\,(950.01)\,(1550.8/1550.4) = 1900.51\,\pi$$

and

$$R = 1.065 \Rightarrow 1/R = 0.94$$

Chapter 11 Fraunhofer Diffraction

11-1. See Figure 11 18 that accompanies the problem in the text for a sketch of the setup. The minima are located as position, y_m determined as,

$$m\,\lambda = b\sin\theta_m = b\,y_m/f \Rightarrow y_m = m\,\lambda\,f/b$$

(a) The first minimum occurs at $y_1 = \lambda\,f/b = (546.1\times10^{-6}\,\text{mm})\,(60\,\text{cm})/(0.015\,\text{cm}) = 2.18\,\text{mm}$.
(b) The separation of the first and second minimum is

$$y_2 - y_1 = (2-1)\lambda\,f/b = 2.18\,\text{mm}$$

11-2. Refer to Figure 11 1 and the surrounding discussion. The ratio I/I_0 is given be Eq. 11 10 is

$$I/I_0 = \frac{\sin^2\beta}{\beta^2}$$

where $\beta = \frac{1}{2}\,k\,b\sin\theta$. The path difference from the edges of the slit to a point on the screen is $\Delta = b\sin\theta$. So when $\Delta = 3/4\,\lambda$,

$$\beta = \frac{1}{2}\,k\,(3/4)\,\lambda = \frac{1}{2}\,\frac{2\,\pi}{\lambda}\,\frac{3}{4}\,\lambda = 3\,\pi/4$$

Under this condition,

$$\frac{I}{I_0} = \frac{\sin^2(3\,\pi/4)}{(3\,\pi/4)^2} = 0.090$$

11-3. See Figure 11 19 that accompanies the problem in the text.
(a) The diffraction minima are located at angles $\theta_m = y_m/L$ where $L = 2$ m is the slit to screen distance, The positions of the minima are given by $m\,\lambda = b\sin\theta_m = by_m/L \Rightarrow y_m = m\lambda L/b$. Then,

$$y_3 - y_{-3} = \Delta y = (3-(-3))\lambda\,L/b \Rightarrow b = \frac{6\,\lambda\,L}{\Delta y} = \frac{6\,(632.8\times10^{-7}\,\text{cm})\,(200\,\text{cm})}{5.625\,\text{cm}} = 0.013\,\text{cm} = 0.13\,\text{mm}$$

(b) $L_{\text{min}} = b^2/2\lambda$, so,

$$\frac{L}{L_{\text{min}}} = \frac{200\,\text{cm}}{(0.0135\,\text{cm})^2/(2\cdot632.8\times10^{-7}\,\text{cm})} = 139$$

The screen is in the far field.

11-4. Let $m_1 = 5$ for λ_1 and $m_2 = 4$ for λ_2. Then,

$$m_1\,\lambda_1 = m_2\,\lambda_2 = b\sin\theta$$
$$5\,\lambda_1 = 4\,\lambda_2 = 4\,(620\,\text{nm}) \Rightarrow \lambda_1 = 496\,\text{nm}$$

11-5. Let the full angle breadth between the first minim on either side of the central maximum be $\varphi = 2\,\theta$, where θ is the angle that locates the first minimum relative to the center of the pattern. For $m = 1$,

$$\lambda = b\sin\theta = b\sin(\varphi/2) \Rightarrow b = \frac{\lambda}{\sin(\varphi/2)} = \frac{550\,\text{nm}}{\sin(\varphi/2)}$$

For $\varphi = 30°$, $b = 2.125\,\mu$m, for $\varphi = 45°$, $b = 1.437\,\mu$m, for $\varphi = 90°$, $b = 0.778\,\mu$m, for $\varphi = 180°$, $b = 0.55\mu$m.

11-6. (a) $\sin\theta = m\,\lambda/b = \dfrac{(1)\,(550)}{2125} \Rightarrow \theta = 15°$

(b) $\beta = \dfrac{k\,b}{2}\sin\theta = \dfrac{\pi\,b}{\lambda}\sin\theta = \dfrac{\pi\,(2.125\ \mu\text{m})}{0.55\ \mu\text{m}}\sin\theta = 12.138\sin\theta$. Then

$$I/I_0 = \sin^2\beta/\beta^2$$

$$\begin{aligned}
\theta = 5°:&\quad \beta = 1.058\,\text{rad}, \quad I/I_0 = 0.678\\
\theta = 10°:&\quad \beta = 2.108\,\text{rad}, \quad I/I_0 = 0.166\\
\theta = 15°:&\quad \beta = 3.142\,\text{rad}, \quad I/I_0 \approx 0\\
\theta = 22.5°:&\quad \beta = 4.645\,\text{rad}, \quad I/I_0 \approx 0.0461
\end{aligned}$$

11-7. (a) The fourth and fifth intersections of $\beta = \tan\beta$ occur at $4.477\,\pi$ and $5.482\,\pi$.

(b) $\dfrac{I_{\beta=4.477\pi}}{I_{\beta=0}} = \dfrac{\sin^2(4.477\,\pi)/(4.477\,\pi)^2}{1} = \dfrac{1}{199};\ \dfrac{I_{\beta=5.482\pi}}{I_{\beta=0}} = \dfrac{\sin^2(5.482\,\pi)/(5.482\pi)^2}{1} = \dfrac{1}{298}$

11-8. For the single slit:

$$\frac{I_1}{I_0} = \frac{\sin^2(1.43\,\pi)/(1.43\pi)^2}{1} = 0.047$$
$$\frac{I_2}{I_0} = \frac{\sin^2(2.46\,\pi)/(2.46\pi)^2}{1} = 0.017$$

For the circular aperture: (See Figure 11 8b)

$$\frac{I_1}{I_0} = 0.0175$$
$$\frac{I_2}{I_0} = 0.00416$$

11-9. $\gamma = \dfrac{k}{2}D\sin\theta = \dfrac{\pi\,D}{\lambda}\sin\theta = \dfrac{\pi\,D}{\lambda}\dfrac{y}{f} \Rightarrow y = \dfrac{f\,\lambda\,\gamma}{\pi\,D} = \gamma\left[\dfrac{(1706.9)\,(5.5\times10^{-5})}{91.44\pi}\right]\text{cm} = (3.268\times10^{-4}\,\text{cm})\,\gamma$

$$1^{\text{st}}\ \text{secondary maximum}\ (\gamma = 5.15)\text{: } y = 1.68\times10^{-3}\,\text{cm}$$

$$2^{\text{nd}}\ \text{secondary maximum}\ (\gamma = 8.42)\text{: } y = 2.75\,10^{-3}\,\text{cm}$$

11-10. $1.22\,D\sin\theta = D\,y/f \Rightarrow y = R = 1.22\,\lambda\,f/D = (1.22)(5.5\times10^{-5}\,\text{cm})(150)/12 = 8.39\times10^{-4}\,\text{cm}$

11-11. Using Eq. (11 21) the angular half width of the Airy disc formed on the moon will be,

$$\Delta\theta_{1/2} = \frac{1.22\,\lambda}{D}$$

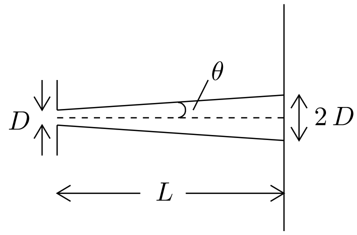
where D is the diameter of the circular aperture. The radius R of the airy disc formed on the moon, which is a distance L from the aperture is

$$R = L\tan\Delta\theta_{1/2} \approx L\,\Delta\theta_{1/2} = \frac{1.22\,\lambda\,L}{D} = \frac{1.22\,(10.6\times10^{-6}\,\text{m})(3.76\times10^{8}\,\text{m})}{10^{-3}\,\text{m}} = 4.86\times10^{6}\,\text{m}$$

The diameter of the laser spot on the moon is about 9.72×10^6 m. The irradiance in the spot (assuming a nearly constant irradiance over the spot (this is not really the best approximation, but it gives an order of magnitude estimate),

$$I = \frac{\Phi}{A} = \frac{\Phi}{\pi\,R^2} = \frac{2000\,\text{W}}{\pi\,(4.86\times10^{6}\,\text{m})^2} = 2.7\times10^{-11}\,\text{W/m}^2$$

11-12.



The angular radius θ of the first minimum in the diffraction pattern is given by the relation, $\sin\theta \approx \theta = 1.22\,\lambda/D$. Then from the geometry illustrated in the figure,

$$\theta \approx \frac{D}{L} = 1.22\,\lambda/D \Rightarrow L = \frac{D^2}{1.22\,\lambda} = \frac{(2\times 10^{-3}\,\text{m})^2}{1.22\,(632.8\times 10^{-9}\,\text{m})} = 5.18\,\text{m}$$

11-13. The distance L for the headlights to be barely resolvable if they are separated by a distance y is given be Eq. (11 22), as,

$$\Delta\theta_{\min} = y/L = 1.22\,\lambda/D \Rightarrow L = \frac{y\,D}{1.22\,\lambda} = \frac{(45\times 2.54\,\text{cm})(0.5\,\text{cm})}{1.22\,(5.5\times 10^{-5}\,\text{cm})} = 8.517\times 10^5\,\text{cm} = 27,900\,\text{ft} = 5.3\,\text{miles}$$

11-14. (a) The minimum and maximum distances are for a line separation of $s = 1\,\text{mm}$ is

Min: $$\Delta\theta = \frac{s}{L} = \frac{1.22\,\lambda}{D} \Rightarrow L = \frac{s\,D_{\min}}{1.22\,\lambda} = \frac{(10^{-3})\left(2\times 10^{-3}\right)}{1.22\cdot 550\times 10^{-9}}\,\text{m} = 3.0\,\text{m}$$

Max: $$L = \frac{s\,D_{\max}}{1.22\,\lambda} = \frac{(10^{-3})\left(7\times 10^{-3}\right)}{1.22\cdot 550\times 10^{-9}}\,\text{m} = 10.4\,\text{m}$$

(b) The pupil diameter will be

$$D = \frac{1.22\,\lambda\,L}{s} = \frac{1.22(550\times 10^{-9})L}{10^{-3}} = 6.71\times 10^{-4}\,L$$

11-15. (a) According to Eq. (11 30) the condition for missing orders is, $a = (p/m)\,b$. The fourth order interference maxima are missing so $p = 4\,m$ and $a = 4\,b = 4\,(0.1\,\text{mm}) = 0.4\,\text{mm}$.
(b) The irradiance is given by,

$$I = 4\,I_0\left(\frac{\sin\beta}{\beta}\right)^2\cos^2\alpha$$

In zeroth order $I = 4\,I_0$. The interference maxima occur for,

$$p\,\lambda = a\sin\theta \Rightarrow \sin\theta = 0, \pm\lambda/a, \pm 2\,\lambda/a, \pm 3\,\lambda/a, \ldots$$

Also $\cos^2\alpha = 1$, so at the interference maxima

$$I = 4\,I_0\left(\frac{\sin\beta}{\beta}\right)^2$$

where $\beta = (k\,b/2)\sin\theta = (\pi\,b/\lambda)\sin\theta$.Then:

$$p=1:\ \sin\theta = \lambda/a;\ \beta = \frac{\pi\,b}{\lambda}\frac{\lambda}{a} = \pi\,(b/a);\ \frac{I}{4\,I_0} = \left(\frac{\sin\beta}{\beta}\right)^2 = \left(\frac{\sin(\pi/4)}{\pi/4}\right)^2 = 0.8106$$

$$p=2:\ \sin\theta = 2\lambda/a;\ \beta = \frac{\pi\,b}{\lambda}\frac{2\lambda}{a} = 2\pi\,(b/a);\ \frac{I}{4\,I_0} = \left(\frac{\sin\beta}{\beta}\right)^2 = \left(\frac{\sin(\pi/2)}{\pi/2}\right)^2 = 0.4053$$

$$p=3:\ \sin\theta = 3\lambda/a;\ \beta = \frac{\pi\,b}{\lambda}\frac{3\lambda}{a} = 3\pi\,(b/a);\ \frac{I}{4\,I_0} = \left(\frac{\sin\beta}{\beta}\right)^2 = \left(\frac{\sin(3\,\pi/4)}{3\,\pi/4}\right)^2 = 0.0.0901$$

11-16. (a) If $a/b = N$ where N is an integer, the the N^{th} fringe is missing on either side of the central maximum. If a/b falls between $N-1$ and N, then the $(N-1)^{\text{th}}$ fringes appear on either side. The total number of fringes is (including the central maximum) is thus $2\,(N-1)+1 = 2\,(a/b-1)+1 = 2\,(a/b)-1$.

(b) For 13 bright fringes in the central diffraction peak,

$$2\,(a/b) - 1 = 13 \Rightarrow a/b = 7 \Rightarrow a = 7\,b = 7\,(0.3\,\text{mm}) = 2.1\,\text{mm}$$

11-17. (a) Diffraction minima occur for,

$$m\,\lambda = b\sin\theta \Rightarrow m\,\lambda \approx b\,\theta$$

Thus the angular width of the central ($m=1$) diffraction peak is $(\Delta\theta_{1/2})_{\text{diff}} = \lambda/b$.

Interference minima occur for,

$$(p+1/2)\,\lambda = a\sin\theta \approx a\,\theta \Rightarrow (\Delta\theta_{1/2})_{\text{int}} = \lambda/2\,a \ (p=0)$$

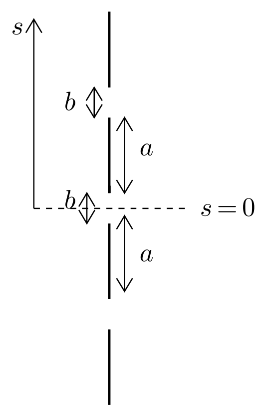
Thus,

$$\frac{(\Delta\theta_{1/2})_{\text{diff}}}{(\Delta\theta_{1/2})_{\text{int}}} = \frac{\lambda/b}{\lambda/2a} = 2\,(a/b)$$

(b) For $a = 10\,b$:

$$\frac{\text{peak width}}{\text{fringe width}} = 2\,(a/b) = 20\,\times$$

11-18.



Consider the geometry shown at left. Begin with an expression like Eq. (11 4) but for three slits. Then,

$$E_P = \frac{E_L}{r_0} e^{i(kr_0-\omega t)} \left\{ \int_{\frac{-b}{2}}^{\frac{b}{2}} e^{iks\sin\theta} ds + \int_{-a-\frac{b}{2}}^{-a+\frac{b}{2}} e^{iks\sin\theta} ds + \int_{a-\frac{b}{2}}^{a+\frac{b}{2}} e^{iks\sin\theta} ds \right\}$$

Taking the integrals in turn:

$$\int_{\frac{-b}{2}}^{\frac{b}{2}} e^{iks\sin\theta} ds = b\sin(\beta)/\beta \text{ as for the single slit in the text. Here } \beta = \frac{k}{2} b\sin\theta.$$

$$\int_{-a-\frac{b}{2}}^{-a+\frac{b}{2}} e^{iks\sin\theta} ds = \frac{1}{i\,k\sin\theta}\Big[e^{ik\sin\theta(a+b/2)} - e^{ik\sin\theta(a-b/2)}\Big] = \frac{e^{2i\alpha}}{k\sin\theta}(2\sin\beta)$$

where $\alpha = \frac{k}{2} a\sin\theta$. The final integral follows from the one just done by letting $a \to -a, \alpha \to -\alpha$. That is

$$\int_{a-\frac{b}{2}}^{a+\frac{b}{2}} e^{iks\sin\theta} ds = \frac{e^{-2i\alpha}}{i\,k\sin\theta}(2\,i\sin\beta)$$

Altogether, then,

$$E_P = \frac{E_L}{r_0} e^{i(kr_0-\omega t)} \left\{ b\sin(\beta)/\beta + \frac{e^{2i\alpha}}{k\sin\theta}(2\sin\beta) + \frac{e^{-2i\alpha}}{k\sin\theta}(2\sin\beta) \right\}$$

$$E_P = \frac{E_L}{r_0} e^{i(kr_0-\omega t)} \left\{ \frac{b\sin(\beta)}{\beta} + \frac{4\sin\beta\cos 2\,\alpha}{k\sin\theta} \right\} = \frac{E_L}{r_0} e^{i(kr_0-\omega t)} \left\{ \frac{b\sin(\beta)}{\beta} + \frac{4\sin\beta\cos 2\,\alpha}{2\,\beta/b} \right\}$$

$$E_P = \frac{E_L b}{r_0} e^{i(kr_0-\omega t)} \frac{\sin\beta}{\beta}(1+2\cos 2\,\alpha)$$

$$I = I_0 \left(\frac{\sin\beta}{\beta}\right)^2 (1+\cos 2\,\alpha)^2$$

This result can be put into the general form by noting,

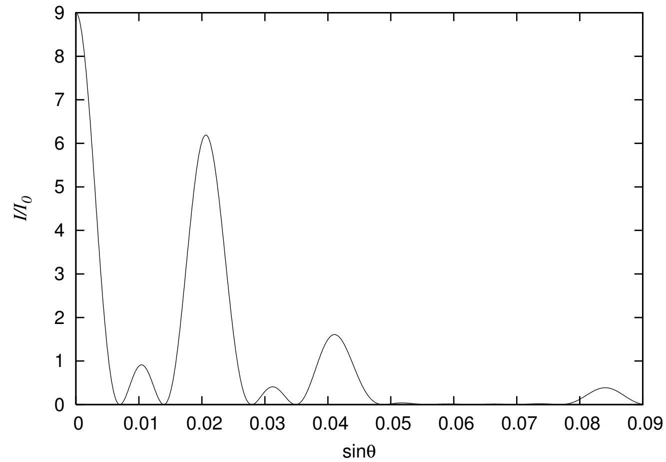
$$1+2\cos 2\,\alpha = 1+2\left(1-2\sin^2\alpha\right) = 3-4\sin^2\alpha = \frac{3\sin\alpha - 4\sin^3\alpha}{\sin\alpha} = \frac{\sin 3\alpha}{\sin\alpha}$$

So,

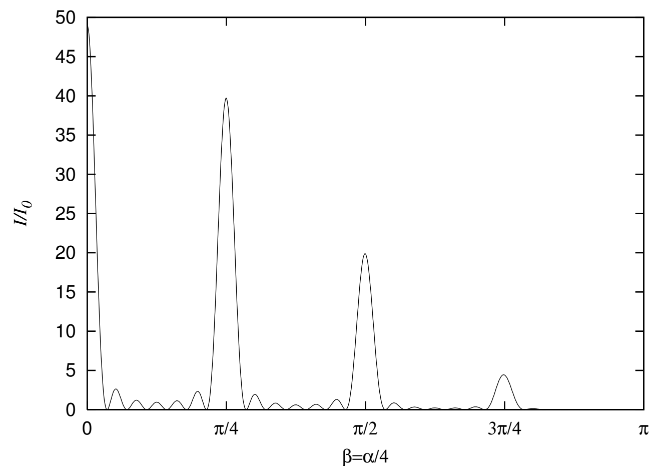
$$I = I_0 \left(\frac{\sin\beta}{\beta}\right)^2 \left(\frac{\sin 3\alpha}{\sin\alpha}\right)^2$$

A plot is shown on the next page.

11-18. *continued*. For $a=3\,b$, and $k\,b=50$, the plot is shown below.



11-19. For $N=7$, $\alpha/\beta=4$



11-20. The irradiance is given by,

$$I=I_0\left(\frac{\sin\beta}{\beta}\right)^2\left(\frac{\sin N\alpha}{\alpha}\right)^2$$

with $N=10$, $a=5\ b$, $b=10^{-4}$ cm, $\lambda=435.8$ nm. Recall that $\beta=\dfrac{\pi}{\lambda}\,b\,\sin\,\theta$, so $\alpha=4\ \beta$. For interference maxima,

$$\left(\frac{\sin N\alpha}{\alpha}\right)^2=1\Rightarrow I=I_0\left(\frac{\sin\beta}{\beta}\right)^2$$

Also, $\sin\theta=m\,\lambda/a$ and $\beta=\dfrac{\pi\,b}{\lambda}\left(\dfrac{m\,\lambda}{a}\right)=m\,\pi\,(b/a)=m\,\pi/5$. Then,

For $m=1$: $I/I_0=\left(\dfrac{\sin\beta}{\beta}\right)^2=\left(\dfrac{\sin(\pi/5)}{\pi/5}\right)^2=0.875$

For $m=2$: $I/I_0=\left(\dfrac{\sin(2\,\pi/5)}{2\,\pi/5}\right)^2=0.573$. For $m=3$: $I/I_0=\left(\dfrac{\sin(3\,\pi/5)}{3\,\pi/5}\right)^2=0.255$

For $m=4$: $I/I_0=\left(\dfrac{\sin(4\,\pi/5)}{4\,\pi/5}\right)^2=0.0547$. For $m=5$: $I/I_0=\left(\dfrac{\sin(5\,\pi/5)}{5\pi/5}\right)^2=0.$

11-21. Consider an N slit aperture, with N an odd number, arranged like the 3 slit aperture of problem 11 18 with the origin at the middle of the central slit. Then, the electric field at point P on the screen has the form,

$$E_P = \frac{E_L}{r_0} e^{i(kr_0 - \omega t)} \left\{ \int_{\frac{-b}{2}}^{\frac{b}{2}} e^{iks\sin\theta} ds + \sum_{j=1}^{(N-1)/2} \left[\int_{-ja-\frac{b}{2}}^{-ja+\frac{b}{2}} e^{iks\sin\theta} ds + \int_{ja-\frac{b}{2}}^{ja+\frac{b}{2}} e^{iks\sin\theta} ds \right] \right\}$$

The first integral is just the single slit integral simplified in the text. That is,

$$\int_{\frac{-b}{2}}^{\frac{b}{2}} e^{iks\sin\theta} ds = \frac{b\sin\beta}{\beta}, \beta = \frac{k}{2} b\sin\theta$$

The sum of the integrals in the square brackets is,

$$\int_{-ja-\frac{b}{2}}^{-ja+\frac{b}{2}} e^{iks\sin\theta} ds + \int_{ja-\frac{b}{2}}^{ja+\frac{b}{2}} e^{iks\sin\theta} ds = \frac{e^{-2ij\alpha}(e^{i\beta} - e^{-i\beta}) + e^{-2ij\alpha}(e^{i\beta} - e^{-i\beta})}{i\,k\sin\theta} = \frac{4\,i\,(\sin\beta)\,(\cos 2\,j\,\alpha)}{i\,k\sin\theta}$$

$$\int_{-ja-\frac{b}{2}}^{-ja+\frac{b}{2}} e^{iks\sin\theta} ds + \int_{ja-\frac{b}{2}}^{ja+\frac{b}{2}} e^{iks\sin\theta} ds = \frac{2\,b\,(\sin\beta)\,(\cos 2\,j\,\alpha)}{\beta}$$

where $\alpha = \frac{k}{2} a\sin\theta$. Then,

$$\sum_{j=1}^{(N-1)/2} \left[\int_{-ja-\frac{b}{2}}^{-ja+\frac{b}{2}} e^{iks\sin\theta} ds + \int_{ja-\frac{b}{2}}^{ja+\frac{b}{2}} e^{iks\sin\theta} ds \right] = \frac{2\,b\,(\sin\beta)}{\beta} \sum_{j=1}^{(N-1)/2} \cos 2j\alpha$$

The sum can be performed as,

$$\sum_{j=1}^{(N-1)/2} \cos 2j\alpha = \mathrm{Re}\left(\sum_{j=1}^{(N-1)/2} e^{2ij\alpha} \right) = \mathrm{Re}\left\{ \sum_{j=1}^{(N-1)/2} (e^{2i\alpha})^j \right\} = \mathrm{Re}\left\{ \frac{(e^{2i\alpha})^{(N-1)/2} - 1}{e^{2i\alpha} - 1} \right\}$$

$$\sum_{j=1}^{(N-1)/2} \cos 2j\alpha = \mathrm{Re}\left(\frac{e^{iN\alpha} - e^{-i\alpha}}{e^{i\alpha} - e^{-i\alpha}} \right) = \mathrm{Re}\left(\frac{\cos N\alpha + i\sin N\alpha - \cos\alpha + i\sin\alpha}{2\,i\sin\alpha} \right) = \frac{\sin N\alpha}{2\sin\alpha} - \frac{1}{2}$$

$$\sum_{j=1}^{(N-1)/2} \left[\int_{-ja-\frac{b}{2}}^{-ja+\frac{b}{2}} e^{iks\sin\theta} ds + \int_{ja-\frac{b}{2}}^{ja+\frac{b}{2}} e^{iks\sin\theta} ds \right] = \frac{2\,b\,(\sin\beta)}{\beta} \sum_{j=1}^{(N-1)/2} \cos 2j\alpha = \frac{b\,(\sin\beta)}{\beta} \left(\frac{\sin N\alpha}{\sin\alpha} - 1 \right)$$

$$E_P = \frac{E_L\,b}{r_0} e^{i(kr_0 - \omega t)} \frac{\sin\beta}{\beta} \left\{ 1 - \frac{\sin N\alpha}{\sin\alpha} + 1 \right\} = \frac{E_L\,b}{r_0} e^{i(kr_0 - \omega t)} \left(\frac{\sin\beta}{\beta} \right) \left(\frac{\sin N\alpha}{\sin\alpha} \right)$$

$$I = I_0 \left(\frac{\sin\beta}{\beta} \right)^2 \left(\frac{\sin N\alpha}{\sin\alpha} \right)^2$$

11-22. (a) See Figure 11 21 that accompanies the problem in the text. Proceeding apace,

$$I = I_0 \left(\frac{\sin\alpha}{\alpha} \right)^2 \left(\frac{\sin\beta}{\beta} \right)^2$$

$$\alpha = \frac{k}{2} a\sin\theta = \frac{\pi\,a}{\lambda}\frac{y}{f} = \frac{\pi\,(0.2)\,y}{(546 \times 10^{-6})\,(1000)\mathrm{mm}} = (1.151/\mathrm{mm})\,y$$

$$\beta = \frac{\pi\,(0.1)\,x}{(546 \times 10^{-6})\,(1000)} \mathrm{mm}^{-1} = (0.575/\mathrm{mm})\,x$$

$$I = I_0 \left[\frac{\sin(1.151y/\mathrm{mm})}{1.151y/\mathrm{mm}} \right]^2 \left[\frac{\sin(0.575\,x/\mathrm{mm})}{0.575\,x/\mathrm{mm}} \right]^2 = I_0 \frac{\sin^2(1.151y/\mathrm{mm})\sin^2(0.575\,x/\mathrm{mm})}{(0.438\,x^2\,y^2)/\mathrm{mm}^2}$$

(b) Minima occur first at $m = 1$ in $y = m\,\lambda\,f/a = 2.73\,\mathrm{mm}$ and $n = 1$ in $x = n\,\lambda\,f/b = 5.46\,\mathrm{mm}$.

(c) As $x \to 0, (\sin\beta)/\beta \to 1$ and as $y \to 0, (\sin\alpha)/\alpha \to 1$. So,

$$\text{At } x = 0, y = 1\,\mathrm{mm}, I = I_0 \left[\frac{\sin(1.151)}{1.151} \right]^2 = 0.629\,I_0 \quad \text{At } x = 1\,\mathrm{mm}, y = 0\,, I = I_0 \left[\frac{\sin(0.575)}{0.575} \right]^2 = 0.0.895 I_0$$

(d) At $x = 2\,\mathrm{mm}, y = 3\,\mathrm{mm},\ I_0 \left[\frac{\sin(3 \times 1.151)}{3 \times 1.151} \right]^2 \left[\frac{\sin(2 \times 0.575\,)}{2 \times 0.575} \right]^2 = 0.005\,I_0$

11-23. Since $m\,\lambda = a\sin\theta$, with $m=1$, $\sin\theta_{1/2} = \lambda/a$.

(a) $\sin\theta_{1/2} = \lambda/\lambda = 1 \Rightarrow \theta_{1/2} = 90°$ (b) $\sin\theta_{1/2} = \lambda/5\lambda = 0.2 \Rightarrow \theta_{1/2} = 11.5°$

(c) $\sin\theta_{1/2} = \lambda/10\lambda = 0.1 \Rightarrow \theta_{1/2} = 5.7°$

11-24. Using the large γ expression,

$$I = I_0\left(\frac{J_1(\gamma)}{\gamma}\right) \approx \frac{I_0}{\gamma^2}\left(\frac{\sin\gamma - \cos\gamma}{\sqrt{\pi}\,\gamma}\right)^2$$

Minima occur at $I=0$ or $\sin\gamma = \cos\gamma$. That is for $\gamma = \pi/4, 5\,\pi/4, 9\,\pi/4... = (4\,m-3)\,\pi/4$, $m = 1, 2, 3, ...$
So minima occur for

$$\gamma = \frac{k}{2} D\sin\theta = \frac{\pi\,D}{\lambda}\sin\theta = (4\,m-3)\,\pi/4 \Rightarrow \sin\theta = (4\,m-3)\,\lambda/4\,D$$

The angular separation between consecutive orders ($\Delta m = 1$) is

$$(\cos\theta)\,\Delta\theta = \frac{4\,\Delta m\,\lambda}{4\,D} = \frac{\Delta m\,\lambda}{D} \Rightarrow \Delta\theta = \frac{\lambda}{D\cos\theta}$$

11-25. We have $I = I_0(\sin^2\beta)/\beta^2$. At secondary maxima, $\beta_1 = 1.43\,\pi$, $\beta_2 = 2.46\,\pi$, $\beta_3 = 3.47\,\pi$ or approximately, $\beta_1 = 1.5\,\pi$, $\beta_2 = 2.5\,\pi$, $\beta_3 = 3.5\,\pi, ... \approx (m+1/2)\,\pi$. So that $\sin\beta \approx \pm 1$.

(a) Then$I = I_0\left(\frac{(\pm 1)}{\beta}\right)^2 = \frac{I_0}{[(m+1/2)\pi]^2}$

(b)

Approximate:	More Accurate
$I_1 \approx \frac{I_0}{(1.5\,\pi)^2} = 0.0450\,I_0$	$I_1 = \left(\frac{\sin(1.43\,\pi)}{1.43\,\pi}\right)^2 I_0 = 0.0472\,I_0$
$I_2 \approx \frac{I_0}{(2.5\,\pi)^2} = 0.0162\,I_0$	$I_2 = \left(\frac{\sin(2.46\,\pi)}{2.46\,\pi}\right)^2 I_0 = 0.0165 I_0$
$I_3 \approx \frac{I_0}{(3.5\,\pi)^2} = 0.00827\,I_0$	$I_3 = \left(\frac{\sin(3.47\,\pi)}{3.47\,\pi}\right)^2 I_0 = 0.00834\,I_0$

11-26. The principal maxima satisfy $m\,\lambda = a\sin\theta$, so that,

$$m_{\max} = \frac{a}{\lambda} = \frac{2/3}{1} = \frac{2}{3}$$

Thus $m=1$ is not allowed and only the central maximum ($m=0$) appears. The beam width is confined by the first minimum, given by $m\,\lambda = a\sin\theta$, with,

$$m = \frac{p}{N} = \frac{\pm 1}{N} = \pm\frac{1}{3}$$

Then

$$\sin\theta = \pm\frac{\lambda}{3}\frac{3}{2} = \pm\frac{1}{2} \Rightarrow \theta = 30°$$

The angular half width of the beam is 30°.

11-27. For three equal slits, the amplitude phasors must be equal and form a closed polygon an equilateral tri angle. Thus the phase difference must be $\varphi = 2\,\pi/3 = 120°$. Or, using Eq. (16 33), $I = 0$ when $\alpha = \ p\pi/N$ with $p = \pm 1, \pm 2, ...$

Then $\alpha = \pm\dfrac{\pi}{3}, \pm\dfrac{2\,\pi}{3}, ...$ and since $\varphi = 2\,\alpha$, $\ \varphi = \pm\dfrac{2\,\pi}{3}, \pm\dfrac{4\,\pi}{3}, ... = \pm 120°, \pm 240° ...$

The additional values of φ also produce equilateral triangles (notice that for $b \approx 0, \beta \approx 0$ and $(\sin\beta)/\beta \approx 1$.)

(b) If $\varphi = \pi$, the phasors alternately add and subtract leaving only one contributing. Thus, at P, $I_p \propto E_0^2$. While at the center the phasors all add and so $I_{\max} \propto (3\,E_0)^2 = 9\,E_0^2$. So $I_p = I_{\max}/9$. Or since $\varphi = 2\,\alpha \Rightarrow \alpha = \pi/2$, $I_P = I_0[(\sin(3\,\pi/2)/(\pi/2)]^2 = I_0$. Compared with $I_{\max} = 9\,I_0$, $I_P = I_{\max}/9$.

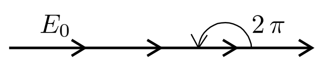
c) At a principal maximum $p = 0, \pm N, \pm 2N, ...$ and so $\alpha = \dfrac{p\,\pi}{N} = 0, \pm\pi, \pm 2\,\pi, ...$

Thus, $\sin(N\alpha)/\sin(\alpha) = \pm N$. Then $I_P = N^2\,I_0 = I_{\max}$ and $I_P/I_{\max} = 1$.

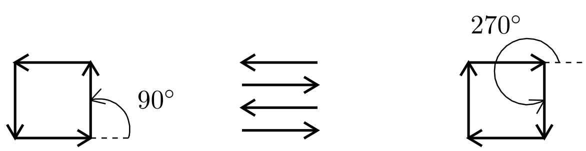
(d) Note that $I_{\text{av}} = I$ on the screen when it is uniformly illuminated with the energy from the three slits. Thus $I_{\text{ave}} = 3\,I_0$, as for the case of incoherent light. On the other hand, $I_P = 9\,I_0$ in the case of coherence. Thus $I_P = 3\,I_{\text{ave}}$.

11-28. Principal maxima and zeros of a 4 slit aperture in terms of phasor diagrams:

Principal maxima: $\alpha = 0, \pi, 2\,\pi, ... \Rightarrow \varphi = 2\,\alpha = 0, 2\,\pi, 4\,\pi, ...$



Zeros: $\alpha = p\,\pi/N = \pm\pi/4, \pm 2\,\pi/4, \pm 3\,\pi/4, ..., p = \pm 1, \pm 2, ... p$ but not equal to $\pm 4, \pm 8, ...$, $\varphi = 2\,\alpha = \pm\pi/2, \pm\pi, \pm 3\,\pi/2, ...$



$\varphi = 90°$ $\qquad \varphi = 180°$ $\qquad \varphi = 270°$

Chapter 12 The Diffraction Grating

12-1. The angular positions of constructive interference are described by $m\,\lambda = a\sin\theta$. For the case at hand $m = 2$, $\lambda_1 = 400\,\mathrm{nm}$, and $\lambda_2 = 600\,\mathrm{nm}$. Then,

$$\theta_1 = \sin^{-1}\left(\frac{m\,\lambda_1}{a}\right) = \sin^{-1}\left(\frac{2\,(400\times 10^{-7})\,\mathrm{cm}}{1/5000\,\mathrm{cm}}\right) = 23.58°$$

$$\theta_2 = \sin^{-1}\left(\frac{m\,\lambda_2}{a}\right) = \sin^{-1}\left(\frac{2\times 600^{-7}\,\mathrm{cm}}{1/5000\,\mathrm{cm}}\right) = 36.87°$$

The angular separation is then,

$$\Delta\,\theta = \theta_2 - \theta_1 = 13.292° = 13°18^{'}$$

12-2. (a) See Figure 12 13 that accompanies the statement of the problem in the text. With $m\ \lambda = a\ \sin\ \theta$, the angular dispersion is found as,

$$\mathfrak{D} \equiv \frac{d\,\theta}{d\,\lambda} = \frac{m}{a\cos\theta} = \frac{m}{a\cos\left(\sin^{-1} m\,\lambda/a\right)} = \frac{3}{\left(\frac{1}{3500}\right)\cos\left(\sin^{-1}\frac{3\times 650\times 10^{-7}}{1/3500}\right)} = 14366\,\mathrm{rad/cm}$$

$$\mathfrak{D} = 14366\,\frac{\mathrm{rad}}{\mathrm{cm}}\times\frac{180}{\pi}\,\frac{\mathrm{deg}}{\mathrm{rad}}\times 10^{-7}\frac{\mathrm{cm}}{\mathrm{nm}} = 0.0823\,\mathrm{deg/nm}$$

The linear dispersion is then,

$$\frac{dy}{d\lambda} = f\,\frac{d\,\theta}{d\,\lambda} = f\,\mathfrak{D} = 1500\,\mathrm{mm}\left(14366\times 10^{-7}\frac{\mathrm{rad}}{\mathrm{nm}}\right) = 2.1549\,\mathrm{mm/nm}$$

So the plate factor is,

$$\frac{d\lambda}{d\,y} = 0.464\,\mathrm{nm/mm}$$

(b) According to Eq. (12 11) he resolving power is, $\mathfrak{R} = m\,N = (3)\,(3500/\mathrm{cm})\,(6\,\mathrm{cm}) = 63,000$

12-3. (a) Principal maxima: $m\,\lambda = a\sin\theta$. So, for the maximum

$$\theta_1 = \sin^{-1}\left(\frac{m\,\lambda}{a}\right) = \sin^{-1}\left(\frac{2\times 600\times 10^{-7}}{10^{-3}}\right) = 6.8921°$$

Minima: $\left(m\pm\frac{1}{N}\right)\lambda = a\sin\theta$. So for the neighboring minimum,

$$\theta_2 = \sin^{-1}\frac{\left(m\pm\frac{1}{N}\right)\lambda}{a} = \sin^{-1}\frac{\left(2+\frac{1}{24}\right)\left(600\times 10^{-7}\right)}{10^{-3}} = 7.0364°$$

Then, $\Delta\,\theta = \theta_2 - \theta_1 = 0.1443° = 8.7'$. For $\left(m - \frac{1}{N}\right)$, we get $\sin\theta = 0.1175, \theta = 6.7478°$ and $\Delta\,\theta = 8.7'$ again.

(b) At the minimum, we found $\theta = 7.0364°$ or $\sin\theta = 0.1225$. Thus,

$$\lambda = \frac{a\sin\theta}{m} = \frac{(10^{-3\mathrm{cm}}\,(0.1225)}{2} = 6.125\times 10^{-5}\,\mathrm{cm} = 612.5\,\mathrm{nm}$$

If we had taken $\left(m - \frac{1}{N}\right) = (2 - 1/24)$ instead, $\sin\theta = 0.1175$ and

$$\lambda = \frac{a\sin\theta}{m} = \frac{(10^{-3}\,\mathrm{cm})\,(0.1175)}{2} = 5.875\times 10^{5}\,\mathrm{cm} = 587.5\,\mathrm{nm}$$

(c) The minimum angular separation is $(\Delta\,\lambda)_{\min} = 612.5\,\mathrm{nm} - 600\,\mathrm{nm} = 12.5\,\mathrm{nm}$. So,

$$\mathfrak{R}_{m=2} = \frac{\lambda}{\Delta\,\lambda_{\min}} = \frac{600}{12.5} = 48$$

Using Eq. (12 11), $\mathfrak{R} = m\,N = (2)\,(24) = 48$.

12-4. $\Re = m\,N = \dfrac{\lambda\,a\,v}{\Delta\,\lambda} = \dfrac{589.2935}{0.597} = 987$; $N = \dfrac{a}{m}$. So, for $m=1$, $N=987$ and for $m=2$, $N=494$.

12-5. The free spectral range is $\text{FSR} = \lambda_2 - \lambda_1 = \lambda_1/m$. Let $\lambda_1 = 350\,\text{nm}$, crown ; $\lambda_1 = 180\,\text{nm}$, quartz.

(a) Then,

$$\lambda_2 = \lambda_1 + \frac{\lambda_1}{m} = 350\,\text{nm} + \frac{350\,\text{nm}}{1} = 700\,\text{nm}, \quad \text{crown}$$
$$\lambda_2 = 180\,\text{nm} + \frac{180\,\text{nm}}{1} = 360\,\text{nm}, \quad \text{quartz}$$

(b) With $a = 1/1200\,\text{mm}$,

$$\theta = \sin^{-1}(m\,\lambda/a) = \sin^{-1}\left[\frac{(1)\,(700\times10^{-6})}{1/1200}\right] = 57.14^\circ, \text{ crown}$$
$$\theta = \sin^{-1}(m\,\lambda/a) = \sin^{-1}\left[\frac{(1)\,(360\times10^{-6})}{1/1200}\right] = 25.59^\circ, \text{ quartz}$$

(c) $\text{FSR} = \lambda_1/m$:

$$m=1\text{: FSR} = 350/1 = 350\,\text{nm, crown; FSR} = 180/1 = 180\,\text{nm, quartz}$$
$$m=2\text{: FSR} = 350/2 = 175\,\text{nm, crown; FSR} = 180/2 = 90\,\text{nm, quartz}$$

12-6. See Figure 12 14 that accompanies the statement of this problem in the text.

$$\Re\,(m=3) = m\,N = (3)\,(16,000\times2.5) = 120,000$$
$$\Re\,(m=2) = m\,N = (2)\,(16,000\times2.5) = 80,000$$
$$\Delta\,\lambda = \lambda/\Re = 550\,\text{nm}/80,000 = 0.006875\,\text{nm} = 0.069\,\text{Å}$$

12-7. (a) $\Re \equiv \dfrac{\lambda}{\Delta\,\lambda} = \dfrac{5893}{6} = 982.16 \Rightarrow 983$, $m = \Re/N = 983/400 = 2.46$; order $= 3$

(b) Any width smaller than the light beam.

12-8. (a) We have $m\,\lambda = a\sin\theta \approx a\,y_m/f$ or $y_m = m\,\lambda\,f/a$. Then,

$$m=0: y_0 = 0$$
$$m=1: y_1 = \frac{(1)\,(546\times10^{-6})\,(2000)}{0.005}\,\text{mm} = 218.4\,\text{mm}$$

Thus $\Delta\,y = y_1 - y_0 = 21.84\,\text{cm}$, independent of N.

(b) The first missing order occurs at $M = a/b = 5$.
Thus, the number of maxima under the central peak is $2\,(M-1)+1$ or $2\,M-1=9$. This is true for all 3 cases.

(c) For minima: $\left(m + \dfrac{1}{N}\right)\lambda = a\sin\theta$. Using zero order $(m=0)$, $\sin\theta \approx \theta = \lambda/a\,N$. So

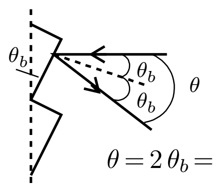
$$\text{Peak width} = 2\,\theta = 2\,\lambda/a\,N;\ \Delta\,y = f\,\Delta\,\theta = 2\,f\,\lambda/a\,N$$
$$N=2\text{: }\Delta\,y = \frac{2\,(200)\,(546\times10^{-7})}{(0.0005)\,(2)}\,\text{cm} = 21.84\,\text{cm},\ N=10\text{: }\Delta\,y = 4.368\,\text{cm},\ N=15,000\text{: }\Delta\,y = 0.0029\,\text{cm}$$

12-9. (a) The resolution is $\mathfrak{R} = \lambda/\Delta\,\lambda = 3500\,\text{Å}/0.02\,\text{Å} = 175,000 = m\,N$. So for $m = 2$, $N = 87,500$ grooves so that there are $87,500/10 = 8750$ grooves/cm.

(b) At normal incidence,

$$\theta_B = \frac{1}{2}\sin^{-1}\left(\frac{m\,\lambda}{a}\right)\frac{1}{2}\sin^{-1}\left(\frac{2\times 350\times 10^{-7}\,\text{cm}}{1/8750\,\text{cm}}\right) = 18.89°$$

(c)



$\theta = 2\,\theta_b = 37.77°$

(d) $\mathfrak{D} = \dfrac{m}{a\cos\theta} = \dfrac{2}{(1/8750)\cos 37.77} = 22139\,\dfrac{\text{rad}}{\text{cm}}$

$$\mathfrak{D} = 22139\,\frac{\text{rad}}{\text{cm}}\times\frac{180}{\pi}\,\frac{\text{deg}}{\text{rad}}\times 10^{-7}\,\frac{\text{cm}}{\text{nm}} = 0.1268\,\text{deg/nm}$$

$$\frac{d\,\lambda}{d\,\theta} = \frac{1}{\mathfrak{D}} = 7.88\,\text{nm/deg}$$

12-10. (a) The resolution is given by $\mathfrak{R} = \lambda/\Delta\lambda_{\min}$. Taking the worst case, or $\lambda = 700$ nm,

$$\mathfrak{R} = \frac{7000\,\text{Å}}{1\,\text{Å}} = 7000 = m\,N = (1)N \Rightarrow N = 7000$$

Then, $a = 2\,\text{cm}/7000$ grooves.

(b) The grating equation gives $m\,\lambda = a\sin\theta \approx a\,\theta \approx a\,y/f$. Thus, $m\,\Delta\lambda = a\,\Delta y/f$. S0,

$$\Delta\,y = \frac{m\,f\,\Delta\lambda}{a} = \frac{(1)\,(50\,\text{cm})\,(1\times 10^{-8}\,\text{cm})}{(2/7000)\,\text{cm}} = 0.00175\,\text{cm}$$

12-11. (a) Using $m\,\lambda = a\,(\sin\theta_i = \sin\theta_m) \Rightarrow \sin\theta_m = \dfrac{m\,\lambda}{a} - \sin\theta_i$,

$$\lambda = 400\,\text{nm: } \theta_m = \sin^{-1}\left[\frac{(1)\,(400)}{1000} - \sin 30°\right] = -\,5.74°$$

$$\lambda = 700\,\text{nm: } \theta_m = \sin^{-1}\left[\frac{(1)\,(700)}{1000} - \sin 30°\right] = +\,11.54°$$

(b) $W = 10\,\text{cm}$: $\mathfrak{R} = m\,N = (1)\,(10,000\,\text{cm}^{-1})\,(10\,\text{cm}) = 100,000$

(c) The plate factor is $d\,\lambda/d\,y = d\,\lambda/f\,d\,\theta = 1/f\,\mathfrak{D}$. The angular dispersion is,

$$\mathfrak{D} = \tan\theta/\lambda = \frac{\tan\,(\sin^{-1} m\,\lambda/a)}{\lambda} = \frac{\tan\,(\sin^{-1}(550/1000))}{550\,\text{nm}} = 0.0011974\,\text{rad/nm}$$

So the plate factor is

$$\frac{d\lambda}{dy} = \frac{1}{(100\,\text{cm})\,(0.00119737)\,\text{nm}^{-1}} = 8.3516\,\text{nm/cm} = 8.35\,\text{Å/mm}$$

(d) $R_{\text{circle}} = R_{\text{grating}}/2 = (1/2)\,(2\,\text{m}) = 1\,\text{m}$

12-12. The plate factor $P.F.$,

$$P.F. = \frac{1}{f\,\mathfrak{D}} = \frac{d\cos\theta}{f m}\,,\ \text{since}\ \mathfrak{D} = \frac{m}{a\cos\theta}\,.$$

Thus,

$$\frac{1}{a}\left(\frac{\text{gr}}{\text{cm}}\right) = \frac{\cos\theta}{m\,f\,(P.F.)}$$

The answer depends on θ. In first order we suppose the $P.F.$ is valid near $\theta = 0°$ so that $\cos\theta \approx 1$. Then,

$$\frac{1}{a} \approx \frac{1}{(1)\,(100\,\text{cm})\,(2\,\text{nm/mm})} = 0.005\,\frac{\text{mm}}{\text{nm}-\text{cm}}\times 10^{6}\,\frac{\text{nm}}{\text{mm}} = 5000\,\text{grooves/cm}$$

12-13. (a) For the Littrow mount: $m\,\lambda = 2\,a\sin\theta_b$, and so,

$$\lambda = \frac{2\,a\sin\theta_b}{m} = 2\,(\text{mm}/300)\sin 10° = 0.00116\,\text{mm} = 1.16\,\mu\text{m}.$$

(b) See the solution to problem 12 12. In this case,

$$P.F. = \frac{a\cos\theta}{m\,f} = \frac{(1\,\text{mm}/300)\,(\cos 20.322°)\,(10^6\,\text{nm/cm})}{(1)\,(170\,\text{cm})(10\,\text{mm/cm})} = 1.84\,\text{nm/mm}$$

(Here, $\sin\theta = m\,\lambda/a = 2\sin\theta_b = 2\sin 10° \Rightarrow \cos\theta = \cos 20.322°$)

12-14. See Figure 12 16 that accompanies the statement of this problem in the text. Generally,

$$\mathfrak{R} = m\,N = m\,(W/a) = (1)\,(15\,\text{cm}/a) \text{ or } a = (15\,\text{cm})/R = (15\,\text{cm})/300,000 = 5\times10^{-5}\text{cm} = 500\text{nm}$$

(a) Littrow: $\theta_b = \sin^{-1}\left(\frac{m\,\lambda}{2\,a}\right) = \sin^{-1}\left(\frac{1\times 200}{2\times 500}\right) = 11.5°$

(b) Normal: $2\,\theta_b = \sin^{-1}\left(\frac{m\,\lambda}{a}\right) = \sin^{-1}\left(\frac{1\times 200}{500}\right) = 23.578°$. So that, $\theta_b = 11.8°$

12-15. As given in the statement of the problem,

$$d = \frac{\lambda}{2\sin\theta} = \frac{488\times10^{-6}\,\text{mm}}{2\sin 60°} = 2.8175\times10^{-4}\,\text{mm} \Rightarrow \frac{1}{d} = 3550\,\text{grooves/mm}$$

If the emulsion is of refractive index n, then $\lambda \to \lambda_0/n$ and $d = \lambda_0/(2\,n\sin\theta)$. As n increases the grating con stant increases.

12-16. (a) $\mathfrak{R} = \frac{\lambda}{\Delta\,\lambda_{\text{min}}} = \frac{6563}{1.8} = m\,N$ or $N = \frac{6563}{1.8\,(1)} = 3647\,\text{grooves}$

(b) Littrow: $\theta_b = \sin^{-1}\left(\frac{m\,\lambda}{2\,a}\right)$ or $22.2° = \sin^{-1}\left(\frac{1\times 6300\,\text{Å}}{2\,a}\right) \Rightarrow \frac{6300\,\text{Å}}{2\,a} = 0.37784 \Rightarrow a = 8.337\times10^{-5}\,\text{cm}.$

Thus, $\frac{1}{a} = 11995\,\text{grooves/cm} \approx 1200\,\text{grooves/mm}.$

(c) $W_{\text{min}} = \frac{3647\,\text{gr}}{1200\,\text{gr/mm}} = 3.04\text{mm}$

12-17. (a) Littrow: $m\,\lambda = 2\,a\sin\theta_b$,

So $m = \frac{2\,a\sin\theta_b}{\lambda} = \frac{2\,(1\,\text{cm}/80)\sin 63°}{\lambda} = \frac{\sin 63°}{40}\frac{\text{cm}}{\lambda}.$

$\lambda = 400\times10^{-7}\,\text{cm}$, $m = 557$

$\lambda = 700\times10^{-7}\,\text{cm}$, $m = 318$

(b) $N = \frac{W}{a} = (12\,\text{cm})\,(80\,\text{gr/cm}) = 960\,\text{grooves}$

(c) Now, $\mathfrak{R} = m\,N$. At $\lambda = 550\,\text{nm}$, $m = \left(\frac{\sin 63}{40}\right)\frac{1}{550\times10^{-7}} = 405$. So $\mathfrak{R} = (405)\,(960) = 388,800$.

Then, $(\Delta\,\lambda)_{\text{min}} = \frac{\lambda}{R} = \frac{5500\,\text{Å}}{388,800} = 0.014\,\text{Å}$

(d) $\mathfrak{D} = \frac{m}{a\cos\theta} = \frac{405}{\frac{\text{cm}}{80}\cos 63°} = 71367\,\frac{\text{rad}}{\text{cm}} \times \frac{180}{\pi}\frac{\text{deg}}{\text{rad}} \times 10^{-7}\frac{\text{cm}}{\text{nm}} = 0.41\,\text{deg/nm}.$

(e) $\text{FSR} = \frac{\lambda_{\text{min}}}{m} = \frac{350}{m}$, where $m = \left(\frac{\sin 63}{40}\right)\frac{1}{350\times10^{-7}} = 636.4$. So, $\text{FSR} = \frac{350\text{nm}}{637} = 0.55\,\text{nm} = 5.5\,\text{Å}.$

Chapter 13 Fresnel Diffraction

13-1. The far field is defined by

$$L \gg \frac{\text{Area}}{\lambda} = \frac{\pi\, d^2}{4\,\lambda} = \frac{\pi (10^{-3}\text{m})^2}{4\,(546 \times 10^{-9}\,\text{m})} = 1.4\,\text{m}$$

Thus $L = 50\,\text{cm}$ is the near field, $L = 1\,\text{m}$ is in the near field and $L = 5\,\text{m}$ is marginally in the far field.

13-2. See Figure 13 9 that accompanies the statement of the problem in the text. From Eq. (13 23),

$$f_n = \frac{R_1^2}{n\,\lambda} = \frac{f_1}{n}$$

Then, the maxima occur for odd values of n at,

$$f_1 = \frac{(0.15\,\text{cm})^2}{(1)\,(5.5 \times 10^{-5}\,\text{cm})} = 409\,\text{cm},\ f_3 = \frac{f_1}{3} = 136\,\text{cm},\ f_5 = \frac{f_1}{5} = 81.8\,\text{cm}$$

The minima occur at even values of n,

$$f_2 = \frac{f_1}{2} = 204.5\,\text{cm},\ f_4 = \frac{f_1}{4} = 102\,\text{cm},\ f_6 = \frac{f_1}{6} = 68.1\,\text{cm}$$

13-3. For a fixed distance f, we have the relation for the maxima and minima, $f = r_0 = R_n^2/n\lambda$, with the maxima occurring for odd n and the minima occurring for even n. Then the hole diameters are, then

$$D_n = 2\,R_n = 2\sqrt{n\,\lambda\,r_0}$$

$$\text{First Max: } n = 1:\quad D_1 = 2\,\sqrt{(1)\,(5.893 \times 10^{-5}\,\text{cm})\,(150\,\text{cm})} = 0.188\,\text{cm}$$

$$\text{Second Max: } n = 3:\quad D_3 = 2\sqrt{(3)\,(5.893 \times 10^{-5}\,\text{cm})\,(150\,\text{cm})} = 0.326\,\text{cm}$$

$$\text{First Min: } n = 2:\quad D_2 = 2\sqrt{(2)\,(5.893 \times 10^{-5}\,\text{cm})\,(150\,\text{cm})} = 0.266\,\text{cm}$$

$$\text{Second Min: } n = 4\quad D_4 = 2\sqrt{(4)\,(5.893 \times 10^{-5}\,\text{cm})\,(150\,\text{cm})} = 0.376\,\text{cm}$$

13-4. (a) $R_1 = \sqrt{n\,\lambda\,r_0} = \sqrt{(1)\,\left(6 \times 10^{-5}\,\text{cm})(20\,\text{cm}\right)} = 0.0346\,\text{cm}$

(b) $R_1 = 1\,\text{cm}$; $N = R_1^2/\lambda\,r_0 = 1^2/(6 \times 10^{-5} \times 20) = 833$

(c) No minima are appear now. Maxima are

$$f_1 = r_0 = \frac{R_1^2}{(1)\lambda} = \frac{(0.0346\,\text{cm})^2}{(6 \times 10^{-5}\,\text{cm})} = 20\,\text{cm};\ f_3 = \frac{f_1}{3} = 6.67\,\text{cm};\ f_5 = \frac{f_1}{5} = 4\,\text{cm}$$

13-5. See Figure 13 20 that accompanies this problem in the text for the geometry of the set up. Let d_1 be the distance from the source point to the indicated point on the wavefront in the aperture and d_2 be the dis tance from the indicated point on the wavefront in the aperture to the observation point P. For the n^{th} zone,

$$(d_1 + d_2) - (p + q) = n\lambda/2$$

From the geometry,

$$d_1 = \sqrt{p^2 + R_n^2} = p\left(1 + \frac{R_n^2}{p^2}\right)^{1/2} \approx p\left(1 + \frac{1}{2}\frac{R_n^2}{p^2}\right);\ d_2 \approx q\left(1 + \frac{1}{2}\frac{R_n^2}{q^2}\right)$$

So the condition becomes,

$$(d_1 + d_2) - (p + q) = \frac{1}{2}R_n^2\left(\frac{1}{p} + \frac{1}{q}\right) \equiv \frac{1}{2}R_n^2\left(\frac{1}{L}\right) = n\lambda/2$$

$$R_n = \sqrt{n\,\lambda\,L}$$

13-6. Using the set up of problem 13 5,

$$\frac{1}{L}=\frac{1}{p}+\frac{1}{q}=\frac{1}{10\,\text{cm}}+\frac{1}{20\,\text{cm}}\Rightarrow L=(20/3)\ \text{cm}$$

(a) $R_1=\sqrt{n\,\lambda\,L}=\sqrt{(1)\,(20/3)\,(6\times10^{-5})}\ \text{cm}=0.2\ \text{mm}$

(b) $N=\dfrac{R_1^2}{\lambda\,L}=\dfrac{(1)^2}{(20/3)\,(6\times10^{-5})}=2500.$

13-7. Using the results of problem 13 5,

$$\frac{1}{L}=\frac{1}{p}+\frac{1}{q}=\frac{1}{50\,\text{cm}}+\frac{1}{50\,\text{cm}}\Rightarrow L=25\,\text{cm}$$

$$R_n=\sqrt{n\,L\,\lambda}=n=\frac{R_n^2}{L\,\lambda}$$

(a) Then,

$$n_{\text{inner}}=\frac{(0.05\,\text{cm})^2}{(25\,\text{cm})\,(5\times10^{-5}\,\text{cm})}=2$$
$$n_{\text{outer}}=\frac{(0.0935\,\text{cm})^2}{(25\,\text{cm})\,(5\times10^{-5}\,\text{cm})}=7$$

So, zones 3, 4, 5, 6, and 7 contribute. Every other mode essentially cancels the preceding zone so only one zone contributes. For the unobstructed wave front, $I_u\propto(a_1/2)^2$ and here $I\propto a_1^2$ assuming $a_1\approx a_2\approx\ldots a_7$. So, $I/I_u=4$.

