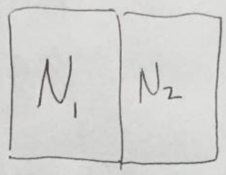
Ch3 Reflection and Refraction

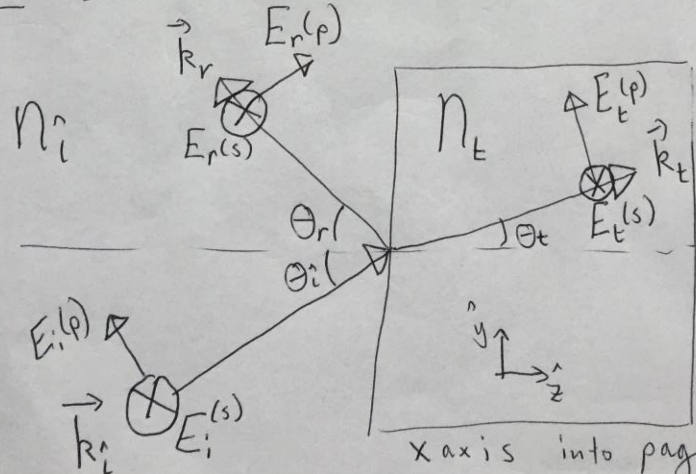


What happens at interface?

Reflection / Transmission

Assumptions: Isotropic media
neglect χ (no absorption).
$\boxed{N = n}$

3.1 Refraction at an interface:



x axis into page.

Assumption: $\vec{k}_i, \vec{k}_r, \vec{k}_t$ lie in a single plane (plane of incidence).

(p) $\longrightarrow$ p-polarized light (parallel to plane of incidence)

(s) $\longrightarrow$ s-polarized light (senkrecht. $\perp$ to plane of incidence).

Electric field confined to plane perpend. to wave-vector.

$$\vec{k}_i = k_i(\hat{y}\sin\Theta_i + \hat{z}\cos\Theta_i)$$
$$\vec{k}_r = k_r(\hat{y}\sin\Theta_r - \hat{z}\cos\Theta_r) \qquad (3.1).$$
$$\vec{k}_t = k_t(\hat{y}\sin\Theta_t + \hat{z}\cos\Theta_t)$$

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$$\vec{E}_i = \left[E_i^{(p)}(\hat{y}\cos\theta_i - \hat{z}\sin\theta_i) + \hat{x}E_i^{(s)}\right] e^{i[k_i(y\sin\theta_i + z\cos\theta_i) - w_i t]}$$

$$\vec{E}_r = \left[E_r^{(p)}(\hat{y}\cos\theta_r + \hat{z}\sin\theta_r) + \hat{x}E_r^{(s)}\right] \times$$
$$\times\ e^{i[k_r(y\sin\theta_r - z\cos\theta_r) - W_r t]}$$

$$\vec{E}_t = \left[E_t^{(p)}(\hat{y}\cos\theta_t - \hat{z}\sin\theta_t) + \hat{x}E_t^{(s)}\right] \times$$
$$\times\ e^{i[k_t(y\sin\theta_t + z\cos\theta_t) - w_t t]}$$

What happens at interface?

$\vec{E}$ parallel to interface is equal on both sides of Interface.

Interface: $z = 0$; $\hat{x}$ and $\hat{y}$ parallel to Interfa

$$\therefore \left[E_i^{(p)}\hat{y}\cos\theta_i + \hat{x}E_i^{(s)}\right] e^{i(k_i y\sin\theta_i - w_i t)}$$
$$+ \left[E_r^{(p)}\hat{y}\cos\theta_r + \hat{x}E_r^{(s)}\right] e^{i(k_r y\sin\theta_r - W_r t)}$$
$$= \left[E_t^{(p)}\hat{y}\cos\theta_t + \hat{x}E_t^{(s)}\right] e^{i(k_t y\sin\theta_t - W_t t)}$$

②

1) 1st deduction:

$$\omega_i = \omega_r = \omega_t \equiv \omega$$

This follows from $(\ )e^{-i\omega_i t} + (\ )e^{-i\omega_r t} = (\ )e^{-i\omega_t t}$

2) 2nd deduction.

$$k_i \sin\theta_i = k_r \sin\theta_r = k_t \sin\theta_t$$

$$k_i = k_r = \frac{n_i \omega}{c} \quad ; \quad k_t = \frac{n_t \omega}{c}$$

$$\therefore \cancel{\frac{n_i \omega}{c}} \sin\theta_i = \cancel{\frac{n_i \omega}{c}} \sin\theta_r$$

$$\boxed{\theta_i = \theta_r}$$

$$k_i \sin\theta_i = k_t \sin\theta_t$$

$n_i \sin\theta_i = n_t \sin\theta_t$; Snell's law.

