

6.4 Linear Polarizers & Jones Matrices (1)

Polarizer can be represented by a matrix that operates on a Jones vector.

$$\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \quad (\text{polarizer with transmission along } x\text{-axis})$$

$$\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \quad \checkmark \text{ allows passage}$$

$$\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \quad \checkmark \text{ kills light.}$$

$$\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \quad (\text{transmission along } y\text{-axis}).$$

$$\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} A \\ B e^{i\delta} \end{bmatrix} = \begin{bmatrix} A \\ 0 \end{bmatrix} \quad \checkmark$$

Compound Polarization Systems:

$$\begin{bmatrix} A' \\ B' \end{bmatrix} = J_{\text{system}} \begin{bmatrix} A \\ B e^{i\delta} \end{bmatrix}$$

↑
output

↓
input.

$$J_{\text{system}} = \bar{J}_n \bar{J}_{n-1} \dots \bar{J}_2 \bar{J}_1$$

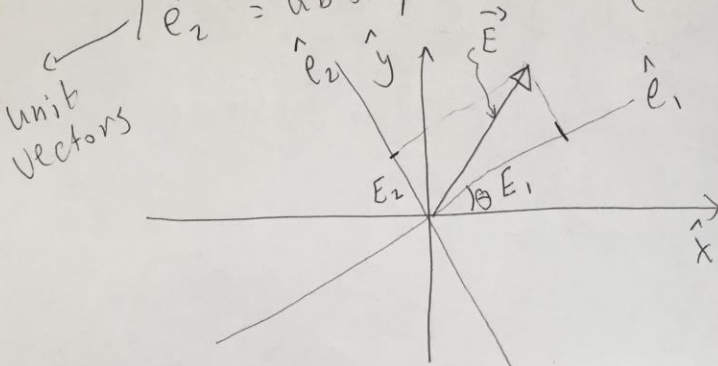
\bar{J}_n = matrix for n^{th} polarizing optical element.

6.5 Jones Matrix for a Polarizer

- Jones matrix for ideal polarizer with transmission axis at an arbitrary θ from x-axis.

Input field: $\vec{E}(z,t) = (E_x \hat{x} + E_y \hat{y}) e^{i(kz - \omega t)}$

let $\begin{cases} \hat{e}_1 = \text{transmission axis} \\ \hat{e}_2 = \text{absorption axis (}\perp \text{ to } \hat{e}_1) \end{cases}$



$$\hat{x} = \hat{e}_1 \cos \theta - \hat{e}_2 \sin \theta$$

$$\hat{y} = \hat{e}_1 \sin \theta + \hat{e}_2 \cos \theta$$

$$\therefore \vec{E}(z,t) = (E_1 \hat{e}_1 + E_2 \hat{e}_2) e^{i(kz - \omega t)}$$

$$E_1 = E_x \cos \theta + E_y \sin \theta$$

$$E_2 = -E_x \sin \theta + E_y \cos \theta$$

E_1 is transmitted.

E_2 is killed (extinguished).

$$\vec{E}_{\text{after}}(z,t) = (E_1 \hat{e}_1 + E_2 \hat{e}_2) e^{i(kz - \omega t)}$$

using $\hat{e}_1 = \cos \theta \hat{x} + \sin \theta \hat{y}$

$$\hat{e}_2 = -\sin \theta \hat{x} + \cos \theta \hat{y}$$

$$\vec{E}_{\text{after}}(z,t) = \left[(E_x \cos \theta + E_y \sin \theta)(\cos \theta \hat{x} + \sin \theta \hat{y}) + (-E_x \sin \theta + E_y \cos \theta)(-\sin \theta \hat{x} + \cos \theta \hat{y}) \right] e^{i(kz - \omega t)}$$

$$= \left[E_x (\cos^2 \theta + \sin^2 \theta) + E_y (\sin \theta \cos \theta - \sin \theta \cos \theta) \right] \hat{x} e^{i(kz - \omega t)}$$

$$+ \left[E_x (\sin \theta \cos \theta - \sin \theta \cos \theta) + E_y (\sin^2 \theta + \cos^2 \theta) \right] \hat{y} e^{i(kz - \omega t)}$$

$\xi = 1 \rightarrow$ no polarizer.