(b) In this case,

$$n_{\text{outer}}=\frac{(0.1\,\text{cm})^2}{(25\,\text{cm})\,(5\times10^{-5}\,\text{cm})}=8$$

In this case zones 3, 4, 5, 6, 7, and 8 contribute and the contributions nearly cancel in pairs giving $I\approx0$.

(c) From above, 5 for part (a) and 6 for part (b).

13-8. From Eq. (13 15), the surface area of the n^{th} Fresnel zone is

$$S_n=\frac{\pi\,r_0'r_0^2}{r_0+r_0'}\left(\frac{\lambda}{r_0}+(2\,n-1)\left(\frac{\lambda}{2\,r_0}\right)^2\right)$$

So that, with $\lambda/r_0=(5\times10^{-5}/50)=10^{-6}$

$$\frac{S_{25}-S_1}{S_1}=\frac{48\,(\lambda/2\,r_0)^2}{(\lambda/r_0)+(\lambda/2r_0)^2}=1.2\times10^{-5}=0.0012\%$$

13-9. (a) Before reduction $R_1=11.25\,\text{cm}$.

The required radius is $R_1=\sqrt{n\,\lambda\,f_1}=\sqrt{(632.8\times10^{-5})\,(200)}=0.1125\,\text{cm}$.

It must be reduced by a factor of 100.

(b) $R_N=\sqrt{N}\,R_1=\sqrt{20}\,(11.25\,\text{cm})=50.31\,\text{cm}$.

13-10. See Figure 13 21 that accompanies the problem in the text. Using $R_n=\sqrt{n\,r_0\,\lambda}$,

$$D_2=2\,R_2=2\,\sqrt{2\,r_0\,\lambda}=2\,\sqrt{2\,(25)\,(5.5\times10^{-5})}\ \text{cm}=0.105\,\text{cm}$$
$$D_4=2\,R_2=2\,\sqrt{4\,r_0\,\lambda}=2\,\sqrt{4(25)\,(5.5\times10^{-5})}\ \text{cm}=0.148\,\text{cm}$$
$$D_6=2\,R_2=2\,\sqrt{6\,r_0\,\lambda}=2\,\sqrt{6(25)\,(5.5\times10^{-5})}\ \text{cm}=0.182\,\text{cm}$$

13-11. The zone areas are given by Eq. (13 15) which goes to Eq. (13 16) in the limit $\lambda/r_0 = \lambda/x \ll 1$. Eq. (13 16) is independent of n, so the areas are approximately constant. For a plane wave, $r_0 \to \infty$, thus from Eq. (13 16),

$$S_n = \frac{\pi r_0' r_0}{r_0 + r_0'} \pi\lambda = \lim_{r_0 \to \infty} \frac{\pi\lambda}{\left(\frac{1}{r_0} + \frac{1}{r_0'}\right)} = \pi \lambda r_0' = \pi \lambda x$$

13-12. The radius is given by $R_4 = \sqrt{4 r_0 \lambda} = \sqrt{(4)(2)(485 \times 10^{-9})}$ m $= 0.00197$ m $= 1.97$ mm. Since 4 zones con tribute and the zones nearly cancel in pairs, the irradiance is nearly zero.

13-13. Since $p \to \infty$,

$$\frac{1}{L} = \cancel{\frac{1}{p}} + \frac{1}{q} = \frac{1}{q} \Rightarrow L = q$$

Also $\Delta z = w$. Using Eq. (13 28),

$$\Delta v = \sqrt{\frac{2}{L\lambda}} \Delta z \Rightarrow L = \frac{2 w^2}{(\Delta v)\lambda} = \frac{2\,(0.0005\,\mathrm{m})^2}{(2.5)^2\,(540 \times 10^{-9}\,\mathrm{m})} = 0.148\,\mathrm{m} = 14.8\,\mathrm{cm}$$

13-14. See Figure 13 22 that accompanies the statement of this problem in the text. Also it may be helpful to examine Figure 13 15 in the body of the text. The parameter L is,

$$\frac{1}{L} = \frac{1}{p} + \frac{1}{q} = \frac{1}{30\,\mathrm{cm}} + \frac{1}{15\,\mathrm{cm}} \Rightarrow L = 10\,\mathrm{cm}$$

(a) For an on axis point, $z_2 = w/2 = 2.5 \times 10^{-4}$ m and $z_1 = -w/2 = -2.5 \times 10^{-4}$ m. Then

$$v_2 = \sqrt{\frac{2}{L\lambda}}\, z_2 = \sqrt{\frac{2}{(0.1)\,(435.8 \times 10^{-9})}}\,(2.5 \times 10^{-4}) = 1.694$$
$$v_1 = \sqrt{\frac{2}{L\lambda}}\, z_1 = \sqrt{\frac{2}{(0.1)\,(435.8 \times 10^{-9})}}\,(-2.5 \times 10^{-4}) = -1.694$$

Then, interpolating in Table 13 1,

$$C(v_2) \approx 0.3265, S(v_2) = 0.5549$$
$$C(v_1) = -0.3265, S(v_1) = -0.5549$$

Using Eq. (13 35),

$$I = I_0 \left\{[C(v_2) - C(v_1)]^2 + [S(v_2) - S(v_1)]^2\right\}$$
$$I = I_0 \left\{(2 \times 0.3265)^2 + (2 \times 0.5549)^2\right\} = 1.658\, I_0 = 1.658\ I_u/2 = 0.829\, I_u$$

(b) The edge of the geometrical shadow on the screen is at $y = 0.375$ mm. The axis from S to P for this point brushes the top edge of the aperture at $z_2 = 0$. Then the bottom edge of the aperture is located at $z = -w = -5 \times 10^{-4}$ m. Then,

$$v_2 = 0, v_1 = \sqrt{\frac{2}{L\lambda}}\, z_1 = \sqrt{\frac{2}{(0.1)\,(435.8 \times 10^{-9})}}\,(-5 \times 10^{-4}) = -3.388$$
$$C(v_2) = 0, S(v_2) = 0; C(v_1) = -0.42746, S(v_1) = -0.49328$$
$$I = I_0 \left\{[C(v_2) - C(v_1)]^2 + [S(v_2) - S(v_1)]^2\right\}$$
$$I = I_0 \left(0.4276^2 + 0.49328^2\right) = 0.4262 I_0 = 0.4262\,(I_u/2) = 0.2131\, I_u$$

13-15. Refer to Figure 13 23 that accompanies the statement of the problem in the text and Figure 13 4a in the body of the text. For both positions,

$$\frac{1}{L}=\frac{1}{60\,\text{cm}}+\frac{1}{120\,\text{cm}}\Rightarrow L=40\,\text{cm}=0.4\,\text{m}$$

(a) Let z' be the coordinate of the point O' in the aperture plane along the straight line from S to the observation point P. (See Figure 13 4a). In the case at hand

$$z'=\frac{p}{p+q}y=\frac{60}{60+120}\,(-2\,\text{mm})=-6.667\times10^{-4}\,\text{m}.$$

The values of the z_1 and z_2 that mark the edges of the unobstructed regions in the aperture plane are to be measured relative to this point so that $z_2=\infty$ and $z_1=+6.667\times10^{-4}\,\text{m}$

$$v_2=\infty,\, v_1=\sqrt{\frac{2}{\lambda L}}z_1=\sqrt{\frac{2}{(589.3\times10^{-9})(0.4)}}\left(6.667\times10^{-4}\right)=1.942$$
$$C(v_2)=0.5, S(v_2)=0.5, C(v_1)=0.4338, S(v_1)=0.3608$$
$$I=I_0\left\{[C(v_2)-C(v_1)]^2+[S(v_2)-S(v_1)]^2\right\}$$
$$I=I_0\left\{(0.5-0.4338)^2+(0.5-0.3608)^2\right\}=0.02376\,I_0=0.02376\,(I_u/2)=0.0119\,I_u$$

(b) In this case $z'=\frac{p}{p+q}y=\frac{60}{60+120}\,(1\,\text{mm})=3.333\times10^{-4}\,\text{m}$. Then, $z_2=\infty$ and $z_1=-3.333\times10^{-4}\,\text{m}$

$$v_2=\infty,\, v_1=\sqrt{\frac{2}{\lambda L}}z_1=\sqrt{\frac{2}{(589.3\times10^{-9})(0.4)}}\left(-3.333\times10^{-4}\right)=-0.9709$$
$$C(v_2)=0.5, S(v_2)=0.5, C(v_1)=-0.7755, S(v_1)=-0.4097$$
$$I=I_0\left\{[C(v_2)-C(v_1)]^2+[S(v_2)-S(v_1)]^2\right\}$$
$$I=I_0\left\{(0.5+0.7755)^2+(0.5+0.4097)^2\right\}=2.454\,I_0=2.454\,(I_u/2)=1.23\,I_u$$

13-16. The parameter L is given by

$$\frac{1}{L}=\frac{1}{30\,\text{cm}}+\frac{1}{60\,\text{cm}}\Rightarrow L=20\,\text{cm}=0.2\,\text{m}$$

(a) The unobstructed region in the upper half of the aperture plane relative to the point in the center of the wire on the line connecting the source and the observation point is bounded by the points $z_{u2}=\infty$ and $z_{u1}=7.5\times10^{-4}$ m. The unobstructed region in the lower half of the aperture plane is bounded by the points $z_{l2}=-7.5\times10^{-4}\,\text{m}$ and $z_{l1}=-\infty$. Then,

$$v_{u2}=\infty,\, v_{u1}=\sqrt{\frac{2}{\lambda L}}z_1=\sqrt{\frac{2}{(546.1\times10^{-9})(0.2)}}\left(7.5\times10^{-4}\right)=3.2094$$
$$C(v_{u2})=0.5, S(v_{u2})=0.5, C(v_{u1})=0.4607, S(v_{u1})=0.5863$$
$$v_{l2}=-3.2094, v_{l1}=-\infty$$
$$C(v_{l2})=-0.4607, S(v_{l2})=-0.5863, C(v_{l1})=-0.5, S(v_{l1})=-0.5$$
$$I=I_0\left\{[C(v_{l2})-C(v_{l1})+C(v_{u2})-C(v_{u1})]^2+[S(v_{l2})-S(v_{l1})+S(v_{u2})-S(v_{u1})]^2\right\}$$
$$I=I_0\left\{\left[-0.4607-(-0.5)+0.5-0.4607\right]^2+\left[-0.5863-(-0.5)+0.5-0.5863\right]^2\right\}=0.03599I_0$$
$$I=0.03599\,(I_u/2)=0.018\,I_u$$

Part (b) is on the next page.

13-16. (b) The point on the screen that is at the edge of the geometrical shadow is at position

$$y = \frac{p+q}{p} z_{\text{wire top}} = \frac{90}{30}(0.75\,\text{mm}) = 2.25\,\text{mm}$$

Following the procedure from part (a):

$$z_{u1} = 0, z_{u2} = \infty, z_{l1} = -\infty, z_{l2} = -1.5 \times 10^{-3}\,\text{m}$$
$$v_{u1} = 0, v_{u2} = \infty, v_{l1} = -\infty, v_{l2} = \sqrt{\frac{2}{(546.1 \times 10^{-9})(0.2)}}\left(-1.5 \times 10^{-3}\right) = -6.42$$
$$C(v_{u2}) = 0.5, S(v_{u2}) = 0.5, C(v_{u1}) = 0, S(v_{u1}) = 0$$
$$C(v_{l2}) = -0.5414\,, S(v_{l2}) = -0.5138, C(v_{l1}) = -0.5, S(v_{l1}) = -0.5$$
$$I = I_0\left\{[C(v_{l2}) - C(v_{l1}) + C(v_{u2}) - C(v_{u1})]^2 + [S(v_{l2}) - S(v_{l1}) + S(v_{u2}) - S(v_{u1})]^2\right\}$$
$$I = I_0\left\{[-0.5414 - (-0.5) + 0.5 - 0]^2 + [-0.5138 - (-0.5) + 0.5 - 0]^2\right\} = 0.4467 I_0 = 0.4467\frac{I_u}{2} = 0.223\,I_u$$

13-17. Refer to the routine used to solve problem 13 15b. Borrowing from that solution,

$$I = I_0\left\{[C(v_2) - C(v_1)]^2 + [S(v_2) - S(v_1)]^2\right\} = I_0\left\{[0.5 - C(v_1)]^2 + [0.5 - S(v_1)]^2\right\}$$

From Table 13 1 the second maximum and minimum occur for,

$$\text{Max: } v_1 = -2.3, C(v_1) = -0.626, S(v_1) = -0.5531$$
$$I_{2^{\text{nd}}\,\text{max}} = 2.378\,I_0 = 2.378\,(I_u/2) = 1.19\,I_u$$
$$\text{Min: } v_1 = -2.75, C(v_1) = -0.4300, S(v_1) = -0.4222$$
$$I_{2^{\text{nd}}\,\text{min}} = 1.715\,I_0 = 1.715\,(I_u/2) = 0.86\,I_u$$

13-18. The parameter L is given by $\frac{1}{L} = \frac{1}{2\,\text{m}} + \frac{1}{3\,\text{m}} \Rightarrow L = 1.2\,\text{m}$. The unobstructed region in the upper half of the aperture plane relative to the point in the center of the wire on the line connecting the source and the observation point is bounded by the points $z_{u2} = \infty$ and $z_{u1} = 1.85 \times 10^{-4}$ m. The unobstructed region in the lower half of the aperture plane is bounded by the points $z_{l2} = -1.85 \times 10^{-4}\,\text{m}$ and $z_{l1} = -\infty$. Then,

$$v_{u2} = \infty, v_{u1} = \sqrt{\frac{2}{\lambda L}} z_1 = \sqrt{\frac{2}{(630 \times 10^{-9})(1.2)}}\left(1.85 \times 10^{-4}\right) = 0.3009$$
$$C(v_{u2}) = 0.5, S(v_{u2}) = 0.5, C(v_{u1}) = 0.3003, S(v_{u1}) = 0.0143$$
$$v_{l2} = -3.2094, v_{l1} = -\infty$$
$$C(v_{l2}) = -0.3003, S(v_{l2}) = -0.0143, C(v_{l1}) = -0.5, S(v_{l1}) = -0.5$$
$$I = I_0\left\{[C(v_{l2}) - C(v_{l1}) + C(v_{u2}) - C(v_{u1})]^2 + [S(v_{l2}) - S(v_{l1}) + S(v_{u2}) - S(v_{u1})]^2\right\}$$
$$I = I_0\left\{\left[-0.3003 - (-0.5) + 0.5 - 0.3003\right]^2 + [-0.0143 - (-0.5) + 0.5 - 0.0143]^2\right\} = 1.103 I_0$$
$$I = 1.103\,(I_u/2) = 0.55 I_u$$

13-19. See Figure 13 25 that accompanies the statement of the problem in the text. The parameter L is given by

$$\frac{1}{L} = \frac{1}{10\,\text{cm}} + \frac{1}{10\,\text{cm}} \Rightarrow L = 5\,\text{cm}$$

Each slit contributes the same electric field at the on axis point and so the phasors associated with each slit will simple add. Then for the upper slit,

$$z_1 = (0.25/2 - 0.02)\,\text{cm} = 0.0105\,\text{cm}, z_2 = (0.25/2 + 0.02)\,\text{cm} = 0.0145\,\text{cm}$$
$$v_1 = \sqrt{\frac{2}{\lambda L}}\, z_1 = \sqrt{\frac{2}{5\,(5.46 \times 10^{-5})}}\,(0.0105\,) = 0.8987, v_2 = 1.2411$$
$$C(v_1) = 0.76426, S(v_1) = 0.33862, C(v_2) = 0.68384, S(v_2) = 0.64925$$
$$I = I_0\left\{[2\,(C(v_2) - C(v_1))]^2 + [2\,(S(v_2) - S(v_1))]^2\right\} = 0.412\,I_0 = 0.412\,I_u/2 = 0.206\,I_u$$

13-20. The parameter L is given by

$$\frac{1}{L}=\frac{1}{25\,\text{cm}}+\frac{1}{25\,\text{cm}}\Rightarrow L=12.5\,\text{cm}$$

(a) The far field condition is

$$p,q\gg b^2/\lambda=(0.075\text{ cm})^2/(4.358\times10^{-5}\,\text{cm})=129\,\text{cm}$$

Since for the case described in the problem statement $p=q=25$cm, far filed approximations are not justi fied.

(b) Using the far field approximations for comparison:

$$\begin{gathered} m\,\lambda=b\sin\theta \qquad\qquad (\text{take } m=1)\\ \sin\theta=\lambda/b=(4.358\times10^{-5})/0.075\Rightarrow\theta=0.03329^\circ\\ y=q\tan\theta=(25\text{ cm})\tan(0.00329^\circ)=0.0145\text{ cm}\end{gathered}$$

(c) See Figure 13 15a for the geometry. In the case at hand, $y=0.0145$ cm so that the point O' in the aper ture plane and connecting the source to the observation point is positioned at,

$$z'=\frac{p}{p+q}\,y=\frac{1}{2}\,y=0.00725\text{ cm}$$

above the center of the aperture. Relative to this point the edges of the slit are at positions

$$z_1=-\left(\frac{0.075}{2}+0.00725\right)\text{cm}=-0.04475\text{ cm},\ z_2=\left(\frac{0.075}{2}-0.00725\right)\text{cm}=0.03025\text{ cm}$$

Then,

$$\begin{gathered} v_1=\sqrt{\frac{2}{L\lambda}}\,z_1=\sqrt{\frac{2}{(12.5)\,(4.358\times10^{-5})}}\,(-0.04475)=-2.7115\\ v_2=\sqrt{\frac{2}{L\lambda}}\,z_1=\sqrt{\frac{2}{(12.5)\,(4.358\times10^{-5})}}\,(0.03025)=1.8329\\ C(v_1)=-0.401,S(v_1)=-0.446;C(v_2)=0.354,S(v_1)=0.425\\ I=I_0\left\{[C(v_2)-C(v_1)]^2+[S(v_2)-S(v_1)]^2\right\}\\ I=I_0\left\{(0.354+0.401)^2+(0.425+0.446)^2\right\}=1.33I_0=1.33(I_u/2)=0.665\,I_u\end{gathered}$$

13-21. According to Babinet's principle, the diffraction pattern is complementary to that due to circular apertures of the same size as the particles, in an otherwise opaque screen. The halo is the complement of the Airy disc. Taking $\lambda=550$ nm,

$$a=\frac{1.22\,\lambda}{\sin\theta}=\frac{1.22\,(550\times10^{-9}\,\text{m})}{\sin(2^\circ)}\approx19\,\mu\text{m}$$

Chapter 14 Matrix Treatment of Polarization

14-1. If the light is polarized along the transmission axis oriented at angle θ the light is transmitted unchanged so that,

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} \cos\theta \\ \sin\theta \end{bmatrix} = \begin{bmatrix} \cos\theta \\ \sin\theta \end{bmatrix}$$

If light is polarized perpendicular to the transmission axis no light is transmitted,

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} \cos(\theta - 90°) \\ \sin(\theta - 90°) \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} \sin\theta \\ -\cos\theta \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

These two matrix equations can be recast as four algebraic equations,

$$a\cos\theta + b\sin\theta = \cos\theta \quad (1)$$
$$c\cos\theta + d\sin\theta = \sin\theta \quad (2)$$
$$a\sin\theta - b\cos\theta = 0 \quad (3)$$
$$c\sin\theta - d\cos\theta = 0 \quad (4)$$

An approach that can be used is multiply Eq. (1) by sin θ and Eq. (3) by cos θ and then subtracting the results. Then,

$$b\left(\sin^2\theta + \cos^2\theta\right) = \cos\theta\sin\theta \Rightarrow b = \cos\theta\sin\theta$$

Proceeding in the same manner with Eqs. (2) and (4) gives,

$$c = \sin\theta\cos\theta$$

Then using these relations for b and c in (3) and (4) leads to,

$$a = \cos^2\theta, d = \sin^2\theta$$

So that,

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} \sin^2\theta & \sin\theta\cos\theta \\ \sin\theta\cos\theta & \cos^2\theta \end{bmatrix}$$

14-2. In general $\tilde{\boldsymbol{E}} = \left[E_{0x}e^{i\varphi_x}\widehat{x\boldsymbol{x}} + E_{0y}\,e^{i\varphi_y}\,\hat{\boldsymbol{y}}\,\right]e^{i(kz-\omega t)} = \begin{bmatrix} E_{0x}e^{i\varphi_x} \\ E_{0y}\,e^{i\varphi_y} \end{bmatrix}e^{i(kz-\omega t)} = \tilde{\boldsymbol{E}}_0$

(a) $\tilde{\boldsymbol{E}} = [E_0\hat{\boldsymbol{x}} - E_0\hat{\boldsymbol{y}}]e^{i(kz-\omega t)} \Rightarrow \tilde{\boldsymbol{E}}_0 = E_0\begin{bmatrix} 1 \\ -1 \end{bmatrix} \Rightarrow \frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ -1 \end{bmatrix}$. Linearly polarized at $-45°$.

(b) $\tilde{\boldsymbol{E}} = [E_0\hat{\boldsymbol{x}} + E_0\hat{\boldsymbol{y}}]e^{i(kz-\omega t)} \Rightarrow \tilde{\boldsymbol{E}}_0 = E_0\begin{bmatrix} 1 \\ 1 \end{bmatrix} \Rightarrow \frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ 1 \end{bmatrix}$. Linearly polarized at 45°.

(c) $\tilde{\boldsymbol{E}} = \left[E_0\hat{\boldsymbol{x}} + E_0e^{-i\pi/4}\hat{\boldsymbol{y}}\right]e^{i(kz-\omega t)} \Rightarrow \tilde{\boldsymbol{E}}_0 = E_0\begin{bmatrix} 1 \\ e^{-i\pi/4} \end{bmatrix} \Rightarrow \frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ \frac{1}{\sqrt{2}}(1-i) \end{bmatrix}$. Then,

$$\tan 2\alpha = \frac{2E_{0x}E_{0y}\cos\varepsilon}{E_{0x}^2 - E_{0y}^2} \rightarrow \infty \Rightarrow 2\,\alpha = 90°, \alpha = 45°$$

Right elliptically polarized at 45°.

(d) $\tilde{\boldsymbol{E}} = \left[E_0\hat{\boldsymbol{x}} + E_0e^{i\pi/2}\hat{\boldsymbol{y}}\right]e^{i(kz-\omega t)} \Rightarrow \tilde{\boldsymbol{E}}_0 = E_0\begin{bmatrix} 1 \\ e^{i\pi/2} \end{bmatrix} \Rightarrow \frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ i \end{bmatrix}$. Left circularly polarized.

14-3. In general $\tilde{\boldsymbol{E}} = [E_{0x}e^{i\varphi_x}\hat{\boldsymbol{x}} + E_{0y}\,e^{i\varphi_y}\,y\hat{\boldsymbol{y}}]e^{i(kz-\omega t)} = \begin{bmatrix} E_{0x}e^{i\varphi_x} \\ E_{0y}\,e^{i\varphi_y}\end{bmatrix}e^{i(kz-\omega t)} = \tilde{\boldsymbol{E}}_0 e^{i(kz-\omega t)}$

(a) $\tilde{\boldsymbol{E}} = (2\,E_0\hat{\boldsymbol{x}} + 0\hat{\boldsymbol{y}})e^{i(kz-\omega t)} \Rightarrow \tilde{\boldsymbol{E}}_0 = 2\,E_0\begin{bmatrix}1\\0\end{bmatrix}$. Linearly polarized along the x direction. Velocity is in the $+z$ direction. The amplitude is $A = 2\,E_0\,\sqrt{1^2+0^2} = 2\,E_0$.

(b) $\tilde{\boldsymbol{E}} = (3E_0\hat{\boldsymbol{x}} + 4E_0\hat{\boldsymbol{y}})e^{i(kz-\omega t)} \Rightarrow \tilde{\boldsymbol{E}}_0 = E_0\begin{bmatrix}3\\4\end{bmatrix}$. The polarization direction makes the angle α with the x axis where,

$$\alpha = \tan^{-1}(4/3) = 53°$$

The wave is traveling in the $+z$ direction with amplitude $A = \sqrt{3^2+4^2}\,E_0 = 5\,E_0$.

(c) $\tilde{\boldsymbol{E}} = 5\,E_0(\hat{\boldsymbol{x}} - i\hat{\boldsymbol{y}})e^{i(kz+\omega t)} \Rightarrow \tilde{\boldsymbol{E}}_0 = 5E_0\begin{bmatrix}1\\-i\end{bmatrix}$. The propagation is in the $+z$ direction. The wave is right circularly polarized with amplitude. The electric field vector traces out a circle of radius $5\,E_0$.

14-4. (a) $\tilde{\boldsymbol{E}}_1 = E_{01}(\hat{\boldsymbol{x}} - \hat{\boldsymbol{y}})e^{i(kz-\omega t)} \Rightarrow \tilde{\boldsymbol{E}}_{01} = 2\,E_{01}\begin{bmatrix}1\\-1\end{bmatrix}$. This is linearly polarized along $-45°$

$\tilde{\boldsymbol{E}}_2 = E_{02}\left(\sqrt{3}\hat{\boldsymbol{x}} + \hat{\boldsymbol{y}}\right)e^{i(kz-\omega t)} \Rightarrow \tilde{\boldsymbol{E}}_{02} = E_{02}\begin{bmatrix}\sqrt{3}\\1\end{bmatrix}$. This is linearly polarized along $\alpha = \tan^{-1}\left(\frac{1}{\sqrt{3}}\right) = 30°$

The angle between the two is 75°.

(b) $\tilde{\boldsymbol{E}}_{01}\cdot\tilde{\boldsymbol{E}}_{02} = E_{01}\,E_{02}(\sqrt{3}-1) = (\sqrt{2}\,E_{01})\,(\sqrt{3+1^2}\,E_{02})\cos(\theta_{12}) \Rightarrow \cos\theta_{12} = \dfrac{\sqrt{3}-1}{2\sqrt{2}} \Rightarrow \theta_{12} = 75°$

14-5. Since we have not determined a matrix for a HWP with its FA at an arbitrary at 45°, we do the problem in two steps. First consider the effect of passing linearly polarized light at $-45°$ through a HWP with FA ver tical,

$$\underbrace{\begin{bmatrix}1 & 0\\0 & -1\end{bmatrix}}_{\text{HWP, FA vert}}\ \underbrace{\begin{bmatrix}1\\-1\end{bmatrix}}_{\text{LP at }-45°} = \begin{bmatrix}1\\1\end{bmatrix}$$

The output is linearly polarized at 45°. Thus the first step rotates the polarization through an angle of 90°. If the linearly polarized light is originally vertical it exits the HWP horizontal. Then the LP and QWP act as,

$$\underbrace{\begin{bmatrix}1 & 0\\0 & -i\end{bmatrix}}_{\text{QWP}}\underbrace{\begin{bmatrix}1 & 1\\1 & 1\end{bmatrix}}_{\text{LP}}\begin{bmatrix}1\\0\end{bmatrix} = \begin{bmatrix}1\\-i\end{bmatrix}$$

The output light has right circular polarization.

14-6. (a) The wave propagates in the $+x$ direction so the phase factor is $e^{i(kx-\omega t)}$. The angle $\alpha = 30°$ so that $\tan\alpha = 30° = \dfrac{1}{\sqrt{3}} = \dfrac{E_{0z}}{E_{0y}}$. Then $\tilde{\boldsymbol{E}} = E_0(\sqrt{3}\,\hat{\boldsymbol{y}} + \hat{\boldsymbol{z}})e^{i(kx-\omega t)}$.

(b) The phase factor is $e^{i(ky-\omega t)}$. Also,

$$\begin{bmatrix}A\\-i\,B\end{bmatrix} \text{ with } A = 2\,B \Rightarrow E_0\begin{bmatrix}2\\-i\end{bmatrix}$$

So, $\tilde{\boldsymbol{E}} = E_0(2\,\hat{\boldsymbol{z}} - i\hat{\boldsymbol{x}})e^{i(ky-\omega t)}$

(c) The propagation vector is $\boldsymbol{k} = \dfrac{|k|}{\sqrt{2}}(\hat{\boldsymbol{x}} + \hat{\boldsymbol{y}})$. Then $\boldsymbol{k}\cdot\boldsymbol{r} = \dfrac{|k|}{\sqrt{2}}(x+y)$.

Then $\tilde{\boldsymbol{E}} = E_0\,\hat{\boldsymbol{z}}\,e^{i\left[|k|(x+y)/\sqrt{2}-\omega t\right]}$.

14-7. The general form is,

$$\tilde{\boldsymbol{E}}_0 = \begin{bmatrix} A \\ B + i\,C \end{bmatrix}$$

(a) Linear polarization: $C = 0$. Phase differences must be multiples of $m\,\pi$.
(b) Elliptically polarized: major axis along a coordinate axis: $B = 0$. The phase difference should be $(m + 1/2)\,\pi$.
(c) Circularly polarized: $B = 0$ and $C = \pm\,A$, with phase difference $(m + 1/2)\,\pi$.

14-8. The form of the ellipse is given as,

$$\left(\frac{E_x}{E_{0x}}\right)^2 + \left(\frac{E_y}{E_{0y}}\right)^2 - 2\left(\frac{E_x}{E_{0x}}\right)\left(\frac{E_y}{E_{0y}}\right)\cos\varepsilon = \sin^2\varepsilon$$

This has the form of a quadratic,

$$E_y^2 + b\,E_y + c = 0$$

with,

$$b = -2\left(\frac{E_{0y}}{E_{0x}}\right)(\cos\varepsilon)\,E_x \text{ and } c = E_{0y}^2\left(\frac{E_x^2}{E_{0x}^2} - \sin^2\varepsilon\right)$$

As shown in the text, when $\tilde{\boldsymbol{E}}_0 = \begin{bmatrix} A \\ B + i\,C \end{bmatrix}$, $E_{0x} = A$, $E_{0y} = \sqrt{B^2 + C^2}$, and $\varepsilon = \tan^{-1}(C/B)$
Solving the quadratic gives,

$$E_y = \frac{-\,b \pm \sqrt{b^2 - 4\,c}}{2}$$

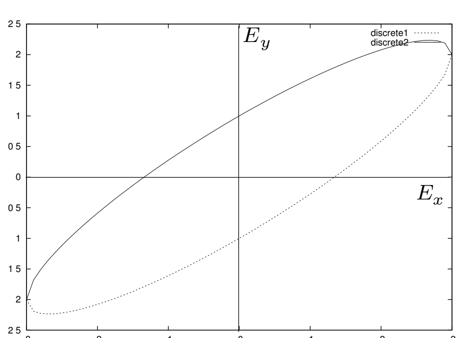
So knowing the form of the Jones vector allows for the construction E_y for given E_x. I'll illustrate this process using the free computer algebra system Maxima:

```
(%i1) (A:3,B:2,C:1)$
(%i2) epsilon:atan(C/B)$
(%i3) (E0x:A,E0y:sqrt(B^2+C^2))$
(%i4) for j from -30 thru 30 do
      (Ex[j]:0.1*j,b[j]:-2*E0y/E0x*cos(epsilon)*Ex[j],c[j]:E0y^2*(Ex[j]^2/E0x^2-(sin(epsilon))^2),
      Ey1[j]:-b[j]/2-1/2*sqrt(b[j]^2-4*c[j]),Ey2[j]:-b[j]/2+1/2*sqrt(b[j]^2-4*c[j]))$
(%i5) for j from 0 thru 0 do ldisplay(Ex[j],Ey1[j],Ey2[j])
```

(%t5) $\mathrm{Ex}_0 = 0$
(%t6) $\mathrm{Ey1}_0 = -1$
(%t7) $\mathrm{Ey2}_0 = 1$

Note that I have displayed the first set of points values. The loops could be extended to form any number of the values. The two E_y values correspond to the two roots of the quadratic. The list so generated can be plotted to form the ellipse. This is done below.

```
(%i7) (Eylist:listarray(Ey1),Exlist:listarray(Ex),Eylist2:listarray(Ey2))$
(%i8) plot2d([[discrete,Exlist,Eylist],[discrete,Exlist,Eylist2]],[gnuplot_term,ps],[gnuplot_ps_term_command,
      "set term postscript eps monochrome"],[gnuplot_out_file, "ellipse.eps"])
```



14-9. (a) $\begin{bmatrix} 3i \\ i \end{bmatrix} \rightarrow i\begin{bmatrix} 3 \\ 1 \end{bmatrix}$ Linear polarization at $\alpha = \tan^{-1}(1/3) = 18.4°$

(b) $\begin{bmatrix} i \\ 1 \end{bmatrix} \rightarrow i\begin{bmatrix} 1 \\ -i \end{bmatrix}$ Right circular polarization, $A = 1$

(c) $\begin{bmatrix} 4i \\ 5 \end{bmatrix} \rightarrow i\begin{bmatrix} 4 \\ -5i \end{bmatrix}$ Right elliptical polarization, axes 5 along y, 4 along x

(d) $\begin{bmatrix} 5 \\ 0 \end{bmatrix}$ Linear polarization along x, $A = 5$

(e) $\begin{bmatrix} 2 \\ 2i \end{bmatrix} \rightarrow 2\begin{bmatrix} 1 \\ i \end{bmatrix}$ Left circular polarization. $A = 2$

(f) $\begin{bmatrix} 2 \\ 3 \end{bmatrix}$ Linear polarization at $\alpha = \tan^{-1}(3/2) = 56.3°, A = \sqrt{2^2 + 3^2} = \sqrt{13}$

(g) $\begin{bmatrix} 2 \\ 6 + 8i \end{bmatrix} = 2\begin{bmatrix} 1 \\ 3 + 4i \end{bmatrix}$ Left elliptical polarization, $E_{0x} = 2, E_{0y} = 2\sqrt{3^2 + 4^2} = 10, \varepsilon = \tan^{-1}(4/3) = 53.1°$

$$\tan 2\alpha = \frac{(2)(1)(5)\cos 53.1°}{1^2 - 5^2} \Rightarrow \alpha = -7.02°$$

14-10. Using the Jones formalism,

$$\underbrace{\begin{bmatrix} 1 & 0 \\ 0 & -i \end{bmatrix}}_{\text{QWP}} \underbrace{\begin{bmatrix} \sqrt{3} \\ 1 \end{bmatrix}}_{\text{LP, }30°} = \begin{bmatrix} \sqrt{3} \\ -i \end{bmatrix}$$

The output light is has right elliptical polarization with $E_{0x}/E_{0y} = \sqrt{3}$.

14-11. Using the Jones formalism,

$$\underbrace{\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}}_{\text{HWP}} \underbrace{\begin{bmatrix} \cos\alpha \\ \sin\alpha \end{bmatrix}}_{\text{LP, }\alpha} = \begin{bmatrix} \cos\alpha \\ -\sin\alpha \end{bmatrix} = \underbrace{\begin{bmatrix} \cos(-\alpha) \\ \sin(-\alpha) \end{bmatrix}}_{\text{LP, }-\alpha}$$

The input light is rotated by 2α.

14-12. Light from the 45° linear polarizer passes through the QWP twice, before and after reflection. Thus it passes through (equivalently) a HWP that rotates the direction of polarization by $2 \times 45° = 90°$. (See problem 14 11.) It then returns to the linear polarizer perpendicular to the transmission axis. As a result no light passes back through the polarizer after being reflected.

14-13. See Figure 14 13 that accompanies the statement of this problem in the text. Using the Jones formalism,

$$\underbrace{\begin{bmatrix} 1 & 0 \\ 0 & -i \end{bmatrix}}_{\substack{\text{QWP} \\ \text{SA, hor}}} \underbrace{\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}}_{\substack{\text{LP} \\ \text{at }45°}} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ -i & -i \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ -i \end{bmatrix} \quad \text{Right circular polarization}$$

14-14. Using the Jones formalism,

$$\underbrace{\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}}_{\substack{\text{LP} \\ \text{TA vert}}} \underbrace{\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}}_{\text{HWP}} \underbrace{\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}}_{\substack{\text{LP} \\ \text{TA hor}}} \underbrace{\begin{bmatrix} 1 & 0 \\ 0 & i \end{bmatrix}}_{\substack{\text{QWP} \\ \text{FA hor}}} \underbrace{\begin{bmatrix} 1 \\ 1 \end{bmatrix}}_{\substack{\text{LP} \\ \text{at }45°}} = \begin{bmatrix} 0 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \quad \text{No light}$$

14-15. Using the Jones formalism,

$$\underbrace{\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}}_{\substack{\text{LP} \\ \text{TA vert}}} \underbrace{\begin{bmatrix} 1 & 0 \\ 0 & -i \end{bmatrix}}_{\substack{\text{QWP} \\ \text{SA hor}}} \underbrace{\begin{bmatrix} \cos 30° \\ \sin 30° \end{bmatrix}}_{\substack{\text{LP} \\ \text{at }30°}} = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \underbrace{\begin{bmatrix} \sqrt{3} \\ -i \end{bmatrix}}_{\substack{\text{(a) Right} \\ \text{Ellipt}}} = \begin{bmatrix} 0 \\ -i \end{bmatrix} = -i \underbrace{\begin{bmatrix} 0 \\ 1 \end{bmatrix}}_{\substack{\text{(b)LP} \\ \text{vert}}}$$

14-16. (a) In all cases the light is linearly polarized light at $\pm 45°$.

$$\underbrace{\begin{bmatrix} 1 & 0 \\ 0 & -i \end{bmatrix}}_{\text{SA}-x}\underbrace{\begin{bmatrix} 1 \\ i \end{bmatrix}}_{\text{LCP}} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}\text{: Linear polarization at at} + 45°$$

$$\underbrace{\begin{bmatrix} 1 & 0 \\ 0 & -i \end{bmatrix}}_{\text{SA}-x}\underbrace{\begin{bmatrix} 1 \\ -i \end{bmatrix}}_{\text{RCP}} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}\text{: Linear polarization at at} - 45°$$

$$\underbrace{\begin{bmatrix} 1 & 0 \\ 0 & i \end{bmatrix}}_{\text{SA}-y}\underbrace{\begin{bmatrix} 1 \\ i \end{bmatrix}}_{\text{LCP}} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}\text{: Linear polarization at at} - 45°$$

$$\underbrace{\begin{bmatrix} 1 & 0 \\ 0 & i \end{bmatrix}}_{\text{SA}-y}\underbrace{\begin{bmatrix} 1 \\ -i \end{bmatrix}}_{\text{RCP}} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}\text{: Linear polarization at at} + 45°$$

(b) For an eighth wave plate: $|\varepsilon_y - \varepsilon_x| = 2\pi/8 = \pi/4$. Then the Jones matrix is,

$$M_{1/8} = \begin{bmatrix} 1 & 0 \\ 0 & e^{\pm i\pi/4} \end{bmatrix}$$

with the $\pm$ for $\varepsilon_y > \varepsilon_x$ or $\varepsilon_y < \varepsilon_x$. Then,

$$\begin{bmatrix} 1 & 0 \\ 0 & e^{\pm i\pi/4} \end{bmatrix}\underbrace{\begin{bmatrix} 1 \\ i \end{bmatrix}}_{\text{LCP}} = \begin{bmatrix} 1 \\ i\,e^{\pm i\pi/4} \end{bmatrix} = \begin{bmatrix} 1 \\ e^{i(\pi/2 \pm \pi/4)} \end{bmatrix} = \begin{bmatrix} 1 \\ e^{i(3\pi/4)} \end{bmatrix} \text{ or } \begin{bmatrix} 1 \\ e^{i(\pi/4)} \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 \\ 0 & e^{\pm i\pi/4} \end{bmatrix}\underbrace{\begin{bmatrix} 1 \\ -i \end{bmatrix}}_{\text{RCP}} = \begin{bmatrix} 1 \\ -i\,e^{\pm i\pi/4} \end{bmatrix} = \begin{bmatrix} 1 \\ e^{-i(\pi/2 \pm \pi/4)} \end{bmatrix} = \begin{bmatrix} 1 \\ e^{-i(3\pi/4)} \end{bmatrix} \text{ or } \begin{bmatrix} 1 \\ e^{-i(\pi/4)} \end{bmatrix}$$

Using Euler's theorem the output light can be expressed as,

$$\begin{bmatrix} A \\ B + iC \end{bmatrix} = \begin{bmatrix} 1 \\ \dfrac{\pm\sqrt{2} \pm i\sqrt{2}}{2} \end{bmatrix}$$

This is of the form of elliptical polarization with $A = 1$, and $|B| = |C| = \sqrt{2}/2$.

14-17. Consider the action of the matrix on a general Jones vector,

$$\begin{bmatrix} 1 & i \\ -i & 1 \end{bmatrix}\begin{bmatrix} A \\ B + iC \end{bmatrix} = \begin{bmatrix} A + iB - C \\ -iA + B + iC \end{bmatrix} = (A - C + iB)\begin{bmatrix} 1 \\ -i \end{bmatrix}\text{: Right circular polarization}$$

For a left circular polarizer try,

$$\begin{bmatrix} 1 & -i \\ i & 1 \end{bmatrix}\begin{bmatrix} A \\ B + iC \end{bmatrix} = \begin{bmatrix} A - iB + C \\ iA + B + iC \end{bmatrix} = (A + C - iB)\begin{bmatrix} 1 \\ +i \end{bmatrix}\text{: Left circular polarization}$$

14-18. Note that,

$$\underbrace{\begin{bmatrix} 1 \\ \pm i \end{bmatrix}}_{\text{Circular}} + \underbrace{\begin{bmatrix} \cos\alpha \\ \sin\alpha \end{bmatrix}}_{\text{Linear}} = \begin{bmatrix} \cos\alpha + 1 \\ \sin\alpha \pm i \end{bmatrix} = \underbrace{\begin{bmatrix} A \\ B \pm iC \end{bmatrix}}_{\text{Elliptical}}$$

14-19. In general, for elliptical polarization, E_{0x} does not equal E_{0y} and ε does not equal 0,

$$E_y = E_{0y}\cos(kz - \omega t) = E_{0y}\cos\alpha$$
$$E_x = E_{0x}\cos(kz - \omega t + \varepsilon) = E_{0x}\cos(\alpha + \varepsilon)$$

Eliminate z and t through α:

$$\frac{E_y}{E_{0y}} = \cos\alpha$$
$$\frac{E_x}{E_{0x}} = \cos(\alpha+\varepsilon) = \cos\alpha\cos\varepsilon - \sin\alpha\sin\varepsilon = \frac{E_y}{E_{0y}}\cos\varepsilon - \sqrt{1-(E_y/E_{0y})^2}\sin\varepsilon$$
$$(E_x/E_{0x}) - (E_y/E_{0y})\cos\varepsilon = -\sqrt{1-(E_y/E_{0y})^2}\sin\varepsilon$$

Squaring gives,

$$(E_x/E_{0x})^2 + (E_y/E_{0y})^2\cos^2\varepsilon - 2\,(E_xE_y)\cos\varepsilon/(E_{0x}E_{0y}) = (1-(E_y/E_{0y})^2\sin^2\varepsilon$$
$$\left(\frac{E_x}{E_{0x}}\right)^2 + \left(\frac{E_y}{E_{0y}}\right)^2(\cos^2\varepsilon + \sin^2\varepsilon) - 2\frac{E_xE_y}{E_{0x}\,E_{0y}}\cos\varepsilon = \sin^2\varepsilon$$
$$\left(\frac{E_x}{E_{0x}}\right)^2 + \left(\frac{E_y}{E_{0y}}\right)^2 - 2\frac{E_xE_y}{E_{0x}\,E_{0y}}\cos\varepsilon = \sin^2\varepsilon$$

14-20. (a) Putting the Jones vector into standard form,

$$\begin{bmatrix} A \\ B+iC \end{bmatrix} = \begin{bmatrix} 2 \\ 3e^{i\pi/3} \end{bmatrix} = \begin{bmatrix} 2 \\ 3\cos(\pi/3) + i\,3\sin(\pi/3) \end{bmatrix} = \begin{bmatrix} 2 \\ 3/2 + i\,3\sqrt{3}/2 \end{bmatrix}$$

So, $A = 2, B = 3/2,$ and $C = 3\sqrt{3}/2$. The normalized standard form is then

$$\frac{1}{\sqrt{A^2+B^2+C^2}}\begin{bmatrix} A \\ B+iC \end{bmatrix} = \frac{1}{\sqrt{13}}\begin{bmatrix} 2 \\ 3/2 + i\,3\sqrt{3}/2 \end{bmatrix} = \frac{1}{\sqrt{13}}\begin{bmatrix} E_{\text{ox}} \\ E_{0y}e^{i\varepsilon} \end{bmatrix}$$

This corresponds to elliptical polarization with

$$E_{0x} = A = 2,\, E_{0y} = \sqrt{B^2+C^2} = 3,\, \varepsilon = \tan^{-1}(C/B) = \pi/3$$

Then,

$$\tan 2\alpha = \frac{E_{0x}\,E_{0y}\cos\varepsilon}{E_{0x}^2 - E_{0y}^2} = \frac{2\,(2)\,(3)\cos(\pi/3)}{2^2-3^2} \Rightarrow \alpha = -25.07°$$

(b) The modification by the rotator is given by the Jones formalism with $\beta = 30°$,

$$\begin{bmatrix} \cos\beta & -\sin\beta \\ \sin\beta & \cos\beta \end{bmatrix}\begin{bmatrix} 2 \\ 3e^{i\pi/3} \end{bmatrix} = \begin{bmatrix} \sqrt{3}/2 & -1/2 \\ 1/2 & \sqrt{3}/2 \end{bmatrix}\begin{bmatrix} 2 \\ 3/2 + i\,3\sqrt{3}/2 \end{bmatrix} = \begin{bmatrix} (\sqrt{3}-3/4) - i\,(3\sqrt{3}/4) \\ (1+3\sqrt{3}/4) + i\,(9/4) \end{bmatrix}$$

In general one can write,

$$\begin{bmatrix} a - ib \\ c + id \end{bmatrix}(a+ib) = \begin{bmatrix} a^2+b^2 \\ (ac - bd) + i\,(bc + ad) \end{bmatrix}$$

For the case at hand, $a = \sqrt{3} - 3/4), b = 3\sqrt{3}/4, c = 1 + 3\sqrt{3}/4,$ and $d = 9/4$. So, the output Jones vector can be written as,

$$\begin{bmatrix} (\sqrt{3}-3/4) - i\,(3\sqrt{3}/4) \\ (1+3\sqrt{3}/4) + i\,(9/4) \end{bmatrix} = \begin{bmatrix} 2.65192 \\ -0.66506 + i\,5.19615 \end{bmatrix} = \begin{bmatrix} A \\ B+iC \end{bmatrix}$$

So, $E_{0x} = A = 2.652, E_{0y} = \sqrt{B^2+C^2} = 5.239, \varepsilon = \tan^{-1}(C/B) = 97.294°$. Then

$$\tan 2\alpha = \frac{E_{0x}\,E_{0y}\cos\varepsilon}{E_{0x}^2 - E_{0y}^2} = \frac{2(2.65)\,(5.24)\cos(97.3°)}{(2.65)^2 - 5.24^2} \Rightarrow \alpha = 4.903°$$

The elliptical polarization is thus rotated by $(4.90° - (-25.07°))$=+30°, as expected.

14-21. According to Eq. (14 12),

$$\left(\frac{E_x}{E_{0x}}\right)^2+\left(\frac{E_y}{E_{0y}}\right)^2-2\frac{E_xE_y}{E_{0x}\,E_{0y}}\cos\varepsilon=\sin^2\varepsilon$$

(a) For $\varepsilon=\pi/2\Rightarrow\cos\varepsilon=0,\sin\varepsilon=1$. Then,

$$\left(\frac{E_x}{E_{0x}}\right)^2+\left(\frac{E_y}{E_{0y}}\right)^2=1$$

This is an ellipse with semi axes of E_{0x} and E_{0y} aligned with the coordinate axes.

(b) If $E_{0x}^2=E_{0y}^2=E_0^2$,

$$E_x^2+E_y^2-2\,E_x\,E_y\cos\varepsilon=E_0^2\sin^2\varepsilon$$

This is an ellipse with principal axes at 45° to the coordinate axes since,

$$\tan 2\alpha=\frac{E_{0x}\,E_{0y}\cos\varepsilon}{E_{0x}^2-E_{0y}^2}=\frac{2\,E_0^2\cos\varepsilon}{E_0^2-E_0^2}=\infty\Rightarrow 2\,\alpha=90°\Rightarrow\alpha=45°$$

(c) If the conditions of both (a) and (b) hold, the equation for the ellipse reduces to

$$E_x^2+E_y^2=E_0^2$$

This is the equation for a circle centered on the origin of radius E_0.

(d) For $\varepsilon=0,\cos\varepsilon=1,\sin\varepsilon=0$. Then,

$$\left(\frac{E_x}{E_{0x}}\right)^2+\left(\frac{E_y}{E_{0y}}\right)^2-2\frac{E_xE_y}{E_{0x}\,E_{0y}}=0$$

$$\left(\frac{E_x}{E_{0x}}-\frac{E_y}{E_{0y}}\right)^2=0\Rightarrow E_y=\left(\frac{E_{0y}}{E_{0x}}\right)E_x$$

This is an equation for a straight line of slope E_{0y}/E_{0x}.

14-22. Take the fast axis of the QWP to be along the x axis. Then the light emerging from the first polarizer has the Jones vector,

$$\begin{bmatrix}\cos\theta\\ \sin\theta\end{bmatrix}$$

The second polarizer (sometimes called the *analyzer*) has its transmission axis at $\alpha=90°+\theta$. Then using the general Jones matrix for a linear polarizer and the relations $\cos\alpha=-\sin\theta$ and $\sin\alpha=\cos\theta$.

$$\begin{bmatrix}\cos^2\alpha & \sin\alpha\cos\alpha\\ \sin\alpha\cos\alpha & \sin^2\alpha\end{bmatrix}=\begin{bmatrix}\sin^2\theta & -\sin\theta\cos\theta\\ -\sin\theta\cos\theta & \cos^2\alpha\end{bmatrix}$$

Then the output light has the Jones vector,

$$\underbrace{\begin{bmatrix}\sin^2\theta & -\sin\theta\cos\theta\\ -\sin\theta\cos\theta & \cos^2\alpha\end{bmatrix}}_{\text{Analyzer}}\underbrace{\begin{bmatrix}1 & 0\\ 0 & i\end{bmatrix}}_{\substack{\text{QWP}\\ \text{FA hor}}}\underbrace{\begin{bmatrix}\cos\theta\\ \sin\theta\end{bmatrix}}_{\text{Amp}=1}=(1-i)\sin\theta\cos\theta\underbrace{\begin{bmatrix}\sin\theta\\ \cos\theta\end{bmatrix}}_{\text{Amp}=1}$$

The amplitude of the output vector (relative to the input vector) is $(1-i)\sin\theta\cos\theta$. Then, with the input irradiance being I_0, the output irradiance is,

$$I=I_0\sin^2\theta\cos^2\theta\,|(1-i)|^2=2\sin^2\theta\cos^2\theta\;I_0$$

CHAPTER 15 PRODUCTION OF POLARIZED LIGHT

15-1. The first polarizer blocks 1/2 of the unpolarized incident light. Then applying Malus' law for the last two polarizers,

$$I_1 = \frac{1}{2} I_0$$
$$I_2 = I_1 \cos^2(30°) = \frac{1}{2} I_0 \cos^2 30°$$
$$I_3 = I_2 \cos^2(60° - 30°) = \frac{1}{2} I_0 \cos^4 30° = 0.2815\, I_0 \Rightarrow 28.15\% \text{ of } I_0$$

15-2. The polarizing angle is given by the relation, $\tan\theta_p = \frac{n_2}{n_1}$. So for $n_{\text{air}} = 1$ and $n_{\text{diam}} = 2.42$

$$\text{Internal reflection:}\quad \theta_p = \tan^{-1}\left(\frac{n_{\text{air}}}{n_{\text{diam}}}\right) = \tan^{-1}\left(\frac{1}{2.42}\right) = 22.5°$$
$$\text{External reflection:}\quad \theta_p = \tan^{-1}\left(\frac{n_{\text{diam}}}{n_{\text{air}}}\right) = \tan^{-1}\left(\frac{2.42}{1}\right) = 67.5°$$

15-3. (a) See Figure 15 22 that accompanies the statement of the problem in the text. Let the amplitude of the unpolarized incident field component parallel to the transmission axis TA_1 be E_{0p} and let the amplitude of the field component perpendicular to TA_1 be E_{0n} with $E_{0p}^2 + E_{0n}^2 = E_0^2$ and $E_{0p}^2 = E_{0n}^2 = E_0^2/2$. Then:

Polarizer 1:
$E_{1p} = \sqrt{\alpha}\, E_{0p}$ through TA_1 since the energy fraction is α. Similarly, $E_{1n} = \sqrt{\beta}\, E_{0n}$ through the extinction axis EA_1.

