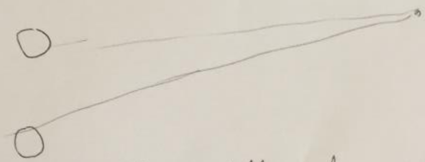
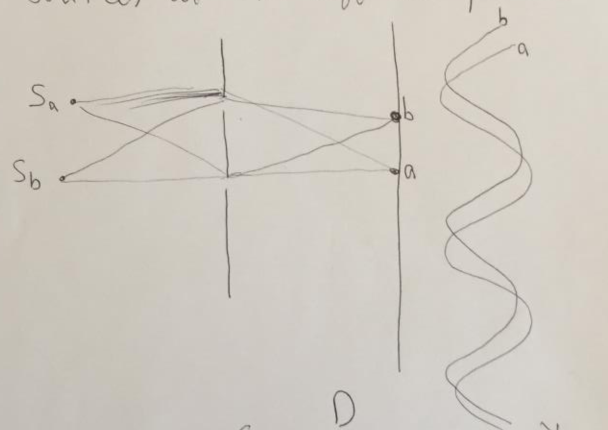
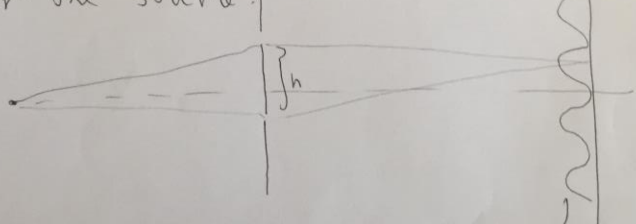
8.5 Young's Two-Slit Setup and Spatial coherence.



Two sources at two different points.

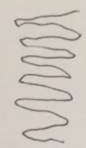


For one source.

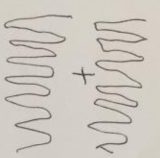


$$I_{tot}(h) = 2 I_0 \left[1 + \cos(kyh/D)\right]$$

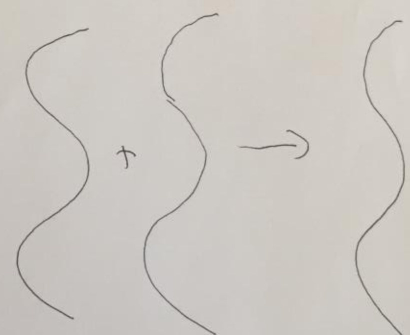
IF $h \to$ large, the fringe pattern becomes dense.



This results in Cancellation.



$\to$ no Signal.



There is interference

<u>9.1</u> The Eikonal Equation ①

Assume light with one frequency isotropic medium; real refractive index.

From wave equation:

$$\nabla^2 \vec{E}(\vec{r},t) + \frac{[n(\vec{r})]^2 w^2}{c^2} \vec{E}(\vec{r},t) = 0$$

<u>Note</u>: $n(\vec{r})$ depends on position

By analogy let the trial solution be

$$\vec{E}(\vec{r},t) = \vec{E}_0(\vec{r})\, e^{i[k_{vac}R(\vec{r}) - wt]}$$

$$k_{vac} = \frac{w}{c} = \frac{2\pi}{\lambda_{vac}}$$

Subbing the solution we get

$$\frac{1}{k_{vac}^2} \nabla^2 \left[\vec{E}_0(\vec{r})\, e^{ik_{vac}R(\vec{r})}\right] + [n(\vec{r})]^2 \vec{E}_0(\vec{r})\, e^{ik_{vac}R(\vec{r})} = 0$$