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$$E_{\text{after}}(z,t) = \begin{bmatrix} \cos^2 \theta + \xi \sin^2 \theta & \sin \theta \cos \theta - \xi \sin \theta \cos \theta \\ \sin \theta \cos \theta - \xi \sin \theta \cos \theta & \sin^2 \theta + \xi \cos^2 \theta \end{bmatrix} \begin{bmatrix} E_x \\ E_y \end{bmatrix} \times e^{i(kz - \omega t)}$$

Case 1: $\xi = 0$

$$\begin{bmatrix} \cos^2 \theta & \sin \theta \cos \theta \\ \sin \theta \cos \theta & \sin^2 \theta \end{bmatrix}$$

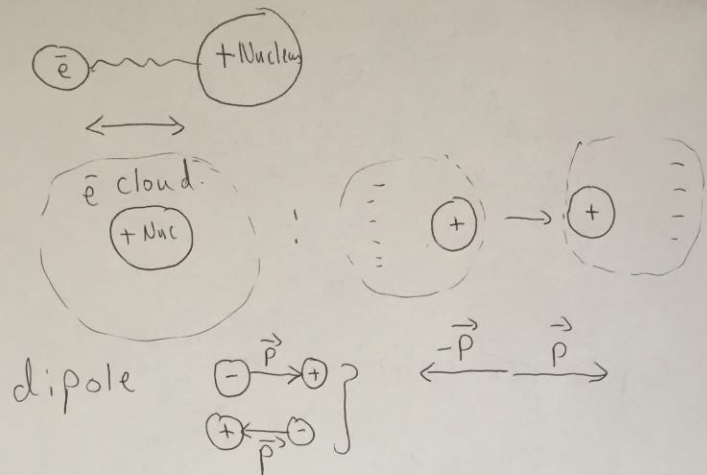
if $\theta = 0 \rightarrow \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$ (\hat{x} transmission)

$\theta = \frac{\pi}{2} \rightarrow \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$ (\hat{y} transmission)

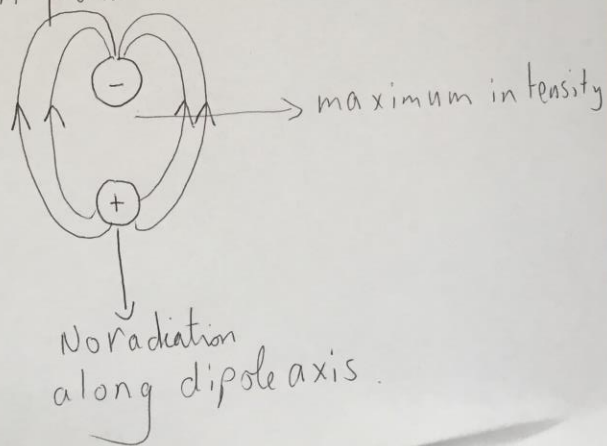
Polarization by Reflection:

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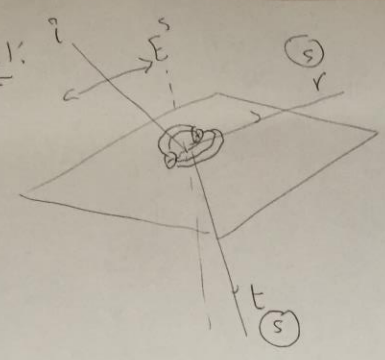
Electron-Oscillator Model.



Radiation pattern:



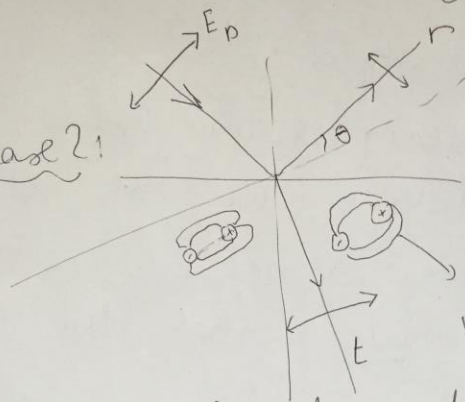
Case 1:



reflected S
transmitted S.

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Case 2:



influenced by refracted wave.

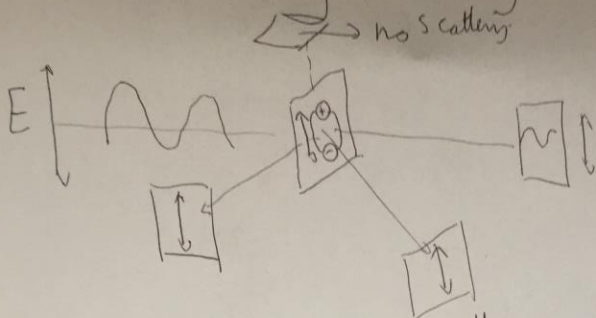
mostly refracted; much less reflected.
For this light, mostly transmitted.

Case 3: Mixture of S, P;

You get S reflected, thus polarization by reflection.

Polarization by Scattering

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Scattering intensity $\propto f^4$

high frequency e.g. blue light is scattered very well. (blue sky)

low frequency e.g. red light is scattered much weaker. (red evenings)