Polarizer 2: Now each component E_{1p} and E_{1n} has a component along both the transmission axis TA_2 and extinction axis EA_2 of the second polarizer. Then

$E_{2p} = \sqrt{\alpha}\,(E_{1p}\cos\theta)$ thru TA_2
or $E_{2p} = \sqrt{\alpha}\,(\sqrt{\alpha}E_{0p})\cos\theta$ thru TA_2
$E_{2p} = \sqrt{\beta}\,(E_{1p}\sin\theta)$ thru EA_2
$E_{2p} = \sqrt{\beta}\,(\sqrt{\alpha}\,E_{0p})\sin\theta$ thru EA_2
$I_p = (\varepsilon_0\, c/2)E_p^2 = (\varepsilon_0\, c/2)[E_{2p}\,(\text{TA})]^2 + [E_{2p}\,(\text{EA})]^2$
$I_p = (\varepsilon_0\, c/2)\{\alpha^2\, E_{0p}^2 \cos\theta + \alpha\,\beta\, E_{0P}^2 \sin^2\theta\}$
$I_p = (\varepsilon_0\, c/2)E_{0p}^2\,(\alpha^2\cos^2\theta + \alpha\,\beta\sin^2\theta)$

$E_{2n} = \sqrt{\alpha}\,(E_{1n}\sin\theta)$ thru TA_2
$E_{2n} = \sqrt{2}\,(\sqrt{\beta}\,E_{0n})\sin\theta$ thru TA_2
$E_{2n} = \sqrt{\beta}\,(E_{1n}\cos\theta)$ thru EA_2
$E_{2n} = \sqrt{\beta}\,(\sqrt{\beta}\,E_{0n})\cos\theta$ thru EA_2
$I_n = (\varepsilon_0\, c/2)E_n^2 = (\varepsilon_0\, c/2)[E_{2n}\,(\text{TA})]^2$
$I_n = (\varepsilon_0\, c/2)\{\alpha\,\beta\, E_{0n}^2\sin^2\theta + \beta^2\, E_{0n}^2\cos^2\theta\}$
$I_n = (\varepsilon_0\, c/2)E_{0n}^2\,(\alpha\,\beta\sin^2\theta + \beta^2\cos^2\theta)$

Using $E_{0p}^2 = E_{0n}^2 = E_0^2/2$ one finds,

$$I = I_p + I_n = \left(\frac{\varepsilon_0\, c}{2}\right)\frac{E_0^2}{2}\{(\alpha+\beta)\cos^2\theta + 2\alpha\,\beta\sin^2\theta\} = \frac{1}{2} I_0\{(\alpha+\beta)\cos^2\theta + 2\alpha\,\beta\sin^2\theta\}$$
$$I = I_0\,\{(1/2)(\alpha+\beta)\cos^2\theta + \alpha\,\beta\sin^2\theta\}$$

In the ideal case, $\beta = 0$ and $\alpha = 1$ so that $I = \frac{1}{2} I_0 \cos^2\theta$.

(b) With $\alpha = 0.95$, $\beta = 0.05$, one finds

θ	ideal	actual	
0°	0.5	0.4525	fractions of I_0
30°	0.375	0.351	
45°	0.250	0.250	
90°	0	0.0475	

15-4. $\frac{\lambda}{2} = t\,(\Delta n)$ or $t = \frac{\lambda}{2\,\Delta n} = \frac{632.8 \times 1 -^{-7}\text{ cm}}{2\,(1.599 - 1.594)} = 0.063\text{mm}$

15-5. See the answers in the back of the text.

15-6. (a) With the flat plate in the position of maximum retardation,

$$\Delta_{\max} = 2\,\lambda = (\Delta n)\,(d_2 - d_1) \text{ or } d_2 - d_1 = \frac{2\lambda}{\Delta n} = \frac{2\,(546.1 \times 10^{-6}\,\text{mm}}{1.555 - 1.546} = 0.12\text{mm}$$

(b) For circularly polarized light, $\Delta = \lambda/4$, so $d_2 - d_1 = \frac{\lambda}{4\,\Delta n} = \frac{546.1 \times 10^{-6}\,\text{mm}}{4\,(1.555 - 1.546)} = 0.015\text{mm}$.

15-7. (a) The sketch is essentially the same as Figure 15 22 that accompanies problem 15 3 in the text. Then,

$$I_1 = I_0/2 \qquad E_2 = E_1 \cos\theta \Rightarrow I_2 = I_1 \cos^2\theta = \frac{1}{2}\,I_0 \cos^2\theta$$

(b) In this case, $I_2 = \frac{1}{2}\,I_0 \cos^2\theta = \frac{1}{2}\,I_0 \cos^2(90° - 0°) = 0$.

(c) The angle between successive polarizers is always 15° so,

$$I_7 = \frac{1}{2}\,I_0 \left[\cos^2(15°)\right]^6 = 0.33\,I_0$$

So 33% of the incident light is transmitted.

15-8. The angular offset between successive polarizers is $\theta = 90°/N$. Applying Malus' law N times in succession,

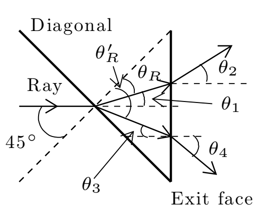
$$I_T = I_0 \left(\cos^2\theta\right)^N = I_0\,[\cos(90°/N)]^{2N} = 0.9\,I_0$$
$$[\cos(90°/N)]^{2N} = 0.9$$

A numerical solution indicates that N is between 23 and 24. For $N = 24, I_T = 0.9022\,I_0$.

15-9. Using, $\lambda/4 = (\Delta n)\ t$,

$$t = \frac{\lambda}{4\,\Delta n} = \frac{589.3 \times 10^{-6}\,\text{mm}}{4\,(1.5534 - 1.5443)} = 0.0162\text{mm}$$

15-10. See Figure 15 24 that accompanies the statement of this problem in the text. Also refer to the figure below for the labeling of the various angles:



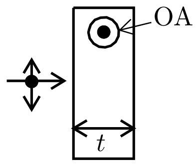
At the diagonal interface:

$$E_p \text{ component from } n_{\parallel} \text{ to } n_{\perp}\text{: } 1.4864 \sin 45 = 1.6584 \sin\theta_R \text{ or } \theta_R = 39.329°$$
$$E_s \text{ component from } n_{\perp} \text{ to } n_{\parallel}\text{: } 1.6584 \sin 45 = 1.4864 \sin\theta'_R \text{ or } \theta'_R = 52.086°$$

On exit:

$$\text{Upper ray: } \theta_1 = 45 - \theta_R = 5.671°;\ 1.6584 \sin 5.671° = (1) \sin\theta_2 \text{ or } \theta_2 = 9.432°$$
$$\text{Lower ray: } \theta_3 = \theta'_R - 45 = 7.086°;\ 1.4864 \sin 7.086° = (1) \sin\theta_4 \text{ or } \theta_4 = 10.566°$$
$$\text{Deviation: } \theta_2 + \theta_4 = 9.432° + 10.566° = 19.997° \approx 20°$$

15-11. The arrangement is as shown below,



$$\Delta\,\varphi = \pi/2 \text{ or } \Delta = \frac{\lambda}{4} = t\,\Delta n$$

$$\Delta n = \frac{\lambda}{4\,t} = \frac{600\times 10^{-6}\,\text{mm}}{4\,(0.03\,\text{mm})} = 0.005$$

15-12. See Figure 15 25 that accompanies the statement of the problem in the text.
(a) The incident angle is the polarizing angle,

$$\tan\theta_p = \frac{n_2}{n_1} = \frac{1.33}{1} \Rightarrow \theta_p = 53.12°$$

(b) The angle θ_R the refracted ray makes with the normal to the air/water interface is

$$\theta_R = \sin^{-1}\left(\frac{\sin\theta_P}{1.333}\right) = 36.877°$$

The polarizing angle for the water/glass interface is, $\theta'_P = \tan^{-1}\left(\frac{1.50}{1.333}\right) = 48.37°$

If the glass surface was parallel to the water surface the angle of incidence on the glass would be $\theta_R = 36.877°$. However, for complete polarization off the glass, θ'_P must be 48.37°. Thus the glass must be tilted by $48.37° - 36.88° = 11.5°$ relative to the water surface.

15-13. (a) With polarizer alone rotating, there is no variation in intensity. This means there can be no linearly polarized, or elliptically polarized, component. The light must be either unpolarized or circularly polar ized, or a mixture of both. If pure circularly polarized, then insertion of a QWP should produce extinction at some position. Since minima do not go to zero, the light is a mixture of circularly polarized and unpo larized light. (See Meyer Arendt, for example, for a convenient diagrammatic analysis of such cases.)

(b) Here the rotating polarizer *does* produce a variation in intensity, so there must be linearly, or ellipti cally, polarized light or a component of both. Since the intensity never goes to zero, it cannot be a case of *pure* linearly polarized light. The use of the QWP now produces zero minima, so the light must be ellipti cally polarized. Any linearly polarized component would not be extinguished under the condition described.

15-14. (a) The angle of incidence at which maximum polarization occurs is,

$$\theta_P = \tan^{-1}\left(\frac{2.42}{1.62}\right) = 56.2°$$

(b) The angle of refraction into the diamond is

$$\theta_R = \sin^{-1}\left[\frac{1.62\sin\theta_P}{2.42}\right] = 33.8°$$

15-15. The rotation of polarized light in an optically active medium is proportional to the inverse of the square of the wavelength: $\beta = \rho\,L\,d \propto 1/\lambda^2$.
(a) For the given parameters, the concentration of the solution is

$$d = \frac{\beta}{p\,L} = \frac{1.23}{(20.5)\,(1.2)}\text{g/cc} = 0.05\,\text{g/cc}$$

(b) Given the stated wavelength dependence,

$$\frac{\beta_{\text{red}}}{\beta_{\text{violet}}} = \frac{\lambda^2_{\text{violet}}}{\lambda^2_{\text{red}}} \Rightarrow \beta_{\text{violet}} = \beta_{\text{red}}\,\lambda^2_{\text{red}}\,/\lambda^2_{\text{violet}} = (15°)\,(700^2/400^2) = 46°$$

15-16. In general the thickness z is related to rotation β by, $\beta = \frac{\pi z}{\lambda_0}(n_{\mathfrak{L}} - n_{\mathfrak{R}})$.

(a) If $\beta = 10° = 0.01745\,\text{rad}$, $z = \frac{\beta\lambda_0}{\pi(\Delta m)} = \frac{(0.1745)(396.8\times10^{-6}\text{mm})}{\pi(0.00011)} = 0.200\,\text{m}$.

(b) Now, $\rho = \beta$ for $z = 1\,\text{mm}$. So,

$$\rho = \frac{\pi z}{\lambda_0}\Delta n = \frac{\pi(1\,\text{mm})}{396.8\times10^{-6}\,\text{mm}}(0.00011) = 0.8709\,\text{rad} = 49.9°$$

Also, by interoplation from Table 15 − 2, $\rho \approx 50°$.

15-17. (a) $\frac{\lambda}{4} = t\,\Delta n \Rightarrow t = \frac{\lambda}{4\,\Delta n} = \frac{589.3\times10^{-6}\,\text{mm}}{4\,|1.4864 - 1.6584|} = 8.57\times10^{-4}\,\text{mm}$

(b) For Zircon, using Table 15 1, $n_{\parallel} = 1.968$ and $n_{\perp} = 1.923$. It is given that $t = 0.0182\text{mm}$. To transmit, it must serve as a HWP so $\frac{\lambda}{z} = t\,\Delta n$, or

$$\lambda = 2\,t\,\Delta n = 2\,(0.00182\,\text{cm})\,(1.968 - 1.923) = 1.638\times10^{-4}\,\text{cm}$$

This is in the infrared. In general

$$(m + 1/2)\,\lambda = t\,\Delta n \Rightarrow \lambda = \frac{t\,\Delta n}{m + 1/2} = \frac{8.19\times10^{-5}\,\text{cm}}{m + 1/2}$$

For $m = 1$, $\lambda = \frac{8.19\times10^{-5}\,\text{cm}}{1.5} = 546$ nm (green). For $m = 2$, $\lambda = 327.6$ nm (ultraviolet). Thus in the visible, green light is transmitted.

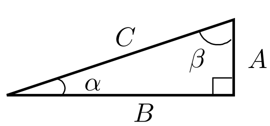
15-18. Let,

$$\tan\theta_p = \frac{n_2}{n_1} \text{ and } \tan\theta_p' = \frac{n_1}{n_2}$$

Then,

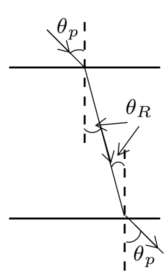
$$\tan\theta_p = \frac{1}{\tan\theta_p'} = \cot\theta_p'$$

So $\theta_p = 90° - \theta_p'$. (See the triangle below.)



$$\tan\alpha = \cot\beta = A/B$$
$$\alpha + \beta = 90°$$

(b) See the figure below.



(1) $\tan\theta_p = n = n_2/n_1$ – Brewster's angle for light in.
(2) $\sin\theta_p = n\sin\theta_R$ – Snell's law.

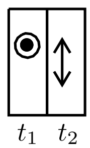
Then from (1): $\sin\theta_p = n\cos\theta_p$.
Substituting 2): $n\sin\theta_R = n\cos\theta_p$: θ_R and θ_p must be complementary.
$n = \tan\theta_p = \cot\theta_R = \frac{1}{\tan\theta_R}$ and $\tan\theta_R = \frac{1}{n} = \frac{n_1}{n_2}$

15-19. (a) $\Delta = t\,\Delta n$. For $t = \lambda$, $\Delta/\lambda = \Delta n = 1.5553 - 1.5462 = 0.0091$

(b) $\dfrac{\lambda}{4} = t\,\Delta n$ or $t = \dfrac{546 \times 10^{-6}\,\text{mm}}{4\,(.0091)} = 0.015\,\text{mm} = 15\mu\text{m}$

(c) $m + 1/4)\,\lambda = t\,\Delta n \Rightarrow m + 1/4 = \dfrac{t\,\Delta n}{\lambda} = \dfrac{(0.735\,\text{mm})\,(0.0091)}{546 \times 10^{-6}\,\text{mm}} = 12.25 \Rightarrow m = 12$

(d)



$$\Delta_1 = -\,\Delta_2 \Rightarrow \Delta_{\text{total}} = t_1\,\Delta n + t_2\,(-\,\Delta n) = \lambda/4$$

$$t_1 - t_2 = \frac{l}{4\,\Delta n} = \frac{546 \times 10^{-6}\,\text{mm}}{4\,(0.0091)} = 0.015\,\text{mm} = 15\;\mu\text{m}$$

15-20. Given that $\lambda = 500\text{nm}$ and $t = 1/16\,\text{in} = 0.0625\,\text{in} = 0.158875\,\text{cm}$,

$$(m + 1/2)\,\lambda\,\Delta n \Rightarrow \Delta n = \frac{(m + 1/2)\,\lambda}{t} = \frac{(m + 1/2)\,(500 \times 10^{-6}\,\text{mm})}{1.5875\text{mm}} = (m + 1/2)\,(3.1496 \times 10^{-4})$$

So for $\Delta m = 1$ between adjacent bands, $\Delta n = 3.15 \times 10^{-4}$.

15-21. From Eq. (15 3), $d = \dfrac{\lambda_0\,\Delta\varphi}{2\,\pi\,\Delta n}$. From Table 15 1, $\Delta n = 1.598 - 1.590 = 0.008$.

(a) Linear polarization: $\Delta\,\varphi = \pi$ and $d = \dfrac{(0.5893\;\mu\text{m})\,(\pi)}{2\,\pi\,(0.008)} = 36.8\,\mu\text{m}$

(b) Circular polarization: $\Delta\,\varphi = \dfrac{\pi}{2}$ and $d = \dfrac{36.8\mu\,m}{2} = 18.4\,\mu\text{m}$

15-22. From Table 15 2, $\rho = 25.535\,\text{deg/mm}$ at 546nm. Then

$$\beta = \rho\,L = +\,25.535\,\text{deg/mm} \times 1.15\,\text{mm} = +\,29.365° \text{ (ccw)}$$

Thus the HWP must be set at $\beta/2 = +\,14.68°$, since it converts linearly polarized light at $+\,\alpha$ to to lin early polarized light at $-\,\alpha$.

15-23. The polarizing angle is $\theta_p = \tan^{-1} n = \tan^{-1} n_2/n_1 = \tan^{-1} 1.5 = 56.31°$.

(a) The reflectance is

$$R = \left[\frac{\cos 56.31 - \sqrt{1.5^2 - \sin^2 56.31}}{\cos 56.31 + \sqrt{1.5^2 - \sin^2 56.31}}\right]^2 = 0.1479 \Rightarrow 14.79\%$$

(b) The incident irradiance divides equally into $I_0/2$ for each mode: Brewster's angle is satisfied at each interface. For the TE mode, 14.8% is reflected, so 85.2% is transmitted at each interface. The TM mode is transmitted throughout at 100%. For N plates, there are $2\,N$ interfaces, so for $N = 10$ we have

$$\%\text{ TE transmitted} = (0.852)^{2N}\,I_0/2 = 0.0203\,I_0 \Rightarrow 2.03\%\,I_0$$

(c) Since $I_{\text{TM}} = 50\%$ of $I_0 \Rightarrow I_{\text{TM}} = 0.50\,I_0$. So,

$$P = \frac{0.5 - 0.0203}{0.5 + 0.0203} = 0.922$$

15-24. See Figure 15 26 that accompanies the statement of this problem in the text. The half wave plate rotates the the polarization of the linearly polarized light so that it is polarized along a line that makes an angle of $2\,\theta$ with the x axis. Therefore the light polarization direction makes an angle of $(90 - 2\,\theta)$ with the trans mission axis of the analyzer. So, using Malus' law, $I = I_0\cos^2(90 - 2\,\theta) \Rightarrow I = I_0\sin^2(2\,\theta)$. his problem can also be worked using matrices, in exactly the manner of problem 14 22, replacing the QWP matrix there by the HWP matrix.

15-25. (a) Using Eq. (15 10) and Table 15 3,

$$\beta = \frac{\pi\, z}{\lambda_0}\,(n_L - n_R) = \beta = \frac{\pi\,(3 \times 10^{-3})}{762 \times 10^{-9}}\,(6 \times 10^{-5}) = 0.742\,\mathrm{rad} = 42.5^\circ$$

(b) From Eq (15 3), with $\Delta\varphi = \pi$,

$$d = \frac{\pi\,\lambda_0}{2\,\pi\,|n_\perp - n_\parallel|} = \frac{\lambda_0}{2\,|n_\perp - n_\parallel|} = z$$

So,

$$\beta = \frac{\pi}{\lambda_0}\,(n_{\mathfrak{L}} - n_{\mathfrak{R}})\,\frac{\lambda_0}{2|n_\perp - n_\parallel|} = \frac{\pi}{2}\,\frac{n_{\mathfrak{L}} - n_{\mathfrak{R}}}{|n_\parallel - n_\perp|} = \frac{\pi}{2}\,\frac{6 \times 10^{-5}}{8.94 \times 10^{-3}} = 0.0105\,\mathrm{rad} = 0.60^\mathrm{c}$$

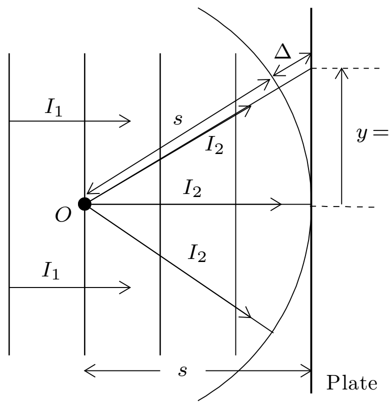
Chapter 16 Holography

16-1. See Figure 16 9 that accompanies the statement of this problem in the text and the sketch below. Using Eq. (7 14),

$$I = I_1 + I_2 + \sqrt{I_1 I_2}\cos\delta = I = I_1 + I_2 + 2\sqrt{I_1 I_2}\left[2\cos^2(\delta/2) - 1\right]$$

with $\delta = 2\pi\,\Delta/\lambda$. From the sketch, $(s+\Delta)^2 = r^2 + s^2 \Rightarrow s^2 + 2\,s\,\Delta + \Delta^2 = r^2 + s^2 \Rightarrow \Delta\,(2s+\Delta) = r^2$

So for $\Delta \ll s$, $\Delta \approx \dfrac{r^2}{2\,s}$ and $\delta \approx \dfrac{\pi\,r^2}{\lambda\,s}$. Then,



$$I = I_1 + I_2 + 2\sqrt{I_1 I_2}\left[2\cos^2\left(\frac{\pi\,r^2}{2\,\lambda\,s}\right)\right.$$

$$I = \underbrace{\left[I_1 + I_2 - 2\sqrt{I_1 I_2}\right]}_{A} + \left[\underbrace{4\sqrt{I_1 I_2}\cos^2}_{B}\left(\underbrace{\frac{\pi\,r^2}{2\,\lambda\,s}}_{\alpha r^2}\right)\right]$$

$$I = A + B\cos^2(\alpha r^2)$$

16-2. (a) As shown in problem 7 4, for $(E_R)^2/(E_S)^2 = N$, the visibility is $\mathcal{V} = 2\sqrt{N}/(N+1)$

(b) For $N = 3$, $\mathcal{V} = 2\sqrt{3}/(3+1) = 0.866$

16-3. If the storage could be done at the same density, the information stored holographically would be,

$$(1\,\text{Gb})\,(1\text{mm}/10^{-3}\text{mm}) = 1000\,\text{Gb}$$

16-4. $d = \dfrac{\lambda\,n}{2\sin\theta} = \dfrac{633\,\text{nm}}{2\sin 10°} = 1823\,\text{nm} = 1.82\,\mu\text{m}$

16-5. $v = \dfrac{x}{t} = \dfrac{\lambda/10}{10^{-9}\,\text{s}} = \dfrac{633\times10^{-10}\,\text{m}}{10^{-9}\,\text{s}} = 63.3\,\text{m/s}$

16-6. $\dfrac{I_{\max}}{I_{\min}} = \left[\dfrac{A_R + A_S}{A_R - A_S}\right]^2 = \left[\dfrac{A_R/A_S + 1}{A_R/A_S - 1}\right]^2 = \left[\dfrac{8+1}{8-1}\right]^2 = 1.65$

16-7. $N_b = \dfrac{V}{\lambda_n^3} = \dfrac{1\,\text{mm}^3}{(492\,\text{nm}/1.30)^3} = \dfrac{(10^{-3})^3\,\text{m}^3}{(492\times10^{-9}/1.30)^3\,\text{m}^3} = 1.8\times10^{10}$ bits

16-8. (a) Refer to Figure 16 5 in the body of the text and note that $2\,\theta = 180°$. Then the spacing is given by

$$d = \frac{\lambda_n}{2} = \frac{\lambda_0}{2\,n} = \frac{500\,\text{nm}}{(2)\,(1.6)} = 156.25\,\text{nm}$$

(b) For $m\,\lambda_n = m\,\lambda_0/n = 2\,d\sin 90° \Rightarrow$. For $m = 1$, $\lambda_0 = 2\,n\,d = 500\,\text{nm}$.

(c) In the film the angle with respect to the normal is $\sin\varphi_F = (1/1.6)\sin(30°) \Rightarrow \varphi_F = 18.21°$

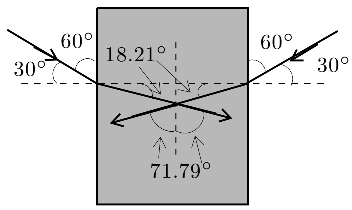
The Bragg angle is then $\theta = 90° - \varphi_F = 71.79°$. The Bragg condition then gives, (for $m = 1$),

$$\lambda_n = \frac{\lambda_0}{n} = 2\,d\sin\theta \Rightarrow \lambda_0 = 2\,n\,d\sin\theta = 2(1.6)(156.25\,\text{nm})\sin(71.79°) = 475\,\text{nm}$$

16-9. (a) See the sketch in Figure 16 7 for the orientation inside the film but note that the film being depicted there has $n = 1$ and so the angle θ is the same inside and outside of the film. In air, the incident angle with respect to the normal to the emulsion is $\varphi_0 = 30°$. The angle with respect to the normal in the film is then

$$\sin\varphi_F = (1/n)\sin\varphi_0 = (1/1.6)\sin(30°) \Rightarrow \varphi_F = 18.21°$$

so that the Bragg angle in the film is $\theta = 90° - \varphi_F = 71.79°$.



(b) The Bragg condition for $m = 1$ gives,

$$d = \frac{\lambda_n}{2\sin\theta} = \frac{\lambda_0}{2\,n\sin\theta} = \frac{633\text{ nm}}{2(1.6)\sin(71.79°)} = 208\,\text{nm}$$

(c) Using the Bragg condition, the correct Bragg angle is,

$$\sin\theta = \frac{\lambda_0}{2\,n\,d} = \frac{450}{2\,(1.6)(208)} = 0.6761 \Rightarrow \theta = 42.54°$$

The correct angle with respect to the normal within the film is then $\varphi_F = 90° - 42.54° = 47.46°$. The angle of incidence from air with respect to the normal is given by Snell's law,

$$\sin\varphi_0 = 1.6\sin\varphi_F = 1.6\sin 47.46° = 1.18$$

No solution is possible for this wavelength and this film.

16-10. Assuming a film of index of refraction unity. The original spacing is $d_0 = \dfrac{\lambda}{2\sin 90} = \dfrac{430}{2} = 215\,\text{nm}$. Upon shrinkage,

$$d = d_0 - 0.15\,d_0 = 182.75\text{ nm}$$

Then, $\lambda = 2\,d\sin 90° = 2\,(182.75\text{ nm})\,(1) = 365$ nm. The blue components shift into the ultraviolet and appear missing.

16-11. (a) $m = \dfrac{\lambda_R}{\lambda_{\text{uv}}} = \dfrac{633}{337} = 1.88\times$ (b) $m = \dfrac{\lambda_R}{\lambda_{\text{x-ray}}} = \dfrac{633}{0.10} = 6330\times$

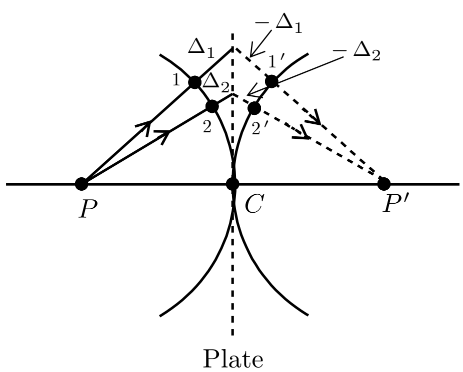
16-12. (a) The plane and spherical waves are of the form $E_R = r\, e^{i\omega t}$ and $E_S = \dfrac{s}{R}\, e^{i(\omega t+\delta)}$ with $E_F = E_R + E_S$. Then,

$$I_F = |E_F|^2 = E_R^2 + E_S^2 + E_R\, E_S + E_R\, E_S = r^2 + \frac{s^2}{R^2} + \frac{r\, s}{R}\, e^{-i\delta} + \frac{r\, s}{R}\, e^{i\delta}$$

$$E_H \propto I_F E_R = \underbrace{\left(r^2 + \frac{s^2}{R^2}\right) E_R}_{E_{H_1}} + \underbrace{\frac{r^2\, s}{R}\, e^{i(\omega t - \delta)}}_{E_{H_3}} + \underbrace{\frac{r^2\, s}{R}\, e^{i(\omega t+\delta)}}_{E_{H_2}}$$

Here E_{H_1} represents the amplitude modulated reference beam, E_{H_3} represents the phase reversed subject beam (real image), and E_{H_2} represents the amplitude modulated subject beam (virtual image).

(b) The result $\delta = \pi\, y^2/\lambda d$ was derived in problem 13 1 with $d = s$.



When the diverging spherical wave front (from P) on the left touches point C on the plate, phases of points 1 and 2 on the expanding wavefront for example show lags related to the path difference: $\Delta_1 > \Delta_2 > \Delta_0 = 0$.

When these differences are reversed, i.e., promoted to advances relative to C, then points such as $1'$ and $2'$, shown along the direction of reinforcement, (toward P') become appropriate. The points $0, 1', 2', \ldots$ are consistent with the spherical wavefront converging toward the (real object) P'.

Chapter 17 Optical Detectors and Displays

17-1. $E_{\text{gap}} = \dfrac{h\,c}{\lambda} = \dfrac{1240\,\text{eV}\cdot\text{nm}}{\lambda} \Rightarrow \lambda = \dfrac{1240\,\text{eV}\cdot\text{nm}}{E_{\text{gap}}} = \dfrac{1240\,\text{eV}\cdot\text{nm}}{0.67\,\text{eV}} = 1850\,\text{nm}$

17-2. (a) 1/10=0.1.

(b) Assume that N electrons are generated per second. Then the responsivity is

$$R = \frac{N\,e/\text{s}}{(10\,N\,h\,c/\lambda)/\text{s}} = 0.1\,\frac{e/\text{s}}{(h\,c/\lambda)/\text{s}} = 0.1\,\frac{1.6\times10^{-19}\,\text{C/s}}{(6.626\times10^{-34}\cdot 3\times10^{8}/0.9\times10^{-6})\,\text{J/s}} = 0.0724\,\text{A/W}$$

17-3. The responsivity is $R = I/P \Rightarrow I = R\,P = (0.8\,\text{A/W})\left(10^{-7}\,\text{W}\right) = 0.08\,\mu\text{A}$

17-4. $P_{\min} = \dfrac{E_{\text{photon}}}{\text{detection time}} = \dfrac{h\,c/\lambda}{t_{\text{det}}} = \dfrac{\left(6.626\times10^{-34}\cdot 3\times10^{8}/1.5\times10^{-6}\right)}{10^{-6}}\,\text{W} = 1.3\times10^{-13}\,\text{W}$

17-5. (a) The number of photons N_{ph} in a detection time t_{det} is,

$$N_{\text{ph}} = \frac{NEP}{(h\,c/\lambda)/t_{\text{det}}} = \frac{4\times10^{-11}}{(6.626\times10^{-34}\cdot 3\times10^{8}/1.5\times10^{-6})/(10^{-6})} = 300$$

(b) Since the number of photons in the detection time is greater than one, the detector is not limited by the intrinsic quantum fluctuations in the incident field.

17-6. The transmission through the liquid crystal cell is dependent on the voltage across the cell as indicated in Figure 17 9. Modulating the voltage then modulates the transmitted irradiance.

17-7. See for example, Saleh, B. E. A., and M. C. Teich. *Fundamentals of Photonics.* New York: John Wiley and Sons, 1991. Section 18 3. Phase modulation is most easily accomplished by using glass plates with parallel scratches both scratched in the same direction. In the absence of a voltage across the cell the molecules align in the direction of the scratches ($\theta = 0$). With an applied electric field (directed from one glass plate toward the other, the molecules tilt toward the direction of the field reaching an equilibrium tilt angle θ_e. Since the liquid crystal is birefringent, the index of refraction seen by light polarized along the $\theta = 0$ direc tion is

$$\frac{1}{n^2(\theta_e)} = \frac{\cos^2\theta_e}{n_e^2} + \frac{\sin^2\theta_e}{n_o^2}$$

Modulating the voltage across the cell thus modulates the index of refraction by changing the tilt angle and in turn the optical path difference and phase shift encountered in a pass through the cell are modulated.

17-8. The magnitude of the magnification is $\left|m\right| = \left|-s'/s\right| = \left|-10\text{ cm}/100\text{ cm}\right| = 0.1$. Thus a feature of object dimension 1 mm will have an image dimension of 0.1mm. This the pixel spacing would have to be less than 0.1 mm. The number of pixels would have to exceed $(20/0.1)^2 = 40000$.

17-9. The saturation power would be

$$P_{\text{sat}} = \frac{I_{\text{sat}}}{R} = \frac{10^{-5}\,\text{A}}{(0.1\,\text{A/W})} = = 0.1\,\text{mW}$$

17-10. The light emerging from the calculator display is polarized. Rotating the polarizer thus changes the irradi ance of the light reaching the eye. In one orientation the calculator display appears unchanged due to the addition of the polarizer. When the polarizer is rotated 90° from this orientation, all of the light from the display is blocked and the display appears dark.

Chapter 18 Matrix Methods in Paraxial Optics

18-1. See the sketch below. Using Eq. (18 1),

$$\frac{1}{f_1} = \frac{n_L - n'}{n\,R_2} - \frac{n_L - n}{n\,R_1} - \frac{(n_L - n)\,(n_L - n')}{n\,n_L}\,\frac{t}{R_1 R_2}$$

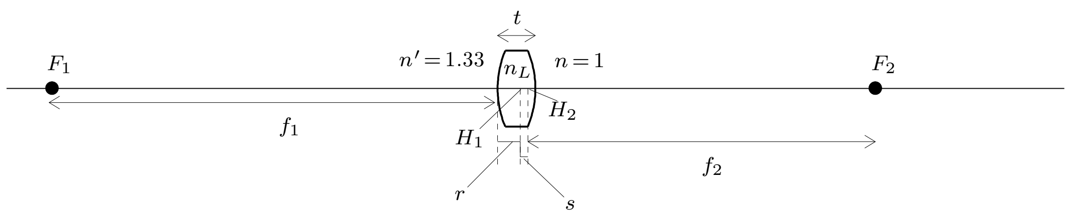
$$\frac{1}{f_1} = \frac{1.6 - 1}{1.33(-40\,\text{cm})} - \frac{1.6 - 1.33}{(1.33)\,(40\,\text{cm})} - \frac{(1.6 - 1.33)(1.6 - 1)}{1.33(1.6)}\,\frac{5\,\text{cm}}{(40\,\text{cm})(-40\,\text{cm})} \Rightarrow f_1 = -62.05\,\text{cm}$$

$$f_2 = -\frac{n'}{n} f_1 = -\frac{1}{1.33}\,(-62.05\,\text{cm}) = 46.66\,\text{cm}$$

Further, using Eq. (18 3),

$$r = \frac{n_L - n'}{n_L R_2}\, f_1\, t = \frac{1.6 - 1}{1.6\,(-40\,\text{cm})}\,(-62.05\,\text{cm})\,(5\,\text{cm}) = 2.91\,\text{cm}$$

$$s = -\frac{n_L - n}{n_L\,R_1} f_2\, t = \frac{1.6 - 1.33}{1.6(+40\,\text{cm})}(46.66\,\text{cm})(5\,\text{cm}) = -0.98\ \text{cm}$$



18-2. (a) Using Eq. (18 1)

$$\frac{1}{f_1} = \frac{n_L - n'}{n\,R_2} - \frac{n_L - n}{n\,R_1} - \frac{(n_L - n)\,(n_L - n')}{n\,n_L}\,\frac{t}{R_1 R_2}$$

$$\frac{1}{f_1} = \frac{1.53 - 1}{12.5\,\text{cm}} - \frac{1.53 - 1}{-20\,\text{cm}} - \frac{(1.53 - 1)^2}{1.53}\,\frac{3\,\text{cm}}{(12.5\text{cm})(-20\,\text{cm})} \Rightarrow f_1 = 14.06\,\text{cm}$$

$$f_2 = -\frac{n'}{n} f_1 = -1\,(-14.06\,\text{cm}) = -14.06\,\text{cm}$$

Further, using Eq. (18 3),

$$r = \frac{n_L - n'}{n_L R_2}\, f_1\, t = \frac{1.53 - 1}{1.53\,(12.5\,\text{cm})}\,(14.06\,\text{cm})\,(3\,\text{cm}) = 1.17\,\text{cm}$$

$$s = -\frac{n_L - n}{n_L\,R_1} f_2\, t = \frac{1.53 - 1}{1.53(-20\text{cm})}(-14.06\,\text{cm})(3\,\text{cm}) = -0.73\ \text{cm}$$

(b) With "object distance" 30 cm from V_1 or $s = 31.17$ cm from H_1,

$$\frac{1}{s_0} + \frac{1}{s_i} = \frac{1}{f} \Rightarrow \frac{1}{31.17\,\text{cm}} + \frac{1}{s_i} = \frac{1}{-14.06\,\text{cm}} \Rightarrow s_i = -9.689\,\text{cm}\,\text{from}\ H_2$$

Since the lens center is $1.50\,\text{cm} - 0.73\,\text{cm} = 0.77\,\text{cm}$ from H_2, the image is $-9.689\,\text{cm} + 0.77\,\text{cm} = -8.92\,\text{cm}$ from the lens center. Treating the lens as a thin lens,

$$\frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) = (1.53 - 1)\left(\frac{1}{-20} + \frac{1}{12.5}\right)\text{cm}^{-1} \Rightarrow f = -14.51\,\text{cm}$$

$$\frac{1}{s_o} + \frac{1}{s_i} = \frac{1}{f} = \frac{1}{30\,\text{cm}} + \frac{1}{s'} = \frac{1}{-14.51\,\text{cm}} \Rightarrow s_i = -9.780\,\text{cm}\,\text{from lens center}$$

The percent error in using the thin lens formula is

$$\%\ \text{error} = \frac{9.78 - 8.92}{8.92} = 9.6\%$$

18-3. Using Eq. (18 1)

$$\frac{1}{f_1} = \frac{n_L - n'}{n\,R_2} - \frac{n_L - n}{n\,R_1} - \frac{(n_L - n)\,(n_L - n')}{n\,n_L}\,\frac{t}{R_1 R_2}$$

$$\frac{1}{f_1} = \frac{1.5 - 1}{10\,\text{cm}} - \frac{1.5 - 1}{-20\,\text{cm}} - \frac{(1.5 - 1)^2}{1.5}\,\frac{5\,\text{cm}}{(10\,\text{cm})(-20\,\text{cm})} \Rightarrow f_1 = 12.63\,\text{cm}$$

$$f_2 = -\frac{n'}{n} f_1 = -1\,(12.63\text{cm}) = -12.63\,\text{cm}$$

Further, using Eq. (18 3),

$$r = \frac{n_L - n'}{n_L R_2}\,f_1\,t = \frac{1.5 - 1}{1.5\,(10\,\text{cm})}\,(12.63\,\text{cm})\,(5\,\text{cm}) = 2.105\,\text{cm}$$

$$s = -\frac{n_L - n}{n_L\,R_1} f_2\,t = \frac{1.5 - 1}{1.5(-20\text{cm})}(-12.63\,\text{cm})(5\,\text{cm}) = -1.0525\ \text{cm}$$

Then $s_0 = 8\,\text{cm} + r = 10.105\,\text{cm}$ and,

$$-\frac{f_1}{s_o} + \frac{f_2}{s_i} = 1 \Rightarrow -\frac{12.63}{10.105} + \frac{(-12.63\,\text{cm})}{s_i} = 1 \Rightarrow s_i = -5.614\,\text{cm}\,\text{left of}\,H_2$$

That is $s_i = -5.614\,\text{cm} + s = -6.67\,\text{cm}$ left of V_2. The magnification is,

$$m = -\frac{n}{n'}\frac{s_i}{s_o} = -\frac{(1)(-5.614)}{(1)\,(10.105)} = +0.556$$

So, $h_i = m\,h_o = 0.556 \times 1\text{in} = 0.556\,\text{in}$.

18-4. Using Eq. (18 1)

$$\frac{1}{f_1} = \frac{n_L - n'}{n\,R_2} - \frac{n_L - n}{n\,R_1} - \frac{(n_L - n)\,(n_L - n')}{n\,n_L}\,\frac{t}{R_1 R_2}$$

$$\frac{1}{f_1} = \frac{1.61 - 1.33}{(1)\,(-10\,\text{cm})} - \frac{1.61 - 1}{(1)\,(10\,\text{cm})} - \frac{(1.61 - 1.33)(1.61 - 1)}{(1)\,(1.61)}\,\frac{2\,\text{cm}}{(10\,\text{cm})(-10\,\text{cm})} \Rightarrow f_1 = -11.51\,\text{cm}$$

$$f_2 = -\frac{n'}{n} f_1 = -\frac{1.33}{1}\,(-11.51\text{cm}) = 15.31\,\text{cm}$$

Further, using Eq. (18 3),

$$r = \frac{n_L - n'}{n_L R_2}\,f_1\,t = \frac{1.61 - 1.33}{1.61\,(-10\,\text{cm})}\,(-11.51\,\text{cm})\,(2\,\text{cm}) = 0.4\,\text{cm}$$

$$s = -\frac{n_L - n}{n_L\,R_1} f_2\,t = \frac{1.5 - 1}{1.61(10\text{cm})}(15.31\,\text{cm})(2\,\text{cm}) = -1.16\ \text{cm}$$

Also, from Eq. 18 4,

$$v = \left(1 - \frac{n'}{n} + \frac{n_L - n'}{n_L\,R_2} t\right) f_1 = \left(1 - \frac{1.33}{1} + \frac{1.61 - 1.33}{1.61\,(-10)}(2)\right)(-11.51\,\text{cm}) = 4.20\,\text{cm}$$

$$w = \left(1 - \frac{n}{n'} - \frac{n_L - n}{n_L\,R_1} t\right) f_2 = \left(1 - \frac{1}{1.33} - \frac{1.61 - 1}{1.61\,(10)}\,2\right)(15.31\,\text{cm}) = 2.64\,\text{cm}$$

Now, $s_o = 60\,\text{cm} + r = 60.4\,\text{cm}$. So,

$$-\frac{f_1}{s_o} + \frac{f_2}{s_i} = 1 \Rightarrow -\frac{-11.51}{60.4} + \frac{15.31\,\text{cm}}{s_i} = 1 \Rightarrow s_i = 18.9\,\text{cm}\,\text{from}\,H_2$$

The magnification is,

$$m = -\frac{n}{n'}\frac{s_i}{s_o} = -\frac{(1)(18.9)}{(1.33)\,(60.4)} = -0.235$$

So, $h_i = m\,h_o = (-0.235)(5\,\text{cm}) = -1.18\,\text{cm}$.

18-5. (a) Using Eq. (18 1)

$$\frac{1}{f_1} = \frac{n_L - n'}{n\,R_2} - \frac{n_L - n}{n\,R_1} - \frac{(n_L - n)\,(n_L - n')}{n\,n_L}\,\frac{t}{R_1 R_2}$$

$$\frac{1}{f_1} = \frac{1.33 - 1}{(1)\,(-10\,\text{cm})} - \frac{1.33 - 1}{(1)\,(10\,\text{cm})} - \frac{(1.33 - 1)^2}{(1)\,(1.33)}\,\frac{20\,\text{cm}}{(10\,\text{cm})(-10\,\text{cm})} \Rightarrow f_1 = -20.15\,\text{cm}$$

$$f_2 = -\frac{n'}{n} f_1 = -\frac{1}{1}(-20.15\,\text{cm}) = 20.15\,\text{cm}$$

Using Eq. (18 3),

$$r = \frac{n_L - n'}{n_L R_2}\,f_1\,t = \frac{1.33 - 1}{1.33(-10\,\text{cm})}\,(-20.15\,\text{cm})\,(20\,\text{cm}) = 10\,\text{cm}$$

$$s = -\frac{n_L - n}{n_L\,R_1}\,f_2\,t = -\frac{1.33 - 1}{1.33(10\text{cm})}(20.15\,\text{cm})(20\,\text{cm}) = -10\ \text{cm}$$

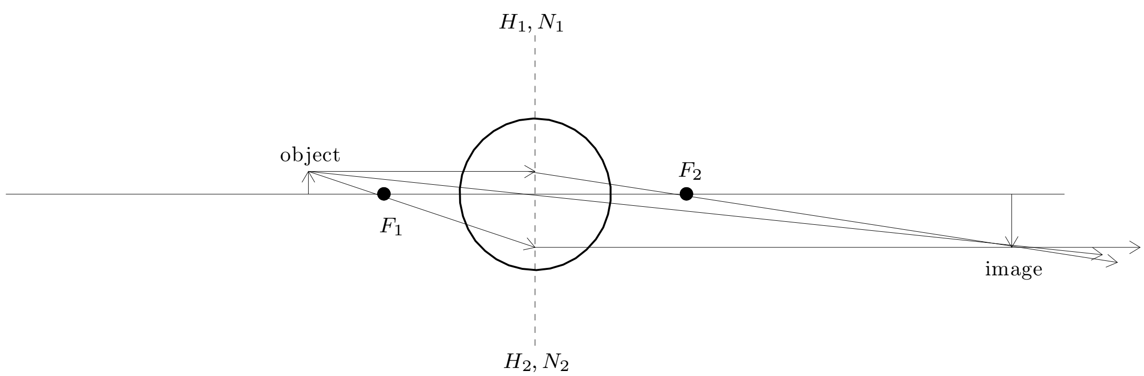
Since $n = n' = 1, v = r = 10\,\text{cm}$ and $w = s = -10\,\text{cm}$.

(b) The object distance is $s_o = 20\,\text{cm} + 10\,\text{cm} = 30\,\text{cm}$ from H_1. So,

$$-\frac{f_1}{s_o} + \frac{f_2}{s_i} = 1 \Rightarrow -\frac{-20.15}{30} + \frac{20.15\,\text{cm}}{s_i} = 1 \Rightarrow s_i = 61.38\,\text{cm from center}$$

The magnification is $m = -\frac{s_i}{s_o} = -\frac{61.38}{30} = -2.05$. The image is inverted, real, and approximately twice the size of the object.

(c)



18-6. (a) The system matrix is

$$M = \mathfrak{R}_{\text{sph}}\,\mathfrak{T}\mathfrak{R}_{\text{plane}} = \begin{bmatrix} 1 & 0 \\ -1/10 & 3/2 \end{bmatrix}\begin{bmatrix} 1 & 5 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 0 & 2/3 \end{bmatrix} = \begin{bmatrix} 1 & 10/3 \\ -1/10 & 2/3 \end{bmatrix}$$

$$y = A\,y_0 + B\,\alpha_0 = (1)(1\,\text{cm}) + (10/3)\,(0) = 1\,\text{cm}$$

$$\alpha = C\,y_0 + D\,\alpha_0 = (-1/10)\,(1\,\text{cm}) + (2/3)\,(0) = -0.1\,\text{rad} = -5.73^\circ$$

With an additional translation,

$$M' = \mathfrak{T}\,M = \begin{bmatrix} 1 & x \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 10/3 \\ -1/10 & 2/3 \end{bmatrix} = \begin{bmatrix} 1 - x/10 & 10/3 + 2\,x/3 \\ -1/10 & 2/3 \end{bmatrix}$$

$$y = (1 - x/10)\,y_0 + (10/3 + 2\,x/3)\,\alpha_0$$

$$\alpha = -0.1\,y_0 + 2\,\alpha_0/3$$

(c) The ray crosses the axis when $y = 0$. For $y_0 = 1$, $\alpha_0 = 0$, $y = 0 \Rightarrow 1 - x/10 = 0 \Rightarrow x = 10\,\text{cm}$

18-7. Use $y_f = A\,y_0 + B\,\alpha_0$ and $\alpha_f = C\,y_0 + D\,\alpha_0$. From Figure 18 12b, $a_0 = 0, y_0 = y_0, y_f = y_f$ so,

$$y_f = A\,y_0, \alpha_f = C\,y_0$$

Proceeding, $y_f = A\,y_0 + B\,\alpha_0, \alpha_f = C\,y_0 + D\,\alpha_0$

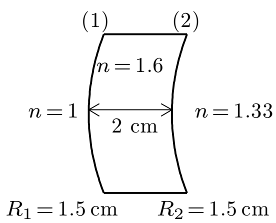
$$q \approx \frac{y_f}{-\alpha_f} = \frac{A\,y_0}{-C\,y_0} = -\frac{A}{C};\ f_2 \approx \frac{y_0}{-\alpha_f} = \frac{y_0}{-C\,y_0} = -\frac{1}{C}$$
$$-s \approx \frac{y_0 - y_f}{-\alpha_f} = \frac{y_0 - A\,y_0}{-C\,y_0} = \frac{1-A}{-C} \Rightarrow s = \frac{1-A}{C}$$

Or, $s = q - f_2 = \frac{-A}{C} + \frac{1}{C} \Rightarrow s = \frac{1-A}{C}$. From Figure 18 12c, $\alpha_0 = \alpha_f = \alpha$. Thus,

$$y_f = A\,y_0 + B\,\alpha, \alpha = C\,y_0 + D\,\alpha \Rightarrow \alpha = \frac{c\,y_0}{1-D}$$
$$-w \approx \frac{y_f}{\alpha} = \frac{A\,y_0 + B\,\alpha}{\alpha} = A\frac{y_0}{\alpha} + B = A\frac{y_0}{C\,y_0/(1-D)} + B = \frac{A - A\,D + B\,C}{C} = \frac{A - \mathrm{Det}(M)}{V} = \frac{A - n_0/n_f}{C}$$
$$w \approx \frac{n_0/n_f - A}{C}$$

18-8. The lens matrix is

$$\begin{bmatrix} 1 & 0 \\ \frac{1.6-1.3}{1.3\,(1.5)} & \frac{1.6}{1.3} \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ \frac{1-1.6}{1.6\,(1.5)} & \frac{1}{1.6} \end{bmatrix} = \begin{bmatrix} 1/2 & 5/4 \\ -3/13 & 25/26 \end{bmatrix}$$



$$p = D/C = -25/6 = -4.17\text{ cm} \quad q = -A/C = 13/6 = 2.17\text{ cm}$$
$$r = \frac{D - 1/1.3}{C} = -0.83\text{ cm} \quad s = (1-A)/C = -2.17\text{ cm}$$
$$f_1 = \frac{n_0/n_f}{C} = -3.33\text{ cm} \quad f_2 = -\frac{1}{C} = 4.33\text{ cm}$$

18-9. The lens matrix is,

$$\begin{bmatrix} 1 & 0 \\ 1/10 & 1 \end{bmatrix} \begin{bmatrix} 1 & 5 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ -1/10 & 1 \end{bmatrix} = \begin{bmatrix} 1/2 & 5/4 \\ -1/20 & 3/2 \end{bmatrix}$$

$$f_1 = 1/C = -20\text{ cm},\ f_2 = -1/C = 20\text{ cm}$$
$$q = -A/C = 10\text{ cm},\ p = D/C = -30\text{ cm}$$
$$r = (D-1)/C = -10\text{ cm}, s = (1-A)/C = -10\text{ cm}$$

18-10. The lens matrix is,

$$\begin{bmatrix} 1 & 0 \\ \frac{0.1}{1.4\,(2)} & \frac{1.5}{1.4} \end{bmatrix} \begin{bmatrix} 1 & 3 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ \frac{-0.5}{1.5\,(5)} & \frac{1}{1.5} \end{bmatrix} = \begin{bmatrix} 4/5 & 2 \\ -3/70 & 11/14 \end{bmatrix}$$

$$f_1 = (n_0/n_f)/C = (1/1.4)/C = -16.67\text{ cm},\ f_2 = -1/C = 25.33\text{ cm}$$
$$q = -A/C = 18.67\text{ cm},\ p = D/C = -18.33\text{ cm}$$
$$r = (D - n_0/n_f)/C = -1.67\text{ cm}, s = (1-A)/C = -4.67\text{ cm}$$

18-11. (a), (b) The lens matrix is,

$$\begin{bmatrix} 1 & 15 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ -1/10 & 1 \end{bmatrix}\begin{bmatrix} 1 & 30 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} -1/2 & 0 \\ -1/10 & -2 \end{bmatrix}$$

$$f_1 = 1/C = -10\,\text{cm},\ f_2 = -1/C = 10\,\text{cm}$$
$$q = -A/C = -5\,\text{cm},\ p = D/C = 20\,\text{cm}$$
$$r = (D-1)/C = 30\,\text{cm},\ s = (1-A)/C = -15\,\text{cm}$$
$$v = (D-1)/C = 30\,\text{cm},\ w = (1-A)/C = -15\,\text{cm}$$

The principal and nodal points fall at the lens center and the focal points fall 10 cm to either side of the lens.

(c) This is case (c) of Figure 18 9. That $B = 0$ signifies that reference planes are object image, or conjugate, planes and A represents the linear magnification.

Check by the thin lens Eq: If an object is at the input plane,

$$\frac{1}{30\,\text{cm}} + \frac{1}{s_i} = \frac{1}{10\,\text{cm}} \Rightarrow s_i = 15\,\text{cm}$$

Then $m = -s_i/s_o = -15/30 = -1/2 = A$.

18-12. (a) The lens matrix is,

$$\begin{bmatrix} 1 & 15 \\ 0.5/(-4) & 3/2 \end{bmatrix}\begin{bmatrix} 1 & 8 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 30 \\ (-0.5)/(4\times 1.5) & 2/3 \end{bmatrix} = \begin{bmatrix} 1/3 & 16/3 \\ -1/6 & 1/3 \end{bmatrix}$$

$$f_1 = 1/C = -6\,\text{in},\ f_2 = -1/C = 6\,\text{in}$$
$$q = -A/C = 2\,\text{in},\ p = D/C = -2\,\text{in}$$
$$r = (D-1)/C = 4\,\text{in},\ s = (1-A)/C = -4\,\text{in}$$

(b) Parallel light focuses at F_2, measured from the output plane (or right surface) by $q = 2\,\text{in}$.

18-13. (a), (b) The lens matrix is,

$$\begin{bmatrix} 1 & 0 \\ 0.5/(-5) & 3/2 \end{bmatrix}\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ \dfrac{-0.5}{(1.5)(-5)} & 2/3 \end{bmatrix} = \begin{bmatrix} 16/15 & 2/3 \\ -1/150 & 14/15 \end{bmatrix}$$

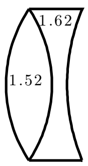
$$f_1 = 1/C = -150\,\text{cm},\ f_2 = -1/C = 150\,\text{cm}$$
$$q = -A/C = 160\,\text{cm},\ p = D/C = -140\,\text{cm}$$
$$r = (D-1)/C = 10\,\text{cm},\ s = (1-A)/C = 10\,\text{cm}$$

18-14. (a), (b) The lens matrix is

$$\begin{bmatrix} 1 & 0 \\ \frac{0.62}{20} & 1.62 \end{bmatrix}\begin{bmatrix} 1 & 1/2 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ \frac{-0.1}{-20\,(1.62)} & \frac{1.52}{1.62} \end{bmatrix}\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ \frac{-0.52}{(20)(1.52)} & \frac{1}{1.52} \end{bmatrix}=\begin{bmatrix} 0.9764 & 0.96755 \\ 0.009182 & 1.0333 \end{bmatrix}$$

$$f_1 = 1/C = -150\,\text{cm},\ f_2 = -1/C = 150\,\text{cm}$$
$$q = -A/C = 160\,\text{cm},\ p = D/C = -140\,\text{cm}$$
$$r = (D-1)/C = 10\,\text{cm},\ s = (1-A)/C = 10\,\text{cm}$$

(c) Consider each lens separately in air:



$$\frac{1}{f} = \frac{n_2 - n_1}{n_1}\left(\frac{1}{R_1} - \frac{1}{R_2}\right) \Rightarrow \frac{1}{f_1} = \frac{1.52 - 1}{1}\left(\frac{1}{20\,\text{cm}} - \frac{1}{-20\,\text{cm}}\right) = 0.052\,\text{cm}^{-1}$$

$$\frac{1}{f_2} = \frac{1.62 - 1}{1}\left(\frac{1}{-20\,\text{cm}} - \frac{1}{20\,\text{cm}}\right) = -0.062\,\text{cm}^{-1}$$

Equivalent focal length: $\frac{1}{f_{\text{eq}}} = \frac{1}{f_1} + \frac{1}{f_2} = 0.052\,\text{cm}^{-1} - 0.062\,\text{cm}^{-1} \Rightarrow f_{\text{eq}} = -100\,\text{cm}$

18-15. (a) Using the matrix calculated in the text,

$$M = \begin{bmatrix} 1 & s' \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 2/3 & 2 \\ -1/6 & 1 \end{bmatrix}\begin{bmatrix} 1 & s \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 2/3 - s'/6 & 2/3\,s + 2 - s\,s'/6 + s' \\ -1/6 & s/6 + 1 \end{bmatrix}$$

(b) Input and output planes correspond to object and image planes when $B = 0$. That is, when,

$$2/3\,s + 2 - s\,s'/6 + s' = 0 \Rightarrow s' = \frac{4\,s + 12}{s - 6}$$

When $B = 0$, $A = m = 2/3 - s'/6$.

(c) When $A = 2/3 - s'/6 = 0$, the output plane at $s' = 4$ cm corresponds to the second focal plane. When $D = -s/6 + 1 = 0$, the input plane at $s = 6$ cm corresponds to the first focal plane.

