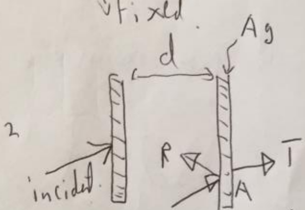
4.4 Fabry - Perot Instrument (6)

- Fabry - Perot Interferometer: $|\overset{\leftrightarrow}{d}|$ adjustable.

- Fabry - Perot Etalon $|\overset{\leftrightarrow}{d}|$ fixed.

- $n_0 \;|\; n_1 \;|\; n_2$



Assumptions: $n_0 = n_2$; No internal reflection.

$$T^{tot} = \frac{T^{max}}{1 + F\sin^2\left(\frac{\Phi}{2}\right)}$$

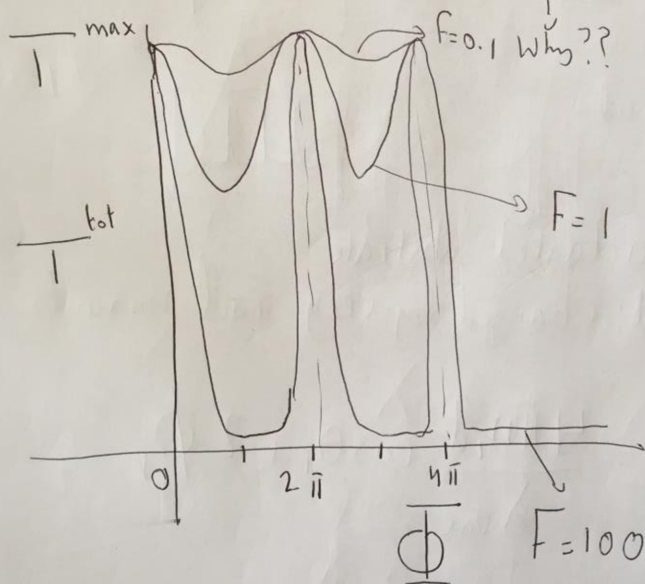
- for $n_0 = n_2$

$$T = T^{0\to1} = T^{1\to2}$$
$$R = R^{0\leftarrow1} = R^{1\to2}$$

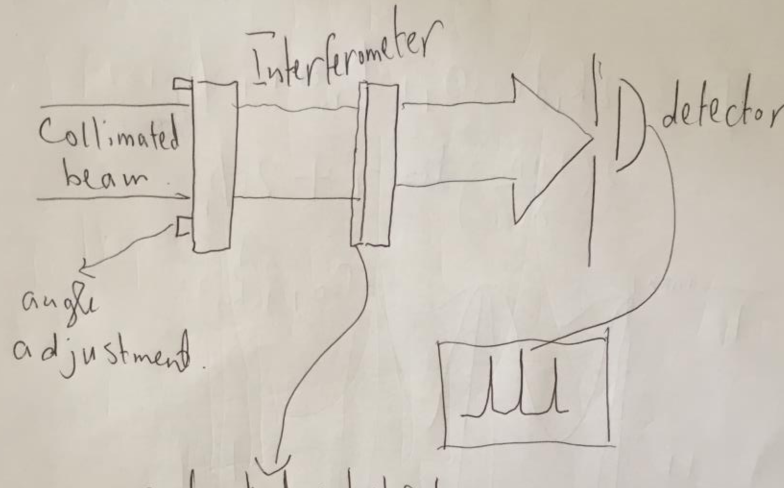
$$\overline{T}^{max} = \frac{\overline{T}^{2}}{(1-R)^2} \; ; \; F = \frac{4R}{(1-R)^2}$$

$$\underline{\Phi}_s = 2k_1 d\cos\theta_1 + \phi_{r_s}^{0\to 1} + \phi_{r_s}^{1\to 2}$$

$$= 2\left(\frac{2\pi n_1}{\lambda_{vac}}\right) d\cos\theta_1 + 2\phi_r$$



4.5 Setup of a Fabry-Perot Instrument (8)



actuated substrate
d: changed by very small amounts.

