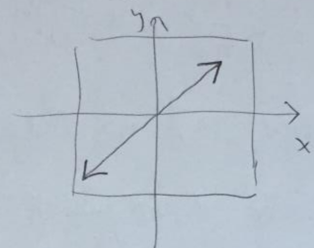
Ch6: Polarization of Light:

1

HW: 4.5, 4.
4.9

- $\uparrow \vec{E}_o$ Direction of $\vec{E}$ field
  Polarization

- If $\vec{E}_o$ oscillates in a plane:
  Linearly polarized.



- If $\vec{E}_o$ oscillates (spirals around a plane) while wave propagates:
  Circular or elliptical polarization

6.1 Linear, Circular, Elliptical Polarization

$$\vec{E}(\vec{r},t) = \vec{E}_0 e^{i(\vec{k}\cdot\vec{r} - wt)}$$

If direction of $\vec{k}$ is $\hat{z}$ direction

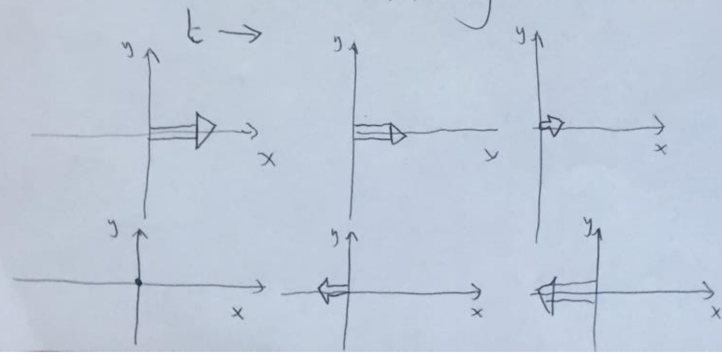
$$\vec{E}(z,t) = (E_x \hat{x} + E_y \hat{y}) e^{i(kz - wt)}$$

We are interested in Real part.

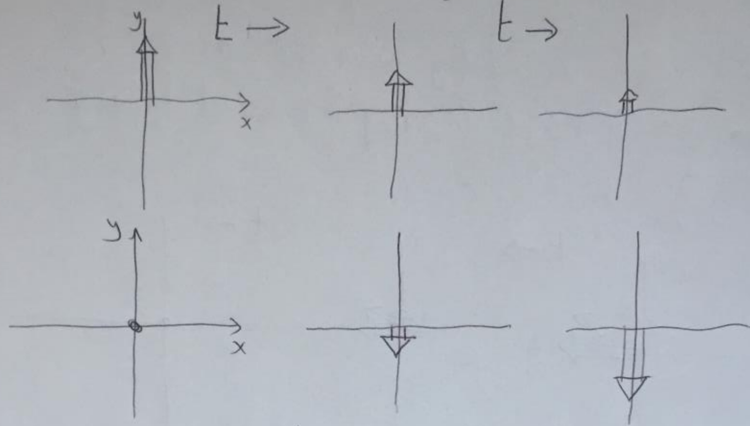
E_x, E_y can be complex.

The relation between E_x, E_y describes the polarization.

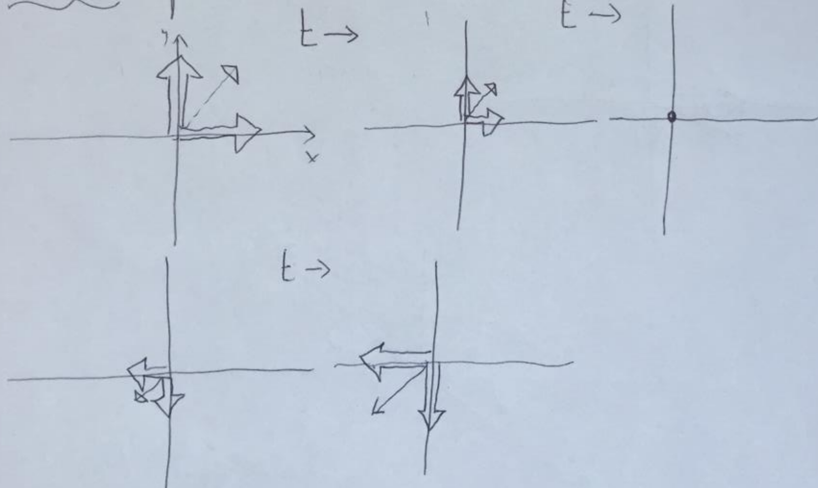
Case 1: $E_y = 0$: Linearly polarized light. Along x-direction.