18-16. It is helpful to use a computer to perform the matrix multiplication in this problem... The sequence of matrices has the form,

$$\begin{bmatrix} 1 & 0 \\ \frac{1 - n_3}{r_6} & n_3 \end{bmatrix}\begin{bmatrix} 1 & t_3 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ \frac{1 - n_3}{n_3 r_5} & \frac{1}{n_3} \end{bmatrix}\begin{bmatrix} 1 & d_2 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ \frac{n_2 - 1}{r_4} & n_2 \end{bmatrix}\begin{bmatrix} 1 & t_2 \\ 0 & 1 \end{bmatrix}\times$$

$$\times\begin{bmatrix} 1 & 0 \\ \frac{1 - n_2}{n_2 r_3} & \frac{1}{n_2} \end{bmatrix}\begin{bmatrix} 1 & d_1 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ \frac{n_1 - 1}{r_2} & n_1 \end{bmatrix}\begin{bmatrix} 1 & t_1 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ \frac{1 - n_1}{n_1 r_1} & \frac{1}{n_1} \end{bmatrix}=\begin{bmatrix} A & B \\ C & D \end{bmatrix}$$

Matrix multiplication and insertion of parameter values gives,

$$A = 0.93935,\ B = 22.2212,\ C = 0.009284,\ D = 0.8448$$

and

$$p = \frac{D}{C} = -90.99\,\text{mm},\ q = -\frac{A}{C} = 101.18\,\text{mm},\ r = v = \frac{D-1}{C} = 16.72\,\text{mm},\ s = w = \frac{1-A}{C} = -6.53\,\text{mm}$$
$$f_1 = 1/C = -107.71\,\text{mm},\ f_2 = -1/C = 107.71\,\text{mm}$$

Since q is the distance of the focal point from the output plane, it is also the distance from the last surface to the film plane.

18-17. The lens matrix is,

$$\begin{bmatrix} 1 & 0 \\ \dfrac{n_L - n'}{n' R_2} & \dfrac{n_L}{n'} \end{bmatrix} \begin{bmatrix} 1 & t \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ \dfrac{n - n_L}{n_L R_1} & \dfrac{n}{n_L} \end{bmatrix} \begin{bmatrix} 1 & d_2 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix}$$

Performing the matrix multiplication gives,

$$A = \frac{n - n_L}{n_L R_1} t + 1, \;\; B = \frac{n t}{n_L}, \;\; D = \frac{n}{n'} + \left(\frac{n_L - n'}{n' R_2} \right) \frac{n t}{n_L}$$

$$C = \frac{n_L - n'}{n' R_2} - \frac{n_L - n}{n' R_1} - \frac{(n_L - n)(n_L - n')}{n' n_L} \frac{t}{R_1 R_2}$$

18-18. From Table 18 2, $f_1 = \dfrac{n}{n'} \dfrac{1}{C} \Rightarrow \dfrac{1}{f_1} = \dfrac{n'}{n} C$. Substituting C from problem 18 17,

$$\frac{1}{f_1} = \frac{n'}{n} \left[\frac{n_L - n'}{n' R_2} - \frac{n_L - n}{n' R_1} - \frac{(n_L - n)(n_L - n')}{n' n_L} \frac{t}{R_1 R_2} \right]$$

$$\frac{1}{f_1} = \frac{n_L - n'}{n R_2} - \frac{n_L - n}{n R_1} - \frac{(n_L - n)(n_L - n')}{n n_L} \frac{t}{R_1 R_2}$$

18-19. From Table 18 2 and using the results of problem 18 17,

$$r = \frac{D - n/n'}{C} = \frac{\left[\frac{n}{n'} + \left(\frac{n_L - n'}{n' R_2} \right) \frac{n t}{n_L} \right] - \frac{n}{n'}}{C}$$

but, $f_1 = \dfrac{n}{n'} \dfrac{1}{C}$. So,

$$r = \left(\frac{n_L - n'}{n' R_2} \right) \left(\frac{n t}{n_L} \right) \left(\frac{n'}{n} \right) f_1 = \frac{n_L - n'}{n_L R_2} f_1 t$$

Similarly,

$$s = \frac{1 - A}{C} = \frac{1 - \left[\frac{n - n_L}{n_L R_1} t + 1 \right]}{C} = \frac{n - n_L}{n_L R_1} t \, (-f_2) = -\frac{n_L - n}{n_L R_1} f_2 t$$

and,

$$v = \frac{D - 1}{C} = \frac{\frac{n}{n'} + \left(\frac{n_L - n'}{n' R_2} \right) \frac{n t}{n_L} - 1}{C} = \left(\frac{n}{n'} + \left(\frac{n_L - n'}{n' R_2} \right) \frac{n t}{n_L} - 1 \right) \left(\frac{n'}{n} f_1 \right) = \left[1 - \frac{n'}{n} + \frac{n_L - n'}{n_L R_2} t \right] f_1$$

$$w = \frac{(n/n') - A}{C} = \left[\frac{n}{n'} - \frac{n - n_L}{n_L R_1} t - 1 \right] (-f_2) = \left[1 - \frac{n}{n'} - \frac{n_L - n}{n_L R_1} t \right] f_2$$

18-20. A typical program in "Basic" which can be easily adapted to other languages:

```
10 REM: MERIDIONAL RAY TRACING.  AFTER THE LAST SURFACE IS CALCULATED, ENTER ANY
20 REM: NEGATIVE NUMBER AS DISTANCE TO NEXT SURFACE TO TERMINATE
30 INPUT "LEFT AND RIGHT REFERENCE INDICES:",N1,N2
40 INPUT "RAY ANGLE DEGREE AND HEIGHT:",A,H
50 INPUT "DISTANCE FROM OBJECT POINT (IF INFINITY ENTER ANYTHING):",D
60 REM: FOR RAY ENTERING PARALLEL TO AXIS, D IS NOT USED IN THE CALCULATION
70 P=3.1415926#:A=P*(A/180):REM ANGLE IS NOW IN RADIANS
80 IF A=0 THEN GOTO 110
90 S=D H/TAN(A):Q= S*SIN(A)
100 GOTO 140
110 Q=H
120 REM: FOR PLANE SURFACE, ENTER R>99990.
130 REM: THE ACTUAL VALUE IS NOT USED IN THE CALCULATION.
140 INPUT "INPUT RAD OF CURV (99999 IF PLANE):",R
150 IF R>99990!  THEN GOTO 210
160 B=Q/R+SIN(A): T=ATN(B/SQR(1 B*B)):REM TAKING ARCSIN(B)
170 C=N1*SIN(T)/N2:T1=ATN(C/SQR(1 C*C)):REM TAKING ARCSIN(C)
180 A1=T1 T+A:A2=(A1/P)*180:REMA2 CONVERTS A1 TO DEGREES AGAIN
190 Q1=(SIN(T1) SIN(A1))*R:S1= Q1/SIN(A1)
200 GOTO 240
210 X=N1*SIN(A)/N2:A1=ATN(X/SQR(1 X*X)):REM TAKING ARCSIN(X)
220 Q1=Q*COS(A1)/COS(A)
230 S1= Q1/SIN(A1):A2=(A1/P)*180:REM A2 IS A1 IN DEGREES
240 PRINT "RAY INTERSECTS AXIS AT";S1;"FROM SURFACE."
250 PRINT "RAY ANGLE IN RADIANS IS ";A1
260 PRINT "RAY ANGLE IN DEGREES IS ";A2
270 PRINT "THE Q PARAMETER IS ";Q1
280 A=A1:N1=N2
290 INPUT "DISTANCE TO NEXT SURFACE:",D1
300 IF D1<0 THEN 360
310 Q=Q1+D1*SIN(A1)
320 INPUT "NEW REFRACTIVE INDEX:",N2
330 INPUT "NEW RADIUS OF CURV (99999 IF PLANE):",R
340 IF R>99990!  THEN GOTO 210
350 GOTO 360
360 END
```

18-21. Use the computer program (problem 18 20) or Table 18 3 in a two step process for each ray, yielding

$$\alpha = 0°\text{: } s' = 3.180\,\text{cm and } \alpha' = -23.51°$$
$$\alpha = -20°\text{: } s' = 16.104\,\text{cm and } \alpha' = 6.081°$$

18-22. Use the computer program (problem 18 20) or Table 18 3 in a 3 step process, giving,

$$s' = -49.525\,\text{cm and } \alpha' = 3.371°$$

18-23. Use the computer program (problem 18 20) or Table 18 3 in a 6 step process, to get,

$$h = 1\text{: } s' = 98.20\,\text{mm and } \alpha' = -0.567°$$
$$h = 5\text{: } s' = 102.45\,\text{mm and } \alpha' = -2.723°$$

Chapter 19 Optics of the Eye

19-1. (a) The conversion between the radiant flux and the luminous flux is

$$\Phi_v = K(\lambda)\,\Phi_e = 685\,(\lambda)\,\Phi_e$$

Estimating from Figure 19 2,

$$\frac{\Phi_{v,\text{He-Cd}}}{\Phi_{v,\text{He-Ne}}} = \frac{685\,(0.02)\,(0.050)}{685\,(0.2)\,(0.004)} = 1.25$$

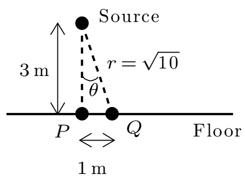
(b) For equal brightnesses,

$$\Phi_{v,\text{Ar}} = \Phi_{v,\text{He-Ne}} \Rightarrow 685\,V(\lambda_{\text{Ar}})\,\Phi_{e,\text{Ar}} = 685\,V\big(\lambda_{\text{He-Ne}}\big)\,\Phi_{e,\text{He-Ne}}$$
$$(0.2)\,\Phi_e = (0.95)\,(0.5\,\text{mW})$$
$$\Phi_{e,\text{Ar}} = 2.4\,\text{mW}$$

19-2. (a) From Table 19 1,

$$\frac{I_v}{E_v} = \frac{d\Phi_v/d\omega}{d\Phi_v/dA} = \frac{\Phi_v/4\,\pi}{\Phi_v/(4\,\pi\,r^2)} \Rightarrow I_v = E_v\,r^2 = (100\,\text{lm/m}^2)\,(3\,\text{m})^2 = 900\text{cd}$$

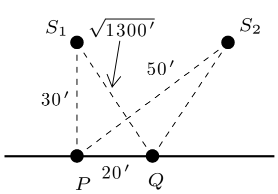
(b) See Eq. (1 13), $I_v(\theta) = I_v(0)\cos\theta$.



Using the general result in (a), at point Q In the diagram,

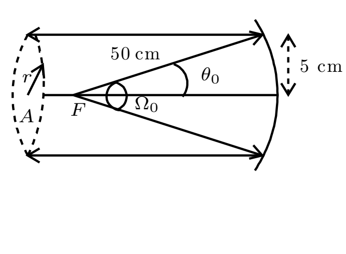
$$E_v = \frac{I_v(0)\cos\theta}{r^2} = \frac{(900\,\text{lm})\,(3/\sqrt{10})}{\left(\sqrt{10}\,\text{m}\right)^2} = 85.4\,\text{lx}$$

19-3. Use the relation between illuminance and luminous intensity given in the solution to problem 19 2a:



$$E_v(P) = \frac{I_v(0)\cos(0)}{(30\,\text{ft})^2} + \frac{I_v(0)\,3/5}{(50\,\text{ft})^2} = \big(1.351\times10^{-3}/\text{ft}^2\big)\,I_v(0)$$
$$E_v(Q) = 2\left[\frac{I_v(0)(\big(30/\sqrt{1300}\big)}{\left(\sqrt{1300}\,\text{ft}\right)^2}\right] = \big(1.280\times10^{-3}/\text{ft}^2\big)\,I_v(0$$
$$\frac{E_v(P)}{E_v(Q)} = 1.055$$

19-4. The arrangement is sketched (not to scale) below,



The indicate angle and solid angle satisfy the relations

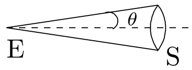
$$\cos\theta_0 = \cos(\sin^{-1}(5/50)\big) = 0.9950, \quad \Omega_0 = 2\,\pi\,(1-\cos\theta_0\big) = 0.01\,\pi\,\text{sr}$$

$$\Phi_{v,\text{inc}} = \Omega_0\,I_v = (0.01\pi)\,(100) = \pi\,\text{lm}, \quad \Phi_{v,\text{ref}} = 0.8\,\Phi_{v,\text{inc}} = 0.8\,\pi\,\text{lm}$$

$$E_v = \frac{\Phi_{v,\text{ref}}}{A} = \frac{0.8\,\pi\,\text{lm}}{\pi\,(0.05\,\text{m})^2} = 320\,\text{lx}$$

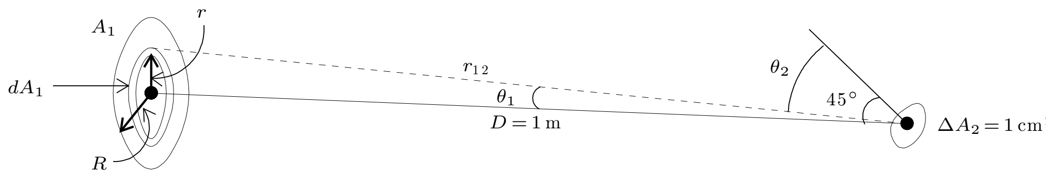
19-5. (a) The solid angle intercepted is $\Omega = 2\pi\,(1-\cos\theta) = 2\pi\,(1-\cos(0.25°)) = 5.98\times 10^{-5}\,\text{sr}$. Here θ is the half angle intercepted by the sun viewed from the earth. The luminance of the sun is about,

$$L_v = \frac{E_v}{\Omega} = \frac{10^5\,\text{lm/m}^2}{5.98\times 10^{-5}\,\text{sr}} = 1.67\times 10^9\,\text{cd/m}^2$$



(b) $E_v = L_v\,\Omega = L\left(\dfrac{\text{lm}}{\text{m}^2\,\text{sr}}\right)\times 2\pi\,\text{sr} = 2\pi\,L\,\text{lm/m}^2$.

19-6. The arrangement is sketched below.



The luminous flux incident on the small surface is

$$\Delta\Phi_{12} = \int_{A_1}\int_{A_2} \frac{L\cos\theta_1\cos\theta_2}{r_{12}^2}\,dA_1\,dA_2 = L\,\Delta A_2\int_{A_1}\frac{\cos\theta_1\cos\theta_2}{r_{12}^2}\,dA_1$$

Consider first illumination of ΔA_2 by an annular ring of incremental area $dA_1 = 2\pi\,r\,dr$, as shown. Then,

$$r_{12} = r^2 + D^2, \cos\theta_1 = D/\sqrt{r^2+D^2}$$

$$\cos\theta_2 = \cos(45° - \theta_1) = \cos(45°)\cos\theta_1 + \sin(45°)\sin\theta_1 = \frac{\sqrt{2}}{2}\left[\frac{D}{\sqrt{r^2+D^2}} + \frac{r}{\sqrt{r^2+D^2}}\right]$$

Adding the contributions from the annular rings that make up the disc,

$$\Delta\Phi_{12} = L\,\Delta A_2\int_0^R \frac{\cos\theta_1\cos(45°-\theta_1)}{r^2+D^2}2\pi\,r\,dr$$

$$\Delta\Phi_{12} = L\,\Delta A_2\int_0^R \frac{D}{\sqrt{r^2+D^2}}\frac{\sqrt{2}}{2}\left(\frac{D}{\sqrt{r^2+D^2}} + \frac{r}{\sqrt{r^2+D^2}}\right)\frac{2\pi\,r}{r^2+D^2}\,dr$$

$$\Delta\Phi_{12} = \sqrt{2}\,\pi\,L\,D\,\Delta A_2\left\{D\int_0^R \frac{r\,dr}{(r^2+D^2)^2} + \int_0^R \frac{r^2\,dr}{(r^2+D^2)^2}\right\}$$

$$\Delta\Phi_{12} = \sqrt{2}\,\pi\,L\,D\,\Delta A_2\left\{D\left[\frac{-1}{2\,(r^2+D^2)}\right]_0^R + \left[\frac{-r}{2\,(r^2+D^2)} + \frac{1}{2\,D}\tan^{-1}\left(\frac{r}{D}\right)\right]_0^R\right\}$$

$$\Delta\Phi_{12} = \sqrt{2}\,\pi\,L\,D\,\Delta A_2\left\{\frac{1}{2\,D}\tan^{-1}\left(\frac{R}{D}\right) + \frac{1}{2\,D} - \frac{D+R}{2(R^2+D^2)}\right\}$$

With $L = 10^5\,\text{lm/sr m}^2, D = 1\,\text{m}, R = 0.2\,\text{m}, dA_2 = 10^{-4}\,\text{m}^2$,

$$\Delta\Phi_{12} = 0.97\,\text{lm}$$

19-7. The power for a refracting surface is defined as $P = \dfrac{n_2 - n_1}{R}$. So,

$$P = \frac{n_{A.H} - 1}{R} = \frac{1.33 - 1}{8\times 10^{-3}\,\text{m}} = 41.6\,\text{m}$$

This value is in agreement with Table 19 2.

19-8. From Table 19 2: $n_{\text{lens}} = 1.45, R_1 = 10\,\text{mm}$, and $R_2 = -6\,\text{mm}$.

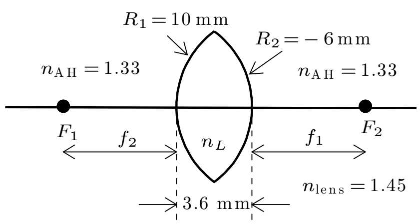
(a) Using the lensmaker's formula,

$$\frac{1}{f} = \frac{n_{\text{lens}} - 1}{1}\left(\frac{1}{R_1} - \frac{1}{R_2}\right) = \frac{1.45 - 1}{1}\left(\frac{1}{10\,\text{mm}} - \frac{1}{-6\,\text{mm}}\right) \Rightarrow f = 8.33\,\text{mm} = P = \frac{1}{f_{\text{in m}}} = \frac{1}{0.00833\,\text{m}} = 120\,\text{D}$$

(b) In its environment,

$$\frac{1}{f} = \frac{n_{\text{lens}} - n_{\text{AH}}}{n_{\text{AH}}}\left(\frac{1}{R_1} - \frac{1}{R_2}\right) = \frac{1.45 - 1}{1}\left(\frac{1}{10} - \frac{1}{-6}\right)\text{mm}^{-1} \Rightarrow f = 41.7\,\text{mm} = P = \frac{1}{f_{\text{in m}}} = 24.0\,\text{D}$$

(c) For a thick lens consider the two successive refractions and the translation across the lens,



For f_1: $s_2' = \infty, s_1 = f_1$ (measured from the first surface). Then,

$$\frac{n_{\text{lens}}}{s_2} + \frac{n_{\text{AH}}}{\infty} = \frac{n_{\text{AH}} - n_{\text{lens}}}{R_2} \Rightarrow \frac{1}{s_2} = \frac{1.33 - 1.45}{(1.33)(-6\,\text{mm})} = s_2 = 66.7\,\text{mm}$$

$$s_1' = (-66.7 - 3.6)\,\text{mm} = -63.1\,\text{mm}.$$

$$\frac{n_{\text{AH}}}{s_1} + \frac{n_{\text{lens}}}{s_1'} = \frac{n_{\text{lens}} - n_{\text{AH}}}{R_1} \Rightarrow s_1 = f_1 = 38\,\text{mm}$$

Proceeding in the same way for f_2:

$$s_1 = \infty, s_2' = f_2 \text{ (measured from the second surface)}$$

$$\frac{n_{\text{AH}}}{\infty} + \frac{n_{\text{lens}}}{s_1'} = \frac{n_{\text{lens}} - n_{\text{AH}}}{R_1} \Rightarrow s_1' = 120.8\,\text{mm}$$

$$s_2 = -(120.8 - 3.6)\,\text{mm} = -117.2\,\text{mm}$$

$$\frac{n_{\text{lens}}}{s_2} + \frac{n_{\text{AH}}}{s_2} = \frac{n_{\text{lens}} - n_{\text{AH}}}{R_2} \Rightarrow s_2 = f_2 = 41.1\,\text{mm}$$

Note that these focal lengths are measured from the lens surfaces. The more formal matrix method leads to *focal lengths measured from the principal planes*. Use the techniques of Chapter 18, The lens matrix is,

$$\underbrace{\begin{bmatrix} 1 & 0 \\ \frac{1.45 - 1.33}{(1.33)(-6)} & \frac{1.45}{1.33} \end{bmatrix}}_{\text{Surface } R_2} \underbrace{\begin{bmatrix} 1 & 3.6 \\ 0 & 1 \end{bmatrix}}_{\text{Translation}} \underbrace{\begin{bmatrix} 1 & 0 \\ \frac{1.33 - 1.45}{(1.45)(10)} & \frac{1.33}{1.45} \end{bmatrix}}_{\text{Surface } R_2} = \begin{bmatrix} 0.9702 & 3.3021 \\ -0.02361 & 0.9503 \end{bmatrix}$$

$$q = -A/C = 41.09\,\text{mm},\; p = D/C = -40.25\,\text{mm}$$
$$r = (D-1)/C = 2.105\,\text{mm}, s = (1-A)/C = -1.262\,\text{mm}$$
$$f_1 = 1/C = -42.355\,\text{mm} \;\; f_2 = -1/C = 42.355\,\text{mm}$$

19-9. Refer to Figure 19 3 and Table 19 2.

(a) For an unaccommodated eye the object is at infinity. Then at the cornea

$$\frac{n_1}{s}+\frac{n_2}{s'}=\frac{n_2-n_1}{R}\Rightarrow s'=\frac{n_2\,R}{n_2-n_1}=\frac{(1.33)\,(8\,\mathrm{mm})}{1.33-1}=32\,\mathrm{mm}\ (\text{right of cornea})$$

At the front surface of the lens which is 3.6 mm from the cornea: $s=(3.6-32)\,\mathrm{mm}=-28.4\,\mathrm{mm}$.

$$\frac{n_{\mathrm{AH}}}{s}+\frac{n_{\mathrm{lens}}}{s'}=\frac{n_{\mathrm{lens}}-n_{\mathrm{AH}}}{R_1}=\frac{1.33}{-28.4\,\mathrm{mm}}+\frac{1.45}{s'}=\frac{1.45-1.33}{10\,\mathrm{mm}}\Rightarrow s'=24.73\,\mathrm{mm}\ (\text{right of }1^{\mathrm{st}}\text{ lens surface})$$

At the rear surface of the lens, $s=(3.6-24.73\,)\mathrm{mm}=-21.13\,\mathrm{cm}$:

$$\frac{n_{\mathrm{lens}}}{s}+\frac{n_{\mathrm{AH}}}{s'}=\frac{n_{\mathrm{AH}}-n_{\mathrm{lens}}}{R_2}\Rightarrow\frac{1.45}{-21.13\,\mathrm{mm}}+\frac{1.33}{s'}=\frac{1.33-1.45}{-6\,\mathrm{mm}}\Rightarrow s'=15.13\,\mathrm{mm}\ (\text{right of }2^{\mathrm{nd}}\text{ lens surface})$$

Thus an object at infinity forms an image at $15.13\,\mathrm{mm}+7.2\,\mathrm{mm}=22.33\,\mathrm{mm}$ from the cornea the back focal plane of the eye. This value is in agreement with Table 19 3.

(b) Repeating for the parameters associated with an eye accommodated for an object that is 25 cm from the cornea:
At the cornea:,

$$\frac{1}{250\,\mathrm{mm}}+\frac{1.33}{s'}=\frac{1.33-1}{8\,\mathrm{mm}}\Rightarrow s'=35.43\,\mathrm{mm}\ (\text{right of cornea})$$

At the front surface of the lens, $s=(3.2-35.43)\,\mathrm{mm}=-32.33\,\mathrm{mm}$:

$$\frac{1.33}{-32.53\,\mathrm{mm}}+\frac{1.45}{s'}=\frac{1.45-1.33}{6\,\mathrm{mm}}\Rightarrow s'=23.83\,\mathrm{mm}\ (\text{right of front lens surface})$$

At the rear lens surface, $s=(4-23.83)\,\mathrm{mm}=--19.83\,\mathrm{mm}$:

$$\frac{1.45}{-19.83}+\frac{1.33}{s'}=\frac{1.33-1.45}{-6\,\mathrm{mm}}\Rightarrow s'=14.39\,\mathrm{mm}\ (\text{right of back lens surface})$$

So, for an eye accommodated for the normal near point, the image is formed at
$(14.39+7.2)\,\mathrm{mm}=21.59\,\mathrm{mm}$ form the cornea.

19-10. (a) The matrix for propagation from the cornea to the back lens surface is,

$$M=\underbrace{\begin{bmatrix}1 & 0\\ \frac{1.45-1.33}{(1.33)\,(-6)} & \frac{1.45}{1.33}\end{bmatrix}}_{\text{Rear lens surface}}\underbrace{\begin{bmatrix}1 & 3.6\\ 0 & 1\end{bmatrix}}_{\text{Lens Trans.}}\underbrace{\begin{bmatrix}1 & 0\\ \frac{1.33-1.45}{(1.45)\,(10)} & \frac{1.33}{1.45}\end{bmatrix}}_{\text{Front lens surface}}\underbrace{\begin{bmatrix}1 & 3.6\\ 0 & 1\end{bmatrix}}_{\text{Trans. to cornea}}\underbrace{\begin{bmatrix}1 & 0\\ \frac{1-1.33}{(1.33)\,(8)} & \frac{1}{1.33}\end{bmatrix}}_{\text{Cornea surface}}$$

$$M=\begin{bmatrix}0.75846 & 5.1050\\ -0.05012 & 0.65180\end{bmatrix}$$

$$q=-A/C=15.133\,\mathrm{mm},\ p=D/C=-13.005\,\mathrm{mm}$$
$$r=(D-1)/C=-1.963\,\mathrm{mm},\ s=(1-A)/C=-4.820\,\mathrm{mm}$$
$$f_1=(n_0/n_f)/C=(1/1.33)/C=-15.00\,\mathrm{mm}\ \ f_2=-1/C=19.95\,\mathrm{mm}$$

The first principal plane (H in Figure 19 3) is a distance $|r|=1.96\,\mathrm{mm}$ to the right of the front corneal sur face. The second principal plane (H' in Figure 19 3) is a distance $|s|=4.82\,\mathrm{mm}$ to the left of the back lens surface which places it a distance of 7.2 mm − 4.82 mm = 2.38 mm to the right of the back lens surface. These positions are in agreement with the values in Table 19 3. The front focal point is a distance of 15.0 mm to the left of the H which places it a distance of 13.0 mm from the front corneal surface. The back focal plane is a distance of 19.95 mm to the right of H' which places to the right of the corneal surface by a distance of $19.95\,\mathrm{mm}+2.38\,\mathrm{mm}=22.3\,\mathrm{mm}$. These positions also agree with the values given in Table 19 3.

19-11. In a Snellen chart letters in the 20/20 row should subtend an angle of $\theta_l = 5' = 5/60° = 0.001454$ rad and details should subtend an angle $\theta_d = 1' = 0.0002909$ rad. Heights on the screen a distance of $D = 5$ ft from the eye are related to these angles by the relation $y = D\,\theta$. So

20/20 row: $y_l = D\,\theta_l = (60\,\text{in})\,(0.001454\,\text{rad}) = 0.0872\,\text{in},\; y_d = y_l/5 = 0.0174\,\text{in}.$

20/15 row: $y_l = (15/20)\,y_{l,20/20} = 0.00654\,\text{in},\; y_d = y_l/5 = 0.00131\,\text{in}.$

20/300 row: $y_l = (300/20)\,y_{l,20/20} = 1.308\,\text{in},\; y_d = y_l/5 = 0.262\,\text{in}.$

20/100 row: $y_l = (100/20)\,y_{l,20/20} = 0.436$ in, $y_d = y_l/5 = 0.0872\,\text{in}.$

20/60 row: $y_l = (60/20)\,y_{l,20/20} = 0.262$ in, $y_d = y_l/5 = 0.0523\,\text{in}.$

19-12. Objects 25 cm from the eye should form images 125 cm from the eye. Since the lens is 25 cm from the eye this implies object and image distances of $s = 23.5\,\text{cm}$ and $s' = -123.5\,\text{cm}$.

(a) The needed power is then

$$P = \frac{1}{f} = \frac{1}{s} + \frac{1}{s'} = \frac{1}{0.235\,\text{m}} - \frac{1}{1.235\,\text{m}} = 3.45/\text{m} = 3.45\,\text{D}$$

(b) Assuming that the eye alone has a far point at infinity, the far point with the lenses is the object position at which the corrective lenses form an image at $-\infty$. The object distance s_{far} from the glasses is found as,

$$P = \frac{1}{f} = \frac{1}{s_{\text{far}}} + \frac{1}{-\infty} \Rightarrow s_{\text{far}} = f = 1/P = (1/3.45)\,\text{m} = 0.290\,\text{m} = 29.0\,\text{cm}$$

This corresponds to a distance of 30.5 cm from the eye. The range of clear vision is expanded if the correction is for a distance further than the normal near point.

19-13. (a) The lens should place the image of an object at infinity at the eye's far point. So,

$$P = \frac{1}{f} = \frac{1}{s} + \frac{1}{s'} = \frac{1}{\infty} + \frac{1}{s'} = \frac{1}{-0.5\,\text{m}} = -2\,\text{D}$$

(b) The new near point is the object position that places an image 15 cm from the eye:

$$P = -2/\text{m} = \frac{1}{s} + \frac{1}{s'} = \frac{1}{s} - \frac{1}{0.15\,\text{m}} \Rightarrow s = 0.214\,\text{m} = 21.4\,\text{cm}$$

(c) Adjusting the procedure for a lens 2 cm from the eye:

$$P = \frac{1}{f} = \frac{1}{s} + \frac{1}{s'} = \frac{1}{\infty} + \frac{1}{s'} = \frac{1}{-0.48\,\text{m}} = -2.083\,\text{D}$$
$$P = -2.083/\text{m} = \frac{1}{s} + \frac{1}{s'} = \frac{1}{s} - \frac{1}{0.13\,\text{m}} \Rightarrow s = 0.178\,\text{m} = 17.8\,\text{cm}$$

The needed power is -2.083 D and the near point is 17.8 cm from the glasses or 19.8 cm from the eye.

19-14. (a) -1.50 D to correct for myopia; -1.50 D to correct for astigmatism, cylinder axis horizontal

(b) -2.00 D to correct for simple myopia

(c) $+2.00$ D to correct for simple hyperopia

(d) $+2.00$ D to correct for hyperopia; -1.50 D for astigmatism, cylinder axis horizontal

19-15. The unaided far point is the image position for objects at infinity. The unaided near point is the image position for objects at the corrected near point.

(a) **Right eye far point:** $P == \frac{1}{f} - 7\,\text{D} = \frac{1}{s} + \frac{1}{s'} = \frac{1}{\infty} + \frac{1}{s'} \Rightarrow s' = f = \frac{1}{P} = \frac{1}{-7\,\text{D}} = -\frac{1}{7}\,\text{m} = -14.3\,\text{cm}$

The unaided far point is 14.3 cm from the eye.

Right eye near point: $P = \frac{1}{f} = -7\,\text{D} = \frac{1}{s} + \frac{1}{s'} = \frac{1}{0.15\,\text{m}} + \frac{1}{s'} \Rightarrow s' = -0.0732\,\text{m} = -7.32\,\text{cm}$

The unaided near point is 7.32 cm from the eye.

Left eye far point: $P = \frac{1}{f} = -5\text{D} = \frac{1}{s} + \frac{1}{s'} = \frac{1}{\infty} + \frac{1}{s'} \Rightarrow s' = f = \frac{1}{P} = \frac{1}{-5\,\text{D}} = -\frac{1}{5}\,\text{m} = -20\,\text{cm}$

The unaided far point is 20 cm from the eye.

Left eye near point: $P = \frac{1}{f} = -5\,\text{D} = \frac{1}{s} + \frac{1}{s'} = \frac{1}{0.15\,\text{m}} + \frac{1}{s'} \Rightarrow s' = -0.0857\,\text{m} = -8.57\,\text{cm}$

The unaided near point is 8.57 cm from the eye.

(b) The corrected far point is the object position that places an image at the unaided far point. The cor rected near point point is the object position that places an image at the unaided near point. With the wrong lens, then:

Right eye far point: $P = \frac{1}{f} = -5\,\text{D} = \frac{1}{s} + \frac{1}{s'} = \frac{1}{s} + \frac{1}{-0.143\,\text{m}} \Rightarrow s = 0.502\,\text{m} = 50.2\,\text{cm}$

The far point with the wrong lens is 50.2 cm.

Right eye near point: $P = \frac{1}{f} = -5\,\text{D} = \frac{1}{s} + \frac{1}{s'} = \frac{1}{s} + \frac{1}{-0.0732\,\text{m}} \Rightarrow s = 0.115\,\text{m} = 11.5\,\text{cm}$

The near point with the wrong lens is 11.5 cm.

Left eye far point: $P = \frac{1}{f} = -7\,\text{D} = \frac{1}{s} + \frac{1}{s'} = \frac{1}{s} + \frac{1}{-0.20\,\text{m}} \Rightarrow s = -0.5\,\text{m}$

This answer is nonsense as the object position can not be negative for a single lens. This means that the eye must strain to see object at infinity. The relaxed eye will not see anything clearly. So the far point with the wrong lens is infinity but the eye must strain to see the far away objects.

Left eye near point: $P = \frac{1}{f} = -7\,\text{D} = \frac{1}{s} + \frac{1}{s'} = \frac{1}{s} + \frac{1}{-0.0857\,\text{m}} \Rightarrow s = 0.214\text{m} = 21.4\,\text{cm}$

The near point with the wrong lens is 21.4 cm.

19-16. Lets choose the lens to eye distance to be 1.5 cm. The lens for far vision should place images of far away object a distance of $(15 - 1.5)$ cm from the lens:

$$P_{\text{far}} = \frac{1}{s} + \frac{1}{s'} = \frac{1}{\infty} + \frac{1}{-0.13.5\,\text{m}} = -7.41\,\text{D}$$

With this lens alone the corrected near point would be the object distance leading to images 11.5 cm from the lens:

$$\frac{1}{s} = P - \frac{1}{s'} = -7.41\,\text{D} - \frac{1}{-0.115\,\text{m}} \Rightarrow s = 0.778\,\text{m}$$

Bifocals are needed. Let's say that the comfortable reading distance is to be 40 cm from the eye. Then the lens for near vision should have a power:

$$P_{\text{near}} = \frac{1}{s} + \frac{1}{s'} = \frac{1}{0.385\,\text{m}} + \frac{1}{-0.115\,\text{m}} = -6.10\,\text{D}$$

Chapter 20 Aberration Theory

20-1. Using Eq. (20 6),

$$a(Q)=(n_1\,l+n_2\,l')-(n_1\,s+n_2\,s') \tag{1}$$

Eqs. (20 16) and (20 17) are,

$$l=s\left[1+\frac{h^2\,(R+s)}{2\,R\,s^2}+\frac{h^4(R+s)}{8\,R^3\,s^2}-\frac{h^4\left(R+s\right)^2}{8\,R^2\,s^4}\right] \tag{2}$$

$$l'=s'\left[1+\frac{h^2\,(R-s')}{2\,R\,s'^2}+\frac{h^4(R-s')}{8\,R^3\,s'^2}-\frac{h^4\left(R-s'\right)^2}{8\,R^2\,s'^4}\right] \tag{3}$$

Using these in (1) and doing some obvious simplification gives,

$$a(Q)=n_1\,s\left[\frac{h^2\,(R+s)}{2\,R\,s^2}+\frac{h^4(R+s)}{8\,R^3\,s^2}-\frac{h^4\left(R+s\right)^2}{8\,R^2\,s^4}\right]+n_2\,s'\left[\frac{h^2\,(R-s')}{2\,R\,s'^2}+\frac{h^4(R-s')}{8\,R^3\,s'^2}-\frac{h^4\left(R-s'\right)^2}{8\,R^2\,s'^4}\right] \tag{4}$$

Consider the terms proportional to h^2:

$$\frac{h^2}{2}\left\{\frac{n_1\,(R+s)}{R\,s}+\frac{n_2(R-s')}{R\,s'}\right\}=\frac{h^2}{2}\left\{\frac{n_1}{s}+\frac{n_2}{s'}+\frac{n_1-n_2}{R}\right\}=0$$

Now the h^4 terms:

$$\frac{h^4}{8}\left\{\frac{n_1}{R^2\,s}+\frac{n_1}{R^3}-\frac{n_1}{s^3}-2\,\frac{n_1}{R\,s^2}-\frac{n_1}{R^2 s}+\frac{n_2}{R^2 s'}-\frac{n_2}{R^3}-\frac{n_2}{s'^3}+2\,\frac{n_2}{R\,s'^2}-\frac{n_2}{R^2 s'}\right\}=$$

$$\frac{h^4}{8}\left\{-\frac{n_1}{s}\left(\frac{1}{s}+\frac{1}{R}\right)^2-\frac{n_1}{s'}\left(\frac{1}{s'}-\frac{1}{R}\right)^2+\frac{1}{R^2}\left(\frac{n_1}{s}+\frac{n_2}{s'}\not+\frac{n_1}{R}-\frac{n_2}{R}\right)\right\}$$

$$a(Q)-\frac{h^4}{8}\left\{\frac{n_1}{s}\left(\frac{1}{s}+\frac{1}{R}\right)^2+\frac{n_1}{s'}\left(\frac{1}{s'}-\frac{1}{R}\right)^2\right\}$$

20-2. (a) If

$$\frac{n_1}{s}+\frac{n_2}{s'}=\frac{n_2-n_1}{R}\text{ and }\frac{1}{s'}=\frac{1}{s}+\frac{1}{R}$$

then,

$$\frac{n_1}{s}+\frac{n_2}{s'}=(n_2-n_1)\left(\frac{1}{s'}-\frac{1}{s}\right)=\frac{n_2}{s'}-\frac{n_2}{s}-\frac{n_1}{s'}+\frac{n_1}{s}$$

$$\frac{n_2}{s}=-\frac{n_1}{s'}\Rightarrow s'=-\,(n_1/n_2)\,s$$

(b) In $a(Q)$ the h^2 term is zero due to Gaussian imaging. For the remainder we must examine

$$a(Q)=-\frac{h^4}{8}\left\{\frac{n_1}{s}\left(\frac{1}{s}+\frac{1}{R}\right)^2+\frac{n_2}{s'}\left(\frac{1}{s'}-\frac{1}{R}\right)^2\right\}$$

Substitute using $1/s'=1/s+1/R$, to give,

$$a(Q)=-\frac{h^4}{8}\left\{\frac{n_1}{s}\left(\frac{1}{s'}-\frac{1}{R}+\frac{1}{R}\right)^2+\frac{n_2}{s'}\left(\frac{1}{s}+\frac{1}{R}-\frac{1}{R}\right)^2\right\}=-\frac{h^4}{8}\left\{\frac{n_1}{s\,s'^2}+\frac{n_2}{s's^2}=\frac{n_1\,s+n_2\,s'}{s'^2s^2}\right\}=0$$

(c) For $s'=R$, s must be $-R$. These values make $1/s+1/R$ and $1/s'-1/R$ vanish independently so $a(Q)=0$. Also $a(Q)=0$ when $h=0$, for rays incident at the vertex ($s=s'=0$).

(d) For $R=8\,\text{cm}$, $\dfrac{1}{s'}=\dfrac{1}{s}+\dfrac{1}{8\,\text{cm}}$ and $s'=-\,(1.36/1.70)\,s=-\,0.8\,s$
Solving these simultaneously gives, $s=-\,18\,\text{cm}$ and $s'=14.4\,\text{cm}$. Thus aplanatic images occur at $s'=0, 8\,\text{cm}$, and $14.4\,\text{cm}$.

20-3. The paraxial image point is found from

$$\frac{n_1}{s}+\frac{n_2}{s'}=\frac{n_2-n_1}{R}$$

since $s=\infty$,

$$0+\frac{1}{s'}=\frac{1-1.5}{-40\,\text{mm}}\Rightarrow s'=80\,\text{cm}$$

Then the ray aberration is becomes, (with $s=\infty$,)

$$a(Q)=-\frac{h^4}{8}\left\{\frac{n_1}{s}\left(\frac{1}{s}+\frac{1}{R}\right)^2+\frac{n_2}{s'}\left(\frac{1}{s'}-\frac{1}{R}\right)^2\right\}=-\frac{h^4}{8}\frac{n_2'}{s'}\left(\frac{1}{s'}-\frac{1}{R}\right)^2=-\frac{25^4}{8}\frac{1}{80}\left(\frac{1}{80}-\frac{1}{-40}\right)^2\text{mm}$$

$$a(Q)=-0.858\,\text{mm}$$

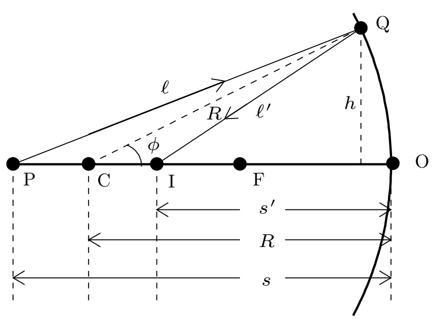
Proceeding,

$$\frac{da}{dh}=-\frac{h^3}{2}\frac{n_2}{s'}\left(\frac{1}{s'}-\frac{1}{R}\right)^2=-\frac{25^3}{2}\frac{1}{80}\left(\frac{1}{80}-\frac{1}{-40}\right)^2=-0.1373$$

$$b_y=\frac{s'}{n_2}\frac{da}{dh}=\frac{80\,\text{mm}}{1}(-0.1373)=-10.98\,\text{mm}$$

$$b_z=\frac{s'\,b_y}{y}=\frac{80}{25}(-10.98\,\text{mm})=-35.14\,\text{mm}$$

20-4. Refer to the figure below:



$a(Q)=\text{PQI}-\text{POI}=\ell+\ell-(s+s')$.
In triangle PQC:
$\ell^2=R^2+(s-R)^2+2R(s-R)+2R(s-R)\cos\phi$
In triangle CQI:
$\ell'^2=R_2+(R-s')^2-2R(R-s')\cos\phi$

Now $\cos\varphi=\sqrt{1-\sin^2\varphi}=\left[1-(h/R)^2\right]^{1/2}\approx 1-\dfrac{h}{2R^2}-\dfrac{h^4}{8R^4}$

This expression can be used in the expressions for ℓ and ℓ' and the resulting expressions expanded. For this task I will ask for help from Maxima:

```
(%i1) (F:1-h^2/2/R^2-h^4/8/R^4,display(F))
```

$$F=-\frac{h^2}{2R^2}-\frac{h^4}{8R^4}+1$$

```
(%i2) (L:sqrt(R^2+(s-R)^2+2*R*(s-R)*F),display(L))
```

$$L=\sqrt{R^2+2\left(-\frac{h^2}{2R^2}-\frac{h^4}{8R^4}+1\right)(s-R)R+(s-R)^2}$$

```
(%i3) (Lp:sqrt(R^2+(R-sp)^2-2*R*(R-sp)*F),display(Lp))
```

$$\text{Lp}=\sqrt{(R-\text{sp})^2+R^2-2\left(-\frac{h^2}{2R^2}-\frac{h^4}{8R^4}+1\right)R(R-\text{sp})}$$

```
(%i4) (Lapp:taylor(L,h,0,4),'L Lapp)
```

(%o4) $L=s+\dfrac{(R-s)h^2}{2sR}-\dfrac{(R^3-2sR^2+s^3)h^4}{8s^3R^3}+\ldots$

```
(%i5) (Lpapp:taylor(Lp,h,0,4),'Lp Lpapp)
```

(%o5) $\text{Lp}=\text{sp}+\dfrac{(R-\text{sp})h^2}{2\,\text{sp}\,R}-\dfrac{(R^3-2\,\text{sp}\,R^2+\text{sp}^3)h^4}{8\,\text{sp}^3R^3}+\ldots$

The last two expressions are ℓ and ℓ' to 4^{th} order in h

continued...

20-4. *continued...*

Using the expressions from Maxima in the expression for $a(Q)$ gives,

$$a(Q) = \ell + \ell' - s - s' = \frac{(R-s)\,h^2}{2\,s\,R} - \frac{\left(R^3 - 2\,s\,R^2 + s^3\right)h^4}{8\,s^3\,R^3} + \frac{(R-s')\,h^2}{2\,s'R} - \frac{\left(R^3 - 2\,s'\,R^2 + s'^3\right)h^4}{8\,s'^3\,R^3}$$

$$a(Q) = \frac{h^2}{2}\left(\frac{1}{s} + \frac{1}{s'} - \frac{2}{R}\right) - \frac{h^4}{8}\left(\frac{1}{s^3} + \frac{1}{s'^3} - 2\,\frac{1}{s^2\,R} - 2\,\frac{1}{s'^2 R} + \frac{1}{R^3} + \frac{1}{R^3}\right)$$

Gaussian imaging gives,

$$\frac{1}{s} + \frac{1}{s'} - \frac{2}{R} = 0$$

So, the term proportional to h^2 vanishes. This relation can be used to eliminate s' in the h^4 term:

$$a(Q) = -\frac{h^4}{8}\left[\frac{1}{s^3} + \left(\frac{2}{R} - \frac{1}{s}\right)^3 - \frac{2}{R}\frac{1}{s^2} - \frac{2}{R}\left(\frac{2}{R} - \frac{1}{s}\right)^2 + \frac{2}{R^3}\right]$$

$$a(Q) = -\frac{h^4}{4R}\left(\frac{1}{s} - \frac{1}{R}\right)^2$$

20-5. Using the result from problem 20 4,

$$a = -\frac{h^4}{4R}\left(\frac{1}{s} - \frac{1}{R}\right)^2$$

With $s \to \infty$, $a = -\dfrac{h^4}{4\,R^3} = -\dfrac{(25)^4}{4\,(400)^3}\,\text{cm} = -0.153\,\text{cm} = 0.0153\,\text{mm}$

Further, $\dfrac{da}{dh}_{s=\infty} = -\dfrac{h^3}{R^3} = -\dfrac{25^3}{400^3} = -2.441 \times 10^{-4}$

Then

$$b_y = \frac{s'}{n_2} = f\,\frac{da}{dy} = (200\,\text{cm})\left(-2.441 \times 10^{-4}\right) = -0.0488\,\text{cm} = -0.488\,\text{mm}$$

$$b_z = \frac{s'}{y}\,b_y = \frac{f\,b_y}{h} = \frac{200\,(-0.0488)}{25}\,\text{cm} = -0.391\,\text{cm} = -3.91\,\text{mm}$$

20-6. As in problem 20 5, with $s = \infty$, $a = -h^4/\left(4R^3\right)$. Also, $h = f/(2 \times 3.75) = 40\,\text{cm}$

(a) $a = -\dfrac{h^4}{4\,R^3} = \dfrac{(40)^4}{-4\,(600)^3}\,\text{cm} = -0.00296\,\text{cm} = -0.0296\,\text{mm}$

(b) $n\,\Delta t = |a| \Rightarrow \Delta t = |a|/n = (0.0296\,\text{mm})/1.4 = 0.021\,\text{mm}$

20-7. Given: $b_z = 1\,\text{cm}$, 4 D ($f = 25\,\text{cm}$), 6 cm diameter lens ($h = 3\,\text{cm}$). For $s = 50\,\text{cm}$, $s' = s\,f/(s-f) = 50\,\text{cm}$.

(a) $b_y = -s'\,\dfrac{da}{dy} = -\dfrac{h\,b_z}{s'} = -\dfrac{(3)\,(1)}{50}\,\text{cm} = -0.6\,\text{mm}$ (b) $d_{\text{blur}} = 2\,|b_x| = 2\,|b_y| = 2\,(0.6\,\text{mm}) = 1.2\,\text{mm}$

20-8. The Coddington shape factor is $\sigma = \dfrac{r_2 + r_1}{r_2 - r_1} = \dfrac{-10 + 10}{-10 - 10} = 0$ and the focal length is,

$$\frac{1}{f} = (1.5 - 1)\left(\frac{1}{10\,\text{cm}} - \frac{1}{-10\,\text{cm}}\right) \Rightarrow f = 10\ \text{cm}$$

For parallel rays, $s'_p = f = 10\,\text{cm}$. Then, from Eq. (20 28)

$$p = \frac{s' - s}{s' + s} = \frac{s/s' - 1}{s/s' + 1} = -1\ (\text{as}\ s \to \infty)$$

$$\frac{1}{s'_h} - \frac{1}{10\,\text{cm}} = \frac{1}{(8)10^3}\,\frac{1}{(1.5)\,(0.5\,)\,\text{cm}}\left\{0 + 0 + [3(1.5) + 2]\,[1.5 - 1]\,(-1)^2 + \frac{1.5^3}{1.5 - 1}\right\} \Rightarrow s'_h = 9.836\,\text{cm}$$

$$b_z = s'_p - s'_h = (10 - 9.836)\,\text{cm} = 0.164\,\text{cm};\ b_y = \left(h/s'\right)b_z = 0.0164\,\text{cm}$$

20-9. The focal length is given by the lensmaker's formula

$$\frac{1}{f} = (1.6-1)\left(\frac{1}{36\,\mathrm{cm}} - \frac{1}{-18\,\mathrm{cm}}\right) \Rightarrow f = 20\,\mathrm{cm} = s'_p$$

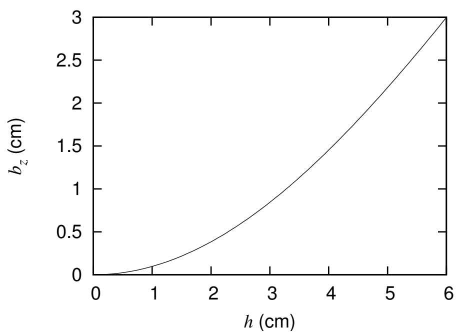
Also, since $s \rightarrow \infty$, $p = \dfrac{s'-s}{s'+s} = -1$ and $\sigma = \dfrac{r_2+r_1}{r_2-r_1} = \dfrac{-18+36}{-18-36} = -1/3$. So, using Eq. (20 28)

$$\frac{1}{s'_h} - \frac{1}{20\,\mathrm{cm}} = \frac{h^2}{8\,(20)^3\,(1.6)\,(0.6)\,\mathrm{cm}}\left\{\frac{3.6}{6}\left(-\frac{1}{3}\right)^2 + 4\,(2.6)\left(-\frac{1}{3}\right)(-1) + 6.8\,(0.6)\,(-1)^2 + \frac{1.6^3}{0.6}\right\}$$

$$s'_h = \frac{1}{0.05/\mathrm{cm} + 0.00024479\,h^2/\mathrm{cm}^3}$$

$$b_z = 20\,\mathrm{cm} - s'_h = 20\,\mathrm{cm} - \frac{1}{0.05/\mathrm{cm} + 0.00024479\,h^2/\mathrm{cm}^3}$$

Plotting gives,



h (cm)	b_z (mm)
1	0.974
2	3.84
3	8.44
4	14.53
5	21.81

20-10. The Coddington shape factor is $\sigma = \dfrac{r_2+r_1}{r_2-r_1} = \dfrac{-10+10}{-10-10} = 0$ and the focal length is,

$$\frac{1}{f} = (1.5-1)\left(\frac{1}{15\,\mathrm{cm}} - \frac{1}{-15\,\mathrm{cm}}\right) \Rightarrow f = 15\,\mathrm{cm}$$

For parallel rays, $s'_p = f = 10\,\mathrm{cm}$. Then, from Eq. (20 28) and with $p = \dfrac{s'-s}{s'+s} = \dfrac{37.5-25}{37.5+25} = 0.2$

$$\frac{1}{s'_h} - \frac{1}{37.5\,\mathrm{cm}} = \frac{1}{(8)15^3}\,\frac{1}{(1.5)\,(0.5\,)\,\mathrm{cm}}\left\{0 + 0 + (6.5)\,(0.5)\,(0.2)^2 + \frac{1.5^3}{0.5}\right\} \Rightarrow s'_h = 35.68\,\mathrm{cm}$$

$$b_z = s'_p - s'_h = (37.5 - 35.68)\,\mathrm{cm} = 1.82\,\mathrm{cm};\; b_y = \left(h/s'\right) b_z = (2/37.5)\,(1.82\,\mathrm{cm}) = 0.0970\,\mathrm{cm}$$

20-11. Since $L = (s'_p - s'_h)/(s'_p s'_h) = b_z\,(s'_p\,s'_h)$, a minimum in L gives a minimum in b_z. From Eq. (20 28),

$$\frac{dL}{d\sigma} = \frac{h^2}{8\,f^3}\frac{1}{n\,(n-1)}\left[\frac{2\,(n+2)}{n-1}\,\sigma + 4\,(n+1)\,p\right] = 0$$

$$\frac{2\,(n+2)}{n-1}\,\sigma = -4\,(n+1)\,p \Rightarrow \sigma = -\frac{2\,(n^2-1)}{n+2}p$$

20-12. The equations,

$$\sigma = \frac{r_2+r_1}{r_2-r_1} \quad \text{and} \quad \frac{1}{f} = (n-1)\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$$

can be solved simultaneously for r_1 and r_2. The general result is,

$$r_1 = \frac{2\,f\,(n-1)}{\sigma+1} \quad \text{and} \quad r_2 = \frac{2\,f\,(n-1)}{\sigma-1}$$

For $\sigma = 0.7, n = 1.7,$ and $f = 30\,\mathrm{cm}$: $r_1 = 17.65\,\mathrm{cm}$ and $r_2 = -100\,\mathrm{cm}$

For $\sigma = 3, n = 1.7,$ and $f = 30\,\mathrm{cm}$: $r_1 = 7.50\,\mathrm{cm}$ and $r_2 = 15.0\,\mathrm{cm}$

20-13. The image position is,

$$s' = \frac{s\,f}{s-f} = \frac{(30)\,(20}{30-20}\,\text{cm} = 60\,\text{cm}$$

The Coddington shape factor is, (see problem 20 11)

$$\sigma = -\frac{2\,(n^2-1)}{n+2}\,\frac{s'-s}{s'+s} = -\frac{2\,(1.6^2-1)}{1.6+2}\,\frac{30-60}{30+60} = 0.289$$

Then, using the results from problem 20 12

$$r_1 = \frac{2\,f\,(n-1)}{\sigma+1} = \frac{2\,(20)\,(1.6)}{1.289}\,\text{cm} = 18.62\,\text{cm} \quad \text{and} \quad r_2 = \frac{2\,f\,(n-1)}{\sigma-1} = \frac{2\,(20)\,(0.6)}{-0.711} = -33.75\,\text{cm}$$

20-14. In either case the radius of curvature of the spherical side found from the lensmaker's formula is ± 50 cm. For the ideal case (see problem 20 11)

$$\sigma = -\frac{2\,(n^2-1)}{n+2}\,\frac{s'-s}{s'+s} = -\frac{2\,(n^2-1)}{n+2}\,\frac{s'/s-1}{s'/s+1} = -\frac{2\,(1.6^2-1)}{1.6+2}\,(-1) = 0.867$$

For light incident on the plane side,

$$\sigma = \frac{r_2+r_1}{r_2-r_1} = \frac{r_2/r_1+1}{r_2/r_1-1} = -1 \text{ as } r_1 \to \infty$$

For light incident on the spherical side,

$$\sigma = \frac{r_2+r_1}{r_2-r_1} = \frac{1+r_1/r_2}{1-r_1/r_2} = +1 \text{ as } r_2 \to \infty$$

Since $\sigma = 1$ is closer to 0.867 than is $\sigma = -1$, the second orientation is better.

20-15. Using the result from problem 20 11,

$$\sigma = -\frac{2\,(n^2-1)}{n+2}\,\frac{s'-s}{s'+s}$$

(a) As $s \to \infty$, $\sigma \to -\dfrac{2\,(n^2-1)}{n+2}\,(-1) = \dfrac{2\,(1.5^2-1)}{1.5+2}\,(-1) = 0.714.$

(b) Using the result found in the solution to problem 20 12:

$$r_1 = \frac{2\,f\,(n-1)}{\sigma+1} = \frac{2\,(30)\,(1.5-1)}{0.714+1}\,\text{cm} = 17.5\,\text{cm} \quad \text{and} \quad r_2 = \frac{2\,f\,(n-1)}{\sigma-1} = \frac{2\,(30)\,(1.5-1)}{0.714-1}\,\text{cm} = -105\,\text{cm}$$

(c) As $s' \to \infty, \sigma \to -\dfrac{2\,(n^2-1)}{n+2}\,(1) = \dfrac{2\,(1.5^2-1)}{1.5+2}\,(1) = -0.714.$ (Reverse the lens.)