In very short λ limit, we get

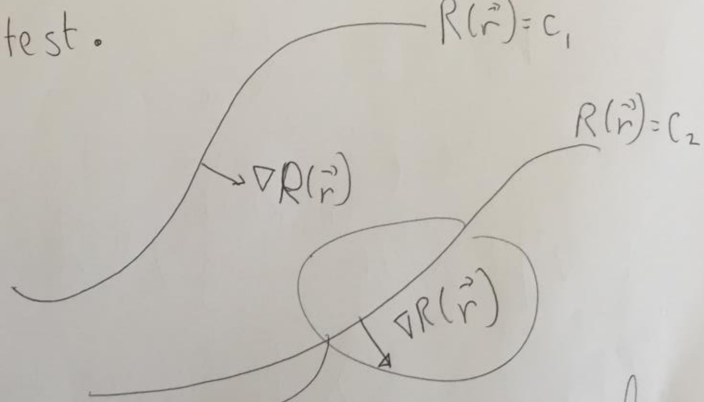
$$\nabla R \cdot \nabla R = [n(\vec{r})]^2$$

$$(9.7) \qquad \nabla R(\vec{r}) = n(\vec{r})\, \hat{s}(\vec{r})$$

$\hat{S}$ = unit vector in direction of $\nabla R(\vec{r})$
= direction normal to wavefront surfaces.

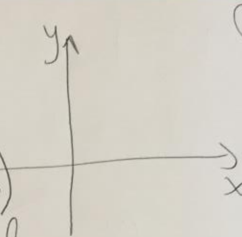
$$\nabla R(\vec{r}) = n(\vec{r})\,\hat{s}(\vec{r}) \quad \text{[Eikonal equation]}.$$

What is $R(\vec{r})$? Real scalar function
$R(\vec{r})$ = constant = wavefront.
This wavefront travels in direction for which $R(\vec{r})$ changes the fastest.



gradient of R gives direction of travel of wavefront.

Example 9.1



let $n(y) = n_0\sqrt{1 + y^2/h^2}$

Verify that $R(x,y) = n_0(x \pm y^2/2h)$ is a solution to the eikonal equation.

$$\nabla R = n_0\left(\hat{x} \pm \frac{y}{h}\hat{y}\right)$$

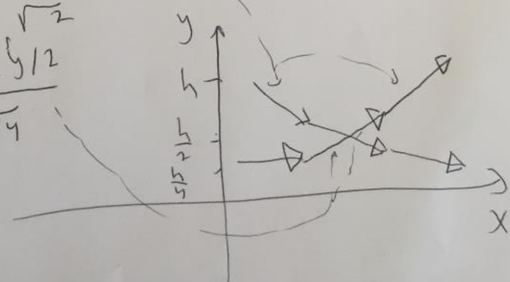
$$\nabla R \cdot \nabla R = n_0^2\left(1 + \frac{y^2}{h^2}\right) = [n(y)]^2 \checkmark$$

The direction of light propagation is

$$\hat{S}(y) = \frac{\nabla R}{|\nabla R|} = \frac{\hat{x} \pm \hat{y}\,\frac{y}{h}}{\sqrt{1 + y^2/h^2}}$$

$$\hat{S}(h) = \frac{\hat{x} \pm \hat{y}}{\sqrt{2}}$$

$$\hat{S}\left(\frac{h}{2}\right) = \frac{\hat{x} \pm \hat{y}/2}{\sqrt{5/4}}$$



9.2 Fermat's Principle

Used to find path of rays.

∴ 

follows a path that takes least time.

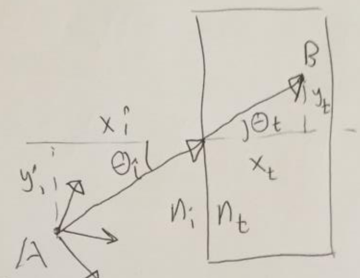
$$OPL\Big|_A^B = \min\left\{\int_A^B n\,dl\right\}$$

$$\Delta t\Big|_A^B = \int_A^B \frac{dl}{v(\vec{r})} = \int_A^B \frac{dl}{c/n(\vec{r})} = \frac{OPL\Big|_A^B}{c}$$

∴ To find correct path of light, we minimize the OPL.

Example 9.2

Derive Snell's law using Fermat's principle.



$$OPL = n_i\sqrt{x_i^2+y_i^2} + n_t\sqrt{x_t^2+y_t^2}$$

This should be minimized.

x_i, x_t = fixed.