3) 3rd deduction:

$$E_i^{(s)} + E_r^{(s)} = E_t^{(s)}$$

$$\left(E_i^{(p)} + E_r^{(p)}\right)\cos\theta_i = E_t^{(p)}\cos\theta_t.$$

We have 4 unknowns: $E_r^{(s)}, E_r^{(p)}, E_t^{(s)}, E_t^{(p)}$.

What about $E_i^{(s)}, E_i^{(p)}$?

$$\vec{B} = \frac{\vec{k} \times \vec{E}}{\omega} = \frac{n}{c} \hat{u} \times \vec{E} \; ; \; \hat{u} = \frac{\vec{k}}{k} \quad (3.10)$$ 3

[we are using only the real part of $\vec{k}$]

[HW: Sub 3.1, 3.2, into 3.10 and show 3.11].

$\vec{B}$ parallel to ~~of~~ interface are equal at $z=0$.

we get (3.12)

$$\frac{n_i}{c}\left[-\hat{x} E_i^{(p)} + E_i^{(s)} \hat{y} \cos\theta_i\right] + \frac{n_i}{c}\left[\hat{x} E_r^{(p)} - E_r^{(s)} \hat{y} \cos\theta_i\right]$$

$$= \frac{n_t}{c}\left[-\hat{x} E_t^{(p)} + E_t^{(s)} \hat{y} \cos\theta_t\right]$$

we get from this:

$$n_i \left(E_i^{(p)} - E_r^{(p)}\right) = n_t E_t^{(p)}$$

$$n_i \left(E_i^{(s)} - E_r^{(s)}\right) \cos\theta_i = n_t E_t^{(s)} \cos\theta_t$$

We have 4 equations and 4 unknowns.
We find $E_r^{(s)}, E_r^{(p)}, E_t^{(s)}, E_t^{(p)}$ in terms of $E_i^{(s)}, E_i^{(p)}$.

3.2 The Fresnel Coefficients

(4)

Example 3.1 Find ratio of transmitted field to incident field, and ratio of reflected field to incident field for s-polarized light.

$$E_i^{(s)} + E_r^{(s)} = E_t^{(s)} \qquad (3.8)$$

$$n_i \left(E_i^{(s)} - E_r^{(s)}\right) \cos\theta_i = n_t E_t^{(s)} \cos\theta_t \qquad (3.14)$$

$$2E_i^{(s)} = E_t^{(s)} + \frac{n_t E_t^{(s)} \cos\theta_t}{n_i \cos\theta_i} = E_t^{(s)}\left[1 + \frac{n_t \cos\theta_t}{n_i \cos\theta_i}\right]$$

$$\boxed{\frac{E_t^{(s)}}{E_i^{(s)}} = \frac{2n_i\cos\theta_i}{n_i\cos\theta_i + n_t\cos\theta_t}}$$

Similarly.

$$\boxed{\frac{E_r^{(s)}}{E_i^{(s)}} = \frac{n_i\cos\theta_i - n_t\cos\theta_t}{n_i\cos\theta_i + n_t\cos\theta_t}}$$

$+\ \theta_i = \theta_r \ ; \ n_i \sin\theta_i = n_t \sin\theta_t$

let us look at

$$r_s = \frac{E_r^{(s)}}{E_i^{(s)}} \quad ; \text{ Fresnel coefficient}$$

$$r_s = \frac{E_r^{(s)}}{E_i^{(s)}} = \frac{n_i \cos\theta_i - n_t \cos\theta_t}{n_i \cos\theta_i + n_t \cos\theta_t}$$

Using $n_i \sin\theta_i = n_t \sin\theta_t$

$$r_s = \frac{n_t \frac{\sin\theta_t}{\sin\theta_i} \cos\theta_i - n_t \cos\theta_t}{n_t \frac{\sin\theta_t}{\sin\theta_i} \cos\theta_i + n_t \cos\theta_t}$$

$$= \frac{n_t \sin\theta_t \cos\theta_i - n_t \cos\theta_t \sin\theta_i \,/\, \cancel{\sin\theta_i}}{n_t \sin\theta_t \cos\theta_i + n_t \cos\theta_t \sin\theta_i \,/\, \cancel{\sin\theta_i}}$$

$$\boxed{r_s = \frac{\sin(\theta_t - \theta_i)}{\sin(\theta_t + \theta_i)}}$$

If for example n_i, n_t are known, then we can find θ_t from θ_i, and plot $\boxed{r_s}$. etc.