20-16. (a) Using Eq. (20 31) $\sigma = \left(\dfrac{2\,n^2-n-1}{n+1}\right)\left(\dfrac{s-s'}{s+s'}\right)_{s\to\infty} = \left(\dfrac{(2\,)1.5^2-1.5-1}{1.5+1}\right)(+1) = 0.8$

(b) Using the result found in the solution to problem 20 12:

$$r_1 = \frac{2\,f\,(n-1)}{\sigma+1} = \frac{2\,(30)\,(1.5-1)}{0.8+1}\,\text{cm} = 16.7\,\text{cm} \quad \text{and} \quad r_2 = \frac{2\,f\,(n-1)}{\sigma-1} = \frac{2\,(30)\,(1.5-1)}{0.8-1}\,\text{cm} = -150\,\text{cm}$$

(c) $\sigma = \left(\dfrac{2\,n^2-n-1}{n+1}\right)\left(\dfrac{s-s'}{s+s'}\right)_{s'\to\infty} = \left(\dfrac{(2\,)1.5^2-1.5-1}{1.5+1}\right)(-1) = -0.8$ (Reverse the lens)

20-17. With $s' = s$, $\sigma = -\dfrac{2\,(n^2-1)}{n+2}\,\dfrac{s'-s}{s'+s} = 0$. So with $\sigma = \dfrac{r_2+r_1}{r_2-r_1} = 0 \Rightarrow r_1 = -r_2$. Then,

$$\frac{1}{f} = (n-1)\left(\frac{1}{r_1} - \frac{1}{r_2}\right) = (1.5-1)\,\frac{2}{r_1} \Rightarrow r_1 = f = 20\,\text{cm}, r_2 = -r_1 = -20\,\text{cm}$$

20-18. In this case,

$$\sigma = \left(\frac{2\,n^2 - n - 1}{n+1}\right)\left(\frac{s - s'}{s + s'}\right)\underset{s=s'}{=} 0$$

Thus the answers are the same as in problem 20 17.

20-19. Generally,

$$\sum \frac{1}{n_i\,f_i} = \frac{1}{R_p}$$

For a flat Petzval surface $R_p \to \infty$, so

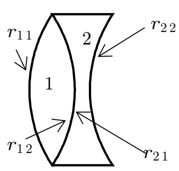
$$\frac{1}{n_1\,f_1} + \frac{1}{n_2\,f_2} = 0 \Rightarrow f_2 = -\,(n_1/n_2)\,f_1 = -\,(1.523/1.72)\,(20)\,\mathrm{cm} = -\,17.71\,\mathrm{cm}$$

20-20. (a) The radius of the Petzval surface is given by Eq. (20 32) as,

$$\frac{1}{R_p} = \sum \frac{1}{n_i\,f_i} = \frac{1}{(1.5736)\,(3.543\,\mathrm{cm})} + \frac{1}{(1.6039)\,(-\,5.391\,\mathrm{cm})} \Rightarrow R_p = 15.7\,\mathrm{cm}$$

(b) $\dfrac{1}{R_p} = 0 = \dfrac{1}{(1.5736)\,(3.543\,\mathrm{cm})} + \dfrac{1}{(1.6039)\,f_2} \Rightarrow f_2 = -\,3.476\,\mathrm{cm}$

20-21. The calculations proceed stepwise as follows given the values of the refractive indices from Table 20 1 and the desired overall focal length f_D of the doublet.



$$P_D = \frac{1}{f_D},\ V_1 = \frac{n_{1D} - 1}{n_{1F} - n_{1C}},\ V_2 = \frac{n_{2D} - 1}{n_{2F} - n_{2C}}$$

$$P_{1D} = P_D\left(\frac{-\,V_1}{V_2 - V_1}\right),\ P_{2D} = P_D\left(\frac{V_2}{V_2 - V_1}\right),\ K_1 = \frac{P_{1D}}{n_{1D} - 1},\ K_2 = \frac{P_{2D}}{n_{2D} - 1}$$

$$r_{11} = 2/K_1,\ r_{12} = -\,r_{11},\ r_{21} = r_{12},\ r_{22} = r_{12}/(1 - K_2\,r_{12})$$

$$f_{1D} = \frac{1}{K_1\,(n_{1D} - 1)},\ f_{2D} = \frac{1}{K_2\,(n_{2D} - 1)},\ f_D = \frac{f_{1D}\,f_{2D}}{f_{1D} + f_{2D}}$$

The last line is repeated for f_C and f_F. A typical Basic program that implements this routine is given below,

```
INPUT "N1C,N1D,N1F ?";MC,MD,MF
INPUT "N2C,N2D,N2F ?";NC,ND,NF
INPUT "DESIRED F ?";FD
P 1/FD
V1 (MD-1)/(MF-MC):V2 (ND-1)/(NF-NC)
P1 -P*V1/(V2-V1):P2 P*V2/(V2-V1)
K1 P1/(MD-1):K2 P2/(ND-1)
R1 2/K1:R2 -R1:S1 R2:S2 R2/(1-K2*R2)
D1 1/(K1*(MD-1)):D2 1/(K2*(ND-1)):D D1*D2/(D1+D2)
C1 1/(K1*(MC-1)):C2 1/(K2*(NC-1)):C C1*C2/(C1+C2)
F1 1/(K1*(MF-1)):F2 1/(K2*(NF-1)):F F1*F2/(F1+F2)
PRINT R1;R2
PRINT S1;S2
PRINT D1;D2
PRINT D
PRINT C1;C2
PRINT C
PRINT F1;F2
PRINT F
END
```

The indices of refraction of the glasses in this problem are given in Table 20 1 as,

	n_C	n_D	n_F
#517/645	1.51461	1.51707	1.52262
#620/380	1.61564	1.62045	1.63198

problem 20 21 continued on the next page...

20-21. *continued...*

The step by step calculations proceed as,

$$f_{\text{eq}} = 20\,\text{cm} \qquad V_1 = \frac{n_{1D} - 1}{n_{1F} - n_{1C}} = 64.5531$$

$$P_D = \frac{1}{f_{\text{in m}}} = \frac{1}{0.2}\,\text{D} = 5\,\text{D} \qquad V_2 = \frac{n_{2D} - 1}{n_{2F} - n_{2C}} = 37.9712$$

$$P_{1D} = P_D \frac{-V_1}{V_2 - V_1} - 0.121423\,\text{cm}^{-1} \qquad P_{2D} = P_D \frac{V_2}{V_2 - V_1} = -0.071423\,\text{cm}^{-1}$$

$$K_1 = \frac{P_{1D}}{n_{1D} - 1} = 0.234830\,\text{cm}^{-1} \qquad K_2 = \frac{P_{2D}}{n_{2D} - 1} = -0.11515\,\text{cm}^{-1}$$

$$r_{11} = \frac{2}{K_1} = 8.5168\,\text{cm} \qquad r_{12} = -r_{11} = -8.5168\,\text{cm} = r_{21}$$

$$r_{22} = \frac{r_{12}}{1 - K_2 r_{12}} = -434.89\,\text{cm}$$

$$\lambda_D\text{:}\; f_{1D} = \frac{1}{K_1(n_{1D} - 1)} = 8.23565\,\text{cm} \qquad f_{2D} = \frac{1}{K_2(n_{2D} - 1)} = -14.0010\,\text{cm}$$

$$f_D = \frac{f_{1D} f_{2D}}{f_{1D} + f_{2D}} = 20.000\,\text{cm}$$

$$\lambda_C\text{:}\; f_{1C} = \frac{1}{K_1(n_{1C} - 1)} = 8.2750\,\text{cm} \qquad f_{2C} = \frac{1}{K_2(n_{2C} - 1)} = -14.1104\,\text{cm}$$

$$f_C = \frac{f_{1C} f_{2C}}{f_{1C} + f_{2C}} = 20.0096\,\text{cm}$$

$$\lambda_F\text{:}\; f_{1F} = \frac{1}{K_1\,(n_{1F} - 1)} = 8.1428\,\text{cm} \qquad f_{2F} = \frac{1}{K_2\,(n_{2F} - 1)} = -13.7456\,\text{cm}$$

$$f_F = \frac{f_{1F} f_{2F}}{f_{1F} + f_{2F}} = 20.0096\,\text{cm}$$

Note: The values listed above are given rounded off in each step, the actual calculations were carried out with the more values stored in the computer.

Problems 20 22 and 20 23 are on the next page.

20-22. The calculations proceed as outlined in detail in the solution to problem 20 21. Data and results are listed here. $f_{\text{eq}} = 5\,\text{cm}, P_D = 0.2\,\text{cm}^{-1}$.

	n_C	n_D	n_F
#517/645	1.63461	1.63810	1.64611
#620/380	1.79608	1.80518	1.82771

(a) $r_{11} = 3.4535\,\text{cm},\ r_{12} = -r_{11} = -3.4535\,\text{cm} = r_{21},\ r_{22} = -12.6576\,\text{cm}$

(b)

	f_1 (cm)	f_2 (cm)	f (cm)
λ_D	2.7061	-5.8985	5.0000
λ_C	2.7210	-5.9659	5.0026
λ_F	2.6726	-5.7380	5.0026

(c) $P_{1D} = 0.369534\,\text{cm}^{-1} = 1/f_{1D},\ P_{2D} = -0.169534\,\text{cm}^{-1} = 1/f_{2D}$.

Dispersive powers: $\Delta_1 = \dfrac{1}{V_1} = 0.018022, \Delta_2 = \dfrac{1}{V_2} = 0.039283$

(d) Check: $V_2\,P_{1D} + V_1\,P_{2D} = (25.4562)\,(0.369534\,\text{cm}^{-1}) + (55.48696)\,(-0.169534\,\text{cm}^{-1}) = 0$

20-23. The calculations proceed as outlined in detail in the solution to problem 20 21. Data and results are listed here. $f_{\text{eq}} = -10\,\text{cm}, P_D = -0.1\,\text{cm}^{-1}$.

	n_C	n_D	n_F
#573/574	1.56956	1.57259	1.57953
#689/312	1.68250	1.68893	1.70462

(a) $r_{11} = -5.2415\,\text{cm},\ r_{12} = -r_{11} = -5.2415\,\text{cm} = r_{21},\ r_{22} = 53.1840\,\text{cm}$

(b)

	f_1 (cm)	f_2 (cm)	f (cm)
λ_D	-4.5770	8.4399	-10.0000
λ_C	-4.6013	8.5194	-10.0050
λ_F	-4.5222	8.2520	-10.0050

Chapter 21 Fourier Optics

21-1. (a) $m\lambda = d\sin\theta = Ym\,d/f$ or $Ym = \dfrac{m\lambda f}{d} = m\left(\dfrac{632.8\times10^{-6}\,\text{mm}\times500\,\text{mm}}{0.5\,\text{mm}}\right) = 0.6328\,\text{mm}$

For $m=1$, $Y_1 = 0.633$ mm. Note that, since $d/b = 2$ for the Ronchi ruling, missing orders are those for which $m = 2, 4, 6, \ldots$ Then:

$m=0, Y_0 = 0$ (DC component)
$m=1, Y_1 = 0.633\,\text{mm}$
$m=3, Y_3 = 1.898\,\text{mm}$
$m=5, Y_5 = 3.164\,\text{mm}$

(b) For the fundamental frequency $m=1$ so $\nu_Y = \dfrac{m}{d} = \dfrac{1}{\lambda_Y} \Rightarrow \lambda Y = \dfrac{d}{m} = \dfrac{0.5\,\text{mm}}{1} = 0.5\,\text{mm}.$

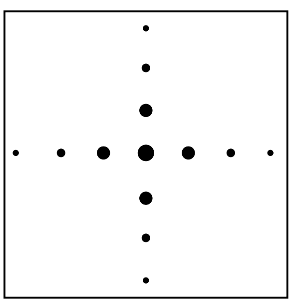
(c) $K_Y = \dfrac{kY}{f} = 2\pi\nu_Y = 2\pi\left(\dfrac{m}{d}\right)$. Thus $K_Y = \left(\dfrac{2\pi}{d}\right)m = m\left(\dfrac{2\pi}{0.5\,\text{mm}}\right)\begin{cases} m=1, K_Y = 12.57\,\text{cycles/mm} \\ m=3, K_Y = 37.70\,\text{cycles/mm} \\ m=5, K_Y = 62.83\,\text{cycles/mm}\end{cases}$

(d) $f(Y) = \dfrac{1}{2} + \dfrac{2}{\pi}\left[\sin\omega Y + \dfrac{1}{3}\sin 3\omega Y + \dfrac{1}{5}\sin 5\omega Y + \ldots\right]$. So the relative fundamental amplitude is 1, the relative amplitude for the second spot is 1/3 and the relative amplitude for the third spot is 1/5. Since irradiance is proportional to the square of the amplitude,

$$I_2/I_1 = 1/9 \text{ and } I_3/I_1 = 1/25$$

21-2. (a) At any point, what is transmitted is a product: If E_0 is the incident amplitude, $E_1 = t_1E_1$ passes the first transparency and $E_2 = t_1E_1 = t_2t_1E_0$ passes the second transparency. Thus the combined transmission function is $t = t_1t_2$.

(b) The pattern in the spectrum plane would look like:



21-3. It is given that the optical density D is given by

$$D = \log_{10}(\text{opacity}) = \log_{10}\left(\frac{1}{T}\right)$$

(a) $D = \log_{10}\left(\dfrac{1}{T}\right) = -\log_{10}(T)$

(b) As in problem 21 2, $T = T_1T_2T_3$. Therefore,

$$D = -\log_{10}(T_1T_2T_3) = D = -(\log T_1 + \log T_2 + \log T_3) = D_1 + D_2 + D_3$$

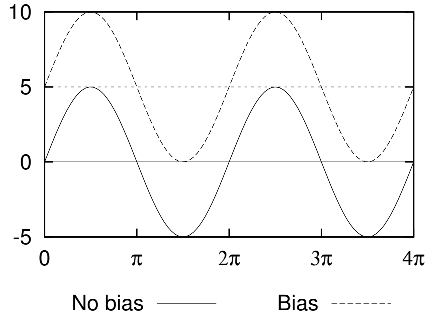
(c) Since the opacity of the i^{th} transparency is $(\text{opacity})_i = 1.25$, the optical density can be formed as

$$D_i = \log_{10}(1.25) = 0.09691$$
$$D_{\text{net}} = 5D_i = 0.48455$$
$$T_{\text{net}} = \log^{-1}(-D) = 10^{-D} = 10^{-.48455} = 0.3277 \quad (32.77\%)$$

21-4. Given $T = \dfrac{E_t}{E_0} = 5\sin(a\,y)$.

(a) A bias of 5 units locates minima at $T = 0$.

(b)



(c) $\dfrac{I_t}{I_0} = \left(\dfrac{E_t}{E_0}\right)^2 = 25\,(1 + \sin a\,y)^2$, For $I_0 = 1$, $I_t = 25\,(1 + \sin a\,y)^2$.

21-5. By definition of a Fourier transform,

$$\mathcal{F}\,[h\,(x)] = \int h\,(x)\,e^{ikx}\;dx$$

Substitute for $h\,(x)$ the convolution of f and g:

$$\mathcal{F}\,[h\,(x)] = \int e^{ikx}\left[\int f\,(x - x')\,g\,(x')\,d\,x'\right]dx$$

Changing the order of integration,

$$\mathcal{F}\,[h\,(x)] = \int g\,(x')\left[\int e^{ikx}\,f\,(x - x')\,d\,x\right]dx'$$

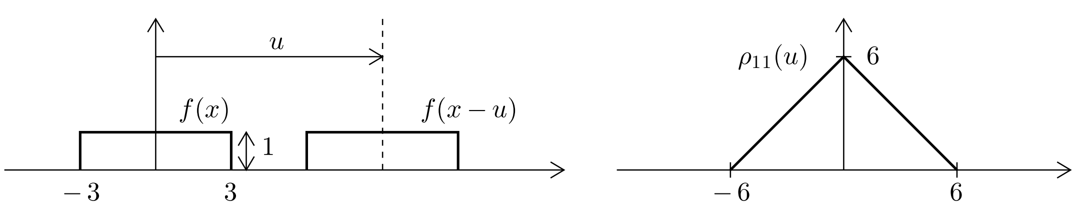
Then let $v \equiv x - x'$ so that $dx = dv$:

$$\mathcal{F}\,[h\,(x)] = \int g\,(x')\left[\int e^{ik(v+x')}\,f\,(v)\,dv\right]dx'$$

$$\mathcal{F}\,[h\,(x)] = \int f(v)\,e^{ikv}\,dv\;\int g\,(x')\,e^{ikx'}\,dx' \quad \text{or}$$

$$\mathcal{F}\,[h\,(x)] = \mathcal{F}\,[f\,(x)] \times \mathcal{F}\,[g\,(x)]$$

21-6. The two functions look like,



Here, $\rho_{11}\,(u) = \displaystyle\int f(x-u)\,f(x)\,dx$. As u varies, $\rho_{11}\,(u)$ measures the area of overlap of the square pulses:

For example,

$$\rho_{11}\,(0) = \int_{-3}^{3}\,[f\,(x)]^2\,dx = \int_{-3}^{3}\,dx = 6$$

$$\rho_{11}\,(3) = \int_{-3}^{3}\,f\,(x)\,f\,(x-3)\,dx = \int_{0}^{3}\,dx = 3$$

21-7. Consider the function $y = A\sin(\omega t + d) = f(t)$. Since this function is periodic we integrate over one period T.

$$\Phi_{11}(\tau) = \int_{-T/2}^{T/2} \underbrace{A\sin(\omega t + \alpha)}_{f(t)} \;\underbrace{A\sin[\omega(t+\tau) + \alpha]}_{f(t+\tau)} dt$$

$$\Phi_{11}(\tau) = A^2 \int_{-T/2}^{T/2} \sin\beta \sin(\beta + \omega\tau) dt = A^2 \int_{-T/2}^{T/2} \sin\beta\,(\sin\beta\cos\omega\tau + \cos\beta\sin\omega\tau)\, d\tau$$

$$\Phi_{11}(\tau) = A^2\cos\omega\tau \int_{-T/2}^{T/2} \underbrace{\sin^2\beta\, dt}_{\text{even}} + A^2 \sin\omega\tau \int_{-T/2}^{T/2} \underbrace{\sin\beta\cos\beta\, dt}_{\text{odd, int}=0}$$

$$\Phi_{11}(\tau) = 2A^2\cos\omega\tau \int_0^{T/2} \sin^2\beta\, dt = 2A^2\cos\omega t \int_0^{T/2} \sin^2(\omega t + \alpha)\, dt$$

$$\Phi_{11}(\tau) = \frac{2A^2\cos\omega\tau}{\omega} \int_0^{T/2} \sin(\omega t + \alpha)\,(\omega\, dt) = \frac{2A^2\cos\omega\tau}{\omega}\left[\frac{\omega t + \alpha}{2} - \frac{\sin 2(\omega t + \alpha}{4}\right]_0^{T/2}$$

$$\Phi_{11}(\tau) = \frac{2A^2\cos\omega\tau}{\omega}\left[\frac{\omega T}{4} - \frac{\sin 2(\omega T/2 + \alpha)}{4} + \frac{\sin 2\alpha}{4}\right]$$

Since $\omega T = 2\pi f T = 2\pi$,

$$\Phi_{11}(\tau) = \frac{2A^2\cos\omega\tau}{\omega}\left[\frac{\pi}{2} - \frac{\sin 2(\pi + \alpha)}{4} + \frac{\sin 2\alpha}{4}\right]$$

$$\Phi_{11}(\tau) = \frac{2A^2\cos\omega\tau}{\omega}\left[\frac{\pi}{2} - \frac{\overbrace{\sin 2\pi}^{0}\cos 2\alpha}{4} - \frac{\overbrace{\cos 2\pi}^{1}\sin 2\alpha}{4} + \frac{\sin 2\alpha}{4}\right]$$

$$\Phi_{11}(\tau) = A^2\pi\left(\frac{\cos\omega\tau}{\omega}\right)$$

21-8. (a) A mirror movement $x\,(=x_2 - x_1)$ of $\Delta x = \lambda/2$ produces a change in I from one peak to the next. So the frequency f_P of the peaks will be,

$$f_P = \frac{\Delta x/\Delta t}{\Delta x} = \frac{V}{\lambda/2} = \frac{2V}{\lambda} = \frac{2\,(5\,\text{mm/s})}{546.1\times 10^{-6}\,\text{mm}} = 18.3\,\text{KHz}$$

(b) As in measuring a wavelength difference with an interferometer, (see Eq. (8 14)), the peak to peak dis tance is

$$\Delta x = \lambda_{\text{ave}}^2/2\,\Delta\lambda$$

$$f_{\text{beat}} = \frac{v}{\Delta x} = \frac{2v\,\Delta\lambda}{\lambda_{\text{ave}}^2} = \frac{2\,(5)\,(5.97\times 10^{-7})}{(5892.94\times 10^{-7})^2}\,\text{Hz} = 17.2\,\text{Hz}$$

21-9. If the total mirror movement is 5 cm, then $x_\omega = 2\times 5 = 10\text{cm}$. According to Eq. (21 43), $\Delta\lambda = \lambda^2/x_w$.

(a) $\Delta\lambda = \frac{(6328)^2}{10\times 10^8}\,\text{Å} = 0.040\,\text{Å}$ (b) $\Delta\lambda = \frac{(10^4)^2}{10\times 10^8}\,\text{Å} = 0.100\,\text{Å}$

21-10. (a) During one pass, the total mirror movement is

$$x = (71.5\,\text{nm}/s)\,(\text{time}) = 71.5\,\frac{\text{nm}}{\text{s}}\,[256\,\text{rdg.}/(1.28\,\text{rdg./s})] = 14300\,\text{nm}$$
$$x_\omega = 2\,x = 28,600\,\text{nm} = 2.86\times 10^3\,\text{cm}$$

(b) $\lambda_{\min} = \dfrac{\lambda^2}{x_\omega} = \dfrac{(400\times 10^{-7}\,\text{cm})^2}{2.86\times 10^{-3}\,\text{cm}} = 5.59\times 10^{-7}\,\text{cm} = 5.59\text{ nm}$

(c) $\lambda_{\min} = \dfrac{2\,x_\omega}{N-1} = \dfrac{2\,(2.86\times 10^{-3}}{255} = 2.24\times 10^{-5}\,\text{cm} = 224\,\text{nm}$

(d) Peak frequency $= \dfrac{2\,V}{\lambda_{\text{shortest}}} = \dfrac{2\,(71.5\,\text{nm/s})}{360\,\text{nm}} = 0.397/\text{s}$. Nyquist: take at least $2\times$ or 0.8 readings/s.

21-11. The given information is,

$$x_w = 2.78\,\text{mm} \qquad \lambda\!: (4400 - 400)\,\text{cm}^{-1}$$
$$4400\,\text{cm}^{-1} = 2.27\,\mu\text{m} \quad 400\,\text{cm}^{-1} = 25\,\mu\text{m} \;\;\text{(both infrared)}$$

(a) We want the wave number $\nu_{\text{wn}} = \dfrac{1}{\lambda}$. For resolution we need, $\Delta\nu_{\text{wn}} = \dfrac{1}{\lambda^2}\,\Delta\lambda$, where $\Delta\lambda = \lambda^2/x_\omega$. Thus,

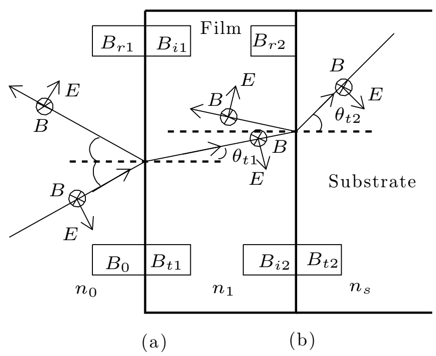
$$\Delta\nu_{\text{wn}} = \frac{1}{\lambda^2}\left(\frac{\lambda^2}{x_\omega}\right) = \frac{1}{x_\omega} = \frac{1}{0.278\,\text{cm}} = 3.597\,\text{cm}^{-1} \approx 3.6\,\text{cm}^{-1}$$

(b) $N - 1 = \dfrac{2\,x_\omega}{\lambda_{\min}} = \dfrac{2\,(2.78\times 10^{-3}\,\text{m})}{2.27\times 10^{-6}\,\text{m}} = 2449$ or $N = 2450$

(c) $V = \dfrac{2.78\,\text{mm}}{30\,\text{s}} = 0.0927\,\text{mm/s}$

Chapter 22 Theory of Multilayer films

22-1. Refer to the figure below for the orientation of the field vectors.



The electric field vectors are all parallel to the plane of incidence and the magnetic field vectors are all into the page. The tangential components are continuous:

$$B_a = B_0 + B_{r1} = B_{t1} + B_{i1}$$
$$B_b = B_{i2} + B_{r2} = B_{t2}$$

$$E_a = E_0 \cos\theta_0 - E_{r1} \cos\theta_0 = E_{t1}\cos\theta_{t1} - E_{i1}\cos\theta_{t1}$$
$$E_b = E_{i2}\cos\theta_{t1} - E_{r2}\cos\theta_{t1} = E_{t2}\cos\theta_{t2}$$

Using $B = n\sqrt{\varepsilon_0\,\mu_0}\,E$, the E equations can be expressed in terms of B:

$$E_a = \frac{B_0\cos\theta_0}{n_0\sqrt{\varepsilon_0\,\mu_0}} - \frac{B_{r1}\cos\theta_0}{n_0\sqrt{\varepsilon_0\,\mu_0}} = \frac{B_{t1}\cos\theta_{t1}}{n_1\sqrt{\varepsilon_0\,\mu_0}} - \frac{B_{i1}\cos\theta_{t1}}{n_1\sqrt{\varepsilon_0\,\mu_0}}$$
$$E_b = \frac{B_{i2}\cos\theta_{t1}}{n_1\sqrt{\varepsilon_0\,\mu_0}} - \frac{B_{r2}\cos\theta_{t1}}{n_1\sqrt{\varepsilon_0\,\mu_0}} = \frac{B_{t2}\cos\theta_{t2}}{n_s\sqrt{\varepsilon_0\,\mu_0}}$$

Define $\gamma_0 \equiv \dfrac{n_0\sqrt{\varepsilon_0\,\mu_0}}{\cos\theta_0}$, $\gamma_1 = \dfrac{n_1\sqrt{\varepsilon_0\,\mu_0}}{\cos\theta_{t1}}$, $\gamma_s = \dfrac{n_s\sqrt{\varepsilon_0\,\mu_0}}{\cos\theta_{t2}}$

Then the pair of equations becomes,

$$E_a = \frac{B_0}{\gamma_0} - \frac{B_{r1}}{\gamma_0} = \frac{B_{t1}}{\gamma_1} - \frac{B_{i1}}{\gamma_1} \Rightarrow E_a = \frac{1}{\gamma_0}(B_0 - B_{r1}) = \frac{1}{\gamma_1}((B_{t1} - B_{i1})$$
$$E_b = \frac{B_{i2}}{\gamma_1} - \frac{B_{r2}}{\gamma_1} = \frac{B_{t2}}{\gamma_s} \Rightarrow E_b = \frac{1}{\gamma_1}(B_{i2} - B_{r2}) = \frac{1}{\gamma_s}B_{t2}$$

Now, $B_{i2} = B_{t1}\,e^{-i\delta}$ and $B_{i1} = B_{r2}\,e^{-i\delta}$. Using these in the expressions developed for E_b and B_b:

$$B_b = B_{t1}\,e^{-i\delta} + B_{i1}\,e^{i\delta} = B_{t2} \quad E_b = \frac{B_{t1}e^{-i\delta}}{\gamma_1} - \frac{B_{i1}\,e^{i\delta}}{\gamma_1} = \frac{B_{t2}}{\gamma_s}$$

Solving these simultaneously for for B_{i1} and B_{t1} give, $B_{t1} = \dfrac{B_b + \gamma_1 E_b}{2}e^{i\delta}$ and $B_{i1} = \dfrac{B_b - \gamma_1 E_b}{2}e^{-i\delta}$

Substituting these into the expressions for E_a and B_a gives,

$$B_a = B_{t1} + B_{i1} \Rightarrow B_a = \frac{B_b + \gamma_1 E_b}{2}e^{i\delta} + \frac{B_b - \gamma_1 E_b}{2}e^{-i\delta}$$
$$E_a = \frac{B_{t1}}{\gamma_1} - \frac{B_{i1}}{\gamma_1} \Rightarrow E_a = B_{t1} = \frac{B_b + \gamma_1 E_b}{2\,\gamma_1}e^{i\delta} - \frac{B_b - \gamma_1 E_b}{2\,\gamma_1}e^{-i\delta}$$

Simplifying using $2\cos\delta = e^{i\delta} + e^{-i\delta}$ and $2\,i\sin\delta = e^{i\delta} - e^{-i\delta}$,

$$B_a = B_b\cos\delta + E_b\,(i\,\gamma_1\sin\delta)$$
$$E_a = B_b\left(\frac{i\sin\delta}{\gamma_1}\right) + E_b\cos\delta$$

In matrix form,

$$\begin{bmatrix} E_a \\ B_a \end{bmatrix} = \begin{bmatrix} \cos\delta & i\,(\sin\delta)/\gamma_1 \\ i\,\gamma_1\sin\delta & \cos\delta \end{bmatrix} \begin{bmatrix} E_b \\ B_b \end{bmatrix}$$

Note that this looks the same as Eq. (22 24) but in the case at hand with the electric field parallel to the plane of incidence $\gamma_1 = n_1\sqrt{\varepsilon_0\,\mu_0}\,/\cos\theta_{t1}$ while for the case treated in detail in the book leading to Eq. (22 24) $\gamma_1 = n_1\sqrt{\varepsilon_0\,\mu_0}\,\cos\theta_{t1}$.

22-2. (a) The film thickness t should satisfy

$$t = \frac{\lambda_F}{4} = \frac{\lambda_0}{4\,n_1}$$

with $n_1 = \sqrt{n_0\,n_s} = \sqrt{(1)\,(1.5)} = 1.22$. Thus

$$t = \frac{500\,\text{nm}}{4\,(1.22)} = 102\,\text{nm}$$

(b) The phase difference is

$$\delta = k_0\,\Delta = \frac{2\,\pi}{\lambda_0}\,(n_1\,t\cos\theta_{t1}) = \frac{2\,\pi}{550\,\text{nm}}\,(1.22)\,(102\,\text{nm})\,(1) = 1.428\,\text{rad}$$

So, $\cos\delta = 0.1423$ and $\sin\delta$=0.9898. Then,

$$R = \frac{n_1^2\,(n_0 - n_s)^2\cos^2\!\delta + (n_0\,n_s - n_1^2)^2\sin^2\!\delta}{n_1^2\,(n_0 + n_s)^2\cos^2\!\delta + (n_0\,n_s + n_1^2)^2\sin^2\!\delta} \qquad \text{with } n_0\,n_s - n_1^2 = 0$$

$$R = \frac{1.5\,(1-1.5)^2\,(0.142315)^2 + 0}{1.5\,(1+1.5)^2\,(0.142315)^2 + (1.5+1.5)^2\,(0.989821)^2}$$

$$R = 8.432\times 10^{-4} \Rightarrow 0.0843\%$$

22-3. From Eq. (22 42) or the relation used in problem 22 2, with $\delta = 2\,\pi\,\Delta/\lambda = 2\,\pi\,(\lambda/2)/\lambda = \pi$, so that $\cos^2\delta = 1$ and $\sin^2\delta = 0$,

$$R = \frac{n_1^2\,(n_0 - n_s)^2\,(1) + 0}{n_1^2\,(n_0 + n_s)^2\,(1) + 0} = \frac{(n_0 - n_s)^2}{(n_0 + n_s)^2}$$

22-4. Using Eq. (22 4),

$$R = \frac{n_1^2\,(n_0 - n_s)^2\cos^2\!\delta + (n_0\,n_s - n_1^2)^2\sin^2\!\delta}{n_1^2\,(n_0 + n_s)^2\cos^2\!\delta + (n_0\,n_s + n_1^2)^2\sin^2\!\delta} = \frac{1.46^2\,(1-1.52)^2\cos^2\delta + (1.52 - 1.46^2)^2\sin^2\delta}{1.46^2\,(1+1.52)^2\cos^2\delta + (1.52 + 1.46^2)^2\sin^2\delta}$$

(a) For $\lambda = 800\,\text{nm}$, $\delta = \dfrac{2\,\pi}{\lambda_0}\,\Delta = \dfrac{2\,\pi}{\lambda_0}\,(n_1\,t) = \dfrac{2\,\pi}{800\,\text{nm}}\,(1.46\,)(137\,\text{nm}) = \pi/2$. So,

$$\Delta/\lambda = 200/800 = 1/4, \cos\delta = 0, \sin\delta = 1$$
$$R = 0.0281 \Rightarrow R = 2.81\%$$

(b) For $\lambda = 600\,\text{nm}$, $\delta = \dfrac{2\,\pi}{600}\,200 = \dfrac{2\,\pi}{3}$. So, $\dfrac{\Delta}{\lambda} = \dfrac{200}{600} = \dfrac{1}{3}$.

$$\cos\delta = -\,0.5, \sin\delta = 0.866$$
$$R = 0.0317 \Rightarrow R = 3.17\%$$

(c) For $\lambda = 400\,\text{nm}$, $\delta = \dfrac{2\,\pi}{400}\,200 = \pi$. So, $\dfrac{\Delta}{\lambda} = \dfrac{200}{400} = \dfrac{1}{2}$.

$$\cos\delta = -\,1, \sin\delta = 0$$
$$R = 0.0426 \Rightarrow R = 4.26\%$$

(Same as glass of $n = 1.52$)

22-5. The phase difference is $\delta = (2\,\pi/\lambda_0)\,(n_1\,t) = (2\,\pi/5600)\,(2.35)(596) = \pi/2$ $(\Delta = \lambda/4)$. This is a quarter wave plate at normal incidence so Eq. (22 43) may be used,

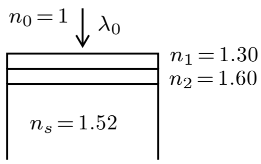
$$R = \left(\frac{n_0\,n_s - n_1^2}{n_0\,n_s + n_1^2}\right)^2 = \left(\frac{(1)(1.52) - (2.35)^2}{(1)(1.52) + (2.35)^2}\right)^2 = 0.323 \Rightarrow R = 32.3\%$$

22-6. The properties of the film for zero reflectance are,

$$n_1 = \sqrt{n_0\, n_s} = \sqrt{(1)\,(4)} = 2$$
$$t = \frac{\lambda_0}{4\, n_1} = \frac{2\,\mu\,\text{m}}{4\,(2)} = 0.25\ \mu\text{m}$$

According to Table 22 1, ZrO_2 has an index of refraction of 2.0 in the near infrared and so could be used as the film.

22-7. The double layer arrangement is as shown.



(a) The layers have quarter wave thickness for $\lambda_0 = 550\,\text{nm}$:

$$t_1 = \frac{\lambda_0}{4\, n_1} = \frac{550\,\text{nm}}{4\,(1.3)} = 105.8\,\text{nm}, \quad t_2 = \frac{\lambda_0}{4\, n_2} = \frac{550\,\text{nm}}{4\,(1.6)} = 85.94\,\text{nm}$$

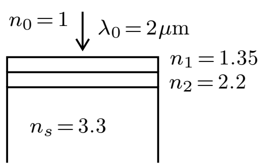
From Eq. (22 45):

$$R = \left(\frac{n_0\, n_2^2 - n_s\, n_1^2}{n_0\, n_2^2 + n_s\, n_1^2}\right)^2 = \left(\frac{(1)\,(1.6)^2 - 1.52\,(1.3)^2}{(1)\,(1.6)^2 + 1.52\,(1.3)^2}\right)^2 = 3 \times 10^{-6} \Rightarrow R = 0.0003\%$$

(b) With the layers reversed:

$$R = \left(\frac{n_0\, n_2^2 - n_s\, n_1^2}{n_0\, n_2^2 + n_s\, n_1^2}\right)^2 = \left(\frac{(1)\,(1.3)^2 - 1.52\,(1.6)^2}{(1)\,(1.3)^2 + 1.52\,(1.6)^2}\right)^2 = 0.1555 \Rightarrow R = 15.55\%$$

22-8. The double layer arrangement is as shown.



(a) The layers have quarter wave thickness for $\lambda_0 = 550\,\text{nm}$:

$$t_1 = \frac{\lambda_0}{4\, n_1} = \frac{2000}{4\,(1.35)}\,\text{nm} = 370\,\text{nm}$$
$$t_2 = \frac{\lambda_0}{4\, n_2} = \frac{2000\,\text{nm}}{4\,(2.2)} = 227\,\text{nm}$$

(b) The ideal index ratio is $\dfrac{n_2}{n_1} = \sqrt{\dfrac{n_s}{n_0}} = \sqrt{\dfrac{3.3}{1}} = 1.8166$

The actual ratio is $n_2/n_1 = 2.2/1.35 = 1.6296$. The % difference is $\dfrac{1.8166 - 1.6296}{1.8166}\,100\% = 10.3\%$

(c) The normal reflectance produced is

$$R = \left(\frac{n_0\, n_2^2 - n_s\, n_1^2}{n_0\, n_2^2 + n_s\, n_1^2}\right)^2 = \left(\frac{(1)\,(2.2)^2 - 3.3\,(1.35)^2}{(1)\,(2.2)^2 + 3.3\,(1.35)^2}\right)^2 = 0.0117 \Rightarrow R = 1.17\%$$

22-9. Refer to Figure 22 5 in the text.

For the $\lambda/4$ layer: $\cos\delta_1 = \cos(\pi/2) = 0$, $\sin\delta_1 = \sin(\pi/2) = 1$.
For the $\lambda/2$ layer: $\cos\delta_2 = \cos(\pi) = -1$, $\sin\delta_2 = \sin(\pi) = 0$.

The transfer matrix is

$$M = \underbrace{\begin{bmatrix} \cos\delta_1 & i\sin\delta_1/\gamma_1 \\ i\,\gamma_1\sin\delta_1 & \cos\delta_1 \end{bmatrix}}_{\lambda/4} \underbrace{\begin{bmatrix} \cos\delta_2 & i\sin\delta_2/\gamma_2 \\ i\,\gamma_2\sin\delta_2 & \cos\delta_2 \end{bmatrix}}_{\lambda/2} = \begin{bmatrix} 0 & i/\gamma_1 \\ i\,\gamma_1 & 0 \end{bmatrix} \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} = \begin{bmatrix} 0 & -i/\gamma_1 \\ -i\,\gamma_1 & 0 \end{bmatrix}$$

Thus the matrix elements of the overall transfer matrix are the negatives of those for the $\lambda/4$ layer acting alone. Thus the reflection coefficient given in Eq. (22 36) for the double layer and the single $\lambda/4$ layer differ by only an overall minus sign. Since $R = |r|^2$, the reflectances of the two layer structures is the same. Note that this is only true at the wavelength λ_0 for which the layers are $\lambda/4$ and $\lambda/2$.

22-10. For a double layer of arbitrary thickness, (to be specified), but normal incidence, the transfer matrix is

$$M = M_1\, M_2 = \begin{bmatrix} \cos\delta_1 & i\sin\delta_1/\gamma_1 \\ i\,\gamma_1\sin\delta_1 & \cos\delta_1 \end{bmatrix} \begin{bmatrix} \cos\delta_2 & i\sin\delta_2/\gamma_2 \\ i\,\gamma_2\sin\delta_2 & \cos\delta_2 \end{bmatrix}$$

$$M = \begin{bmatrix} \cos\delta_1\cos\delta_2 - (\gamma_2/\gamma_1)\sin\delta_1\sin\delta_2 & i(\cos\delta_1\sin\delta_2/\gamma_2 + \sin\delta_1\cos\delta_2/\gamma_1) \\ i\,(\gamma_2\cos\delta_1\sin\delta_2 + \gamma_1\sin\delta_1\cos\delta_2) & \cos\delta_1\cos\delta_2 - (\gamma_1/\gamma_2)\sin\delta_1\sin\delta_2 \end{bmatrix} \equiv \begin{bmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{bmatrix}$$

The general form of the reflection coefficient is, (since in our case m_{12} and m_{21} are pure imaginary)

$$r = \frac{\gamma_0\, m_{11} + \gamma_0\,\gamma_s\, m_{12} - m_{21} - \gamma_s\, m_{22}}{\gamma_0\, m_{11} + \gamma_0\,\gamma_s m_{12} + m_{21} + \gamma_s\, m_{22}}$$

Now, let us put this result in a form convenient for programming. Since in our case m_{12} and m_{21} are pure imaginary, write $m_{12} = i\,\mu_{12}$, $m_{21} = i\,\mu_{21}$ where μ_{12} and μ_{21} are real. Then the reflection coefficient can be recast as,

$$r = \frac{(\gamma_0\, m_{11} - \gamma_s\, m_{22}) + i\,(\gamma_0\,\gamma_s\,\mu_{12} - \mu_{21})}{(\gamma_0\, m_{11} + \gamma_s\, m_{22}) + i\,(\gamma_0\,\gamma_s\,\mu_{12} + \mu_{21})}$$

Making the substitutions,

$\gamma_0 = n_0\,\sqrt{\varepsilon_0\mu_0}$, $\gamma_1 = n_1\,\sqrt{\varepsilon_0\mu_0}$, $\gamma_2 = n_2\,\sqrt{\varepsilon_0\mu_0}$, and $\gamma_s = n_s\sqrt{\varepsilon_0\mu_0}$ in this expression (including in the matrix elements) leads to

$$r = \frac{(n_0\, M - n_s\, P) + i\,(n_0\, n_s\, N - O)}{n_0\, M + n_s\, P) + i\,(n_0\, n_s\, N + O)} \equiv \frac{A + i\,B}{C + i\,F}$$

Here

$$M = \cos\delta_1\cos\delta_2 - (n_2/n_1)\sin\delta_1\sin\delta_2 \quad P = \cos\delta_1\cos\delta_2 - (n_2/n_1)\sin\delta_1\sin\delta_2$$
$$N = \cos\delta_1\sin\delta_2/n_2 + \sin\delta_1\cos\delta_2/n_1 \qquad O = n_2\cos\delta_1\sin\delta_2 + n_1\sin\delta_1\cos\delta_2$$

The reflectance is,

$$R = |r|^2 = \frac{A^2 + B^2}{C^2 + F^2}$$

To calculate $M, N, O,$ and P, the phase differences must be formed. For a given wavelength λ, $\delta = \frac{2\pi}{\lambda_0}\, n\, t$, where t is the film thickness. Write, $t = Q\ (\lambda_0/n)$ where, for example, $Q = 1/4$ for a $\lambda/4$ layer. Q will be variable to account for different film thicknesses. We will do this for each layer. In addition to the variables M, N, O, P and A, B, C, F defined above the programming variables are:

$$n_1 \to G \qquad n_2 \to H \qquad n_s \to I$$
$$\lambda_0 \to L \qquad t_1 \to T = Q\ \lambda_0/n_1 \qquad t_2 \to U = S\,\lambda_0/n_2$$
$$\lambda_0 \to W\,(\text{in loop}) \qquad \delta_1 \to D \qquad \delta_2 \to E$$

A program to calculate the reflectance for different parameters is given below:

```
5  PRINT ″2-LAYER FILM″
10 INPUT ″N1  ″;G,″N2 ″;H,″NS  ″;I,″WLO(MICRON)  ″;L
15 INPUT ″THICK1  ″;Q,″THICK2  ″;S
20 T Q*L/G:U S*L/H
25 FOR W 0.3 TO 0.8 STEP 0.05
30 D 2*PI*G*T/W:E 2*PI*H*U/W
35 M COS(D)*COS(E)-H*SIN(D)*SIN(E)/G
40 N COS(D)*SIN(E)/H+SIN(D)*COS(E)/G
45 O G*SIN(D)*COS(E)+H*COS(D)*SIN(E)
50 P COS(D)*COS(E)-G*SIN(D)*SIN(E)/H
55 A M-I*P:B I*N-O:C M+I*P:F I*N+O
60 R 100*((A*A+B*B)/(C*C+F*F))
65 PRINT ″WL  ″;W,″REF  ″;R
70 NEXT W
75 END
```

Sample output is given on the next page.

22-10. *Continued...*

To generate Figure 22 4 in the text, use the following input data:

Case a	Case b	Case c
$n_1 = 1.65$	$n_1 = 1.38$	$n_1 = 1.38$
$n_2 = 2.1$	$n_2 = 1.6$	$n_2 = 1.85$
$Q = 0.25$	$Q = 0.25$	$Q = 0.25$
$S = 0.25$	$S = 0.25$	$S = 0.4$

In all cases $n_s = 1.52$ and $\lambda_0 = 0.55\ \mu$m.

The program output is:

$\lambda\ \mu$m	R_a	R_b	R_c
0.3	7.84%	4.91%	7.96%
0.35	16.19	4.48	10.63
0.40	14.02	1.80	3.11
0.45	7.24	0.90	0.013
0.5	1.84	1.11	0.74
0.55	0.10	1.26	1.26
0.60	1.32	1.15	0.88
0.65	3.95	0.97	0.26
0.70	6.84	0.90	0.0032
0.75	9.45	0.98	0.32
0.80	11.6	1.22	1.16

22-11. Quarter wave matrices (as in the solution to problem 22 9) for normal incidence give a composite transfer matrix of

$$M = \begin{bmatrix} 0 & i/\gamma_1 \\ i\gamma_1 & 0 \end{bmatrix}\begin{bmatrix} 0 & i/\gamma_2 \\ i\gamma_2 & 0 \end{bmatrix}\begin{bmatrix} 0 & i/\gamma_3 \\ i\gamma_3 & 0 \end{bmatrix} = \begin{bmatrix} 0 & -i\gamma_2/(\gamma_1\gamma_3) \\ -i\gamma_1\gamma_3/\gamma_2 & 0 \end{bmatrix} \equiv \begin{bmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{bmatrix}$$

Using Eq. (22 36),

$$r = \frac{\gamma_0 m_{11} + \gamma_0\gamma_s m_{12} - m_{21} - \gamma_s m_{22}}{\gamma_0 m_{11} + \gamma_0\gamma_s m_{12} + m_{21} + \gamma_s m_{22}}$$

$$r = \frac{\gamma_0\gamma_s[-i\gamma_2/(\gamma_1\gamma_3)] + i\gamma_1\gamma_3/\gamma_2}{\gamma_0\gamma_s[-i\gamma_2/(\gamma_1\gamma_3)] - i\gamma_1\gamma_3/\gamma_2} = \frac{\frac{\gamma_1\gamma_3}{\gamma_2} - \frac{\gamma_0\gamma_s\gamma_2}{\gamma_1\gamma_3}}{-\frac{\gamma_1\gamma_3}{\gamma_2} - \frac{\gamma_0\gamma_s\gamma_2}{\gamma_1\gamma_3}}$$

Now $r = 0$ when

$$\frac{\gamma_1\gamma_3}{\gamma_2} = \frac{\gamma_0\gamma_s\gamma_2}{\gamma_1\gamma_3} \Rightarrow \gamma_0\gamma_s\gamma_2^2 = \gamma_1^2\gamma_3^2$$

Since for each case $\gamma = m\sqrt{\varepsilon_0\mu_0}$. Substituting this for each γ and simplifying gives

$$n_0 n_s n_2^2 = n_1^2 n_3^2$$

$$\frac{n_1 n_3}{n_2} = \sqrt{n_0 n_s}$$

22-12. We need , for $R = 0$,

$$\frac{n_1 n_3}{n_2} = \sqrt{n_0 n_s} = \sqrt{1 \times 4} = 2$$

Using the data in Table 22 1, we see that a good approximations can be formed by using MgF_2 for layer 3 so that $n_3 = 1.35$, SiO for layer 2 so that $n_2 = 1.5$, and ZnS for layer 1 so that $n_1 = 2.2$. For these choices,

$$\frac{n_1 n_3}{n_2} = \frac{(2.2)(1.35)}{1.5} = 1.98 \approx 2$$

22-13. Using Eq. (22 53),

$$R = \left[\frac{\left(\frac{n_0}{n_s}\right)\left(\frac{n_L}{n_H}\right)^{2N} - 1}{\left(\frac{n_0}{n_s}\right)\left(\frac{n_L}{n_H}\right)^{2N} + 1}\right]^2 = \left[\frac{\left(\frac{1}{1.52}\right)\left(\frac{1.38}{2.6}\right)^{2N} - 1}{\left(\frac{1}{1.52}\right)\left(\frac{1.38}{2.6}\right)^{2N} + 1}\right]^2$$

(a) $N = 2, R = 81.1\%$ (b) $N = 4, R = 98.4\%$ (c) $N = 8, R = 99.99\%$

22-14. Substituting in Eq. (22 53) as in problem 22 13, one finds for $N=4, n_H=4, n_L=1.35,$ and $n_s=1.50$.

22-15. In Eq. (22 53), let $x=n_0/n_s$ and $z=n_L/n_H$. Then

$$\pm\sqrt{R}=\frac{1-x\,z^{2N}}{1+x\,z^{2N}}$$

since $x<1$ and $z<1$ we select the positive square root by writing,

$$+\sqrt{R}=\frac{1-x\,z^{2N}}{1+x\,z^{2N}}$$

Then,

$$\begin{aligned}\sqrt{R}+x\,z^{2N}\sqrt{R}&=1-x\,z^{2N}\\ x\,z^{2N}\left(1+\sqrt{R}\right)&=1-\sqrt{R}\\ z^{2N}&=\frac{1-\sqrt{R}}{x\,(1+\sqrt{R})}\end{aligned}$$

In the case at hand, this becomes,

$$\begin{aligned}z^4=\frac{1-\sqrt{0.9}}{(1/1.5)\,(1+\sqrt{0.9})}&=0.03950\\ z&=0.4458\\ \frac{n_H}{n_L}=\frac{1}{z}&=2.243\end{aligned}$$

22-16. As in the solution to problem 22 15, with $x=n_0/n_s$ and $z=n_L/n_H$,

$$R=\left(\frac{x\,z^{2N}-1}{x\,z^{2N}+1}\right)^2$$

As $N\to\infty$, $z^{2N}\to 0$ since $z<1$. In this case $R\to\left(\frac{-1}{1}\right)^2=1$

As $z\to 0$, $R\to\left(\frac{-1}{1}\right)^2=1.$

Chapter 23 Fresnel equations

23-1. From Eq. (23 28),

$$r_{\mathrm{TM}} = \frac{-n^2\cos\theta + \sqrt{n^2 - \sin^2\theta}}{n^2\cos\theta + \sqrt{n^2 - \sin^2\theta}}$$

The numerator vanishes when,

$$n^2\cos\theta = \sqrt{n^2 - \sin^2\theta}$$

Squaring gives,

$$n^4\cos^2\theta = n^2 - \sin^2\theta \Rightarrow n^4\cos^2\theta - n^2 + \sin^2\theta = 0$$

This is a quadratic in n^2,

$$n^2 = \frac{+1 \pm \sqrt{1 - 4\sin^2\theta\cos^2\theta}}{2\cos^2\theta} = \frac{1 \pm \sqrt{1 - 4\sin^2\theta + 4\,\sin^4\theta}}{2\cos^2\theta} = \frac{1 \pm \sqrt{(2\sin^2\theta - 1)^2}}{2\cos^2\theta}$$

$$n^2 = \frac{1 \pm (2\sin^2\theta - 1)}{2\cos^2\theta}$$

$$+\,\text{sign}: n^2 = \frac{\sin^2\theta}{\cos^2\theta} = \tan^2\theta \Rightarrow n = \tan\theta \qquad \text{Brewster's law}$$

$$-\,\text{sign}: n^2 = \frac{2\,(1 - \sin^2\theta)}{2\cos^2\theta} = 1 \qquad \text{Trivial Solution}$$

23-2. The critical angle is,

$$\sin\theta_c = \frac{n_1}{n_2} = \frac{1}{n} \Rightarrow n = \frac{1}{\sin\theta_c} = \frac{1}{\sin(33°33')} = 1.8094$$

External Reflection:

$$\tan\theta_p = \frac{n_2}{n_1} = n \Rightarrow \theta_p = \tan^{-1}(1.8094) = 61°4'$$

Internal Reflection:

$$\tan\theta'_p = \frac{n_1}{n_2} = \frac{1}{n} \Rightarrow \theta'_p = \tan^{-1}\left(\frac{1}{1.8094}\right) = 28°56'$$

23-3. The critical angle is,

$$\theta_c = \sin^{-1}(1/n) = \sin^{-1}(1/1.84) = 32.9°$$

The polarizing angles are,

$$\text{External:} \quad \theta_p = \tan^{-1}(n) = \tan^{-1}(1.84) = 61.5°$$
$$\text{Internal:} \quad \theta'_p = \tan^{-1}(1/n) = \tan^{-1}(1/1.84) = 28.5°$$

23-4. Given,

$$\sin\theta_c \equiv \sin\theta = \frac{1}{n} \text{ and } \tan\theta_p \equiv \tan\theta = n$$

Then,

$$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{1/n}{\sqrt{1 - (1/n)^2}} = n$$
$$\frac{1}{n^2} = \sqrt{1 - (1/n)^2}$$
$$\frac{1}{n^4} = 1 - (1/n)^2$$
$$1 = n^4 - n^2 \Rightarrow n^4 - n^2 - 1 = 0$$
$$n^2 = \frac{1 \pm \sqrt{1+4}}{2} = \frac{1+\sqrt{5}}{2}, \frac{1-\sqrt{5}}{2} \quad \text{Take positive value}$$
$$n^2 = \frac{1+\sqrt{5}}{2} \Rightarrow n = 1.272 = n_2/n_1 = n_2$$

23-5. In each of the 4 equations the desired form results from introducing θ_t through the relation $n = \sin\theta/\sin\theta_t$. For example:

$$r_{\mathrm{TE}} = \frac{\cos\theta - \sqrt{n^2 - \sin^2\theta}}{\cos\theta + \sqrt{n^2 - \sin^2\theta}} = \frac{\cos\theta - \sin\theta\sqrt{1/\sin^2\theta_t - 1}}{\cos\theta + \sin\theta\sqrt{1/\sin^2\theta_t - 1}} = \frac{\cos\theta - \sin\theta\sqrt{(\cos^2\theta_t)/\sin^2\theta_t}}{\cos\theta + \sin\theta\sqrt{(\cos^2\theta_t)/\sin^2\theta_t}}$$

$$r_{\mathrm{TE}} = \frac{\cos\theta\sin\theta_t - \sin\theta\cos\theta_t}{\cos\theta\sin\theta_t + \sin\theta\cos\theta_t} = -\frac{\sin(\theta - \theta_t)}{\sin(\theta + \theta_t)}$$

The other 4 relations are similarly shown to be true. Proceeding by first noting from above that,

$$\sqrt{n^2 - \sin^2\theta} = \sin\theta\cos\theta_t/\sin\theta_t$$

Then,

$$r_{\mathrm{TM}} = \frac{-n^2\cos\theta + \sqrt{n^2 - \sin^2\theta}}{n^2\cos\theta + \sqrt{n^2 - \sin^2\theta}} = \frac{-\frac{\sin^2\theta\cos\theta}{\sin^2\theta_t} + \frac{\sin\theta\cos\theta_t}{\sin\theta_t}}{\frac{\sin^2\theta\cos\theta}{\sin^2\theta_t} + \frac{\sin\theta\cos\theta_t}{\sin\theta_t}}$$

$$r_{\mathrm{TM}} = \frac{-\sin\theta\cos\theta + \cos\theta_t\sin\theta_t}{\sin\theta\cos\theta + \cos\theta_t\sin\theta_t} = \frac{-\sin\theta\cos\theta\overbrace{(\sin^2\theta_t + \cos^2\theta_t)}^{1} + \cos\theta_t\sin\theta_t\overbrace{(\sin^2\theta + \cos^2\theta)}^{1}}{\sin\theta\cos\theta(\sin^2\theta_t + \cos^2\theta_t) + \cos\theta_t\sin\theta_t(\sin^2\theta + \cos^2\theta)}$$

$$r_{\mathrm{TM}} = \frac{-\sin\theta\cos\theta\sin^2\theta_t - \sin\theta\cos\theta\cos^2\theta_t + \cos\theta_t\sin\theta_t\sin^2\theta + \cos\theta_t\sin\theta_t\cos^2\theta}{+\sin\theta\cos\theta\sin^2\theta_t + \sin\theta\cos\theta\cos^2\theta_t + \cos\theta_t\sin\theta_t\sin^2\theta + \cos\theta_t\sin\theta_t\cos^2\theta}$$

$$r_{\mathrm{TM}} = \frac{(\sin\theta\cos\theta_t - \cos\theta\sin\theta_t)(\sin\theta\sin\theta_t - \cos\theta\cos\theta_t)}{(\sin\theta\cos\theta_t + \cos\theta\sin\theta_t)(\sin\theta\sin\theta_t + \cos\theta\cos\theta_t)} = -\frac{\sin(\theta - \theta_t)\cos(\theta + \theta_t)}{\sin(\theta + \theta_t)\cos(\theta - \theta_t)}$$

$$r_{\mathrm{TM}} = -\frac{\tan(\theta - \theta_t)}{\tan(\theta + \theta_t)}$$

and,

$$t_{\mathrm{TE}} = \frac{2\cos\theta}{\cos\theta + \sqrt{n^2 - \sin^2\theta}} = \frac{2\cos\theta}{\cos\theta + \sin\theta\cos\theta_t/\sin\theta_t} = \frac{2\cos\theta\sin\theta_t}{\cos\theta\sin\theta_t + \sin\theta\cos\theta_t} = \frac{2\cos\theta\sin\theta_t}{\sin(\theta + \theta_t)}$$

Finally,

$$t_{\mathrm{TM}} = \frac{2n\cos\theta}{n^2\cos^2\theta + \sqrt{n^2 - \sin^2\theta}} = \frac{2\cos\theta\sin\theta/\sin\theta_t}{\frac{\sin^2\theta\cos\theta}{\sin^2\theta_t} + \frac{\sin\theta\cos\theta_t}{\sin\theta_t}} = \frac{2\cos\theta\sin\theta_t}{\cos\theta\sin\theta + \sin\theta_t\cos\theta_t}$$

The denominator in this expression is the same as the one that appeared in the work leading to the final form of r_{TM}. Using the result derived there,

$$t_{\mathrm{TM}} = \frac{2\cos\theta\sin\theta_t}{\sin(\theta + \theta_t)\cos(\theta - \theta_t)}$$

23-6. Write,

$$r_{\mathrm{TM}} = \frac{-n^2\cos\theta + i\sqrt{\sin^2\theta - n^2}}{n^2\cos\theta + i\sqrt{\sin^2\theta - n^2}} \equiv \frac{-a + ib}{a + ib} = -\frac{a - ib}{a + ib} = -\frac{\sqrt{a^2 + b^2}e^{-i\tan^{-1}(b/a)}}{\sqrt{a^2 + b^2}e^{i\tan^{-1}(b/a)}}$$

$$r_{\mathrm{TM}} = (1)\,e^{i\Phi_{\mathrm{TM}}} = -1\,e^{-2i\tan^{-1}(b/a)} = e^{-2i\tan^{-1}(b/a) + i\pi}$$

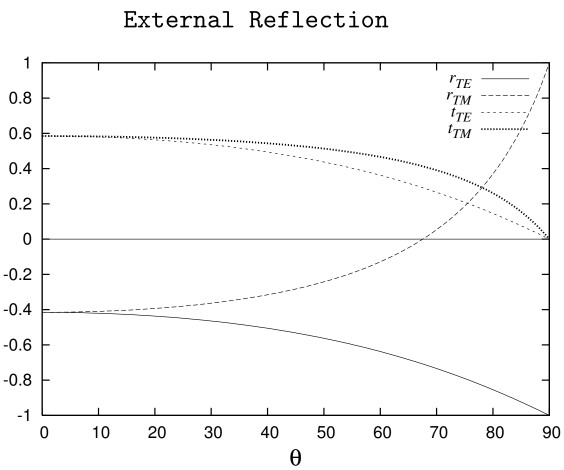
$$\Phi_{\mathrm{TM}} = -2\tan^{-1}(b/a) + \pi$$

$$\tan\left(\frac{\pi}{2} - \Phi_{\mathrm{TM}}\right) = \frac{b}{a} = \frac{\sqrt{\sin^2\theta - n^2}}{n^2\cos\theta}$$

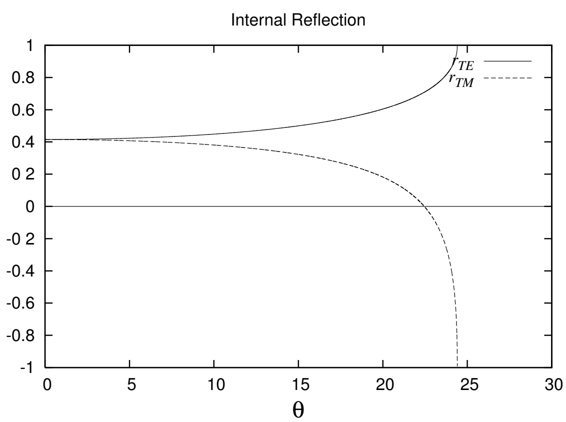
$$\tan\left(\Phi_{\mathrm{TM}} - \frac{\pi}{2}\right) = -\frac{\sqrt{\sin^2\theta - n^2}}{n^2\cos\theta}$$

23-7. The plots below are for diamond ($n = 2.42$). They are produced with the free computer algebra system Gnuplot under the TeXmacs front end.