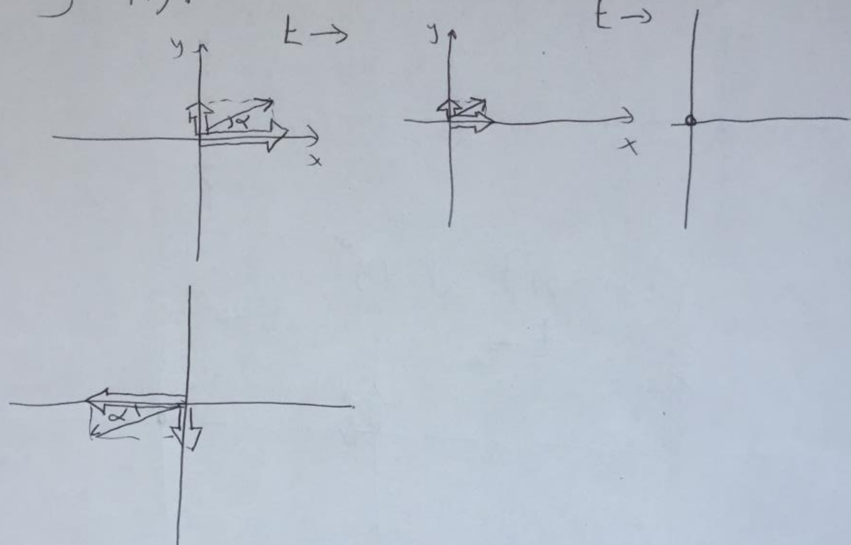
Case 2: $\hat{y}$ polarized light. (3)

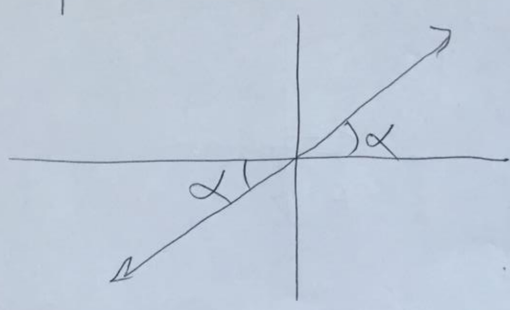


Case 3: polarized at 45°



In general: Linearly polarized light if E_x & E_y have the same complex phase (or phases differing by π).





Circular Polarized light:

Suppose $E_y = iE_x = \left(e^{i\frac{\pi}{2}}\right)E_x$; (delay of $\frac{\pi}{2}$)

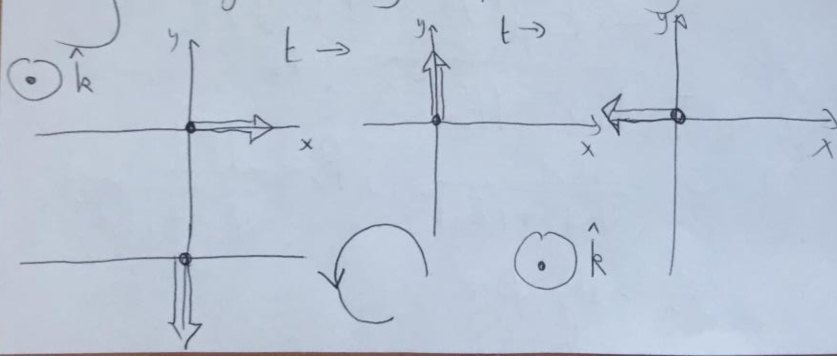
let E_x be real.

$$\vec{E}(z,t) = \mathrm{Re}\left[E_x e^{i(kz-wt)}\right]\hat{x} + \mathrm{Re}\left[e^{i\pi/2} E_x e^{i(kz-wt)}\right]\hat{y}$$

$$= E_x \cos(kz-wt)\,\hat{x} + E_x \cos\left(kz-wt+\frac{\pi}{2}\right)\hat{y}$$

$$= E_x\left[\cos(kz-wt)\,\hat{x} - \sin(kz-wt)\,\hat{y}\right]$$

$\hat{y}$ lags $\hat{x}$ by a quarter cycle.