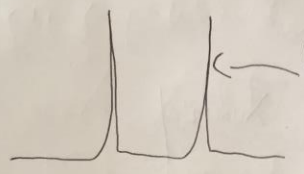
$$\Phi = \frac{4\pi n_1 d}{\lambda_{vac}} \cos\theta_1 + 2\phi_r$$

1 ??

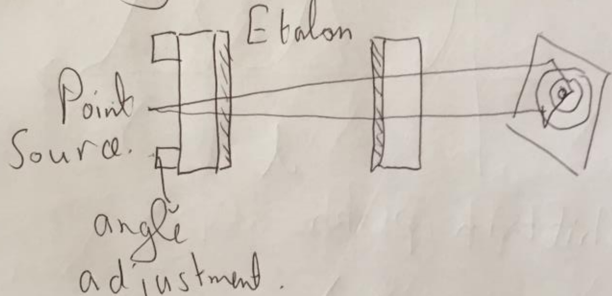
$n_1 = 1$ (air).

$d = \frac{\lambda_{vac}}{2} \rightarrow \Delta\Phi = 2\pi$

$$T^{tot} = \frac{T^{max}}{1 + F\sin^2\left(\frac{\Phi}{2}\right)}$$



The Fabry-Perot Etalon Setup



d: fixed.

Angle of light is varied and not distance d.

$\Phi \propto d, \theta,$

4.6 Distinguishing Nearby λ's in a FP instrument.

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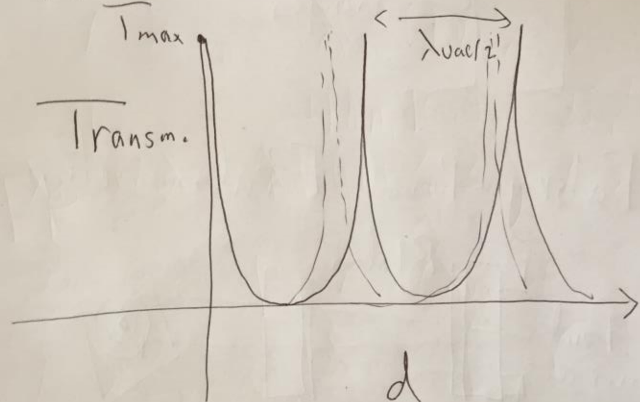
- FP measures small changes in λ.



Shift in peaks

$\lambda_{vac} + \Delta\lambda$

11

$$\bar{\Phi} = \frac{4\pi n_1 d}{\lambda_{vac}} \cos\theta_1 + 2\phi_r$$

$$\bar{\Phi} \overset{??}{\div} \Delta\bar{\Phi} = \frac{4\pi n_1 d \cos\theta_1}{\lambda_{vac} + \Delta\lambda} + 2\phi_r$$

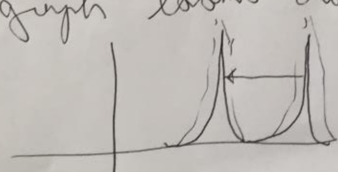
$$\frac{1}{\lambda_{vac} + \Delta\lambda} = \frac{1}{\lambda_{vac}\left(1 + \frac{\Delta\lambda}{\lambda_{vac}}\right)} \cong$$

$$\cong \frac{1 - \Delta\lambda/\lambda_{vac}}{\lambda_{vac}}$$

$$\Delta\bar{\Phi} = \frac{4\pi n_1 d \cos\theta_1}{\lambda_{vac}^2} \Delta\lambda$$

What happens if $\Delta\bar{\Phi} = 2\pi$?

The graph looks the same: So what do we do?



Free Spectral range ; FSR (12)

FSR : $\Delta\lambda_{FSR}$ causes fringes to shift through one period.

$$\Delta\lambda_{FSR} = \frac{\lambda_{vac}^2 (2\pi)}{4\pi n_1 d\cos\theta_1} = \frac{\lambda_{vac}^2}{2n_1 d\cos\theta_1}$$

FP not suited to measure $\Delta\lambda$'s larger than this $\Delta\lambda_{FSR}$.

