4.2 Transmittance through Double Interface at Subcritical angles

(1)

How Much Power is transmitted or reflected from double-Interface?

$$t_s^{tot} = \frac{t_s^{0\to1} e^{ik_1 d\cos\theta_1} t_s^{1\to2}}{1 - r_s^{0\leftarrow1} r_s^{1\to2} e^{2ik_1 d\cos\theta_1}}$$

Using (3.33) $\quad T_s^{tot} = \frac{n_t \cos\theta_t}{n_i \cos\theta_i} |t_s|^2$; θ_2 is real $\theta_1 < \theta_c$

$$= \frac{n_2 \cos\theta_2}{n_0 \cos\theta_0} \left| t_s^{tot} \right|^2$$

$$T_s^{tot} = \frac{n_2 \cos\theta_2}{n_0 \cos\theta_0} \frac{|t_s^{0\to1}|^2 \, |t_s^{1\to2}|^2}{\left| e^{-ik_1 d\cos\theta_1} - r_s^{0\leftarrow1} r_s^{1\to2} e^{ik_1 d\cos\theta_1} \right|^2}$$

This can be written as

$$T_s^{tot} = \frac{T_s^{max}}{1 + F_s \sin^2\left(\frac{\Phi_s}{2}\right)}$$

(2)

$$\mathcal{T}_s^{max} \equiv \frac{\mathcal{T}_s^{0\to1}\,\mathcal{T}_s^{1\to2}}{\left(1-\sqrt{R_s^{0\leftarrow1}R_s^{1\to2}}\right)^2}$$

$$\Phi_s = 2k_1 d\cos\theta_1 + \phi_{r_s}^{0\leftarrow1} + \phi_{r_s}^{1\to2}$$

$$F_s = \frac{4\sqrt{R_s^{0\leftarrow1}R_s^{1\to2}}}{\left(1-\sqrt{R_s^{0\leftarrow1}R_s^{1\to2}}\right)^2}$$

$\phi_{r_s}^{0\leftarrow1}$, $\phi_{r_s}^{1\to2}$: possible phase shifts upon reflection.

$$r_s^{0\leftarrow1} = |r_s^{0\leftarrow1}|\,e^{i\phi_{r_s}^{0\leftarrow1}}$$

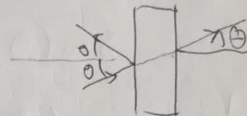
$$r_s^{1\to2} = |r_s^{1\to2}|\,e^{i\phi_{r_s}^{1\to2}}$$

<u>Note</u>: If all n's are real, $\phi_{r_s}^{0\leftarrow1}$ & $\phi_{r_s}^{1\to2}$ are 0 or π.

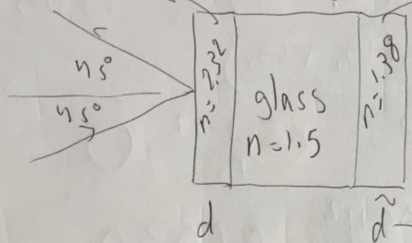
F_s = coefficient of finesse; How is $\mathcal{T}^{tot}$ changed by Φ_s?

Example 4.2

Beam Splitter



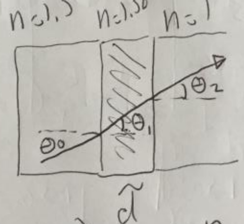
Partial Reflection coating



anti-reflection coating.

$\lambda_{vac} = 633\,nm$

$\tilde{d}$ → smallest value to give highest Transmitt. through back?



$n_0 = 1.5$ (glass) ; $n_1 = 1.38$; $n_2 = 1$ (air)

$\Theta_2 = 45^\circ$?

$n_1 \sin\Theta_1 = n_2 \sin\Theta_2 = \sin\Theta_2 \Rightarrow \Theta_1 = 30.82^\circ$

$n_0 \sin\Theta_0 = n_1 \sin\Theta_1 = \sin\Theta_2 \Rightarrow \Theta_0 = 28.13^\circ$

4

$$r_s^{1\to 2} = \frac{-\sin(\theta_1 - \theta_2)}{\sin(\theta_1 + \theta_2)} = 0.253$$

$$r_s^{0\leftarrow 1} = \frac{-\sin(\theta_1 - \theta_0)}{\sin(\theta_1 + \theta_0)} = -0.0549$$

$$\Theta_{rs}^{0\leftarrow 1} = \pi \ ? \qquad \Theta_{rs}^{1\to 2} = 0$$

$$R_s^{0\leftarrow 1} = |r_s^{0\leftarrow 1}|^2 = 0.0030$$

$$R_s^{1\to 2} = |r_s^{1\to 2}|^2 = 0.0640$$

$$T_s^{0\to 1} \overset{??}{=} T_s^{0\leftarrow 1} = 1 - R_s^{0\leftarrow 1} = 0.997$$

$$T_s^{1\to 2} = 1 - R_s^{1\to 2} = 1 - 0.0640 = 0.936$$

$$F = \frac{4\sqrt{R_s^{0\leftarrow 1} R_s^{1\to 2}}}{\left(1 - \sqrt{R_s^{0\leftarrow 1} R_s^{1\to 2}}\right)^2} = 0.0570.$$

$$T_s^{max} = 0.960$$

$$\overline{T}_s^{tot} = \frac{0.960}{1 + 0.0570 \sin^2\left(\frac{2k_1 \tilde{d} \cos\theta_1 + \pi}{2}\right)}$$ 5

$\overline{T}_s^{tot}$ is max when $\frac{2k_1 \tilde{d} \cos\theta_1 + \pi}{2} = 0$

$\overline{T}_s^{tot}$ (max) $= 0.960$

if we remove $\tilde{d}$ then $\overline{T}_s^{tot} = 0.908$

smallest $\tilde{d}$:

let $2k_1 \tilde{d} \cos\theta_1 + \pi = 2\pi$??(Why not zero)

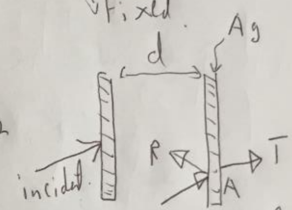
$k_1 = 2\pi n_1 / \lambda_{vac}$

$$\boxed{\tilde{d} = 134\,nm}$$

4.4 Fabry - Perot Instruments (6)

- Fabry - Perot Interferometer: $|\overleftrightarrow{d}|$ adjustable

- Fabry - Perot Etalon $|\overleftrightarrow{d}|$ fixed



- $n_0 \;|\; n_1 \;|\; n_2$

Assumptions: $n_0 = n_2$; No internal reflection.

$$\overline{T}^{tot} = \frac{\overline{T}^{max}}{1 + F\sin^2\left(\frac{\Phi}{2}\right)}$$

- for $n_0 = n_2$

$$\overline{T} = \overline{T}^{0\to1} = \overline{T}^{1\to2}$$

$$R = R^{0\to1} = R^{1\to2}$$

$$\overline{T}^{max} = \frac{\overline{T}^2}{(1-R)^2} \quad ; \quad F = \frac{4R}{(1-R)^2}$$

$$\Phi_s = 2k_1 d\cos\theta_1 + \phi_{r_s}^{0\leftarrow 1} + \phi_{r_s}^{1\to 2}$$

$$= 2\left(\frac{2\pi n_1}{\lambda_{vac}}\right) d\cos\theta_1 + 2\phi_r$$