HW: Plot r_s as in Fig 3.3 for θ_i from $0 \to 90$.

3.3 Reflectance and Transmittance:

Energy Conservation : $P_i = P_r + P_t$

$$P_i^{(s)} = P_r^{(s)} + P_t^{(s)} \quad ; \quad P_i^{(p)} = P_r^{(p)} + P_t^{(p)}$$

Power $\propto$ Intensity ;

Intensity $\propto |\text{Amplitude}|^2$;

Reflectance = Fraction of reflected power

$$R_s = \frac{P_r^{(s)}}{P_i^{(s)}} = \frac{\bar{I}_r^{(s)}}{\bar{I}_i^{(s)}} = \frac{|E_r^{(s)}|^2}{|E_i^{(s)}|^2} = |r_s|^2$$

$$R_p = |r_p|^2$$

The total reflected intensity $\bar{I}_r = \bar{I}_r^{(s)} + \bar{I}_r^{(p)}$

$$= R_s \bar{I}_i^{(s)} + R_p \bar{I}_i^{(p)}$$

$$P_t^{(s)} = P_i^{(s)} - P_r^{(s)} = (1 - R_s) P_i^{(s)}$$

$$P_t^{(p)} = P_i^{(p)} - P_r^{(p)} = (1 - R_p) P_i^{(p)}$$

Transmittance $T_s = \dfrac{P_t^{(s)}}{P_i^{(s)}} = 1 - R_s \; ; \; T_p = \dfrac{P_t^{(p)}}{P_i^{(p)}} = 1 - R_p$

3.4 Brewster's Angle

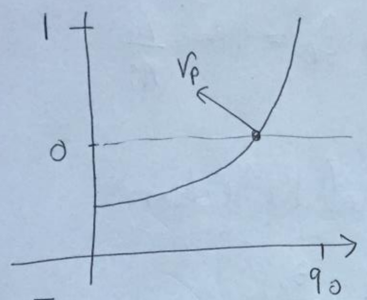

Fig 3.3 Θ_i

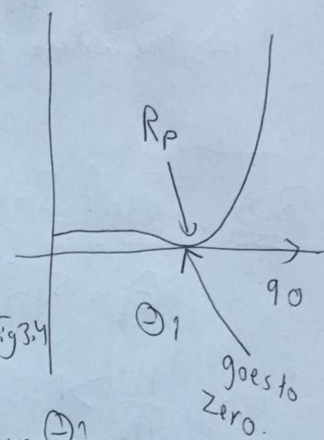

Fig 3.4 Θ_i

r_p & $R_p \longrightarrow 0$ at certain Θ_i.

$\therefore$ No p-polarized light is reflected at this Θ_i.

From (3.22) $r_p = \dfrac{E_r^{(p)}}{E_i^{(p)}} = \dfrac{\tan(\Theta_t - \Theta_i)}{\tan(\Theta_t + \Theta_i)}$

at $\Theta_t + \Theta_i = \frac{\pi}{2} \rightarrow \tan(\Theta_t + \Theta_i) \longrightarrow \infty$ and $r_p = 0$

$\Theta_i + \Theta_t = \frac{\pi}{2}$ [No p-polarized reflection)

Using Snell's law: $n_i \sin\Theta_i = n_t \sin\Theta_t$

$= n_t \sin\left(\frac{\pi}{2} - \Theta_i\right) = n_t \cos\Theta_i$

$\therefore \tan\Theta_B = \frac{n_t}{n_i}$

$\Rightarrow$ brewsters angle $\Theta_B = \tan^{-1}\frac{n_t}{n_i}$

3.5 Total Internal Reflection:

$$\Theta_t = \sin^{-1}\left(\frac{n_i \sin\Theta_i}{n_t}\right)$$

if $\frac{n_i}{n_t} > 1$ - there is a critical angle

where $\sin\Theta_i = \frac{n_t}{n_i}$

$$\Theta_c \equiv \sin^{-1}\frac{n_t}{n_i}$$

If $\Theta_i > \Theta_c$, we get total internal reflection.