This is left handed circularly polarized light.

[Rotates CW looking in $\hat{k}$ direction].

6.2 Jones Vectors

- Deals with light that has a definite Polarization, such as plane waves.
- Two-Dimensional Matrix algebra.

$$\vec{E}(z,t) = \left(|E_x| e^{i\phi_x} \hat{x} + |E_y| e^{i\phi_y} \hat{y} \right) e^{i(kz-\omega t)}$$

$$= E_{eff} \left(A \hat{x} + B e^{i\delta} \hat{y} \right) e^{i(kz-\omega t)}$$

$$E_{eff} \equiv \sqrt{|E_x|^2 + |E_y|^2} \; e^{i\phi_x}$$

$$A \equiv \frac{|E_x|}{\sqrt{|E_x|^2 + |E_y|^2}} \; ; \; B \equiv \frac{|E_y|}{\sqrt{|E_x|^2 + |E_y|^2}}$$

$$\delta = \phi_y - \phi_x$$

$A^2 + B^2 = 1$

Case 1: $E_y = 0$; $B = 0$; everything well defined.

Case 2: $E_x = 0$, then $e^{i\phi_x}$ is indeterminate.

We let $E_{eff} = |E_y| e^{i\phi_y}$, $B = 1$, $\delta = 0$

Using 2.62: $\bar{I} = \frac{1}{2} n c \varepsilon_0 \vec{E} \cdot \vec{E}^*$

we get $\bar{I} = \frac{1}{2} n c \varepsilon_0 |E_{eff}|^2$??

Jones Vector: $A\hat{x} + Be^{i\delta}\hat{y}$

$(A\hat{x} + Be^{i\delta}\hat{y}) \cdot (A\hat{x} + Be^{i\delta}\hat{y})^* = 1$ (unit vector)

we use matrix notation

$$\begin{bmatrix} A \\ Be^{i\delta} \end{bmatrix}$$

This describes the polarization state of any plane wave field.

① Case 1: linear Polarization along $\hat{x}$. (8)

$E_y = 0 \quad A = 1 \; ; \; B = 0$

(we don't care about δ).

$\begin{bmatrix} 1 \\ 0 \end{bmatrix}$

② Linear Polarization along $\hat{y}$.

$E_x = 0 ; \quad B = 1 \; ; \; \delta = 0 \; ; A = 0$

special case $\longrightarrow \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

③ Linearly polarized at angle α (measured from x-axis). e.g $\alpha = 45°$

$|E_x| = |E_y| \qquad \phi_x = \phi_y = 0$

$A = \frac{1}{\sqrt{2}} \; ; \; B_y = \frac{1}{\sqrt{2}} \qquad \delta = 0 \Rightarrow \begin{bmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{bmatrix}$

$|E_x| = \cos\alpha \; ; \; |E_y| = \sin\alpha$

$A = \cos\alpha \; ; \; B = \sin\alpha$

$\longrightarrow \begin{bmatrix} \cos\alpha \\ \sin\alpha \end{bmatrix}$

Right circularly polarized.

$$|E_x| = |E_y| \qquad \phi_x = \phi_y + \frac{\pi}{2}$$

$$A = \frac{1}{\sqrt{2}} \; ; \; B = \frac{1}{\sqrt{2}} \; ; \; \delta = \frac{-\pi}{2}$$

$$\begin{bmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} e^{-i\frac{\pi}{2}} \end{bmatrix} = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ -i \end{bmatrix}$$

6.3 Elliptically Polarized Light

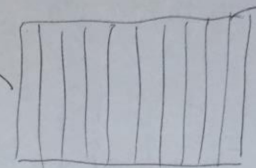
Elliptically Polarized: A state between Linear and circular polarization.

6.4 Linear Polarizers & Jones Matrices

- History: 1928 Land.; Polarizer.

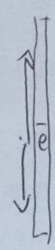
conductive.

Polaroid Sheet.

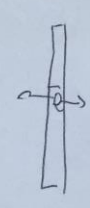


$\updownarrow \vec{E}$ gets absorbed.

$\longleftrightarrow \vec{E}$ gets through.



absorption.



can't oscillate, passes though.