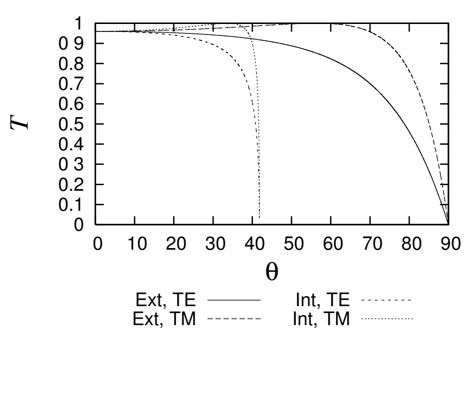
```
gnuplot] n1 2.42~set title "External Reflection"~k pi/180~set xrange [0:90]~set xzeroaxis lt 1~
         set size 0.7,0.7~set xlabel "{/Symbol q}" font "Times,20"
         plot (cos(k*x)-sqrt(n1**2-sin(k*x)*sin(k*x)))/(cos(k*x)+sqrt(n1**2-sin(k*x)*sin(k*x))) title "{/Italic
         r_{TE}",(-n1*n1*cos(k*x)+sqrt(n1**2-sin(k*x)*sin(k*x)))/(n1*n1*cos(k*x)+sqrt(n1**2-sin(k*x)*sin(k*x)))
         title "{/Italic r_{TM}",2*cos(k*x)/(cos(k*x)+sqrt(n1**2-sin(k*x)*sin(k*x))) title "{/Italic
         t_{TE}",2*n1*cos(k*x)/(n1*n1*cos(k*x)+sqrt(n1**2-sin(k*x)*sin(k*x))) lw 3 title "{/Italic t_{TM}"
```




```
gnuplot] n1 1/2.42~k pi/180~set xrange [0:30]~set xzeroaxis lt 1~set samples 10000
         set size 0.7,0.7~set xlabel "{/Symbol q}" font "Times,20"~set title "Internal Reflection"
         plot (cos(k*x)-sqrt(n1**2-sin(k*x)*sin(k*x)))/(cos(k*x)+sqrt(n1**2-sin(k*x)*sin(k*x))) title "{/Italic
         r_{TE}",(-n1*n1*cos(k*x)+sqrt(n1**2-sin(k*x)*sin(k*x)))/(n1*n1*cos(k*x)+sqrt(n1**2-sin(k*x)*sin(k*x)))
         title "{/Italic r_{TM}"
```



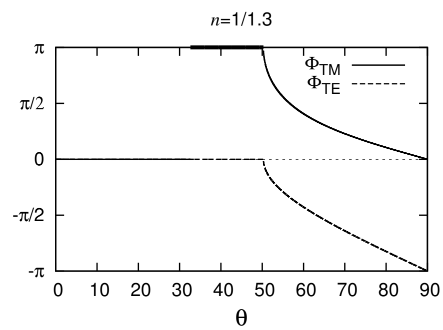
23-8. The transmittance is $T = 1 - R$. With Gnuplot scripts similar to those of the last solution,

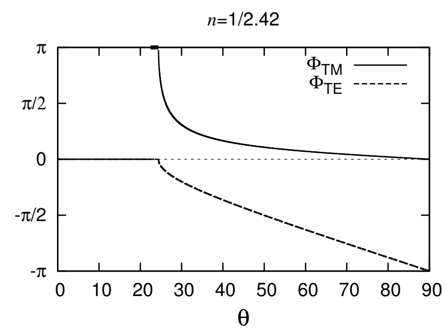


23-9. For $n=1/1.3$:
$\theta_c=\sin^{-1}(1/1.3)=0.878\,\text{rad}=50.3°$, $\theta_p'=\tan^{-1}(1/1.3)=0.6557=37.6°$.
For $n=1/2.42$:
$\theta_c=\sin^{-1}(1/2.42)=0.426=24.41°$, $\theta_p'=\tan^{-1}(1/2.42)=0.392=22.46°$.

Then plotting directly the expression in Eqs. (23 38) and (23 39) for these two cases. The Gnuplot script is shown.

```
GNUplot] k pi/180~n1 1/2.42~set xrange [0:90]~jp 22.46~jc 24.41~set samples 100000~
        set size 0.5,0.5~set xlabel "{/Symbol q}" font "Times,18"
        set title "{/Italic n} 1/2.42"
        set ytics ("-{/Symbol p}" -pi,"-{/Symbol p}/2" -pi/2,"0" 0,"{/Symbol p/2}" pi/2,"{/Symbol p}" pi )~set
        xzeroaxis lt 3
        f1(x)  0< x && x<jp ? 0:1/0
        f2(x)  jp< x && x<jc ? pi:1/0
        f3(x)  jc< x && x<90 ? -2*atan(sqrt(sin(k*x)*sin(k*x)-n1**2)/n1/n1/cos(k*x))+pi:1/0
        g1(x) 0< x && x<jc ? 0:1/0
        g2(x) jc< x && x<90 ? -2*atan(sqrt(sin(k*x)*sin(k*x)-n1**2)/cos(k*x)):1/0
        plot f1(x) lw 2 notitle, f2(x) lt 1 lw 6 notitle, f3(x) lt 1 lw 2 title "{/Symbol F}_{TM}", g1(x) lt 2
        lw 2 notitle, g2(x) lt 2 lw 2 title "{/Symbol F}_{TE}"
```





23-10. The reflectance is

$$R=\left(\frac{1-n}{1+n}\right)^2$$

(a) Air film: $n=1.38/1$, $R=0.0255\Rightarrow 2.55\%$

(b) Film glass: $n=1.52/1.38$, $R=0.00233\Rightarrow 0.233\%$

(c) Air Glass: $n=1.52/1$, $R=0.0426\Rightarrow 4.26$

(d) Using Eq. (22 43)

$$R=\left(\frac{n_0\,n_s-n_1^2}{n_0\,n_s+n_1^2}\right)^2=\left(\frac{1.52-1.38^2}{1.52+1.38^2}\right)^2=0.0126=1.26\%$$

23-11. Using

$$R_{\text{TE}}=\left(\frac{\cos\theta-\sqrt{n^2-\sin^2\theta}}{\cos\theta-\sqrt{n^2-\sin^2\theta}}\right)^2\quad R_{\text{TM}}=\left(\frac{-n^2\cos\theta+\sqrt{n^2-\sin^2\theta}}{n^2\cos\theta+\sqrt{n^2-\sin^2\theta}}\right)^2$$

θ	R_{TE}	R_{TM}
0°	2.01%	2.01%
10°	2.10%	1.91%
45°	5.23%	0.274%
90°	100%	100%

23-12. For diamond $n = 2.42$.

(a) **External:**
TM: $\theta_p = \tan^{-1}(n) = \tan^{-1}(2.42) = 67.55^{\circ}$, no critical angle.
TE: No Brewster angle, no critical angle.

(b) **Internal:**
TM: $\theta_c = \sin^{-1}(1/2.42) = 0.426 = 24.41°$, $\theta_p' = \tan^{-1}(1/2.42) = 0.392 = 22.46°$.
TE: $\theta_c = \sin^{-1}(1/2.42) = 0.426 = 24.41°$, no Brewster angle.

23-13. The reflectance and transmission are given by

$$R_{\mathrm{TE}} = \left(\frac{\cos\theta - \sqrt{n^2 - \sin^2\theta}}{\cos\theta - \sqrt{n^2 - \sin^2\theta}}\right)^2 = \left(\frac{\cos(50°) - \sqrt{1.6^2 - \sin^2(50°)}}{\cos(50°) + \sqrt{1.6^2 - \sin^2(50°)}}\right)^2 = 0.1385 = 13.85\%,$$

$$T_{\mathrm{TE}} = 1 - R_{\mathrm{TE}} = 86.15\%$$

$$R_{\mathrm{TM}} = \left(\frac{-n^2\cos\theta + \sqrt{n^2 - \sin^2\theta}}{n^2\cos\theta + \sqrt{n^2 - \sin^2\theta}}\right)^2 = \left(\frac{-1.6^2\cos(50°) + \sqrt{1.6^2 - \sin^2(50°)}}{1.6^2\cos(50°) + \sqrt{1.6^2 - \sin^2(50°)}}\right)^2 = 0.00623 = 0.623\%$$

$$T_{\mathrm{TM}} = 1 - R_{\mathrm{TM}} = 99.38\%$$

23-14. (a) For the TE case begin with

$$E + E_r = E_t \tag{23 20}$$
$$n_1 E\cos\theta - n_1 E_r\cos\theta = n_2 E_t\cos\theta_t \tag{23 21}$$

Solve Eq. (23 20) for E_r and use the result in Eq. (23 21),

$$n_1 E\cos\theta - n_1(E_t - E)\cos\theta = n_2 E_t\cos\theta_t$$

Rearrangement gives,

$$t_{\mathrm{TE}} = \frac{E_t}{E} = \frac{2n_1\cos\theta}{n_1\cos\theta + n_2\cos\theta_t} = \frac{2\cos\theta}{\cos\theta + n\cos\theta_t} = \frac{2\cos\theta}{\cos\theta + \sqrt{n^2 - \sin^2\theta}}$$

Similarly for the TM mode

$$-n_1 E + n_1 E_r = -n_2 E_t \tag{23 22}$$
$$E\cos\theta + E_r\cos\theta = E_t\cos\theta_t \tag{23 23}$$

Substitution of E_r from Eq. (23 22) into Eq. (23 33) and subsequent rearrangement gives,

$$E\cos\theta + \big(E - n E_t\big)\cos\theta = E_t\cos\theta_t$$
$$t_{\mathrm{TM}} = \frac{E_t}{E} = \frac{2\cos\theta}{n\cos\theta + \cos\theta_t} = \frac{2n\cos\theta}{n^2\cos\theta + n\cos\theta_t} = \frac{2n\cos\theta}{n^2\cos\theta + \sqrt{n^2 - \sin^2\theta}}$$

(b) From Eq. (23 20), after dividing through by E, one finds

$$1 + r_{\mathrm{TE}} = t_{\mathrm{TE}}$$
$$t_{\mathrm{TE}} = 1 + \frac{\cos\theta - \sqrt{n^2 - \sin^2\theta}}{\cos\theta + \sqrt{n^2 - \sin^2\theta}} = \frac{\cos\theta + \sqrt{n^2 - \sin^2\theta} + \cos\theta - \sqrt{n^2 - \sin^2\theta}}{\cos\theta + \sqrt{n^2 - \sin^2\theta}} = \frac{2\cos\theta}{\cos\theta + \sqrt{n^2 - \sin^2\theta}}$$

Similarly after dividing Eq. (23 22) by E and by n_1,

$$-1 + r_{\mathrm{TM}} = -n\,t_{\mathrm{TM}}$$

$$t_{\mathrm{TM}} = \frac{1}{n}(1 - r_{\mathrm{TM}}) = \frac{1}{n}\left(\frac{n^2\cos\theta + \sqrt{n^2 - \sin^2\theta} - \left(-n^2\cos\theta + \sqrt{n^2 - \sin^2\theta}\right)}{n^2\cos\theta + \sqrt{n^2 - \sin^2\theta}}\right) = \frac{2n\cos\theta}{n^2\cos\theta + \sqrt{n^2 - \sin^2\theta}}$$

23-15. (a) The critical angle is $\theta_c = \sin^{-1}(n_2/n_1) = \sin^{-1}(1/1.458) = 43.3°$

The polarizing angle for external reflection is is $\theta_p = \tan^{-1}(n_2/n_1) = \tan^{-1}(1.458/1) = 55.6°$

The polarizing angle for internal reflection is $\theta_p' = \tan^{-1}(1/1.458) = 34.4°$

(b) At normal incidence: $R_{TE} = r_{TE}^2 = \left(\frac{1-n}{1+n}\right)^2 = \left(\frac{1-1.1458^2}{1+1.1458^2}\right)^2 = 0.0347, T_{TE} = 1 - R_{TE} = 0.9653$

At $\theta = 45°$: $R_{TE} = \left(\frac{\cos\theta - \sqrt{n^2 - \sin^2\theta}}{\cos\theta + \sqrt{n^2 - \sin^2\theta}}\right)^2 = \left(\frac{\cos 45° - \sqrt{(1.458)^2 - \sin^2 45°}}{\cos 45° + \sqrt{(1.458)^2 - \sin^2 45°}}\right)^2 = 0.0821$

$T_{TE} = 1 - R_{TE} = 0.9179.$

(c) At normal incidence: $R_{TM} = R_{TE} = 0.0347, T_{TE} = 0.9653$

At $\theta = 45°$: $R_{TM} = \left(\frac{-n^2\cos\theta + \sqrt{n^2 - \sin^2\theta}}{n^2\cos\theta + \sqrt{n^2 - \sin^2\theta}}\right)^2 = \left(\frac{-(1.458)^2\cos 45° + \sqrt{1.458^2 - \sin^2 45°}}{(1.458)^2\cos 45° + \sqrt{1.458^2 - \sin^2 45°}}\right)^2$

$R_{TM} = 0.0067, T_{TM} = 0.9933$

(d) Use Eqs. (23 28) and (23 29):

For incident angles less than θ_p', $\phi_{TM} = \phi_{TE} = 0$. So for $\theta = 0°$ and $20°$, $\phi_{TM} - \phi_{TE} = 0$.

For $\theta = 40°$ which lies between θ_p' and θ_c, $\phi_{TM} - \phi_{TE} = \pi - 0 = \pi$.

For $\theta > \theta_c$: with $n = 1/1.458$,

$$\phi_{TM} - \phi_{TE} = -2\tan^{-1}\left(\frac{\sqrt{\sin^2\theta - n^2}}{n^2\cos\theta}\right) + \pi + 2\tan^{-1}\left(\frac{\sqrt{\sin^2\theta - n^2}}{\cos\theta}\right)$$

$\theta = 50°$: $\quad \phi_{TM} - \phi_{TE} = 2.43\,\text{rad} = 139°$

$\theta = 70°$ $\quad \phi_{TM} - \phi_{TE} = 2.65\,\text{rad} = 152°$

$\theta = 90°$ $\quad \phi_{TM} - \phi_{TE} = \pi = 180°$

23-16. (a) By trial and error, $\theta = \theta_{\text{rhomb}} = 59.857°$, so that with $n = 1/1.65$

$$2(\phi_{TE} - \phi_{TM}) = -2\tan^{-1}\left(\frac{\sqrt{\sin^2\theta - n^2}}{\cos\theta}\right) + 2\tan^{-1}\left(\frac{\sqrt{\sin^2\theta - n^2}}{n^2\cos\theta}\right) - \pi = -1.5\,\pi \Rightarrow \pi/2$$

(b) If $\theta = (1 + 0.05)\,59.857° = 62.850°$:

$$2(\phi_{TE} - \phi_{TM}) = -2\tan^{-1}\left(\frac{\sqrt{\sin^2\theta - n^2}}{\cos\theta}\right) + 2\tan^{-1}\left(\frac{\sqrt{\sin^2\theta - n^2}}{n^2\cos\theta}\right) - \pi = -1.543\,\pi \Rightarrow 0.457\,\pi$$

If $\theta = (1 - 0.05)\,59.857° = 56.864°$:

$$2(\phi_{TE} - \phi_{TM}) = -2\tan^{-1}\left(\frac{\sqrt{\sin^2\theta - n^2}}{\cos\theta}\right) + 2\tan^{-1}\left(\frac{\sqrt{\sin^2\theta - n^2}}{n^2\cos\theta}\right) - \pi = -1.461\,\pi \Rightarrow 0.539\,\pi$$

23-17. Using Eq. (23 65) for the TE case:

$$R_{\rm TE}=\left|\frac{\cos\theta-\sqrt{\left(n_R^2-n_I^2-\sin^2\theta\right)+i\,(2\,n_R\,n_I)}}{\cos\theta-\sqrt{\left(n_R^2-n_I^2-\sin^2\theta\right)+i\,(2\,n_R\,n_I)}}\right|^2$$

and for the TM case,

$$R_{\rm TM}=\left|\frac{-\left[n_R^2-n_I^2+i\,(2\,n_R\,n_I)\right]\cos\theta+\left(n_R^2-n_I^2-\sin^2\theta\right)+i\,(2\,n_R\,n_I)}{\left[n_R^2-n_I^2+i\,(2\,n_R\,n_I)\right]\cos\theta+\left(n_R^2-n_I^2-\sin^2\theta\right)+i\,(2\,n_R\,n_I)}\right|^2$$

The given parameters are $n_R=2.485$ and $n_I=1.381$. The free computer algebra system Maxima with the TeXmacs front end, can be used to determine the reflectances. Below I show the syntax used and give a table with the output.

```
(%i1) (nR:2.484,nI:1.381,k:%pi/180)$
(%i2) for j from 0 step 10 thru 90 do (RTE[j]:float(abs((cos(k*j)-sqrt(nR^2-nI^2-
      sin(k*j)*sin(k*j)+%i*2*nR*nI))/(cos(k*j)+sqrt(nR^2-nI^2-
      sin(k*j)*sin(k*j)+%i*2*nR*nI)))^2),RTM[j]:float(abs(((-nR^2-nI^2+2*%i*nR*nI)*cos(k*j)+sqrt(nR^2-nI^2-
      sin(k*j)*sin(k*j)+%i*2*nR*nI))/((nR^2-nI^2+2*%i*nR*nI)*cos(k*j)+sqrt(nR^2-nI^2-
      sin(k*j)*sin(k*j)+%i*2*nR*nI)))^2))
  (%o3) done
(%i4) for j from 0 step 10 thru 90 do display(RTE[j],RTM[j])
```

θ	0°	30°	50°	70°	90°
$R_{\rm TE}$	29.3	34.5	45.4	65.7	100
$R_{\rm TM}$	29.3	24.2	14.9	5.4	100

23-18. Using the Maxima program from the last problem with the input values $n_R=1.5$ and $n_I=5.3$ I find,

θ	$R_{\rm TE}$	$R_{\rm TM}$
0°	82.5%	82.5%
30°	84.7%	80.1%
60°	90.9%	69.5%

23-19. Given $n_I=5.3$ at $\lambda=589.3\,\text{nm}$. (a) $\alpha=4\,\pi\,n_I/\lambda=4\,\pi\,(5.3)/(589.3\,\text{nm})=0.113\,\text{nm}^{-1}$

(b) $I=I_0\,e^{-\alpha s}$. For $I=0.01\,I_0$, $e^{-\alpha s}=0.01\Rightarrow s=(-1/\alpha)\ln(0.01)=40.75\,\text{nm}=0.069\,\lambda$

23-20. Consider Eq. (23 47)

$$1=r^2+n\left(\frac{\cos\theta_t}{\cos\theta}\right)t^2\quad n=n_2/n_1$$

(a) **External Reflection:** $n>1$; $\theta_t<\theta_i$; $\cos\theta_t>\cos\theta$. Thus, $1-r^2=n\left(\frac{\cos\theta_t}{\cos\theta}\right)t^2>t^2\Rightarrow t^2<1-r^2$
Since $r^2<1$, $t^2<1$.

Internal Reflection: $n<1$; $\theta_t>\theta_i$; $\cos\theta_t<\cos\theta$. Thus, $1-r^2=n\left(\frac{\cos\theta_t}{\cos\theta}\right)t^2<t^2\Rightarrow t^2>1-r^2$
No upper limit is imposed.

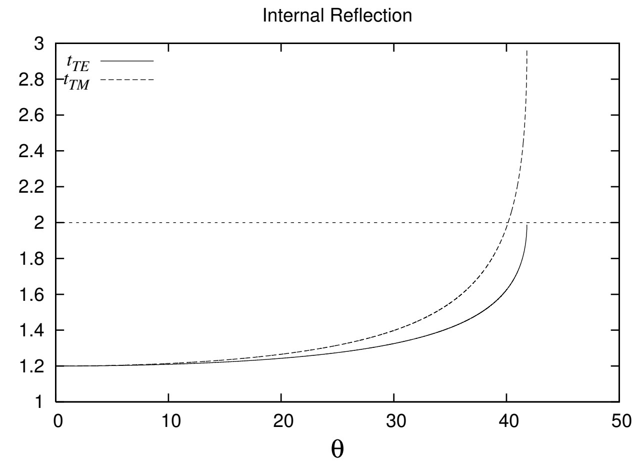
(c) For the angle of incidence equal to the critical angle, $\sin\theta=\sin\theta_c=n$. Then,

$$t_{\rm TE}=\frac{2\cos\theta}{\cos\theta+\sqrt{n^2-\sin^2\theta}}=\frac{2\cos\theta}{\cos\theta}=2$$
$$t_{\rm TM}=\frac{2\,n\cos\theta}{n^2\cos\theta+\sqrt{n^2-\sin^2\theta}}=\frac{2}{n}$$

Continued on the next page...

23-20. *Continued...*

(c)



23-21. (a) The penetration depth is

$$|z|_{1/e} = \frac{1}{\alpha} = \frac{\lambda}{2\,\pi}\frac{1}{\sqrt{\sin^2\theta/n^2 - 1}} = \frac{0.546\,\mu\text{m}}{2\,\pi}\frac{1}{\sqrt{\sin^2(45°)/(1/1.6)^2 - 1}} = 0.164\,\mu\text{m}$$

(b) Since irradiance is proportional to the square of the field amplitude and with $\alpha = \dfrac{1}{|z|_{1/e}} = 6.089\,\mu\text{m}^{-1}$,

$$\frac{I}{I_0} = e^{-2\alpha|z|} = e^{-2(6.089\,\mu\text{m}^{-1})(1\mu\text{m})} = 5.1\times 10^{-6}$$

Chapter 24 Nonlinear optics and the modulation of light

24-1. For a single field,

$$E=\frac{1}{2}E_0\,(e^{i\omega t}+e^{-i\omega t})=E_0\cos\omega\,t$$

The third order polarization is

$$P_3=\varepsilon_0\,\chi_3\,E^3=\varepsilon_0\,\chi_3\,\frac{E_0^3}{8}\big[\,e^{i3\omega t}+e^{-i3\omega t}+3\,e^{i\omega t}+3\,e^{-i\omega t}\big]=\frac{\varepsilon_0\,\chi_3\,E_0^3}{4}\,(\cos 3\,\omega\,t+3\cos\omega\,t)$$

Frequencies ω and $3\,\omega$ appear.

24-2. The total electric field is

$$E=\frac{1}{2}\,E_{01}\,(e^{i\omega_1 t}+e^{-i\omega_1 t})+\frac{1}{2}\,E_{02}\,(e^{i\omega_2 t}+e^{-i\omega_2 t})\equiv A+B$$
$$P_3=\varepsilon_0\,\chi_3\,E^3=\varepsilon_0\,\chi_3\,(A^3+B^3+3\,A^2\,B+3\,A\,B^2)$$

Here,

$$A^3=\frac{E_{01}^3}{8}\left(e^{i3\omega_1 t}+e^{-i3\omega_1 t}+3\,e^{i\omega_1 t}+3\,e^{-i\omega_1 t}\right)=\frac{E_{01}^3}{4}\,(\cos 3\,\omega_1\,t+3\cos\omega_1 t)$$
$$B^3=\frac{E_{02}^3}{8}\,[\cos 3\,\omega_2\,t+3\cos\omega_2\,t]$$
$$3\,A^2\,B=\frac{3\,E_{01}^2\,E_{02}}{8}\left[\left(e^{i2\omega_1 t}+e^{-i2\omega_1 t}\right)\left(e^{i\omega_2 t}+e^{-i\omega_2 t}\right)\right]$$
$$3\,A^2\,B=\frac{3E_{01}^2\,E_{02}}{8}\left[e^{i(2\omega_1+w_2)t}+e^{-i(2\omega_1+w_2)t}+e^{i(2\omega_1-w_2)t}+e^{-i(2\omega_1-\omega_2)t}+2\,(e^{i\omega_2 t}+e^{-i\omega_2 t})\right]$$
$$3\,A^2\,B=\frac{3\,E_{01}^2\,E_{02}}{4}\,[\cos\,2\,\omega_1+\omega_2)\,t+\cos\,(2\,\omega_1-w_2)\,t+2\cos\omega_2\,t]$$

Similarly, interchanging subscripts 1 and 2,

$$3\,A\,B^2=\frac{3\,E_{02}^2\,E_{01}}{4}\,[\cos\,(2\,\omega_2+\omega_1)\,t+\cos\,(2\,\omega_2-\omega_1)\,t+2\cos\omega_1\,t]$$

Frequencies appearing: $3\,\omega_1$, $3\,\omega_2$, $2\,\omega_1+\omega_2$, $2\,\omega_1-\omega_2$, $2\,\omega_2+\omega_1$, $2\,\omega_2-\omega_1$, ω_1, ω_2.

24-3. The total electric field is,

$$E=E_{01}\cos\omega_1\,t+E_{02}\cos\omega_2\,t+E_{03}\cos\omega_3\,t$$
$$E=\frac{E_{01}}{2}\left(e^{i\omega_1 t}+e^{-i\omega_1 t}\right)+\frac{E_{02}}{2}\left(e^{i\omega_2 t}+e^{-i\omega_2 t}\right)+\frac{E_{03}}{2}\left(e^{i\omega_3 t}+e^{-i\omega_3 t}\right)$$

The second order polarization is $P_2=\varepsilon_0\,\chi_2\,E^2$. Algebra similar to that of the previous two problems leads to

$$P_2=\frac{\varepsilon_0\,\chi_2}{2}\Big\{(E_{01}^2+E_{02}^2+E_{03}^2)+E_{01}^2\cos 2\,\omega_1\,t+E_{02}^2\cos 2\,\omega_2\,t+E_{03}^2\cos 2\,\omega_3\,t+$$
$$+2\,E_{01}\,E_{02}\,[\cos\,(\omega_1+\omega_2)\,t+\cos\,(\omega_1-\omega_2)\,t+\cos\,(\omega_2+\omega_3)\,t+\cos\,(\omega_2-\omega_3)\,t+\cos\,(\omega_1+\omega_3)\,t+\cos\,(\omega_1-\omega_3)\,t\}$$

Frequencies include: $0, 2\,\omega_1, 2\,\omega_2, 2\,\omega_3, \omega_1\pm\omega_2, \omega_1\pm\omega_3, \omega_2\pm\omega_3$

24-4. For crystals with inversion symmetry, $1/n^2$ cannot change by reversing E. From Eq. (24 7):

For $+E$: $\dfrac{1}{n^2}=\dfrac{1}{n_0^2}+r\,(+E)+R\,(+E)^2$

For $-E$: $\dfrac{1}{n^2}=\dfrac{1}{n_0^2}+r\,(-E)+R(-E)^2$

For these to be equal for all E, $r\,E=-r\,E\Rightarrow r=0$.

24-5. (a) $L_C = \dfrac{\lambda_0}{4\,\Delta n} = \dfrac{694\,\text{nm}}{4\,(1.534 - 1.505)} = 5893\,\text{nm} = 8.62\,\lambda_0$ (b) $\Delta n = \dfrac{\lambda_0}{4\,L_C} = \dfrac{1.06\,\mu\text{m}}{4\,(5.8\,\mu\,\text{m})} = 0.046$

24-6. Using Eq. (24 8) and Table 24 2, the half wave voltage is independent of length and found to be

$$V_{HW} = \frac{\lambda_0}{2r\,n_0^3} = \frac{546 \times 10^{-9}\,\text{m}}{2\,(8.56 \times 10^{-12}/\text{V})\,(1.48)^3} = 9.84\,\text{kV}$$

24-7. For this cell,

$$\Delta\,n = \frac{r}{2}\,n_0^3\,E = \frac{r}{2}\,n_0^3\,\frac{V}{L} = \frac{30.9 \times 10^{-12}}{2}\,(2.29)^3\,\frac{426}{.01} = 7.9 \times 10^{-6}$$

$$\Phi = \frac{2\,\pi}{\lambda_0}\,r\,n_0^3\,V = \frac{2\,\pi}{632.8 \times 10^{-9}}\,(30.9 \times 10^{-12})\,(2.29)^3\,(426) = 1.57\,\text{or}\,\frac{\pi}{2}\,\text{rad}$$

Thus, the cell behaves as a quarter wave plate.

24-8. According to Eq. (24 9)

$$I = I_{\max}\,\sin^2\!\left(\frac{\pi}{2}\,\frac{V}{V_{HW}}\right)$$

Combining $V_{HW} = \dfrac{\lambda_0}{2\,r\,n_0^3}$ and $\Phi = \dfrac{2\pi}{\lambda_0}\,r\,n_0^3$, we have the ratio

$$\frac{V}{V_{HW}} = \frac{\Phi\,\lambda_0}{2\,\pi\,r\,n_0^3} \cdot \frac{2\,r\,n_0^3}{\lambda_0} = \frac{\Phi}{\pi}$$

So,

$$I = I_{\max}\,\sin^2\left(\frac{\pi}{2}\,\frac{\Phi}{\pi}\right) = I_{\max}\,\sin^2\left(\frac{\Phi}{2}\right)$$

(a) Now $I = 0$ when the argument of $\sin^2$ is $0, \pi, 2\,\pi, \ldots$ The first such value above zero makes

$$\Phi/2 = \pi \Rightarrow \Phi = 2\,\pi \Rightarrow \frac{\pi}{2}\,\frac{V}{V_{HW}} = \pi \Rightarrow V = 2\,V_{HW}$$

That is for $\Phi = 2\,\pi, 4\,\pi, \ldots, V = 2\,V_{HW}, 4\,V_{HW}, \ldots$, respectively.

(b) With an initial HWP, the $\sin^2$ curve in Figure (24 6) is simply translated to the left by the angle π. Thus, at $V = 0, I = I_{\max}$, and at $V = V_{HW}, I = I_{\max}$.

24-9. Both effects are present in media lacking inversion symmetry. The ratio of retardations is

$$\frac{\Phi_{\text{pock}}}{\Phi_{\text{kerr}}} = \frac{2\,\pi\,r\,n_0^3\,V}{\lambda_0} \cdot \frac{d^2}{2\,\pi\,K\,V^2\,L} = \frac{r}{K}\,\frac{n_0^3\,d^2}{\lambda_0\,VL} = \frac{10}{1}\,\frac{2^3\,(0.01)^2}{(550 \times 10^{-9})\,(1 \times 10^4)\,(0.02)} = 73$$

24-10. The length of the cell is found as,

$$V_{HW} = \frac{d}{\sqrt{2\,K\,L}} \Rightarrow L = \frac{d^2}{2\,K\,V_{HW}^2} = \frac{(0.015)^2}{2\,(0.036 \times 10^{-12})\,(30 \times 10^3)^2}\,\text{m} = 3.47\,\text{m}$$

The required cell is too long to be practical.

24-11. Starting with the given Doppler effect relation,

$$\Delta\,v = \frac{2\,\nu\,u_p}{v}$$

where, where $u_p = \pm\,v_S\,\sin\theta$ and $2\,\lambda_S\sin\theta = \lambda$ so that $u_p = \pm\,v_S\,\dfrac{\lambda}{2\,\lambda_S} = \pm\,\dfrac{\nu_S\,\lambda}{2}$. Then,

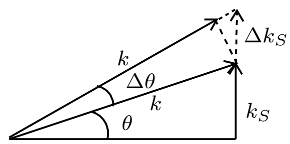
$$\Delta\,\nu = \frac{\cancel{2}\nu}{v}\left(\pm\,\frac{\nu_S\,\lambda}{\cancel{2}}\right) = \pm\,\nu\,\nu_S\left(\frac{\lambda}{v}\right) = \pm\,\cancel{\nu}\,\nu_S\left(\frac{1}{\cancel{\nu}}\right) = \pm\,\nu_S$$

$$\nu' - \nu = \pm\,\nu_S \Rightarrow \nu' = \nu \pm \nu_S \Rightarrow \omega' = \omega \pm \omega_S$$

24-12. The distance moved by the sound wave is $x = v_S t = v_S \left(\frac{W}{v_{\text{light}}}\right)$.

So, $x = v_S \left(\frac{nW}{c}\right) = 3000\left[\frac{1.5\,(0.01)}{3\times 10^8}\right] = 150\,\text{nm}$. The sound wave advances by $\lambda/3.3$ for $\lambda = 500\,\text{nm}$.

24-13. Adapting the relevant part of Figure 24 11 leads to the diagram below,



(a) From the vector diagram, for small $\Delta\theta, \Delta\theta \approx \frac{\Delta k_S}{k}$

(b) $k_S = \frac{2\pi}{\lambda_S} = \frac{2\pi\nu_S}{v_S}$ so $\Delta k_S = \frac{2\pi\,\Delta\nu_S}{v_S}$. Also, $K = \frac{2\pi}{\lambda}$.

Then, $\Delta\theta = \frac{\Delta k_S}{k} = \frac{2\pi\,\Delta\nu_S}{v_S}\frac{\lambda}{2\pi} = \frac{\lambda}{v_S}\Delta\nu_S$

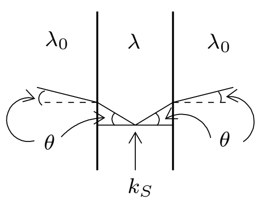
(c) The number of resolvable spots is $N = \frac{\Delta\theta}{\theta_D} = \frac{\lambda}{v_S}\Delta\nu_S\left(\frac{D}{\lambda}\right) = \left(\frac{D}{v_s}\right)\Delta\nu_S = \tau\,\Delta v_S$

(d) $N = \left(\frac{D}{v_S}\right)\Delta\nu_S = \frac{0.01}{5.95\times 10^3}(120-80)\,10^6 = 67$

24-14. The required acoustic frequency is found as, $\lambda = \lambda_0/n = 2\lambda_s\sin\theta = 2\,(v_S/\nu_S)\sin\theta$

$$\nu_S = (2\,n/\lambda_0)\,v_s\sin\theta = \frac{2\,(1.6)}{632.8\times 10^{-9}\,\text{m}}\left(2500\,\frac{\text{m}}{\text{s}}\right)\sin 1^\circ = 221\,\text{MHz}$$

24-15. Using Eq. (24 18), $\lambda = (\lambda_0/n) = 2\lambda_S\sin\theta$ and from Snell's law $\sin\theta_0 = n\sin\theta$. Therefore,



$$\frac{\lambda_0}{n} = 2\lambda_S\sin\theta_0/n \Rightarrow \lambda_0 = 2\lambda_S\sin\theta_0$$

24-16. Generally, $\sin\theta = \frac{\lambda}{2\lambda_S} = \left(\frac{\lambda}{2\,v_S}\right)\nu_S = \frac{632.8\times 19^{-9}\,\text{m}}{2\,(11\times 10^3\,\text{m/s})}\nu_S = (2.876\times 10^{-11}\,\text{s})\,\nu_S$

At $\nu_S = 50\,\text{MHz}$: $\sin\theta = 0.001438; \theta = 0.0824^\circ = 4.94'$ At $\nu_S = 80\,\text{MHz}$: $\sin\theta = 0.002301; \theta = 0.1218^\circ = 7.91'$

So, $\Delta\theta = 2.97'$

24-17. The rotation of the plane of polarization is $\beta = VB\,d$ where for a solenoid $B = \mu_0\,n\,i$. For an optical isolator $\beta = \pi/4$. For Zn S, $V = 0.225$ min /G cm $= 65.45$ rad/T m. It is given that the number of turns per meter in the solenoid is $n = 6000$/m. The design parameters are then the current i and length d. So,

$$\beta = \frac{\pi}{4} = VB\,d = V\mu_0\,n\,i\,d \Rightarrow i\,d = \frac{\pi/4}{V\mu_0\,n} = \frac{\pi/4}{(65.45)\,(4\pi\times 10^{-7})\,(6000)}\,\text{A}\cdot\text{m} = 1.59\,\text{A}\cdot\text{m}$$

For example for $d = 5\,\text{cm} = 0.05\,\text{m}$, the necessary current is $i = 31.8\,\text{A}$.

24-18. (a) At 632.8 nm, $V = \frac{\beta}{B\,d} = \frac{900\,\text{min}}{(5098\,\text{G})\,(2.73\,\text{cm})} = 0.0647\,\frac{\text{min}}{\text{G cm}}$

(b) At 543.5 nm, $V = \frac{1330\,\text{min}}{(5098\,\text{G})\,(2.73\,\text{cm})} = 0.0956\,\frac{\text{min}}{\text{G cm}}$

24-19. $\beta = VB\,d = \left(0.0423\,\frac{\text{min}}{\text{G cm}}\right)(4000\,\text{G})\,(5\,\text{cm}) = 846' = 14.1^\circ$

$\frac{dn}{d\lambda} = \frac{V}{1.0083\,\lambda} = \frac{0.0423}{1.0083\,(0.589\,\mu\text{m})} = 0.0712\,\mu\text{m}^{-1}$

Chapter 25 Optical Properties of Materials

25-1. (a) Consider,

$$K \equiv n^2 = (n_R + i\, n_I)^2 = n_R^2 - n_I^2 + 2\, i\, n_I n_R = K_R + i\, K_I$$
$$K_R = n_R^2 - n_I^2,\ K_I = 2\, n_I n_R$$

Solving these two relations for n_R and n_I proceeds as,

$$K_I = 2\, n_I n_R = 2\, n_I \sqrt{K_R^2 + n_I^2}$$
$$K_I^2 = 4\, n_I^2 \,(K_R^2 + n_I^2)$$
$$4\, n_I^4 + 4 K_R^2 n_I^2 - K_I^2 = 0$$
$$n_I^2 = \frac{-\,4\, K_R \pm \sqrt{16\, K_R^2 + 16\, K_I^2}}{8} = \frac{-\, K_R \pm \sqrt{K_R^2 + K_I^2}}{2}$$

To make $n_I^2 > 0$, choose the $+$ sign. Thus,

$$n_I = \left[\frac{-\, K_R + \sqrt{K_R^2 + K_I^2}}{2}\right]^{1/2}$$

$$n_R^2 = K_R + n_I^2 = \frac{2\, K_R}{2} + \frac{-\, K_R + \sqrt{K_R^2 + K_I^2}}{2} = \frac{K_R + \sqrt{K_R^2 + K_I^2}}{2}$$

$$n_R = \left[\frac{K_R + \sqrt{K_R^2 + K_I^2}}{2}\right]^{1/2}$$

(b) If $K_I \approx K_R$,

$$n_R = \left[\frac{K_I + \sqrt{2\, K_I^2}}{2}\right]^{1/2} = \sqrt{K_I}\left(\frac{1 + \sqrt{2}}{2}\right)^{1/2} = 1.099\sqrt{K_I}$$

$$n_I = \left[\frac{-\, K_I + \sqrt{2\, K_I^2}}{2}\right]^{1/2} = \sqrt{K_I}\left(\frac{-\,1 + \sqrt{2}}{2}\right)^{1/2} = 0.455\sqrt{K_I}$$

25-2. "Nearly transparent": We assume ω is not too large and use the results of Section 25 5. As explained in the text between Eqs. (25 29) and (25 30), $\alpha = 2\, K_I$. Then using Eq. (25 55),

$$2\, K_I = 2\left(\frac{\sigma \mu_0\, \omega}{2}\right)^{1/2}$$

Eliminate ω in favor of n_R, using Eq. (25 56)

$$n_R = \left(\frac{\sigma}{2\, \omega\, \varepsilon_0}\right)^{1/2}$$

Then,

$$K_I n_R = \left(\frac{\sigma\, \mu_0\, \omega}{2}\right)^{1/2}\left(\frac{\sigma}{2\, \omega\, \varepsilon_0}\right)^{1/2} = \frac{\sigma}{2}\left(\frac{\mu_0}{\varepsilon_0}\right)^{1/2}$$

$$\alpha = 2\, K_I = \frac{\sigma}{n_R}\left(\frac{\mu_0}{\varepsilon_0}\right)^{1/2} = 377\,\frac{\sigma}{n_R}$$

25-3. Using Eqs. (25 37) and (25 38) we have,

$$n_R^2 - n_I^2 \equiv K_R = 1 + \frac{N e^2}{m\,\varepsilon_0}\left[\frac{\omega_0^2 - \omega^2}{(\omega_0^2 - \omega^2)^2 + \gamma^2\,\omega^2}\right] \quad 2\,n_I\,n_R \equiv K_I = \frac{N e^2}{m\,\varepsilon_0}\left[\frac{\gamma\,\omega}{(\omega_0^2 - \omega^2)^2 + \gamma^2\,\omega^2}\right]$$

Solving these simultaneously as in problem 25 1 gives,

$$n_R = \left[\frac{K_R + \sqrt{K_R^2 + K_I^2}}{2}\right]^{1/2} \quad n_I = \left[\frac{-K_R + \sqrt{K_R^2 + K_I^2}}{2}\right]^{1/2}$$

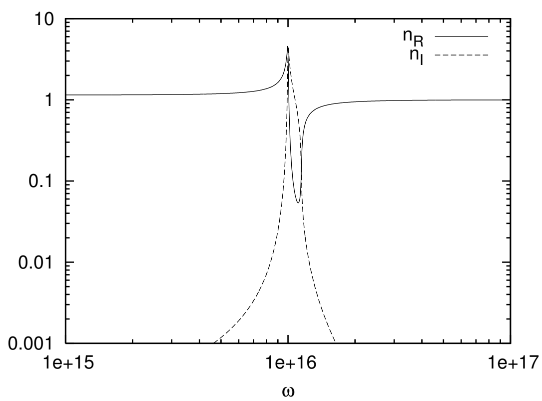
To compare to Figure 25 2 use the parameters,

$$\omega_0 = 10^{16}\,\mathrm{s}^{-1} \qquad \gamma = 10^{14}\,\mathrm{s}^{-1} \qquad N = 10^{28}\,\mathrm{m}^{-3}$$
$$m = 9.109 \times 10^{-31}\,\mathrm{kg} \qquad e = 1.6 \times 10^{-19}\,\mathrm{C} \qquad \varepsilon_0 = 8.854 \times 10^{-12}\,\mathrm{F/m}$$

Using the free plotting program GNUplot:

```
This is a TeXmacs interface for GNUplot.

GNUplot] w0 1e16~g 1e14~N 1e28~eps0 8.854e-12~e 1.6e-19~m 9.109e-31
         KR(x) 1+N*e**2/m/eps0*(w0**2-x**2)/((w0**2-x**2)**2+g**2*x**2)
         KI(x) N*e**2/m/eps0*g*x/((w0**2-x**2)**2+g**2*x**2)
         nR(x) sqrt((KR(x)/2+sqrt(KR(x)*KR(x)+KI(x)*KI(x))/2))~nI(x) KI(x)/2/nR(x)
         set xrange [1e15:1e17]~set samples 1000~set yrange [0.001:10]
         set logscale xy~set size 0.6,0.6~set xlabel "{/Symbol w}"
         plot nR(x) title "n_R",nI(x) title "n_I"
```



25-4. Given

$n = 1$ electron/atom $\quad N = \dfrac{N_A\,\rho}{M} = \dfrac{(6.02 \times 10^{26})\,(2.70 \times 10^3)}{26.982}\,\mathrm{m}^{-3} = 6.027 \times 10^{28}\,\mathrm{m}^{-3} \quad \sigma_0 = 3.54 \times 10^7/\Omega\ \mathrm{m}$

(a) $\sigma_0 = \dfrac{N e^2}{m\,\gamma} \Rightarrow \gamma = \dfrac{N e^2}{m\,\sigma_0} = \dfrac{(6.027 \times 10^{28})\,(1.602 \times 10^{-19})^2}{(9.109 \times 10^{-31})\,(3.54 \times 10^7)}\mathrm{s}^{-1} = 4.80 \times 10^{13}\,s^{-1}$

(b) $\omega_p^2 = \dfrac{N e^2}{m\,\varepsilon_0} = \dfrac{(6.027 \times 10^{28})\,e^2}{m\,\varepsilon_0} = 1.918 \times 10^{32}\,\mathrm{s}^{-2} \Rightarrow \omega_p = 1.38 \times 10^{16}\,\mathrm{s}^{-1}$

(c) At $\lambda = 550\,\mathrm{nm}, \omega = 2\,\pi\,f = 2\,\pi\,c/\lambda = 3.425 \times 10^{15}\,\mathrm{Hz}$. Now,

$$n_R^2 - n_I^2 \equiv K_R = 1 - \frac{\omega_p^2}{\omega^2 + \gamma^2} = -15.347 \quad 2\,n_I\,n_R \equiv K_I = \frac{\gamma}{\omega}\left(\frac{\omega_p^2}{\omega^2 + \gamma^2}\right) = 0.2291$$

Solving these simultaneously as in problem 25 1 gives,

$$n_R = \left[\frac{K_R + \sqrt{K_R^2 + K_I^2}}{2}\right]^{1/2} = 0.0292 \quad n_I = \frac{K_I}{2\,n_R} = 3.92$$

25-5. Before approximation, $k^2 = \frac{\omega^2}{c^2} + i\left(\frac{\sigma\,\omega\,\mu_0}{1 - i\,\omega/\gamma}\right)$. If $\omega \ll \gamma$ or $\frac{\omega}{\gamma} \ll 1$, we can neglect $i\omega/\gamma$ compared with the real part, 1, in the denominator: $1 - i\,\omega/\gamma$. Then

$$k^2 \approx \frac{\omega^2}{c^2} + i\,\sigma\omega\,\mu_0 = \frac{w^2}{c^2}\left(1 + \frac{i\,\sigma\,c^2\,\mu_0}{\omega}\right). \qquad \text{Use } c^2 \equiv \frac{1}{\varepsilon_0\,\mu_0}$$

$$k^2 \approx \frac{w^2}{c^2}\left(1 + i\,\frac{\sigma}{\varepsilon_0\,\omega}\right).$$

But if $\omega \ll \frac{\sigma}{\varepsilon_0}$, or $\Rightarrow \frac{\sigma}{\varepsilon_0\,\omega} \gg 1$, we can neglect the real part, compared with the imaginary, so

$$k^2 \approx i\,\frac{w^2}{c^2}\,\frac{\sigma}{\varepsilon_0\,\omega} = i\,(\omega\,\varepsilon\,\mu_0) = i\,\omega\,\sigma\,\mu_0$$

This is the approximation stated at the beginning of Section 25 5 in the text. Equation (25 58) then fol lows as detailed there.

25-6. Use Eq. (25 58)

$$\delta = \frac{1}{K_I} = \sqrt{\frac{2}{\sigma\,\mu_0\,\omega}} = \sqrt{\frac{\lambda}{\sigma\,\mu_0\,\pi\,c}}$$

(a) For $\nu = 60\,\mathrm{Hz} \Rightarrow \omega = 2\,\pi\,(60)\,s^{-1}$

$$\delta_{\mathrm{cu}} = \left(\frac{2}{\sigma\,\mu_0\,\omega}\right)^{1/2} = \left(\frac{2}{(5.76\times10^7)\,(4\,\pi\times10^{-7})\,(2\,\pi)\,(60)}\right)^{1/2} \mathrm{m} = 0.00856\,\mathrm{m} = 0.856\,\mathrm{cm}$$

(b) For $\lambda = 3\,\mathrm{m}$

$$\delta_{\mathrm{cu}} = \left(\frac{\lambda}{\sigma\,\mu_0\,\pi\,c}\right)^{1/2} = \left(\frac{3}{(5.76\times10^7)\,(4\,\pi\times10^{-7})\,\pi\,(3\times10^8)}\right)^{1/2} \mathrm{m} = 6.63\,\mu\mathrm{m}$$

25-7. (a) $\delta_{\mathrm{Al}} = \left(\frac{2}{\sigma\,\mu_0\,\omega}\right)^{1/2} = \left(\frac{2}{3.54\times10^7\,(4\,\pi\times10^{-7})\,2\,\pi\times6\times10^4}\right)^{1/2} \mathrm{m} = 0.345\,\mathrm{mm}$

(b) $\delta_{\mathrm{s.w.}} = \left(\frac{3.54\times10^7}{4.3}\right) \times \delta_{\mathrm{Al}} = 0.991\,\mathrm{m} \approx 1\,\mathrm{m}$

25-8. $\delta_{\mathrm{Ag}} = \left(\frac{\lambda}{\sigma\,\mu_0\,\pi\,c}\right)^{1/2} = \left(\frac{0.1}{3\times10^7(4\,\pi\times10^{-7})\,\pi\,(3\times10^8)}\right)^{1/2} \mathrm{m} = 1.68\times10^{-6}\,\mathrm{m} = 1.68\,\mu\mathrm{m}$

As long as the silver coating is thicker than this the silver plated brass component would work.