$y_{tot} = y_i + y_t$ = constant.

$$OPL = n_i\sqrt{x_i^2+y_i^2} + n_t\sqrt{x_t^2+(y_{tot}-y_i)^2}$$

Only variable is y_i

$$\frac{d(OPL)}{dy_i} = n_i\frac{y_i}{\sqrt{x_i^2+y_i^2}} + n_t\frac{\left[-(y_{tot}-y_i)\right]}{\sqrt{x_t^2+(y_{tot}-y_i)^2}} = 0$$

6

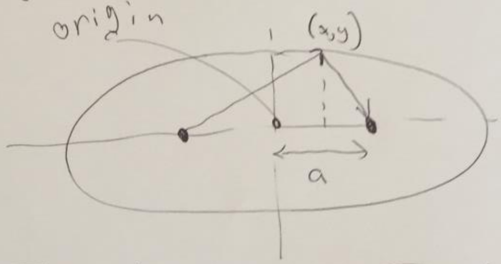
$$\sin\theta_i = \frac{y_i}{\sqrt{x_i^2 + y_i^2}} \; ; \; \sin\theta_t = \frac{y_t}{\sqrt{x_t^2 + y_t^2}}$$

$$n_i \sin\theta_i - n_t \sin\theta_t = 0$$

$$\boxed{n_i \sin\theta_i = n_t \sin\theta_t}$$

7

Example 9.3 Derive equation of curvature for a reflective surface that causes all rays leaving one point to image to another. (In 2D).



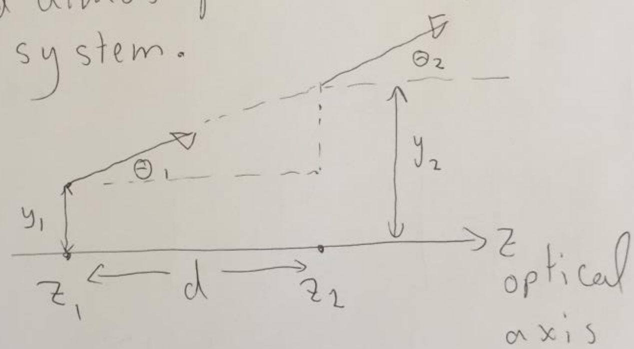
$$\sqrt{(x+a)^2 + y^2} + \sqrt{(x-a)^2 + y^2} = b$$

[after some algebra] [
we get

$$\frac{x^2}{\left(\frac{b^2}{4}\right)} + \frac{y^2}{\left(\frac{b^2}{4} - a^2\right)} = 1 \text{ ; ellipse.}$$

9.3 Paraxial Rays and ABCD matrices (8)

- Ray Optics: Describe light as rays and not waves.
  Good approximation if $\lambda <<$ features.
- Paraxial Approximation: rays near and almost parallel to optical axis of system.



$\sin\theta \cong \theta$
$\tan\theta \cong \theta$

let n be uniform everywhere.

$$\theta_2 = \theta_1$$
$$y_2 = y_1 + d\tan\theta_1 = y_1 + \theta_1 d$$

$$\therefore \begin{bmatrix} y_2 \\ \theta_2 \end{bmatrix} = \begin{bmatrix} 1 & d \\ 0 & 1 \end{bmatrix} \begin{bmatrix} y_1 \\ \theta_1 \end{bmatrix}$$

ABCD matrix for propagation through a distance d.

Example 9.4 let $d = a + b$

show that applying ABCD matrix for d, or for a then b, gives same result.

$$\begin{bmatrix} y_{mid} \\ \theta_{mid} \end{bmatrix} = \begin{bmatrix} 1 & a \\ 0 & 1 \end{bmatrix} \begin{bmatrix} y_1 \\ \theta_1 \end{bmatrix}$$

$$\begin{bmatrix} y_2 \\ \theta_2 \end{bmatrix} = \begin{bmatrix} 1 & b \\ 0 & 1 \end{bmatrix} \begin{bmatrix} y_{mid} \\ \theta_{mid} \end{bmatrix}$$

$$\therefore \begin{bmatrix} y_2 \\ \theta_2 \end{bmatrix} = \begin{bmatrix} 1 & b \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & a \\ 0 & 1 \end{bmatrix} \begin{bmatrix} y_1 \\ \theta_1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & a+b \\ 0 & 1 \end{bmatrix} \begin{bmatrix} y_1 \\ \theta_1 \end{bmatrix} \checkmark$$