$\Delta\lambda_{FSR}$ in terms of ν (frequency).

$$\nu = \frac{c}{\lambda_{vac}} \Rightarrow \Delta\nu = \frac{-c}{\lambda_{vac}^2}\Delta\lambda$$

$$\therefore \boxed{|\Delta\nu_{FSR}| = \frac{c}{\lambda_{vac}^2}\Delta\lambda_{FSR}}$$

Example 4.4:

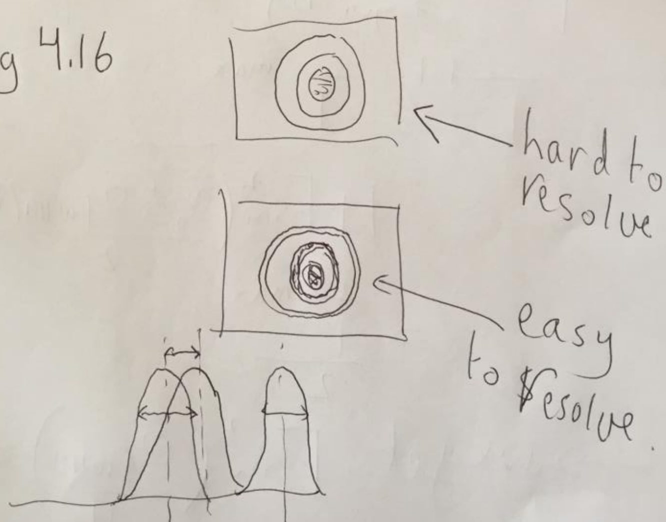
FP interferometer. : $d = 1\,cm$ $n_1 = 1$

$\lambda_{vac} = 500\,nm$. Find FSR?

$$\Delta\lambda_{FSR} = \frac{\lambda_{vac}^2}{2n_1 d_0 \cos\theta_1} = 0.0125\,nm$$

Next Question: What is the smallest change in λ that can be resolved?

Fig 4.16



(14)

FWHM: Full Width at half Maximum

let $\overline{I}^{tot} = \dfrac{\overline{I}^{max}}{1+F\sin^2\left(\frac{\Phi}{2}\right)}$

for $\Phi = m(2\pi) \longrightarrow \overline{I}^{tot} = \overline{I}^{max}$

If $\Phi \longrightarrow \Phi \pm \Phi_{FWHM}/2$, then $\overline{I}^{tot}$ drops by one half.

$$\overline{I}^{tot} = \frac{\overline{I}^{max}}{1+F\sin^2\left(\frac{\Phi_0 \pm \Phi_{FWHM}/2}{2}\right)} = \frac{\overline{I}^{max}}{2}$$

$\longrightarrow$ we get $F\sin^2\left(\frac{\Phi_{FWHM}}{4}\right) = 1$

let $\bar{\Phi}_{FWHM}/4$ be small 15

then $\sin\left(\frac{\bar{\Phi}_{FWHM}}{4}\right) \cong \frac{\bar{\Phi}_{FWHM}}{4}$

This is a good approximation if F is large (>100)

then $\bar{\Phi}_{FWHM} \cong \frac{4}{\sqrt{F}}$

We define $f = \frac{2\pi}{\bar{\Phi}_{FWHM}} = \frac{\pi\sqrt{F}}{2}$; f = reflecting finesse.

Finally, Resolving power RP≡

$$RP \equiv \frac{\lambda_{vac}}{\Delta\lambda_{FWHM}}.$$

$$\Delta\lambda_{FWHM} = \frac{\Delta\lambda_{FSR}}{f} = \frac{\lambda_{vac}^2}{\pi n_1 d \cos\theta_1 \sqrt{F}}$$

Example 4.5

FP interferometer. $R = 0.85$

Find finesse, minimum distinguishable wavelength separation, RP?

a) $F = \dfrac{4R}{(1-R)^2} = 151$

$f = \dfrac{\pi\sqrt{F}}{2} = 19.3$

$\Delta\lambda_{FWHM} = \dfrac{\Delta\lambda_{FSR}}{f} = 0.00065\,nm$

$RP = \dfrac{\lambda_{vac}}{\Delta\lambda_{FWHM}} = 772000.$