25-9. (a) $I = I_0\,e^{-\alpha x} \Rightarrow (I/I_0) = (1/4) = e^{-\alpha x} = e^{-\alpha(3.42\mathrm{m})} \Rightarrow 3.42\,\alpha = \ln(4) \Rightarrow \alpha = 0.405\,\mathrm{m}^{-1}$

(b) $(I/I_0) = (1/100) = e^{-(0.405\mathrm{m}^{-1})x} \Rightarrow (0.405\,\mathrm{m}^{-1}) = \ln(100) \Rightarrow x = 11.37\,\mathrm{m}$

25-10. For a metal we have,

$$n_R^2 - n_I^2 \equiv K_R = 1 - \frac{\omega_p^2}{\omega^2+\gamma^2} \quad 2\, n_I\, n_R \equiv K_I = \frac{\gamma}{\omega}\left(\frac{\omega_p^2}{\omega^2+\gamma^2}\right)$$

Solving these simultaneously as in problem 25 1 gives,

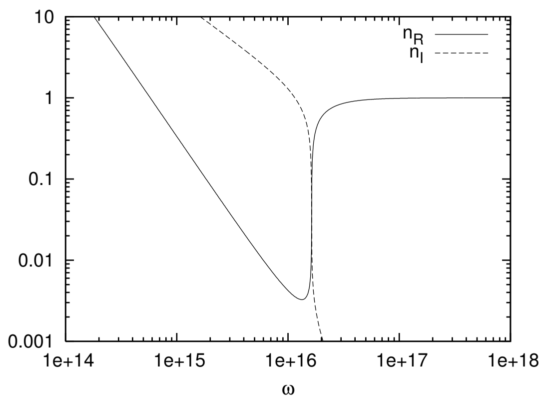
$$n_R = \left[\frac{K_R + \sqrt{K_R^2 + K_I^2}}{2}\right]^{1/2} \quad n_I = \frac{K_I}{2\, n_R}$$

To compare to Figure 25 3 use the parameters,

$$\omega_p = 1.63 \times 10^{16}\,\mathrm{s}^{-1} \qquad \gamma = 4.1 \times 0^{13}\,\mathrm{s}^{-1}$$

Using the free plotting program GNUplot:

```
This is a TeXmacs interface for GNUplot.
GNUplot] wp 1.63e16~g 4.1e13
         KR(x) 1-wp*wp/(x*x+g*g)
         KI(x) g/x*wp*wp/(x*x+g*g)
         nR(x) sqrt((KR(x)/2+sqrt(KR(x)*KR(x)+KI(x)*KI(x))/2))~nI(x) KI(x)/2/nR(x)
         set xrange [1e14:1e18]~set samples 1000~set yrange [0.001:10]
         set logscale xy~set size 0.6,0.6~set xlabel "{/Symbol w}"
         plot nR(x) title "n_R",nI(x) title "n_I"
```



25-11. Referring to page 543,

$$n^2 \approx 1 + \frac{N e^2}{m\, \varepsilon_0\, \omega_0^2}\left(1 + \frac{\omega^2}{\omega_0^2} + \frac{\omega^4}{\omega_0^4}\right) \equiv 1 + \alpha\left(1 + \frac{\omega^2}{\omega_0^2} + \frac{\omega^4}{\omega_0^4}\right) = (1+\alpha) + \frac{\alpha}{\omega_0^2}\frac{(2\,\pi\, c)^2}{\lambda^2} + \frac{\alpha}{\omega_0^4}\frac{(2\,\pi\, c)^4}{\lambda^4}$$

$$n^2 = A' + \frac{B'}{\lambda^2} + \frac{C'}{\lambda^4} \quad \text{where} \quad A' = 1+\alpha \qquad B' = \alpha\,(2\,\pi\, c)^2/\omega_0^2 \qquad C' = \alpha\,(2\,\pi\, c)^4/\omega_0^2$$

$$n = \left[A' + \frac{B'}{\lambda^2} + \frac{C'}{\lambda^4}\right]^{1/2} = A^{1/2}\left[1 + \left(\frac{B'/A'}{\lambda^2} + \frac{C'/A'}{\lambda^4}\right)\right]^{1/2} \approx \sqrt{A'}\left[1 + \frac{B'/2\,A'}{\lambda^2} + \frac{C'/2\,A'}{\lambda^4} - \frac{1}{8}\left(\frac{8'/A'}{\lambda^2}\right)^2\right]$$

$$n = \sqrt{A'}\left\{1 + \frac{1}{2}\frac{B'/A'}{\lambda^2} + \frac{1}{\lambda^4}\left(\frac{C'}{2\,A'} - \frac{B'^2}{8\,A'^2}\right)\right\} = A + \frac{B}{\lambda^2} + \frac{C}{\lambda^4}$$

$$A \equiv \sqrt{A'} = (1+\alpha)^{1/2}$$

$$B \equiv B'/2\sqrt{A'} = \frac{B'}{2\,A} = \frac{\alpha\,(2\,\pi\, c)^2}{2\,\omega_0^2\,(1+\alpha)^{1/2}} = 2\left(\frac{\pi\, c}{\omega_0}\right)^2 \frac{1}{(1+\alpha)^{1/2}}$$

$$C \equiv \sqrt{A'}\left[\frac{C6'}{2\,A'} - \frac{B'^2}{8\,A'^2}\right] = \frac{(2\,\pi\, c)^4\,\alpha}{2\,\omega_0^4\, A'^{1/2}} - \frac{(2\,\pi\, c)^4\,\alpha^2}{8\,\omega_0^4\, A'^{3/2}} = \frac{(2\,\pi\, c)^4\,\alpha}{2\,\omega_0^4\, A'^{1/2}}\left[1 - \frac{\alpha}{4\,(1+\alpha)}\right] = C = \frac{(2\,\pi\, c)^4\,\alpha}{2\,\omega_0^4\,(1+\alpha)^{1/2}}\left(\frac{4+3\,\alpha}{4\,(1+\alpha)}\right)$$

$$C = 2\left(\frac{\pi\, c}{\omega_0}\right)^4 \frac{\alpha\,(4+3\,\alpha)}{(1+\alpha)^{3/2}}$$

25-12. The Lorentz force law is,

$$\boldsymbol{F} = \boldsymbol{F}_E + \boldsymbol{F}_B = q\,\boldsymbol{E} + q\,\boldsymbol{v} \times \boldsymbol{B}$$

where $\boldsymbol{v}$ is the velocity of the charged particle experiencing the force

$$\frac{|\boldsymbol{F}_B|}{|\boldsymbol{F}_E|} = \frac{|q\,\boldsymbol{v} \times \boldsymbol{B}|}{|q\,\boldsymbol{E}|} \leqslant \frac{|q\,v\,B|}{|q\,E|} = \frac{|v(E_0/c)|}{|E_0|} = v/c$$

The magnitude of the magnetic force is much less the the magnitude of the electric force so long as the speed of the charged particle is much less than the speed of light.

Chapter 26 Laser Operation

26-1. (a) According to Eq. (26 6), the spectral energy density has the form

$$\rho(\nu)=\frac{8\,\pi\,h\,\nu^3}{c^3}\frac{1}{e^{h\nu/k_BT}-1}$$

The energy per unit volume per unit wavelength interval $\rho(\lambda)$ is related to $\rho(\nu)$ by the relation,

$$\int_{\lambda=0}^{\lambda=\infty}\rho(\lambda)d\,\lambda=\int_{\nu=0}^{\nu=\infty}\rho(\nu)\,d\,\nu$$

so that integration over all wavelengths or frequencies gives the total energy density. Since, $\nu=c/\lambda$,

$$d\nu=-\frac{c}{\lambda^2}\,d\lambda$$

So, using Eq. (16 6) and changing variables in the frequency integral proceeds as,

$$\int_{\nu=0}^{\nu=\infty}\rho(\nu)\,d\,\nu=\int_{\nu=0}^{\nu=\infty}\frac{8\,\pi\,h\,\nu^3}{c^3}\frac{1}{e^{h\nu/k_BT}-1}d\,\nu=\int_{\lambda=\infty}^{\lambda=0}\frac{8\,\pi\,h}{\lambda^3}\frac{1}{e^{h\nu/k_BT}-1}\Big(-\frac{c}{\lambda^2}\Big)d\,\lambda$$
$$\int_{\nu=0}^{\nu=\infty}\rho(\nu)\,d\,\nu=-\int_{\lambda=0}^{\lambda=\infty}\frac{8\,\pi\,h}{\lambda^3}\frac{1}{e^{h\nu/k_BT}-1}\Big(-\frac{c}{\lambda^2}\Big)d\,\lambda=\int_{\lambda=0}^{\lambda=\infty}\frac{8\,\pi\,h\,c}{\lambda^5}\frac{1}{e^{h\nu/k_BT}-1}d\,\lambda$$

So that

$$\rho(\lambda)=\frac{8\,\pi\,h\,c}{\lambda^5}\frac{1}{e^{h\nu/k_BT}-1}$$

(b) Generally the time averaged energy density $\langle u\rangle$ of radiation moving in a given direction is related to the exitance in that direction by the relation $M=v\,\langle u\rangle$. (See problem 26 2). If the energy has a variety of propagation directions the relation becomes

$$M=(\text{fraction of energy moving in given direction})\,(\text{average velocity component in that direction})\,u$$

The exitance per wavelength energy is the power per area leaving a surface. Let us place the small area in question so that its surface normal points in the z direction. In black body radiation the energy is dis tributed isotropically. On average then, half of the "photons" making up the field will have a velocity com ponent in the z direction. The z component of the velocity vector for a given photon can be calculated as,

$$v_z=\boldsymbol{v}\cdot\hat{\boldsymbol{z}}=c\cos\theta$$

Here θ is the azimuthal angle. The average z velocity component of the photons with a component in the positive z direction is then found by summing (integrating) the velocity z components associated with the vectors pointing into a given elemental solid angle $d\omega$ and then dividing by the total solid angle $(2\,\pi)$ in a hemisphere,

$$v_{z,\text{ave}}=\frac{1}{2\,\pi}c\int\cos\theta d\omega=\frac{c}{2\,\pi}\int_0^{2\pi}d\varphi\int_0^{\pi/2}\cos\theta\sin\theta\,d\theta=\frac{c}{2}$$

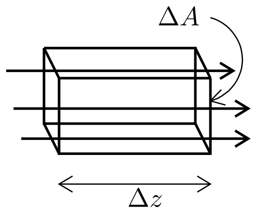
Then the relationship between the energy density and exitance becomes,

$$M=\frac{1}{2}\frac{c}{2}\langle u\rangle$$

This relation also holds in each wavelength range so that,

$$M_\lambda=\frac{c}{4}\,\rho(\lambda)=\frac{c}{4}\frac{8\,\pi\,h\,c}{\lambda^5}\frac{1}{e^{h\nu/k_BT}-1}=\frac{2\pi\,h\,c^2}{\lambda^5}\frac{1}{e^{h\nu/k_BT}-1}$$

26-2. Consider the small volume shown below.



Let the total energy in the volume $\Delta V = \Delta A\, \Delta z$ is W then

The energy density in the volume is then $\langle u \rangle = \dfrac{W}{\Delta V}$.

In a time $\Delta t = \Delta z / c$ all of the energy in the volume will pass

through the right face of the volume. So the irradiance I in

can be formed as $I = \dfrac{W}{\Delta A\, \Delta t} = \dfrac{W}{\Delta A\, \Delta z / c} = c\, \langle u \rangle$.

26-3. Equations (26 14) (in steady state) and (26 16) are,

$$0 = -A_{21} N_2 - \frac{\sigma I}{h\nu'}(N_2 - N_1) \tag{1}$$
$$N_T = N_1 + N_2 \tag{2}$$

From (2), $N_1 = N_T - N_2$. Using this in (1) and rearranging proceeds as,

$$0 = -A_{21} N_2 - \frac{\sigma I}{h\nu'}(N_2 - N_T + N_2)$$
$$N_2 \left(A_{21} + 2\,\sigma I / h\nu' \right) = (\sigma I / h\nu')\, N_T$$
$$N_2 = \frac{\sigma I / (h\nu')\, N_T}{(A_{21} + 2\,\sigma I / h\nu')} = \frac{\sigma I / (h\nu' A_{21})}{1 + 2\,\sigma I / (h\nu' A_{21})} N_T \tag{3}$$

Using (3) in (2) gives,

$$N_1 = N_T - N_2 = \left(\frac{1 + 2\,\sigma I / (h\nu' A_{21})}{1 + 2\,\sigma I / (h\nu' A_{21})} - \frac{\sigma I / (h\nu' A_{21})}{1 + 2\,\sigma I / (h\nu' A_{21})} \right) N_T = \frac{1 + \sigma I / (h\nu' A_{21})}{1 + 2\,\sigma I / (h\nu' A_{21})}$$

Then,

$$N_{\rm inv} = N_2 - N_1 = \frac{\sigma I / (h\nu' A_{21})}{1 + 2\,\sigma I / (h\nu' A_{21})} N_T - \frac{1 + \sigma I / (h\nu' A_{21})}{1 + 2\,\sigma I / (h\nu' A_{21})} = -\frac{N_T}{1 + 2\,\sigma I / (h\nu' A_{21})}$$

26-4. (a) From Section 26 2,

$$\alpha = -\sigma(N_2 - N_1) = \frac{\sigma N_T}{1 + 2\,\sigma I / (h\nu' A_{21})} \equiv \frac{\alpha_0}{1 + 2\,\sigma I / (h\nu' A_{21})}$$

Then $\alpha = \alpha_0 / 2$ for

$$2\,\sigma I_{S,\rm abs} / (h\nu' A_{21}) = 1 \Rightarrow I_{S,\rm abs} = \frac{h\nu'}{2\,\sigma} A_{21} = \frac{h\nu'}{2\sigma\tau_2}$$

(b) In an ideal four level gain medium the saturation irradiance is given by Eq. (26 39) as

$$I_S = \frac{h\nu'}{\sigma\tau_2}$$

The difference of the factor of 2 arises because the ideal four level gain medium has a lower lasing level with a zero lifetime so that it is never populated. Each photon transition from upper to lower level then changes the population inversion by $(N_2 - \cancel{N_1}) - (N_2 - 1 - \cancel{N_1}) = 1$ since it decreases the population of the upper level by 1 but does not change the population of the lower level which is always zero. In the absorptive medium each absorption changes the inversion by $(N_2 - N_1) - [(N_2 + 1) - (N_1 - 1)] = -2$ since it decreases the population of the lower level by one and increases the population of the upper level by 1.

26-5. (a) The rate equations are given by Eqs (26 26) (26 29). Setting the indicated parameters to zero:

$$\frac{dN_3}{dt} = -\kappa_{32}\, N_3 - \frac{\sigma_p\, I_p}{h\nu_p}\,(N_3 - N_0)$$
$$\frac{dN_2}{dt} = \kappa_{32}\, N_3 - \kappa_{21}\, N_2 - \frac{\sigma I}{h\nu'}\,(N_2 - N_1)$$
$$\frac{dN_1}{dt} = \kappa_{21}\, N_2 + \frac{\sigma I}{h\nu'}\,(N_2 - N_1) - \kappa_{10}\, N_1$$
$$\frac{dN_0}{dt} = -\left(\frac{dN_3}{dt} + \frac{dN_2}{dt} + \frac{dN_1}{dt}\right) \Rightarrow N_T = N_0 + N_1 + N_2 + N_3$$

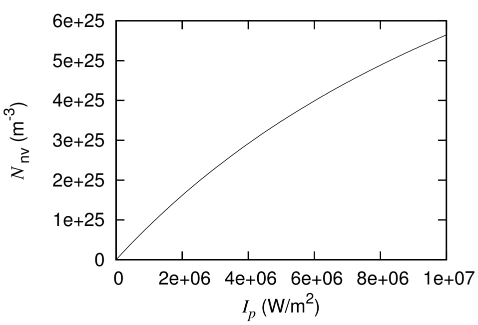
(b) In steady state and with $I = 0$ these become,

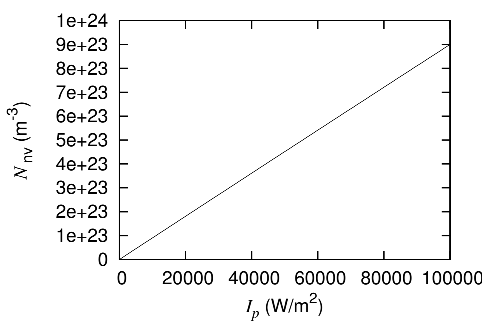
$$0 = -\kappa_{32}\, N_3 - \frac{\sigma_p\, I_p}{h\nu_p}\,(N_3 - N_0) \quad (1)$$
$$0 = \kappa_{32}\, N_3 - \kappa_{20}\, N_2 \quad (2)$$
$$0 = \kappa_{21}\, N_2 - \kappa_{10}\, N_1 \quad (3)$$
$$N_T = N_0 + N_1 + N_2 + N_3 \quad (4)$$

These are four equations in four unknowns. Solving them simultaneously using the free computer algebra system Maxima I find, (in mks units)

$$N_{\text{inv}} = N_2 - N_1 = \frac{1.499985 \times 10^{31}\, I_p}{100000\, I_p + 1.6565 \times 10^{12}} \quad (5)$$

Plotting this expression gives





(c) From the plot or from (5), the inversion is greater than or equal to zero for all pump irradiances.

(d) The small signal gain coefficient is

$$\gamma_0 = \sigma\, N_{\text{inv}} = 0.01/\text{cm} = 1/\text{m} \Rightarrow N_{\text{inv}} = \gamma_0/\sigma = 10^{22}/\text{m}^3 = \frac{1.499985 \times 10^{31}\, I_p}{100000\, I_p + 1.6565 \times 10^{12}}$$

Numerically I find $I_p = 1104\,.4\text{W/m}^2$.

(e) For $\gamma_0 = 1/\text{cm} = 100/\text{m}$. $N_{\text{inv}} = 10^{24}/\text{m}^3, I_p = 1.11 \times 10^5\ \text{W/m}^2$.

(f) Solving (1) (4) in steady state for the two pump irradiances, I find,

For $I_p = 1104\ \text{W/m}^2$:
$N_0 = 1.49990 \times 10^{26}/\text{m}^3$
$N_1 = 1.000 \times 10^{17}/\text{m}^3$
$N_2 = 1.000 \times 10^{22}/\text{m}^3$
$N_3 = 1.000 \times 10^{17}/\text{m}^3$

For $I_p = 1.1 \times 10^5\ \text{W/m}^2$:
$N_0 = 1.490 \times 10^{26}/\text{m}^3$
$N_1 = 1.000 \times 10^{19}/\text{m}^3$
$N_2 = 1.000 \times 10^{24}/\text{m}^3$
$N_3 = 1.000 \times 10^{19}/\text{m}^3$

It is reasonable to set $N_0 \approx N_T$ in both cases.
(g) Using the suggested relations for an ideal 4 level gain medium

$$\gamma_0 = \sigma R_{p2}\, \tau_2 = \sigma\, \frac{\kappa_{32}}{\kappa_3}\left(\frac{\sigma_p I_p}{h\,\nu_p}\right) N_T \tau_2 \Rightarrow I_p = \frac{\gamma_0}{\sigma}\frac{\kappa_3}{\kappa_{32}}\frac{h\,\nu_p}{\sigma_p \tau_2}\frac{1}{N_T} = \gamma_0\, \frac{1}{1}\, 1104.3\ \text{W/m}$$

For $\gamma_0 = 0.01/\text{cm} = 1/\text{m}$; $I_p = 1104.3\ \text{W/m}^2$, For $\gamma_0 = 1/\text{cm} = 1/\text{m}$; $I_p = 1.1043 \times 10^5\ \text{W/m}^2$

26-6. This is conveniently done with the help of the free computer algebra system Maxima:

```
(%i1) eq1:0 Rp2-k2*N2-Q*(N2-N1)
```

(%o1) $0 = -(\mathrm{N2}-\mathrm{N1})\,Q - \mathrm{k2}\,\mathrm{N2} + \mathrm{Rp2}$

```
(%i2) eq2:0 Rp1+k21*N2-k10*N1+Q*(N2-N1)
```

(%o2) $0 = (\mathrm{N2}-\mathrm{N1})\,Q + \mathrm{k21}\,\mathrm{N2} - \mathrm{k10}\,\mathrm{N1} + \mathrm{Rp1}$

```
(%i3) sols:solve([eq1,eq2],[N1,N2])
```

(%o3) $\left[\left[\mathrm{N1} = \frac{(\mathrm{Rp2}+\mathrm{Rp1})\,Q + \mathrm{k21}\,\mathrm{Rp2} + \mathrm{k2}\,\mathrm{Rp1}}{(-\mathrm{k21}+\mathrm{k2}+\mathrm{k10})\,Q + \mathrm{k10}\,\mathrm{k2}}, \mathrm{N2} = \frac{(\mathrm{Rp2}+\mathrm{Rp1})\,Q + \mathrm{k10}\,\mathrm{Rp2}}{(-\mathrm{k21}+\mathrm{k2}+\mathrm{k10})\,Q + \mathrm{k10}\,\mathrm{k2}}\right]\right]$

```
(%i4) Ninv:ratsimp(-rhs(sols[1][1])+rhs(sols[1][2]))
```

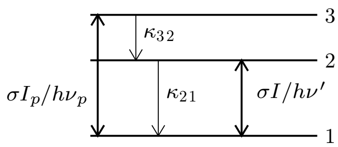
(%o4) $-\frac{(\mathrm{k21}-\mathrm{k10})\,\mathrm{Rp2} + \mathrm{k2}\,\mathrm{Rp1}}{(\mathrm{k21}-\mathrm{k2}-\mathrm{k10})\,Q - \mathrm{k10}\,\mathrm{k2}}$

Here $Q = \sigma I / h\nu'$, so,

$$N_2 - N_1 = N_{\rm inv} = -\frac{(\kappa_{21}-\kappa_{10})\,R_{p2} + \kappa_2\,R_{p1}}{(\kappa_{21}-\kappa_2-\kappa_{10})\,(\sigma I/h\nu') - \kappa_{10}\,\kappa_2} = \frac{(1-\kappa_{21}/\kappa_{10})\,R_{p2} - (\kappa_2/\kappa_{10})\,R_{p1}}{\kappa_2 + (1+\kappa_{20}/\kappa_{10})\,(\sigma I/h\nu')}$$

26-7.

(a)



(b)

$$\frac{dN_3}{dt} = -\kappa_{32}\,N_3 - \frac{\sigma_p\,I_p}{h\nu_p}\,(N_3 - N_1)$$

$$\frac{dN_2}{dt} = \kappa_{32}\,N_3 - \kappa_{21}\,N_2 - \frac{\sigma I}{h\nu'}\,(N_2 - N_1)$$

$$\frac{dN_1}{dt} = -\left(\frac{dN_3}{dt} + \frac{dN_2}{dt}\right) \Rightarrow N_T = N_1 + N_2 + N_3$$

(c) In steady state and for $I = 0$ these relations become,

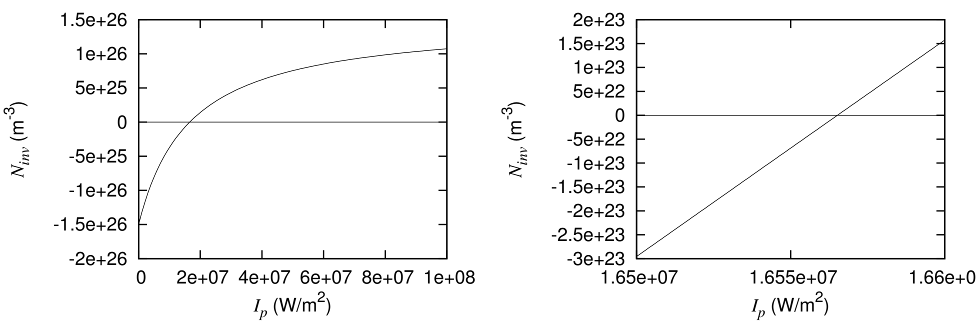
$$0 = \kappa_{23}\,N_3 - \frac{\sigma_p\,I_p}{h\nu_p}\,(N_3 - N_1) \tag{1}$$

$$0 = \kappa_{32}\,N_3 - \kappa_{21}\,N_2 \tag{2}$$

$$N_T = N_1 + N_2 \tag{3}$$

Solving these, using Maxima, for the given parameters leads to (in mks units),

$$N_2 - N_1 = N_{\rm inv} = \frac{7.499925 \times 10^{30}\,I_p - 1.242375 \times 10^{38}}{50001\,I_p + 8.2825 \times 10^{11}}$$



(d) Using the expression for the inversion form part (c), I find $N_{\rm inv} = 0$ for $I_p = 1.6565 \times 10^7\,\mathrm{W/m^2}$.

(e) Setting $\gamma_0 = 0.01/\mathrm{cm} = 1/\mathrm{m} = \sigma N_{\rm inv} = (10^{-22}\,\mathrm{m}^2)\,N_{\rm inv}$ leads to $I_p = 1.6567 \times 10^7\,\mathrm{W/m^2}$.
(f) Setting $\gamma_0 = 0.01/\mathrm{cm} = 100/\mathrm{m} = \sigma N_{\rm inv} = (10^{-22}\,\mathrm{m}^2)\,N_{\rm inv}$ leads to $I_p = 1.6788 \times 10^7\,\mathrm{W/m^2}$.

(g) In a three level system a large pump is needed just to reach a population inversion. Pump thresholds for lasing are typically not too much more than the pump needed for inversion. In a ideal four level system an inversion occurs for any nonzero pump.

26-8. Consider Eq. (26 41),

$$\ln\left(\frac{I_L}{I_0}\right) + \frac{1}{I_S}(I_L - I_0) = \gamma_0 L$$

If $I_L \ll I_S$ and $I_0 \ll I_S$, this relation becomes,

$$\ln\left(\frac{I_L}{I_0}\right) \approx \gamma_0 L$$
$$I_L \approx I_0 e^{\gamma_0 L}$$

26-9. Consider Eq. (26 40),

$$\frac{dI}{dz} = \frac{\gamma_0}{1 + I/I_S} I$$

If $I \gg I_S$, this becomes,

$$\frac{dI}{dz} = \frac{\gamma_0}{I/I_S} I \approx \gamma_0 I_S$$
$$\int_{I_0}^{I_L} dI = \gamma_0 I_S \int_0^L dz$$
$$I_L - I_0 = \gamma_0 I_S L$$
$$I_L = I_0 + \gamma_0 I_S L$$

26-10. (a) Using Eq. (26 38) and (26 39) for ideal 4 level gain media in the expression from problem 26 9 gives,

$$I_L - I_0 = \gamma_0 I_S L = \sigma R_{p2} \tau_2 \left(\frac{h\nu'}{\sigma \tau_2}\right) L = h\nu' R_{p2} L$$

(b) The number of photons added to the field per time due to a gain medium of cross sectional area A and length L is,

$$\frac{N_{\text{output}}}{t} = (I_L - I_0) A / h\nu' = R_{p2} A L = R_{p2} V = \frac{N_{\text{pump}}}{t}$$

Here R_{p2} is the pump rate per volume and $V = A L$ is the volume of the gain medium. Thus in the large input irradiance limit every pump event leads to an output photon.
(c) For lower input irradiance some pump events lead to nonradiative decay rather than stimulated emis sion. (Of course for non ideal gain media there are many other processes that reduce the pump efficiency in both limits.)

26-11. The relation between the input and output irradiance is,

$$\ln\left(\frac{I_L}{I_0}\right) + \frac{1}{I_S}(I_L - I_0) = \gamma_0 L$$

For the parameters given in the problem:

$$\ln\left(\frac{I_L}{I_0}\right) + \frac{1}{100\,\text{W/cm}^2}(I_L - I_0) = 2$$

One must solve the relation numerically. I find,

(a) When $I_0 = 1\,\text{W/cm}^2$, $I_L = 6.96\,\text{W/cm}^2$. (b) When $I_0 = 10\,\text{W/cm}^2$, $I_L = 49.7\,\text{W/cm}^2$

(c) When $I_0 = 100\,\text{W/cm}^2$, $I_L = 221\text{W/cm}^2$ (d) When $I_0 = 1000\,\text{W/cm}^2$, $I_L = 1180\,\text{W/cm}^2$

(e) When $I_0 = 10,000\,\text{W/cm}^2$, $I_L = 10,200\,\text{W/cm}^2$

26-12. Using the results from problem 26 11:

(a) $I_L - I_0 = 5.96\,\mathrm{W/cm^2}$, $I_L/I_0 = 6.96$ (b) $I_L - I_0 = 39.7\,\mathrm{W/cm^2}$, $I_L/I_0 = 4.97$

(c) $I_L - I_0 = 121\,\mathrm{W/cm^2}$, $I_L/I_0 = 2.21$ (d) $I_L - I_0 = 180\,\mathrm{W/cm^2}$, $I_L/I_0 = 1.18$

(e) $I_L - I_0 = 200\,\mathrm{W/cm^2}$, $I_L/I_0 = 1.02$

Note that as the input irradiance is increased, the added irradiance becomes a smaller and smaller fraction of the input irradiance. This is what is meant by gain saturation.

26-13. The relation between the input and output irradiance is,

$$\ln\left(\frac{I_L}{I_0}\right) + \frac{1}{I_S}(I_L - I_0) = \gamma_0\, L$$

When detuned by $\Delta\nu/2$, the lineshape function is 1/2 its peak value. The small signal gain coefficient is proportional to $g(\nu')$ and the saturation irradiance is inversely proportional to $g(\nu')$. So for the case at hand $\gamma_0(\nu') = \gamma_0(\nu_0)/2 = 1/\mathrm{cm}$ and $I_S(\nu') = 2I_S(\nu_0) = 200\,\mathrm{W/cm^2}$. For these parameters

$$\ln\left(\frac{I_L}{I_0}\right) + \frac{1}{200\,\mathrm{W/cm^2}}\,(I_L - I_0) = 1$$

One must solve the relation numerically. I find:

(a) When $I_0 = 1\,\mathrm{W/cm^2}$, $I_L = 2.70\,\mathrm{W/cm^2}, I_L - I_0 = 1.70\,\mathrm{W/cm^2}$, $I_L/I_0 = 2.70$

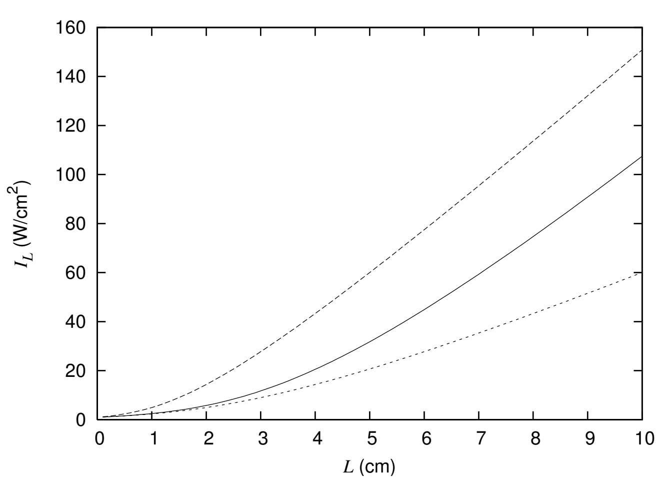
(b) When $I_0 = 10\,\mathrm{W/cm^2}$, $I_L = 25.2\,\mathrm{W/cm^2}$, $I_L - I_0 = 15.2\,\mathrm{W/cm^2}$, $I_L/I_0 = 2.520$

(c) When $I_0 = 100\,\mathrm{W/cm^2}$, $I_L = 181\mathrm{W/cm^2}$, $I_L - I_0 = 81\,\mathrm{W/cm^2}$, $I_L/I_0 = 1.81$

(d) When $I_0 = 1000\,\mathrm{W/cm^2}$, $I_L = 1170\,\mathrm{W/cm^2}$, $I_L - I_0 = 170\,\mathrm{W/cm^2}$, $I_L/I_0 = 1.17$

(e) When $I_0 = 10,000\,\mathrm{W/cm^2}$, $I_L = 10,200\,\mathrm{W/cm^2}$, $I_L - I_0 = 200\,\mathrm{W/cm^2}$, $I_L/I_0 = 1.02$

26-14. The plot is shown below. I used Maxima to numerically solve Eq. (26 41) for the given parameters and then plotted the generated list of points. In the plot the top (dashed) curve is for $\gamma_0 = 2/\mathrm{cm}$ and $I_S = 10\,\mathrm{W/cm^2}$, the middle (solid) curve is for $\gamma_0 = 1/\mathrm{cm}$ and $I_S = 20\,\mathrm{W/cm^2}$. The bottom (dotted) curve is for $\gamma_0 = 1/\mathrm{cm}$ and $I_S = 10\,\mathrm{W/cm^2}$.



26-15. (a) Use Eq. (26 45) with $S = R_3 = 1 - T_3$. Then

$$I_{\text{out}} = T_3\, I_S \left(\frac{(\gamma_0 - \gamma_{\text{th}})\, L}{1 - S} \right) = T_3\, I_S \left(\frac{(\gamma_0 - \gamma_{\text{th}})\, L}{1 - (1 - T_3)} \right) = T_3\, I_S \left(\frac{(\gamma_0 - \gamma_{\text{th}})\, L}{T_3} \right) = I_S (\gamma_0 - \gamma_{\text{th}})\, L$$

(b) If $\gamma_0 \gg \gamma_{\text{th}}$,

$$I_{\text{out}} \approx I_S\, \gamma_0\, L$$

As in problem 26 10, using Eq. (26 38) and (26 39) for ideal 4 level gain media in the expression above

$$I_{\text{out}} = \gamma_0\, I_S\, L = \sigma\, R_{p2}\, \tau_2 \left(\frac{h\nu'}{\sigma \tau_2} \right) L = h\nu' R_{p2}\, L$$

The number of photons emitted into the output field per time due to a gain medium of cross sectional area A and length L is then,

$$\frac{N_{\text{output}}}{t} = (I_L - I_0)\, A / h\nu' = R_{p2}\, A\, L = R_{p2}\, V = \frac{N_{\text{pump}}}{t}$$

Here R_{p2} is the pump rate per volume and $V = A\, L$ is the volume of the gain medium. Thus, in the large input irradiance limit every pump event leads to an output photon.

(c) The output efficiency is the ratio of output power to pump power not the ratio of the rate of photons leaving the cavity to the pump rate. That is the efficiency for the case of part (b) would be,

$$\text{efficiency} = \frac{(N_{\text{output}}/t)\, h\, \nu'}{(N_{\text{pump}}/t)\, h\nu_p} = \frac{\nu'}{\nu_p} < 1$$

26-16. (a) $\gamma_{\text{th}} = \dfrac{1}{L} \ln(1/R_3) = 0.00513/\text{cm}$

(b) $I_{\text{out}} = T_3\, I_S \dfrac{(\gamma_0 - \gamma_{\text{th}})\, L}{1 - S} = T_3\, I_S \dfrac{(2\, \gamma_{\text{th}} - \gamma_{\text{th}})\, L}{1 - R_3} = (0.05)\, (2000)\, \dfrac{0.00513 \times 10}{0.05} \text{W/cm}^2 = 102.6\, \text{W/cm}^2$

26-17. The output irradiance has the form,

$$I_{\text{out}} = T_3\, I_S \left(\frac{(\gamma_0 - \gamma_{\text{th}})\, L}{1 - S} \right)$$

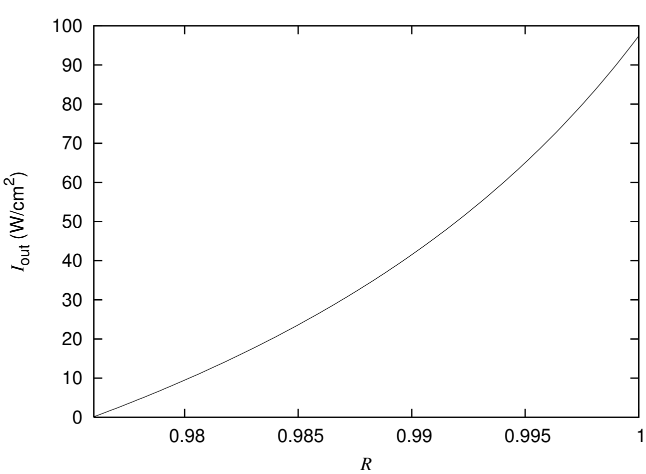
It is given that $R_3 = 0.95 = 1 - T_3$ so $A_3 = 0$. Further $R_1 = R_2 \equiv R$. Then

$$S = R_1\, R_2\, R_3 = R^2\, (0.95);\ \gamma_{\text{th}}\, L = \ln(1/S)$$

So,

$$I_{\text{out}} = (0.05)\, (2000\, \text{W/cm}^2)\, \frac{(0.01)\, (10) - \ln(1/(0.95 R^2))}{1 - 0.95\, R^2} = (100\, \text{W/cm}^2)\, \frac{0.1 - \ln(1/(0.95 R^2))}{1 - 0.95\, R^2}$$

Plotting gives,



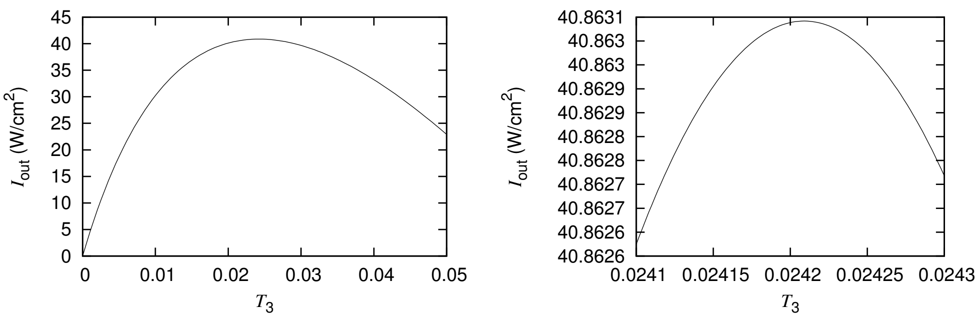
26-18. The output irradiance has the form,

$$I_{\text{out}} = T_3\, I_S \left(\frac{(\gamma_0 - \gamma_{\text{th}})\, L}{1 - S} \right)$$

From the given information, $R_3 = 1 - T_3 - 0.01 = 0.99 - T_3$, $R_1 = R_2 = 0.99$, $\gamma_0 = 0.01/\text{cm}$, $I_S = 2000\ \text{W/cm}^2$ and $L = 10\,\text{cm}$. Then $S = 0.99)^2\,(0.99 - T_3)$

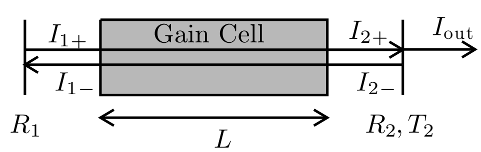
$$I_{\text{out}} = \left(2000\ \text{W/cm}^2\right) T_3\, \frac{\left[0.1 + \ln((0.99)^2\,(0.99 - T_3))\right]}{1 - 0.99)^2\,(0.99 - T_3)}$$

The plot looks like,



(b) From the plot on the right it is evident that the optimum T_3 is just a bit more that 0.0242. Such an optimum value exists whenever there are losses other than output losses in the cavity. If the transmission coefficient is too small the light generated in the cavity primarily ends up wasted into the other loss mecha nisms. If the transmission coefficient is too large, the net gain per pass is reduced and less of the energy stored in the population inversion is extracted by stimulated emission.

26-19. Consider the linear cavity shown below,



Saturation is now due to both the right and left going fields in the cavity. Equation (26 40) can be rewritten as,

$$\frac{1}{I_+}\frac{dI_+}{dz} = \frac{\gamma_0}{1 + (I_+ + I_-)/I_S} = -\frac{1}{I_-}\frac{dI_-}{dz}$$

The minus sign is due to the fact the the left going wave propagates in the $-z$ direction.

This relation leads to,

$$\frac{1}{I_+}\frac{dI_+}{dz} + -\frac{1}{I_-}\frac{dI_-}{dz} = 0 \Rightarrow \frac{d}{dz}[\ln(I_+\, I_-)] \Rightarrow I_+\, I_- = C$$

where C is a constant with dimensions of squared irradiance. Using this fact, one can write,

$$\frac{1}{I_+}\frac{dI_+}{dz} = \frac{\gamma_0}{1 + (I_+ + C/I_+)/I_S} \Rightarrow \frac{1}{I_+}(1 + (I_+ + C/I_+)/I_S)\, dI_+ = \gamma_0 dz$$

$$\int_{I_{1+}}^{I_{2+}} \left(\frac{1}{I_+} + \frac{1}{I_S} + \frac{C}{I_+^2\, I_S} \right) dI_+ = \gamma_0 \int_0^L dz \Rightarrow \ln\!\left(\frac{I_{2+}}{I_{1+}} \right) + \frac{1}{I_S}(I_{2+} - I_{1+}) - \frac{C}{I_S}\!\left(\frac{1}{I_{2+}} - \frac{1}{I_{1+}} \right) = \gamma_0\, L$$

Now the relation between I_{2+} and I_{1+} can be found by examining the irradiance at the ends of the gain cell and including the effects of the mirrors:

$$I_{1+}\, I_{1-} = I_{2+}\, I_{2-} = C \Rightarrow I_{1+}\,(\, I_{1+}/R_1) = I_{2+}\,(I_{2+} R_2) \Rightarrow I_{1+} = I_{2+}\, \sqrt{R_2 R_1} \quad \text{and } C = I_{2+}^2 R_2$$

Using these in the relations above gives,

$$\frac{1}{2}\ln\!\left(\frac{1}{R_1\, R_2} \right) + \frac{I_{2+}}{I_S}\left(1 - \sqrt{R_1\, R_2}\right) - \frac{I_{2+}^2\, R_2}{I_S}\left(\frac{1}{I_{2+}} - \frac{1}{\sqrt{R_2\, R_1} I_{2+}} \right) = \gamma_0\, L$$

$$I_{\text{out}} = T_2\, I_{2+} = T_2\, I_S\, \frac{\gamma_0\, L - \frac{1}{2}\ln(1/(R_1\, R_2))}{1 - \sqrt{R_1\, R_2} - R_2 + \sqrt{R_2/R_1}} = \frac{T_2 I_S}{2} \frac{\gamma_0\,(2L) - \ln(1/(R_1\, R_2))}{\left(1 - \sqrt{R_1\, R_2}\right)\left(1 + \sqrt{R_2/R_1}\right)}$$

26-20. When $R_1 = 1$, Eq. (26 47) becomes,

$$I_{\text{out}} = \frac{T_2\, I_S}{2} \frac{\gamma_0(2\,L) - \ln(1/R_2)}{(1 - \sqrt{R_2})\,(1 + \sqrt{R_2})} = \frac{T_2\, I_S}{2} \frac{\gamma_0(2\,L) - \ln(1/R_2)}{1 - R_2} = \frac{T_2\, I_S}{2} \frac{\gamma_0(2\,L) - \ln(1/S)}{1 - S}$$

This differs from the ring cavity result given in Eq. (26 43) by the presence of two different factors of 2. The linear cavity result has $\gamma_0(2\,L)$ instead of $\gamma_0\,L$ because the field encounters the gain medium twice in each round trip through the cavity. The linear cavity result has $I_S/2$ rather than I_S because in the linear cavity the gain medium is saturated by both the right and left going fields but the output is proportional only to the right going field.

26-21. a)At line center,

$$\sigma = B_{21}\, g(\nu_0)\, h\,\nu_0/c = \left(\frac{A_{21}\, c^3}{8\,\pi\, h\,\nu_0^3}\right) \frac{2}{\pi \Delta\nu_H} \frac{h\,\nu_0}{c} = \frac{A_{21}\, c^2}{4\,\pi^2 \nu_0^2} \frac{1}{\Delta\nu_H} = \frac{A_{21} \lambda_0^2}{4\,\pi^2 \Delta\nu_H} = \frac{(0.34)\,\left(10.6 \times 10^{-6}\right)^2}{4\,\pi^2\,(10^9)}$$

$$\sigma = 9.69 \times 10^{-22}\,\text{m}^2$$

(b) $N_{\text{inv}} = \gamma/\sigma = (3/\text{m})/(9.69 \times 10^{-22}\,\text{m}^2) = 3.1 \times 10^{21}/\text{m}^3$

(c) $I_S \approx \dfrac{h\,\nu_0}{\sigma\,\tau_2} = \dfrac{\left(6.626 \times 10^{-34}\right)\,(3 \times 10^8/10.6 \times 10^{-6})}{(9.69 \times 10^{-22})\,(10 \times 10^{-6})} = 1.94 \times 10^6\,\text{W/m}^2$

26-22. The Doppler width is

$$\Delta\nu_D = \left(\frac{8\,k_B\,T}{M\,c^2} \ln(2)\right)^{1/2} \nu_0 = \left(\frac{8 \cdot 1.38 \times 10^{-23} \cdot 400}{(20.2 \cdot 1.66 \times 10^{-27})\, 9 \times 10^{16}} \ln(2)\right)^{1/2} \left(\frac{3 \times 10^8}{6.33 \times 10^{-7}}\right) = 1.5 \times 10^9\,\text{Hz}$$

26-23. Consider the ring cavity shown in Figure (26 8) in the text. Let the field just before M_3 in that figure at time $t + \tau$, where $\tau = P/c$ is the cavity round trip time be

$$E(t+\tau) = E_0(t+\tau)\, e^{i\omega(t+\tau)}$$

Without a gain medium or a field injected into the cavity, this field is formed by the propagation of the field that exists just before mirror three at time t around the cavity. That is,

$$E_0(t+\tau)\, e^{i\omega(t+\tau)} = \sqrt{S}\, e^{i(\omega\tau - kP)} E_0(t) e^{i\omega t}$$
$$E_0(t+\tau) = \sqrt{S} e^{-ikP} E_0(t)$$

If the field is resonant with the cavity, $e^{-ikP} = 1$. So, with a Taylor series expansion of the left hand side,

$$E_0(t) + \tau \frac{dE_0(t)}{dt} = \sqrt{S} E_0(t)$$

Rearranging, integrating, and forming the irradiance gives,

$$\frac{dE_0(t)}{dt} = -\frac{(1 - \sqrt{S})}{\tau} E_0(t) \equiv -(\Gamma/2) E_0(t)$$
$$E_0(t) = E_0(0)\, e^{-\Gamma t/2}$$
$$I \propto E_0^2 \Rightarrow I = I_0 e^{-\Gamma t}$$

Here

$$\Gamma = 2\frac{1 - \sqrt{S}}{\tau} = 2\frac{1 - \sqrt{S}}{P}\, c$$

Now the expansion leading to this result is only good for S close to 1. So let $S = 1 - \mathcal{L}$ where $\mathcal{L} \ll 1$. Then

$$1 - \sqrt{S} = 1 - \sqrt{1 - \mathfrak{L}} \approx 1 - (1 - \mathcal{L}/2) = \mathcal{L}/2 = (1 - S)/2$$

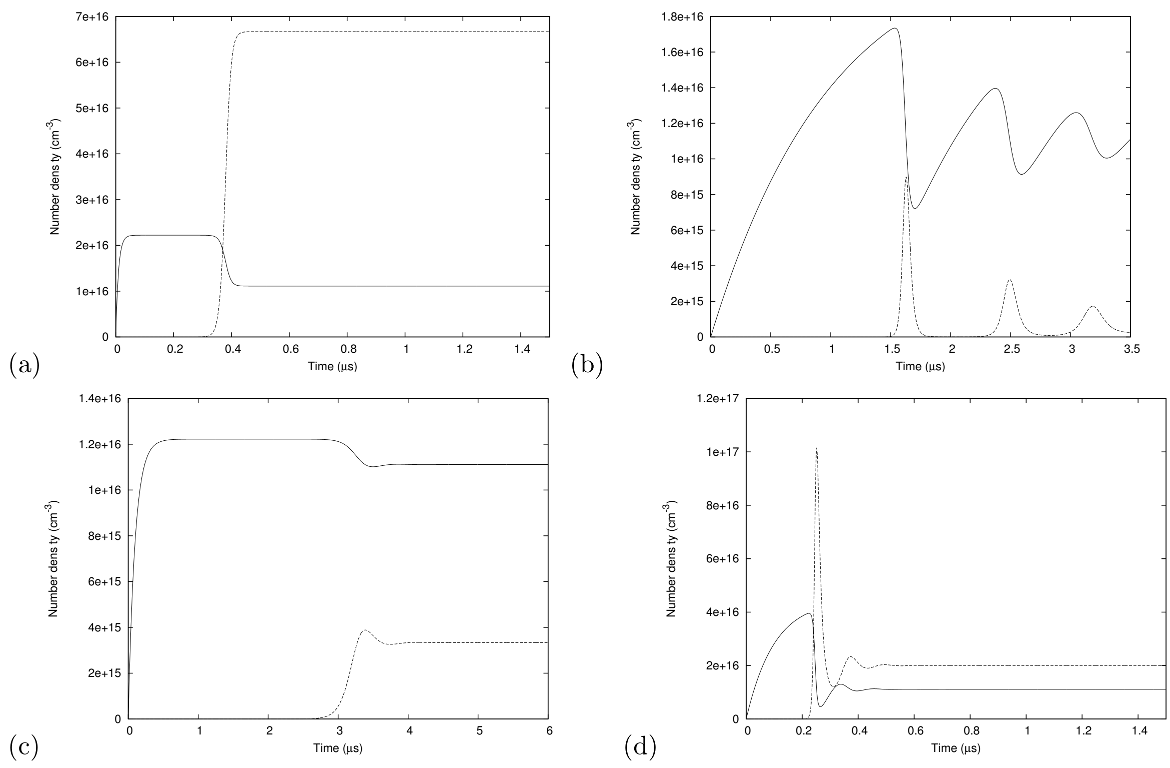
So,

$$\Gamma = 2\frac{1 - \sqrt{S}}{P}\, c \approx \frac{1 - S}{P} c$$

26-24. Figure 26 13 was made by using the free computer algebra system Maxima to numerically integrate Eqs. (26 54) and (26 55). The script I used is shown below.

```
(%i1) (sc:3e-8,d:1e-9,Vr:0.3,G:1e8,A:1e7,Rth:G*A/sc/Vr,y:2)$
(%i2) R:y*Rth$
(%i3) (Np[1]:0.0,N2[1]:0.0)$
(%i4) for j:2 thru 1500 do
      (N2[j]:N2[j-1]+d*(R-A*N2[j-1]-sc*Np[j-1]*N2[j-1]),
      Np[j]:Np[j-1]+d*(sc*Vr*(Np[j-1]+1)*N2[j-1]-G*Np[j-1]))$
(%i5) (N2list:listarray(N2),Nplist:listarray(Np),t:makelist(d*(k-1)/1e-6,k,1,1500))$
(%i6) plot2d([[discrete,t,N2list],[discrete,t,20*Nplist]],[gnuplot_curve_styles, ["with lines 1", "with lines
      1"]],[gnuplot_preamble, "set nokey;set samples 2000;set xlabel ’Time ({/Symbol m}s)’;set ylabel ’Number
      density (cm^{-3})’;set xrange [0:1.5];set label 1 ’{/Italic N}_{inv}’ at 1.2,1.2e16;set label 2 ’{/Italic
      N_p}’ at 1.2,0.73e16 "])$
(%i8) plot2d([[discrete,t,N2list],[discrete,t,20*Nplist]],[gnuplot_term, ps], [gnuplot_out_file,
      "fig26_13.eps"],[gnuplot_curve_styles, ["with lines 1", "with lines 2"]],[gnuplot_ps_term_command, "set
      term postscript eps enhanced monochrome 14"],[gnuplot_preamble, "set nokey;set samples 2000;set xlabel
      ’Time ({/Symbol m}s)’;set ylabel ’Number density (cm^{-3})’;set xrange [0:1.5];set label 1 ’{/Italic
      N}_{inv}’ at 0.73,1.2e16;set label 2 ’20{/Italic N_p}’ at 0.78,0.8e16;set size 0.7,0.7"]);
```

26-25. In figure (a), (b), and (d) below the solid curve represents N_{inv} and the dotted curve is $20\,N_p$. In figure (c) the solid curve is N_{inv} and the dotted curve is $100\,N_p$.



26-26. (a) The fields constructively interfere when they differ in phase by an integer multiple of $2\,\pi$. This occurs for $t = n/\delta\nu$ where n is an integer.

(b) In a mode locked laser adjacent cavity modes are phase locked and differ in frequency by the inverse of the round trip cavity time. Thus these waves will constructively interfere once every round trip.

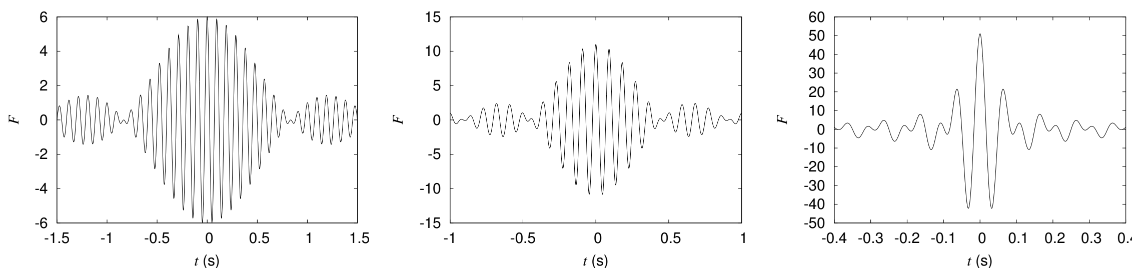
26-27. The pulse width is roughly the inverse of the gain bandwidth $\Delta\nu = \nu\,\Delta\lambda/\lambda = c\,\Delta\lambda/\lambda^2$.

$$\text{Ar}^+\text{:} \qquad \Delta t_p \approx \frac{1}{\Delta\nu} = \frac{\lambda^2}{c\,\Delta\lambda} = \frac{488^2}{3\times10^{17}\cdot 0.004}\,\text{s} = 1.98\times10^{-10}\,\text{s}$$

$$\text{He Ne} \qquad \Delta t_p \approx \frac{1}{\Delta\nu} = \frac{\lambda^2}{c\,\Delta\lambda} = \frac{633^2}{3\times10^{17}\cdot 0.002}\,\text{s} = 6.68\times10^{-10}\,\text{s}$$

$$\text{Dye} \qquad \Delta t_p \approx \frac{1}{\Delta\nu} = \frac{\lambda^2}{c\,\Delta\lambda} = \frac{590^2}{3\times10^{17}\cdot 80}\,\text{s} = 1.45\times10^{-14}\,\text{s}$$

26-28. The plots for the three cases are shown below. I'll take the pulse width is to be approximately the full width at half maximum of these waveforms.



(a) (b) (c)

(a) $\Delta\nu \sim (10 + 5\,(0.2) - 10)\text{Hz} = 1\,\text{Hz}$, $\Delta t_p \sim 1\,\text{s}$, $\Delta\nu \sim 1/\Delta t_p$.

(b) $\Delta\nu \sim (10 + 10\,(0.2) - 10)\text{Hz} = 2\,\text{Hz}$, $\Delta t_p \sim 0.5$, $\Delta\nu \sim 1/\Delta t_p$.

(c) $\Delta\nu \sim (10 + 50\,(0.2) - 10)\text{Hz} = 10\,\text{Hz}$, $\Delta t_p \sim 0.1$, $\Delta\nu \sim 1/\Delta t_p$.

26-29. Let $E = \text{Re}(\tilde{E})$, where,

$$\tilde{E} = \left(E_0\,e^{i(2\pi\nu_0 t + \varphi_0)}\right) \sum_{j=-(N-1)/2}^{(N-1)/2} \left(e^{i(2\pi j \nu_{\text{fsr}} t}\right)$$

For this formula to be sensible N must be odd. Now let $x = e^{i(2\pi\nu_{\text{fsr}} t)}$. Then,

$$\tilde{E} = E_0\,e^{i(2\pi\nu_0 t + \varphi_0)} \left(\sum_{j=-(N-1)/2}^{(N-1)/2} \left(x^j\right)\right) = E_0\,e^{i(2\pi\nu_0 t + \varphi_0)} \frac{x^{(N-1)/2+1} - x^{-(N-1)/2}}{x - 1}$$

$$\tilde{E} = E_0\,e^{i(2\pi\nu_0 t + \varphi_0)} \frac{e^{i(N\pi\nu_{\text{fsr}} t)} e^{i\pi\nu_{\text{fst}} t} - e^{-iN\pi\nu_{\text{fsr}} t} e^{i\pi\nu_{\text{fst}} t}}{e^{i2\pi\nu_{\text{fsr}} t} - 1} = E_0\,e^{i(2\pi\nu_0 t + \varphi_0)} \frac{e^{i\pi\nu_{\text{fst}} t}}{e^{i\pi\nu_{\text{fst}} t}} \, \frac{e^{i(N\pi\nu_{\text{fsr}} t)} - e^{-iN\pi\nu_{\text{fsr}} t}}{e^{i\pi\nu_{\text{fst}} t} - e^{-i\pi\nu_{\text{fst}} t}}$$

$$\tilde{E} = E_0\,e^{i(2\pi\nu_0 t + \varphi_0)} \frac{\sin(N\pi\,\nu_{\text{fsr}}\,t)}{\sin(\pi\,\nu_{\text{fsr}}\,t)}$$

$$E = \text{Re}\left(\tilde{E}\right) = E_0 \cos(2\,\pi\nu_0\,t + \varphi_0) \frac{\sin(N\pi\,\nu_{\text{fsr}}\,t)}{\sin(\pi\,\nu_{\text{fsr}}\,t)}$$

26-30. The power for the electric field of problem 26 29 is given by $P \propto \langle E^2 \rangle$ where the brackets indicate a time average over a period long compared to an optical period but short compared to a round trip cavity time. So,

$$P \propto E_0^2 \frac{\sin^2(N\pi\,\nu_{\rm fsr}\,t)}{\sin^2(\pi\,\nu_{\rm fsr}\,t)} \Rightarrow P = P_0 \frac{\sin^2(N\pi\,\nu_{\rm fsr}\,t)}{\sin^2(\pi\,\nu_{\rm fsr}\,t)}$$

This function peaks when its denominator tends to zero. At these times $t = n/\nu_{\rm fsr}$ where n is an integer. At these times the numerator also tends to zero. L'Hopital's rule gives,

$$\lim_{\alpha \to n\pi} \frac{\sin(N\alpha)}{\sin(\alpha)} = \lim_{\alpha \to n\pi} \frac{N\cos(N\alpha)}{\cos(\alpha)} = \pm N$$

so that,

$$\lim_{\alpha \to n\pi} \frac{\sin^2(N\alpha)}{\sin^2(\alpha)} = N^2$$

and thus,

$$P_p = P_0\,N^2$$

The power is periodic in time and so the width of the pulse near $t = 0$ is the same as the width of all of the pulses. The first minimum zero occurs when the denominator goes to zero for finite t. This occurs when

$$N\pi\,\nu_{\rm fsr}\,t_1 = \pi \Rightarrow t_1 = \frac{1}{N\nu_{\rm fsr}}$$

The width of the pulse from zero to zero on either side of the central maximum is $2\,t_1$ and so $\Delta t_p \sim t_1$ is a reasonable measure of the pulse width. Thus,

$$\Delta t_p \sim \frac{1}{N\,\nu_{\rm fsr}} = \frac{1}{\Delta\nu}$$

where $\Delta\nu = N\,\nu_{\rm fsr}$ is the range of frequencies that make up the signal.

26-31. The pulse repetition rate PRR is once per cavity round trip:

$$\mathrm{PRR} = 1/(2\,d/c) = \frac{c}{2\,d} = \frac{3 \times 10^8}{2(1.5)}\,\mathrm{Hz} = 10^8\,\mathrm{Hz}$$

The peak power is

$$P_p = \frac{P_{\rm ave}}{\mathrm{PRR}\,\Delta t_p} = \frac{10\,\mathrm{W}}{(10^8)(70 \times 10^{-12})} = 1430\,\mathrm{W}$$

26-32. Using the rough relation for the **full-width** angular spread due to diffraction from a slit, $\Delta\theta = 2\,\lambda/b$ where b is the slit width,

"vertical spread" $$\Delta\theta \approx \frac{2\,\lambda}{b} = \frac{2 \times 8 \times 10^{-7}}{10^{-6}} = 1.6\,\mathrm{rad} \approx 90^\circ$$

"horizontal spread" $$\Delta\theta \approx \frac{2\,\lambda}{b} = \frac{2 \times 8 \times 10^{-7}}{10 \times 10^{-6}} = 0.16\,\mathrm{rad} \approx 9^\circ$$

26-33. The band gap energy is the energy of the emitted photon,

$$E = \frac{h\,c}{\lambda} = \frac{1240\,\mathrm{eV} \cdot \mathrm{nm}}{800} = 1.55\,\mathrm{eV}$$

26-34. Assuming equal reflectance R from each cleaved end leads to the threshold condition

$$R^2\,e^{2\gamma_0 L} = 1$$

Solving for the R,

$$R = e^{-\gamma_0 L} = e^{-(40)(0.02)} = 0.45$$

26-35. (a) In steady state Eqs. (26 54) and (26 55) become,

$$0 = R_{p2} - \kappa_2 N_2 - \sigma c N_p N_2 \tag{1}$$
$$0 = -\Gamma N_p + (V_g/V_c)\,\sigma\, c\, N_p N_2 \tag{2}$$

From (2),

$$N_2 = \frac{\Gamma}{\sigma c}\frac{V_c}{V_g}$$

Using this in (1) gives,

$$N_p = \frac{R_{p2} - \kappa_2 N_2}{\sigma c N_2} = \frac{R_p - \frac{\kappa_2 \Gamma}{\sigma c}\frac{V_c}{V_g}}{\Gamma V_c/V_g} = \frac{\sigma c\,(V_g/V_c)\,R_{p2} - \kappa_2 \Gamma}{\sigma\, c\,\Gamma}$$

(b) In problem 26 23 it is shown that $\Gamma = \frac{c}{P}(1-S)$. Also $V_g/V_c = L/P$ where I have taken the cross sec tional area A of the beam to be the same as that of the gain medium. The rate that photons leave the cavity is given by

$$\frac{d(N_p V_c)}{dt} = T_3 \frac{c}{P} V_c N_p \Rightarrow I_{\text{out}} = \frac{1}{A} h\nu' \frac{d(N_p V_c)}{dt} = \frac{1}{A} h\nu' T_3\,(c/P)\; V_c N_p = T_3 h\nu' c N_p$$

$$I_{\text{out}} = h\nu' c T_3 \frac{\sigma c\,(L/P)\,R_{p2} - \kappa_2\,(1-S)\,c/P}{\sigma c\,(1-S)\,c/P} = T_3 \frac{\kappa_2 h\nu'}{\sigma} \frac{(\sigma R_{p2} L/\kappa_2) - (1-S)}{1-S}$$

(c) Now, $\kappa_2 = 1/\tau_2$ so that

$$\frac{\kappa_2 h\nu'}{\sigma} = \frac{h\nu'}{\sigma \tau_2} = I_S \text{ and } \frac{\sigma R_{p2}}{\kappa_2} = \sigma R_{p2}\tau_2 = \gamma_0$$

Then,

$$I_{\text{out}} = T_3 I_S \frac{\gamma_0 L - (1-S)}{1-S}$$

This agrees with Eq. (26 43) only for parameters such that

$$1 - S \approx \ln(1/S)$$

This is approximately valid if $S = 1 - \mathcal{L}$ is close to one so that $\mathcal{L} \ll 1$. Then

$$\ln(1/S) = \ln\left(\frac{1}{1-\mathcal{L}}\right) = -\ln(1-\mathcal{L}) \approx -(-\mathcal{L}) \approx \mathcal{L} = 1 - S \quad (\mathcal{L} \ll 1)$$

So for $1 - S \ll 1$, (low losses) the approach taken in this problem agrees with the approach taken in the body of the text leading to Eq. (26 43).

(d) If S is not close to one the approach leading to Eq. (26 43) remains valid. The formation of the rate equation for N_p given as Eq. (26 55) is not valid for large losses as the approach borrowed the cavity loss term from the approach taken in Chapter 8 (see also the solution to problem 26 23) which requires that the loss per round trip be small for its validity.

Chapter 27 Characteristics of Laser Beams

27-1. (a) The TEM_{00} Gaussian beam is similar to a plane wave in that the direction of energy flow is primarily in a given direction and at the beam waist and in the far field the phase fronts are planar. The irradiance in a TEM_{00} beam dies off in the radial direction whereas a plane wave represents the (impossible) situation for which the wave exists in all space.

(b) TEM_{00} Gaussian beams have nearly spherical phase fronts but the center of curvatures of these phase fronts changes with distance from the beam waist. The center of curvature of all of the phase fronts of a spherical wave is at the position of the source of the wave. A TEM_{00} beam is confined in the (cylindrical) radial direction and propagates primarily in a given linear direction. Spherical waves propagate radially outward from the source.

27-2. Eq. (27 3) has the form,

$$\nabla^2 \tilde{E} - \frac{n^2}{c^2}\frac{\partial^2 \tilde{E}}{\partial t^2} = 0$$

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}\right)\tilde{E} - \frac{k^2}{\omega^2}\frac{\partial^2 \tilde{E}}{\partial t^2} = 0$$

Take,

$$\tilde{E} = U(x,y,z)\, e^{i(kz-\omega t+\varphi)}$$

The necessary x, y and t derivatives are easy to obtain.

$$\frac{\partial^2}{\partial x^2}\tilde{E} = e^{i(kz-\omega t+\varphi)}\frac{\partial^2}{\partial x^2}U$$

$$\frac{\partial^2}{\partial y^2}\tilde{E} = e^{i(kz-\omega t+\varphi)}\frac{\partial^2}{\partial y^2}U$$

$$\frac{\partial^2 \tilde{E}}{\partial t^2} = e^{i(kz-\omega t+\varphi)}\left(-\omega^2 U\right)$$

For z and t a bit more work is required.

$$\frac{\partial \tilde{E}}{\partial z} = e^{i(kz-\omega t+\varphi)}\frac{\partial U}{\partial z} + i\,k\,U\,e^{i(kz-\omega t+\varphi)}$$

$$\frac{\partial^2 \tilde{E}}{\partial z^2} = \frac{\partial}{\partial z}\left(e^{i(kz-\omega t+\varphi)}\frac{\partial U}{\partial z} + i\,k\,U\,e^{i(kz-\omega t+\varphi)}\right)$$

$$\frac{\partial^2 \tilde{E}}{\partial z^2} = i\,k\,e^{i(kz-\omega t+\varphi)}\frac{\partial U}{\partial z} + e^{i(kz-\omega t+\varphi)}\frac{\partial^2 U}{\partial z^2} + i\,k\,e^{i(kz-\omega t+\varphi)}\frac{\partial U}{\partial z} - k^2\,U\,e^{i(kz-\omega t+\varphi)}$$

$$\frac{\partial^2 \tilde{E}}{\partial z^2} = e^{i(kz-\omega t+\varphi)}\left[2\,i\,k\,e^{i(kz-\omega t+\varphi)}\frac{\partial U}{\partial z} - k^2 U\,e^{i(kz-\omega t+\varphi)} + \frac{\partial^2 U}{\partial z^2}\right]$$

Similarly,

$$\frac{\partial^2 \tilde{E}}{\partial t^2} = e^{i(kz-\omega t+\varphi)}\left[-2\,i\,\omega\,e^{i(kz-\omega t+\varphi)}\frac{\partial U}{\partial t} - \omega^2 U\,e^{i(kz-\omega t+\varphi)} + \frac{\partial^2 U}{\partial t^2}\right]$$

Using these all in the wave equation gives

$$e^{i(kz-\omega t+\varphi)}\left\{\frac{\partial^2 U}{\partial x^2} + \frac{\partial^2 U}{\partial y^2} + \frac{\partial^2 U}{\partial z^2} + 2\,i\,k\,\frac{\partial U}{\partial z} - \left(k^2 - \frac{k^2}{\omega^2}\omega^2\right)U\right\} = 0$$

$$\frac{\partial^2 U}{\partial x^2} + \frac{\partial^2 U}{\partial y^2} + \frac{\partial^2 U}{\partial z^2} + 2\,i\,k\,\frac{\partial U}{\partial z} = 0$$

27-3. We wish to integrate,

$$\frac{dp}{dz} = \frac{i}{z - i\,z_0}$$

Separation of variables and subsequent integration proceeds as,

$$-\,i\int_{p(0)}^{p(z)} dp = \int_0^z \frac{1}{z' - i\,z_0} dz'$$

$$-\,i\,(p(z) - p(0)) = \ln(z' - i\,z_0)|_0^z = \ln\!\left(\frac{z - i\,z_0}{-\,i\,z_0}\right) = \ln(1 + i\,z/z_0)$$

$$e^{-ip(z)}\,e^{ip(0)} = 1 + i\,z/z_0 = \sqrt{1 + z^2/z_0^2}\;e^{i\tan^{-1}(z/z_0)} = \sqrt{\frac{z^2 + z_0^2}{z_0^2}}\,e^{i\tan^{-1}(z/z_0)}$$

$$e^{ip(z)} = \sqrt{\frac{z_0^2}{z^2 + z_0^2}}\,e^{-i\tan^{-1}(z/z_0)} e^{ip(0)}$$

The phase factor $e^{ip(0)}$ can be absorbed into the overall phase factor in the waveform.

27-4. (a) The beam waist occurs at $z = 0$. So, using Eqs. (27 14) and (27 21)

$$q(0) = -\,i\,z_0 = -\,i\,\frac{\pi\,w_0^2}{\lambda} = -\,i\,\frac{\pi\,(5 \times 10^{-4})^2}{632.8 \times 10^{-9}}\,\mathrm{m} = -\,i\,(1.24\,\mathrm{m})$$

(b) Using the result from (a),

$$q(z) = z - i\,z_0 \Rightarrow q(50\,\mathrm{m}) = 50\,\mathrm{m} - i\,(1.24\,\mathrm{m})$$

It would be a waste of effort to use

$$\frac{1}{q} = \frac{1}{R} + i\,\frac{\lambda}{\pi\,w^2(z)}$$

directly since it would simply lead algebraically to the $q = z - i\,z_0$ through the use of Eqs (21 29) and (27 20). However it is instructive to note that since $z = 50\,\mathrm{m} \gg z_0 = 1.24\,\mathrm{m}$ and so $R \approx z$. Further the spot size in the far field is given approximately by,

$$w(z) = z\,\frac{\pi}{\lambda\,w_0} = z\,\theta_{FF} = \left(50 \times 4 \times 10^{-4}\,\mathrm{m}\right) = 0.02\,\mathrm{m}$$

$$\frac{1}{q} = \frac{1}{R} + i\,\frac{\lambda}{\pi\,w^2(z)} \approx \frac{1}{50\,\mathrm{m}} + i\,\frac{632.8 \times 10^{-9}}{\pi\,(0.02)^2\,\mathrm{m}} \Rightarrow q \approx \frac{1}{0.02 + i(5.036 \times 10^{-4})}\,\mathrm{m} = 49.97\,\mathrm{m} - i\,(1.26\,\mathrm{m})$$

This gives a feel for the nature and appropriateness of the far field approximations.

27-5. (a), (b) Using Eqs. (27 19) and (27 20) together with some of the results from the solution to problem 27 4,

$$R(z) = z\left(1 + \frac{z_0^2}{z^2}\right) = (50\,\mathrm{m})\left[1 + \left(\frac{1.24}{50}\right)^2\right] = 50.03\,\mathrm{m} \approx z$$

$$w(z) = \sqrt{\frac{\lambda z_0}{\pi}\left(1 + z^2/z_0^2\right)} = 0.02016 \approx z\,\theta_{FF} = 0.02\,\mathrm{m}$$

27-6. (a) From symmetry considerations the beam waist must be at the center of the cavity. More formally, using Eqs (27 32) and (27 33) as follows,

$$\frac{R_{M2}}{R_{M1}} = -1 = \frac{z_2}{z_1}\frac{1+z_0^2/z_2^2}{1+z_0^2/z_2^2} = \frac{z_2/z_0}{z_1/z_0}\frac{1+z_0^2/z_2^2}{1+z_0^2/z_2^2} \equiv \frac{u_2}{u_1}\frac{1+1/u_2^2}{1+1/u_1^2} \qquad u_{1,2} \equiv z_{1,2}/z_0$$

$$-1 = \frac{u_1}{u_2}\frac{1+u_2^2}{1+u_1^2} \Rightarrow u_2^2\,u_1 + u_2(1+u_1^2) + u_1$$

$$u_2 = \frac{-1-u_1^2 \pm \sqrt{(1+u_1^2)^2 - 4\,u_1^2}}{2\,u_1} = \frac{-1-u_1^2 \pm (1-u_1^2)}{2\,u_1} = -u_1 \text{ or } -1/u_1$$

For $u_2 = -u_1$, $z_2 = -z_1$. Then using Eq. (27 34), $z_2 - z_1 = 2\,z_2 = d \Rightarrow z_2 = d/2$, $z_1 = -d/2$. This is the solu tion of interest.

But what about the other solution? For this case $z_1 = -z_0^2/z_2$ and Eq. (27 34) gives

$$z_2 + z_0^2/z_2 = z_2(1 + z_0^2/z_2^2) = d$$

But $z_2(1 + z_0^2/z_2^2) = R_{M2}$ and since in this case R_{M2} does not equal d, the second solution is not valid for this case.

All in all, it is best to use the symmetry argument when it is available.

(b) Use, $R(z) = R_{M2} = z_2\left(1 + z_0^2/z_2^2\right) \Rightarrow z_0 = \sqrt{\left(\frac{R_{M1}}{z_1} - 1\right) z_1^2} = \sqrt{\left(\frac{10}{0.17} - 1\right)}(0.17\,\text{m})$,

$$z_0 = 1.293\,\text{m} = \pi\,w_0^2/\lambda \Rightarrow w_0 = \sqrt{z_0\,\lambda/\pi} = \sqrt{(1.293)\,(632.8\times10^{-9})/\pi} = 5.10\times10^{-4}\,\text{m} = 0.510\,\text{mm}$$

(c) The spot sizes at the mirrors are equal to each other and each have the value,

$$w(z_2) = w_0\sqrt{1 + z_2^2/z_0^2} = (0.510\,\text{mm})\,\sqrt{1 + 0.17^2/1.293^2} = 0.514\,\text{mm}$$

(d) $\theta_{FF} = \dfrac{\lambda}{\pi\,w_0} = \dfrac{632.8\times10^{-9}}{\pi\,(5.10\times10^{-4})} = 0.395\,\text{mrad}$

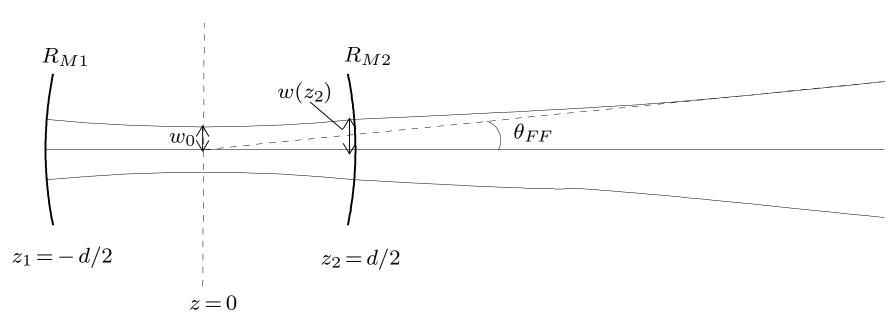
(e) $z_{FF} \gg 50\,(1.293\,\text{m}) = 64.6\,\text{m}$

(f) The irradiance at the center of the beam waist is found (see problem 27 13) from,

$$I_0 = \frac{2\,\Phi_{\text{tot}}}{\pi\,w_0^2} = \frac{(0.01\,\text{W})}{\pi\,(0.051\,\text{cm})^2} = 1.22\,\text{W/cm}^2$$

Using Eq. (27 26),

$$I(\rho = 0, z = z_{FF}) = I_0\frac{z_0^2}{(z_{FF})^2 + z_0^2} = \left(1.22\,\text{W/cm}^2\right)\left(\frac{1.293^2}{64.6^2 + 1.293^2}\right) = 4.89\times10^{-4}\,\text{W/cm}^2$$

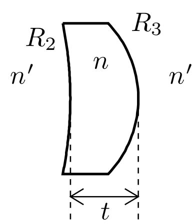


27-7. (a) The $ABCD$ matrix is,

$$\begin{bmatrix} 1 & 0 \\ \frac{n-n'}{R_3 n'} & \frac{n}{n'} \end{bmatrix}\begin{bmatrix} 1 & t \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ \frac{n-n'}{R_2 n'} & \frac{n}{n'} \end{bmatrix}=\begin{bmatrix} 1 & 0 \\ \frac{0.5}{-0.64} & \frac{1.5}{1} \end{bmatrix}\begin{bmatrix} 1 & 0.004 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ \frac{-0.5}{-1.5(-2)} & \frac{1}{1.5} \end{bmatrix}=\begin{bmatrix} 1.0007 & 0.0027 \\ -0.5318 & 0.9979 \end{bmatrix}$$

(b) For $t \to 0$:

$$=\begin{bmatrix} 1 & 0 \\ \frac{0.5}{-0.64} & \frac{1.5}{1} \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ \frac{-0.5}{-1.5(-2)} & \frac{1}{1.5} \end{bmatrix}=\begin{bmatrix} 1 & 0 \\ -0.53125 & 1 \end{bmatrix}$$



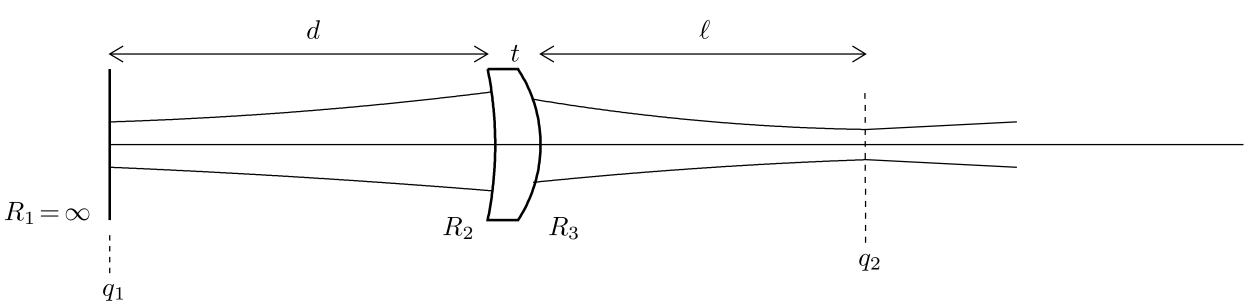
27-8. (a) From the overall matrix in the solution to problem 27 7b,

$$C = -0.53125 = -\frac{1}{f} \Rightarrow f = 1.88\,\text{m}$$

(b) Using the lensmaker's formula for a thin lens,

$$\frac{1}{f} = \left(\frac{n_2 - n_1}{n_1}\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) = \left(\frac{1.5-1}{1}\right)\left(\frac{1}{-2\,\text{m}} - \frac{1}{-0.64\,\text{m}}\right) = 1.88\,\text{m}$$

Figure for Problems 27-9 and 27-10



27-9. (a) The plane mirror has $R = \infty$ so that.

$$\frac{1}{q_1} = \frac{1}{R_1} + i\frac{\lambda}{\pi w_1^2} = \frac{1}{\infty} + i\frac{\lambda}{\pi w_1^2} \Rightarrow q_1 = -i\frac{\pi w_1^2}{\lambda} = -i\, z_{01}$$

Note that what is denoted as w_1 here is the spot size at a the beam waist of the beam in the cavity.

(b) Using Eq. (27 19), with $z = d = 0.7\,\mathrm{m}$, $R = R(z) = 2\,\mathrm{m}$

$$R_2 = z\left(1 + z_{01}^2/z^2\right) \Rightarrow z_0 = z\sqrt{\frac{R}{z} - 1} = (0.7\,\mathrm{m})\sqrt{\frac{2}{0.7} - 1} = 0.954\,\mathrm{m}$$

$$z_{01} = \frac{\pi w_1^2}{\lambda} \Rightarrow w_1 = \sqrt{\frac{\lambda z_{01}}{\pi}} = \sqrt{\frac{\left(632.8 \times 10^{-9}\right)(0.954)}{\pi}} = 4.38 \times 10^{-4}\,\mathrm{m} = 0.438\,\mathrm{mm}$$

(c) Combining the results of parts (a) and (b),

$$q_1 = -i\, z_{01} = -i\,(0.954\,\mathrm{m})$$

(d) At the beam waist outside the cavity,

$$\frac{1}{q_2} = \frac{C q_1 + D}{A q_1 + B}\,\frac{A q_1^* + B}{A q_1^* + B} = \frac{A C q_1 q_1^* + B C q_1 + A D\, q_1^* + B D}{A^2 q_1 q_1^* + B^2}$$

Here, in mks units, $q_1 = -i\,(0.954)$ and,

$$A = 1 - 0.53\,\ell \quad B = 0.7 + 0.63\,\ell$$
$$C = -0.53 \qquad D = 0.63$$

(e) Using Eq. (27 17), $R_2 = \infty$ and the result from (d),

$$\frac{1}{q_2} = \frac{1}{R_2} + i\frac{\lambda}{\pi w_2^2} = \cancel{\frac{1}{R_2}} + i\frac{1}{z_{02}} = \frac{A C q_1 q_1^* + B C q_1 + A D\, q_1^* + B D}{A^2 q_1 q_1^* + B^2}$$

The values for the various parameters listed in part (d). Equating real parts, (in mks units)

$$0 = \frac{A C q_1 q_1^* + B D}{A^2 q_1 q_1^* + B^2} \Rightarrow A C q_1 q_1^* = -B D$$
$$(1 - 0.53\,\ell)\,(-0.53)(0.954)^2 = -(0.7 + 0.63\,\ell)\,(0.63) \Rightarrow \ell = 0.063\,\mathrm{m} = 6.3\,\mathrm{cm}$$

With this, $A = 0.967$, $B = 0.740$, $C = -0.53$, and $D = 0.63$. Equating imaginary parts,

$$z_{02} = \frac{\pi w_2^2}{\lambda} = \frac{A^2 q_1 q_1^* + B^2}{(-BC + AD)\,|q_1|} = 1.464\,\mathrm{m} \Rightarrow w_2 = \sqrt{\frac{\lambda z_{02}}{\pi}} = 5.4 \times 10^{-4}\,\mathrm{m} = 0.54\,\mathrm{mm}$$

27-10. (a) If one treats the output mirror/lens as a thin lens Eqs. (27 46) and (27 47) can be used to predict the location and size of the beam waist. From the solution to problem 27 8a, $f = 1.88$ m. Then using some of the results of the previous series of problems,

$$\frac{1}{w_{02}^2} = \frac{1}{w_{01}^2}\left(1 - \frac{Z_1}{f}\right)^2 + \frac{1}{f^2}\left(\frac{\pi w_{01}}{\lambda}\right)^2 = \frac{1}{(4.38 \times 10^{-4}\,\mathrm{m})^2}\left(1 - \frac{0.7}{1.88}\right)^2 + \frac{1}{(1.88\,\mathrm{m})^2}\left(\frac{\pi\left(4.38 \times 10^{-4}\right)}{632.8 \times 10^{-9}}\right)^2$$
$$w_{02} = 5.43 \times 10^{-4}\,\mathrm{m} = 0.543\,\mathrm{mm}$$

and $Z_2 = f + \dfrac{f^2\,(Z_1 - f)}{(Z_1 - f)^2 + z_{01}^2} = 1.88\,\mathrm{m} + \dfrac{(1.88)^2(0.7 - 1.88)}{(0.7 - 1.88)^2 + 0.954^2}\,\mathrm{m} = 0.069\,\mathrm{m} \approx 7\,\mathrm{cm}$

These answers correspond reasonably well to the answers found with the more exact and exacting for malism of the previous solution.

(b) These approximate formulas are good only if (see the discussion following Eq. (27 48)) $z_{01}^2 \gg (Z_1 - f)^2$. For the case at hand,

$$z_{01}^2 = (0.954\,\mathrm{m})^2 = 0.910\,\mathrm{m}^2,\ (Z_1 - f)^2 = (0.7 - 1.88)^2 = 1.3924\,\mathrm{m}^2$$

So the validity criterion is not met.

27-11. (a) The far field distance for the external beam is

$$z_{FF} = 50\, z_{02} = 50\,(1.464\,\text{m}) = 73.2\,\text{m}$$

Note that z_{02} was calculated in the solution to problem 27 9e.

(b) The far field divergence angle is, $\theta_{FF} = \dfrac{\lambda}{\pi\, w_{02}} = \dfrac{632.8\times 10^{-9}}{\pi\,(5.4\times 10^{-4})} = 3.73\times 10^{-4}\ \text{rad} = 0.373\,\text{mrad}$

(c) The spot size at the entrance to the beam expander is,

$$w_{\text{Ent}}(z=30) = w_{02}\left(1+(z/z_{02})^2\right)^{1/2} = (0.54\,\text{mm})\left(1+30^2/1.464^2\right)^{1/2} = 11.08\,\text{mm}$$

Assume that 30 m is in the far field (it is not according to our adopted criterion but the ratio $z/z_{02} = (30/1.464)$ is large enough that the far field formulas are reasonably good. For example using the far field divergence angle to estimate w_{Ent} gives,

$$w_{\text{Ent}} \approx z\,\theta_{FF} = (30\,\text{m})\left(3.73\times 10^{-4}\right) = 0.01119\,\text{m} = 11.19\,\text{mm}$$

which is a good approximation. The expanded beam has a radius of $10\times w_{\text{Ent}} \approx 111\,\text{mm} = 11.1\,\text{cm}$.

(d) The Rayleigh range for the beam exiting the beam expander is

$$z_{0\text{BE}} = \frac{\pi\omega_{0\text{BE}}^2}{\lambda} = \frac{\pi\,(0.111)^2}{632.8\times 10^{-9}}\,\text{m} = 6.12\times 10^4\,\text{m}$$

Using Eq. (22 47),

$$Z_2 = f + \frac{f^2\,(Z_1 - f)}{(Z_1 - f)^2 + z_{0\text{BE}}^2} = 0.1\,\text{m} + \frac{(0.1)^2(0.2-0.1)}{(0.2-0.1)^2 + (6.12\times 10^4)^2}\,\text{m} \approx 0.1\,\text{m}$$

Using Eq. (22 46),

$$\frac{1}{w_{02}^2} = \frac{1}{w_{01}^2}\left(1 - \frac{Z_1}{f}\right)^2 + \frac{1}{f^2}\left(\frac{\pi\, w_{01}}{\lambda}\right)^2 = \frac{1}{(11.1\times 10^{-2}\,\text{m})^2}\left(1 - \frac{0.2}{0.1}\right)^2 + \frac{1}{(0.1\,\text{m})^2}\left(\frac{\pi\left(11.1\times 10^{-2}\right)}{632.8\times 10^{-9}}\right)^2$$

$$\frac{1}{w_{02}^2} \approx \frac{1}{(0.1\,\text{m})^2}\left(\frac{\pi\left(11.1\times 10^{-2}\right)}{632.8\times 10^{-9}}\right) \Rightarrow w_{02} = 1.81\times 10^{-7}\text{m} = 0.18\,\mu\text{m}$$

This spot size is less than λ and so one must question the validity of using the Gaussian beam analysis.

Since $z_{0\text{BE}}^2 \gg (Z_1 - f)^2$, the approximate formulas would work well.

27-12. (a), (b) Equations (22 47) and (22 46) with $Z_1 = 0$ gives directly,

$$Z_2 = f + \frac{f^2(-f)}{(-f)^2 + z_{01}^2} = f\left(1 - \frac{f^2/z_{01}^2}{1 + f^2/z_{01}^2}\right) = \frac{f}{1 + f^2/z_{01}^2}$$

$$\frac{1}{w_{02}^2} = \frac{1}{w_{01}^2} + \frac{1}{f^2}\left(\frac{\pi\, w_{01}}{\lambda}\right)^2 = \frac{1}{w_{01}^2}\left[1 + \frac{1}{f^2}\left(\frac{\pi\, w_{01}^2}{\lambda}\right)^2\right] = \frac{1}{w_{01}^2}\left[1 + \frac{z_{01}^2}{f^2}\right]$$

(c) For example, for a lens of focal length 20 cm and for light of wavelength 500 nm, $Z_2 \approx f$ if,

$$\frac{f^2}{z_{01}^2} \ll 1 \Rightarrow f^2 \ll \left(\pi\, w_{01}^2/\lambda\right)^2$$

$$\frac{f^2\,\lambda^2}{\pi^2} \ll w_{01}^4$$

$$1.01\times 10^{-15}\,\text{m}^4 \ll w_{01}^4$$

$$w_{01} > 1.783\times 10^{-4}\,\text{m}$$

So for these parameters as long as w_{01} is suitable larger than about 0.2 mm, $Z_2 \approx f$.

27-13. The total power is the irradiance integrated over the transverse plane,

$$\Phi_{\text{tot}} = \int_A I\,dA = I_0 \left(\frac{w_0}{w(z)}\right)^2 \int_0^{2\pi} d\varphi \int_0^{\infty} e^{-2\rho^2/w^2(z)} \rho\,d\rho$$

$$\text{Let } u = 2\rho^2/w^2(z) \Rightarrow d\,u = 4\rho\,d\rho/w^2(z) \Rightarrow \rho\,d\rho = du\,w^2(z)/4$$

$$\Phi_{\text{tot}} = 2\pi I_0 \left(\frac{w_0}{w(z)}\right)^2 \left(\frac{w^2(z)}{4}\right) \int_0^{\infty} e^{-u} du = \left(\frac{\pi}{2} w_0^2\right) I_0$$

27-14. Using the change of variable introduced in the solutions to problem 27 13, the fraction of transmitted power can be formed as

$$F = \frac{\Phi(a)}{\Phi_{\text{tot}}} = \frac{\int_0^{2a^2/w^2} e^{-u} du}{\int_0^{\infty} e^{-u} du} = 1 - e^{-2a^2/w^2}$$

27-15. Use the result of problem 27 14 and compare the powers transmitted with the iris all the way open and opened to some radius a, then the ratio F of these powers is,

$$F = 1 - e^{-2a^2/w^2}$$

so that,

$$e^{-2a^2/w^2} = 1 - F \Rightarrow -2a^2/w^2 = \ln(1-F) = -\ln\left(\frac{1}{1-F}\right)$$

$$w = \frac{\sqrt{2}\,a}{\sqrt{\ln\left(\frac{1}{1-F}\right)}}$$

27-16. The collimated beam length is taken to be $2z_0 = 2(\pi w_0^2/\lambda)$. Also $d = 4.5(\sqrt{2}\,w_0)$ so $w_0 = d/(4.5\sqrt{2})$. Thus,

$$2z_0 = \frac{\pi d^2}{20.25\,\lambda} = \left(\frac{\pi}{20.25\,(1.064\times10^{-6}\,\text{m}}\right) d^2 = \left(1.458\times10^{-5}\,\text{m}^{-1}\right) d^2$$

$$(2z_0)_{d=1\text{cm}} = 14.58\,\text{m}$$
$$(2z_0)_{d=2\text{cm}} = 58.3\,\text{m}$$
$$(2z_0)_{d=3\text{cm}} == 131.2\,\text{m}$$
$$(2z_0)_{d=5\text{cm}} = 364.5\,\text{m}$$

27-17. Equation (27 58) gives,

$$H_m(\xi) = (-1)^m e^{\xi^2} \frac{d^m}{d\xi^m}\left(e^{-\xi^2}\right)$$

$$H_0(\xi) = (-1)^0 e^{\xi^2} \frac{d^0}{d\xi^0}\left(e^{-\xi^2}\right) = (-1)^0 e^{\xi^2} e^{-\xi^2} = 1$$

$$H_1(\xi) = (-1)^1 e^{\xi^2} \frac{d}{d\xi}\left(e^{-\xi^2}\right) = -e^{\xi^2}\left(-2\xi e^{-\xi^2}\right) = 2\xi = \frac{2\sqrt{2}\,x}{w}$$

$$H_2(\xi) = (-1)^2 e^{\xi^2} \frac{d^2}{d\xi^2}\left(e^{-\xi^2}\right) = e^{\xi^2} \frac{d}{d\xi}\left(-2\xi e^{-\xi^2}\right) = e^{\xi^2}\left(-2e^{-\xi^2} - 2\xi(-2\xi)e^{-\xi^2}\right)$$

$$H_2(\xi) = 4\xi^2 - 2 = 4\left(\frac{\sqrt{2}\,x}{w}\right)2 - 2 = \frac{8x^2}{w^2} - 2$$

27-18. Given,

$$p(z) = \frac{i}{2}\ln\left[\frac{\lambda^2 z^2 + \left(\pi w_0^2\right)^2}{\left(\pi w_0^2\right)^2}\right] - (m+n+1)\tan^{-1}\left(\frac{\lambda z}{\pi w_0^2}\right)$$

Now, $z_0 = \pi w_0^2/\lambda$, so,

$$i\,p(z) = -\frac{1}{2}\ln\left[\frac{z^2 + \left(\pi w_0^2/\lambda\right)^2}{\left(\pi w_0^2/\lambda\right)^2}\right] - i(m+n+1)\tan^{-1}\left(\frac{\lambda z}{\pi w_0^2}\right)$$

$$i\,p(z) = -\frac{1}{2}\ln\left(\frac{z^2 + z_0^2}{z_0^2}\right) - i(m+n+1)\tan^{-1}(z/z_0)$$

Recall that,

$$w^2(z) = w_0^2\left(1 + z^2/z_0^2\right) = w_0^2\left(\frac{z^2+z_0^2}{z_0^2}\right) \Rightarrow \left(\frac{z^2+z_0^2}{z_0^2}\right) = \frac{w^2(z)}{w_0^2}$$

Then,

$$i\,p(z) = -\frac{1}{2}\ln\left(\frac{w^2(z)}{w_0^2}\right) - i(m+n+1)\tan^{-1}(z/z_0)$$

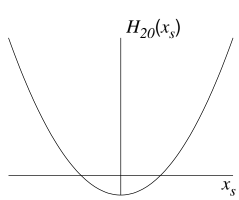
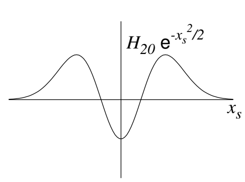
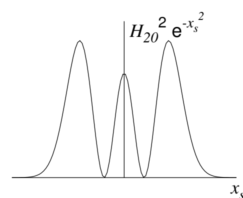
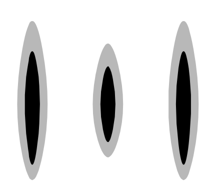
$$i\,p(z) + i(m+n+1)\tan^{-1}(z/z_0) = \ln\left[\left(\frac{w^2(z)}{w_0^2}\right)^{-1/2}\right]$$

$$i\,p(z) + i(m+n+1)\tan^{-1}(z/z_0) = \ln\left(\frac{w_0}{w(z)}\right)$$

$$e^{ip(z)+i(m+n+1)\tan^{-1}(z/z_0)} = \frac{w_0}{w(z)}$$

$$e^{ip(z)} = \frac{w_0}{w(z)}e^{-i(m+n+1)\tan^{-1}(z/z_0)}$$

27-19. The row appears as

$H_2(x_s)$	$H_2\,e^{-x_s^2/2}$	$\left(H_2\,e^{-x_s^2/2}\right)^2$	Burn Pattern
			

27-20. (a) For the TEM_{00} mode the fraction transmitted is given in problem 27 14 as $F_{00} = \dfrac{\Phi(a)}{\Phi_{\text{tot}}} = 1 - e^{-2a^2/w^2}$

(b) For TEM_{01}: The irradiance is

$$I_{01} = I_0\big(w_0^2/w^2\big)\, H_0^2(x_s)\, H_1^2(y_s)\, e^{-(\rho^2/w^2)} = I_0\big(w_0^2/w^2\big)\left(2\sqrt{2}\, y/w\right) e^{-\rho^2/w^2}$$

Do the integrals in cylindrical coordinates with $y = \rho \sin\varphi$. The fraction transmitted is

$$F_{01} = \frac{\Phi(a)}{\Phi_{\text{tot}}} = \frac{\int_0^{2\pi}\int_0^a I_{01}\,\rho\, d\rho\, d\varphi}{\int_0^{2\pi}\int_0^\infty I_{01}\,\rho\, d\rho\, d\varphi}$$

Further make the change of variables used in problem 27 14: $u = 2\,\rho^2/w^2(z) \Rightarrow \rho\, d\rho = du\, w^2(z)/4$
After canceling common terms, in the expression for F_{01},

$$F_{01} = \frac{\int_0^{2\pi}\sin^2\varphi\, d\varphi \int_0^{2a^2/w^2} u\, e^{-u} du}{\int_0^{2\pi}\sin^2\varphi\, d\varphi \int_0^\infty u\, e^{-u} du}$$

I'll use the free computer algebra system,

```
(%i1) T integrate((sin(j))^2,j,0,2*%pi)*integrate(u*exp(-u),u,0,2*a^2/w^2)/%pi/integrate(u*exp(-u),u,0,inf);
Is a zero or nonzero?nonzero
```

(%o7) $F_{01} = 1 - \dfrac{(w^2 + 2\,a^2)\, e^{-\frac{2a^2}{w^2}}}{w^2}$

Simplification gives, $F_{01} = 1 - \big(1 + 2\,a^2/w^2\big)\, e^{-2a^2/w^2}$

(c) Similarly, for the TEM_{11} mode with $y = \rho\sin\varphi, x = \rho\cos\varphi, u = 2\,\rho/w^2$

$$F_{11} = \frac{\int_0^{2\pi}\int_0^\infty \left(2\sqrt{2}y/w\right)^2 \left(2\sqrt{2}x/w\right)^2 e^{-2\rho^2/w^2}\rho\, d\rho d\varphi}{\int_0^{2\pi}\int_0^\infty \left(2\sqrt{2}y/w\right)^2 \left(2\sqrt{2}x/w\right)^2 e^{-2\rho^2/w^2}\rho\, d\rho\, d\varphi} = \frac{\int_0^{2\pi}\sin^2\varphi\cos^2\varphi\, d\varphi \int_0^{2a^2/w^2} u^2 e^{-u} du}{\int_0^{2\pi}\sin^2\varphi\cos^2\varphi\, d\varphi \int_0^\infty u^2 e^{-u} du}$$

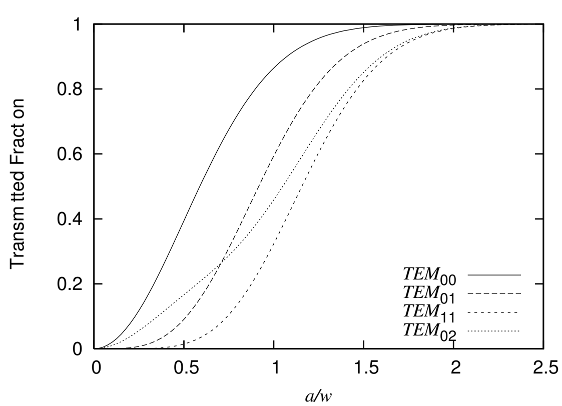
Integration under Maxima and subsequent simplification gives, $F_{11} = 1 - \left(1 + \dfrac{2\,a^2}{w^2} + \dfrac{1}{2}\left(\dfrac{2\,a^2}{w^2}\right)^2\right) e^{-2a^2/w^2}$

(d) The same procedure gives

$$F_{02} = \frac{\int_0^{2\pi}\int_0^\infty \left(4\,(\sqrt{2}y/w)^2 - 2\right)^2 e^{-2\rho^2/w^2}\rho\, d\rho d\varphi}{\int_0^{2\pi}\int_0^\infty \left(4\,(\sqrt{2}y/w)^2 - 2\right)^2 e^{-2\rho^2/w^2}\rho\, d\rho d\varphi} = \frac{\int_0^{2\pi}\int_0^{2a^2/w^2}\big(16u^2\sin^4\varphi - 8\,u\sin^2\varphi + 4\big)e^{-u} du\, d\varphi}{\int_0^{2\pi}\int_0^\infty \big(16u^2\sin^4\varphi - 8\,u\sin^2\varphi + 4\big)e^{-u} du\, d\varphi}$$

Simplification gives, $F_{02} = 1 - \left(\dfrac{1}{2}\big(2\,a^2/w^2\big)^2 + \dfrac{2}{3}\big(2\,a^2/w^2\big) + 1\right) e^{-2a^2/w^2}$

27-21.



27-22. From the transmittance curves in problem 27 21 it is evident that the transmittance of the TEM_{00} mode is significantly higher than that of the higher order modes for apertures $a/w<2$. It is somewhat higher at all finite aperture sizes. The spot size w is determined by the nature of the cavity and is the same for all of the modes. One could place an adjustable aperture in the laser cavity and change the size of the aperture until only the TEM_{00} mode has low enough loss to lase.

27-23. (a) $\theta_{FF}=\dfrac{\lambda}{\pi w_0}\Rightarrow w_0=\dfrac{\lambda}{\pi\theta_{FF}}=\dfrac{488\times 10^{-9}}{\pi\,(0.001)}\,\mathrm{m}=1.55\times 10^{-4}\,\mathrm{m}=0.155\,\mathrm{mm}$

(b) $I_0=\dfrac{2\,\Phi_{\mathrm{tot}}}{\pi w_0^2}=\dfrac{10}{\pi\,(1.55\times 10^{-4})^2}\mathrm{W/m^2}=1.32\times 10^8\ \mathrm{W/m^2}=1.32\times 10^4\ \mathrm{W/cm^2}$

(c) The Rayleigh range is $z_0=\dfrac{\pi w_0^2}{\lambda}=\dfrac{\pi\left(1.55\times 10^{-4}\right)^2}{488\times 10^{-9}}\mathrm{m}=0.155\,\mathrm{m}$. The irradiance is then

$$I(\rho=0,z=10\,\mathrm{m})=\frac{z_0^2}{z^2+z_0^2}I_0=\frac{0.155^2}{10^2+0.155^2}\left(1.32\times 10^4\ \mathrm{W/cm^2}\right)=3.17\ \mathrm{W/cm^2}$$

27-24. The Rayleigh range is needed:
The beam waist is, by symmetry, in the center of the cavity and so the $z_1=-\,0.1$ m and $z_2=0.1$ m. Also $R_2=1\,\mathrm{m}$. Then,

$$R(z_2)=z_2\left(1+\frac{z_0^2}{z_2^2}\right)\Rightarrow z_0=z_2\sqrt{\frac{R(z_2)}{z_2}-1}=0.3\,\mathrm{m}$$

Equation (27 67) gives the cavity mode frequencies as

$$\nu_{mnq}=\left[q+\frac{m+n+1}{\pi}\left(\tan^{-1}(z_2/z_0)-\tan^{-1}(z_1/z_0)\right)\right]$$

Using this relation,

(a) $\nu_{0,0,q+1}-\nu_{0,0,q}=c/2d=\dfrac{3\times 10^8}{0.4}\mathrm{Hz}=750\,\mathrm{MHz}$

(b) $\nu_{m,n,q+1}-\nu_{m,n,q}=c/2d=\dfrac{3\times 10^8}{0.4}\mathrm{Hz}=750\,\mathrm{MHz}$

(c) $\nu_{0,1,q}-\nu_{0,0,q}=\dfrac{c}{2d}\dfrac{1}{\pi}\left(\tan^{-1}\left(\dfrac{z_2}{z_0}\right)-\tan^{-1}\left(\dfrac{z_1}{z_0}\right)\right)=154\,\mathrm{MHz}$

(d) $\nu_{1,0,q}-\nu_{0,0,q}=\dfrac{c}{2d}\dfrac{1}{\pi}\left(\tan^{-1}\left(\dfrac{z_2}{z_0}\right)-\tan^{-1}\left(\dfrac{z_1}{z_0}\right)\right)=154\,\mathrm{MHz}$

(e) $\nu_{1,1,q}-\nu_{0,0,q}=\dfrac{c}{2d}\dfrac{2}{\pi}\left(\tan^{-1}\left(\dfrac{z_2}{z_0}\right)-\tan^{-1}\left(\dfrac{z_1}{z_0}\right)\right)=308\,\mathrm{MHz}$

Chapter 28 Selected Modern Applications

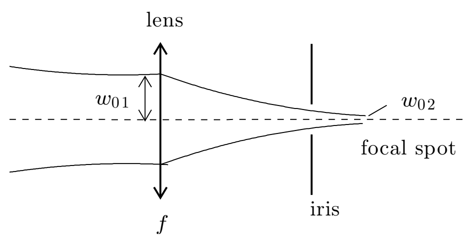
Note: Many of the problems in this chapter are open-ended research problems. We do not provide solutions to those problems.

28-1. (a) Excimer lasers emit ultraviolet electromagnetic radiation. The short wavelength emission can be focused to a smaller spot size and so is useful in micro surgical applications where precise small cuts need to be made. Recall Eq. 27 50 in which it is indicated that the spot size of a focused beam is directly pro portional to the wavelength of the radiation.

(b), (c) Hemoglobin remains strongly absorbing in the UV range and the absorption coefficient of water begins to increase below about 300 nm making excimer lasers useful as scalpels and for photo coagulation.

28-3. The Nd:YAG laser is typically used in this procedure. Possible design parameters are:

Wavelength:	$1.06\,\mu$m
Pulse width:	10 ns
Energy per pulse:	4 mJ
Spot size of beam incident on focusing lens	3.4 mm
Power of focusing lens	20 D



Assume that the beam is incident on the lens at a waist of radius $w_{01} = 3.4$ mm. The Eq. (27 46) (with Z_1=0) give the focal spot radius as,

$$\frac{1}{w_{02}^2} = \frac{1}{w_{01}^2} + \frac{1}{f^2}\left(\frac{\pi\, w_{01}}{\lambda}\right) = \frac{1}{(3.4\times10^{-3})^2} + \frac{1}{(0.05\,\mathrm{m})^2}\left(\frac{\pi\,(3.4\times10^{-3}\,\mathrm{m})}{1.06\times10^{-6}\,\mathrm{m}}\right) \Rightarrow w_{02} = 5\ \mu\mathrm{m}$$

The confocal parameter for the focused beam is,

$$z_{02} = \pi\, w_{02}^2/\lambda = \frac{\pi\,(5\times10^{-6})^2}{1.06\times10^{-6}}\,\mathrm{m} = 7.40\times10^{-5}\,\mathrm{m} = 0.074\,\mathrm{mm}$$

Twice this distance is the length of focal region of the focused spot.

28-4. During the pulse the power in the beam is

$$P = \Phi = \frac{0.004\,\mathrm{J}}{10^{-8}\,\mathrm{s}} = 4.0\times10^5\,\mathrm{W}$$

For a TEM_{00} beam, the irradiance at the center of the spot is given by problem 27 13 as,

$$I = \frac{2\,\Phi}{\pi\, w_{02}^2} = \frac{8\times10^5}{\pi\,(5\times10^{-6})^2}\,\mathrm{W/m^2} = 1.0\times10^{16}\,\mathrm{W/m^2} = 10^{12}\,\mathrm{W/cm^2}$$

This value is in rough agreement with the estimate given in Example 28 1.

28-5. $I = \frac{\varepsilon_0 c}{2}\,E_0^2 \Rightarrow E_0 = \sqrt{\frac{2\,I}{\varepsilon_0\, c}} = \sqrt{\frac{2\cdot5.7\times10^{15}}{8.85\times10^{-12}\cdot3\times10^8}} = 2.1\times10^9\,\mathrm{V/m}$

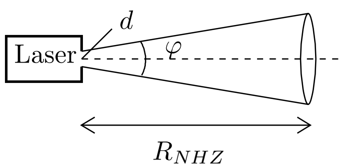
28-6. Using Eq. (27 48) one can estimate the beam diameter on the earth as,

$$D = f\,(2\,\theta_{FF}) = (3.6205\times10^7\,\mathrm{m})\left(5\times10^{-6}\right) = 180\,\mathrm{m}$$

The average irradiance over the beam is found roughly as, $I = \Phi/\left(\pi\, D^2/4\right) = 7700\,\mathrm{W/m^2}$. Using the result from problem 28 5,

$$E_0 = I = \frac{\varepsilon_0 c}{2}\,E_0^2 \Rightarrow E_0 = \sqrt{\frac{2\,I}{\varepsilon_0\, c}} = \sqrt{\frac{2\cdot7700}{8.85\times10^{-12}\cdot3\times10^8}}\,\mathrm{V/m} = 2400\,\mathrm{V/m}$$

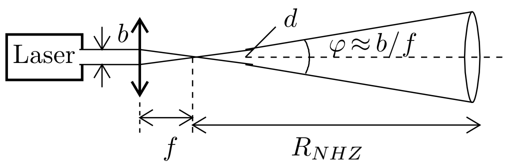
28-7. (a), (b)



$$R_{NHZ}=\frac{1}{\varphi}\left[\left(\frac{4\,P}{\pi\,\mathrm{MPE}}\right)^{1/2}-d\right]=\frac{1}{0.003}\left[\left(\frac{4\times 50}{\pi\,5.1\times 10^{-4}}\right)-0.3\right]\mathrm{cm}$$

$$R_{NHZ}=37,100\,\mathrm{cm}=371\,\mathrm{m}$$

28-8. (a), (b)



$$R_{NHZ}=\frac{f}{b}\left[\left(\frac{4\,P}{\pi\,\mathrm{MPE}}\right)^{1/2}\right]=\frac{7.5}{0.5}\left[\left(\frac{4\times 50}{\pi\,1.6\times 10^{-3}}\right)\right]\mathrm{cm}$$

$$R_{NHZ}=2990\ \mathrm{cm}=29.9\ \mathrm{m}$$

28-10. (a) $\nu_p=\nu_s+\nu_i\Rightarrow\nu_i=\nu_p-\nu_s\Rightarrow\dfrac{1}{\lambda_i}=\dfrac{1}{\lambda_p}-\dfrac{1}{\lambda_s}=\dfrac{1}{1060\,\mathrm{nm}}-\dfrac{1}{1500\,\mathrm{nm}}\Rightarrow\lambda_i=3610\,\mathrm{nm}$

(b) The following two relations must both be satisfied,

$$d=m_s\,\lambda_s/2=m_i\lambda_i/2$$
$$d\approx 1\,\mathrm{m}$$

with d being the cavity length. These conditions are met for, for example, by

$$d=1.001775\,\mathrm{m}=370\,(3610)\,(1500\times 10^{-9}\,\mathrm{m}\big)/2=370\,(1500)\,(3610\,\mathrm{nm})/2$$

In practice one would need active control of the cavity length to maintain the dual resonance.

(c) If both beams are TEM_{00} modes the resonant frequencies are given by Eq. (27 67) as,

$$\nu_{00\,q}=\left[q+\frac{1}{\pi}\big(\tan^{-1}(z_2/z_0)-\tan^{-1}(z_1/z_0)\big)\right]\frac{c}{2\,d}$$
$$d=\frac{c}{2\,\nu_{00\,q}}\left[q+\frac{1}{\pi}\big(\tan^{-1}(z_2/z_0)-\tan^{-1}(z_1/z_0)\big)\right]=\frac{\lambda_{00\,q}}{2}\left[q+\frac{\tan^{-1}(z_2/z_0)}{\pi}\right]$$

Here I have noted that the beam waist is at the flat mirror at $z_1=0$. The confocal parameter can be found from the relation,

$$R_2=R(z_2=d)=d\,(1+z_0^2/d^2)$$
$$\left(\frac{R_2}{d}-1\right)^{-1/2}=d/z_0=z_2/z_0$$

Although d will not be exactly 1 m it is a good approximation to set it to the nominal value in the argu ment inside the arctan function. Then,

$$\frac{\tan^{-1}(z_2/z_0)}{\pi}\approx\frac{\tan^{-1}(2/1-1)^{-1/2}}{\pi}=\frac{1}{4}$$

Then the relations that must be satisfied for dual resonance are,

$$d=\frac{\lambda_s}{2}\left[q_s+\frac{1}{4}\right]=\frac{\lambda_i}{2}\left[q_i+\frac{1}{4}\right]$$

The cavity length could therefore be,

$$d\approx 0.9991366875\,\mathrm{m}=369\,[\,(3610)+1/4](1500\times 10^{-9}\,\mathrm{m}\big)/2=369\,[\,(1500)+1/4](3610\times 10^{-9}\,\mathrm{m}\big)/2$$

Again, one would not set the cavity length to this value but rather scan the cavity length until a dual reso nance is reached and this is the length corresponding to one of the possible dual resonances.

28-11. If the field is resonant with the cavity the and there are no absorption losses all of the incident power is transmitted through the cavity. In that case the intracavity power co propagating with the incident light will be

$$P_{\text{coprop}} = \frac{P_{\text{tansmitted}}}{T} = \frac{P_{\text{inc}}}{(1-R)} = \frac{0.1\,\text{W}}{1-0.97} = 3.33\,\text{W}$$

The counter propagating intracavity field will have a power of,

$$P_{\text{counterprop}} = R\,P_{\text{coprop}} = (0.97)\,(3.33\,\text{W}) = 3.23\,\text{W}$$

28-16. From thermodynamics: $v_{\text{rms}} = \sqrt{\frac{3\,k_B T}{m}}$.

(a) For the oxygen molecule at $T = 68°$ F $= 20°$ C $= 293$ K, $v_{\text{rms}} \approx \sqrt{\frac{3 \cdot 1.38 \times 10^{-23} \cdot 293}{32 \cdot 1.67 \times 10^{-27}}}\,\text{m/s} = 480\,\text{m/s}$

(b) For rubidium at $T = 0.002$ K, $v_{\text{rms}} = \sqrt{\frac{3\,k_B T}{m}} \approx \sqrt{\frac{3 \cdot 1.38 \times 10^{-23} \cdot 0.002}{85.5 \cdot 1.67 \times 10^{-27}}}\,\text{m/s} = 0.8\,\text{m/s}$

28-17. As in the solution to problem 28 16b but with $T = 0.003$ K,

$$v_{\text{rms}} = \sqrt{\frac{3\,k_B T}{m}} \approx \sqrt{\frac{3 \cdot 1.38 \times 10^{-23} \cdot 0.003}{85.5 \cdot 1.67 \times 10^{-27}}}\,\text{m/s} = 0.98\,\text{m/s}$$

The detuning should match the Doppler shift. So,

$$\Delta\nu \approx \nu\, v_{\text{rms}}/c = v_{\text{rms}}/\lambda = (0.98\,\text{m/s})/\left(800 \times 10^{-9}\,\text{m}\right) = 1.225 \times 10^{6}\,\text{Hz}$$