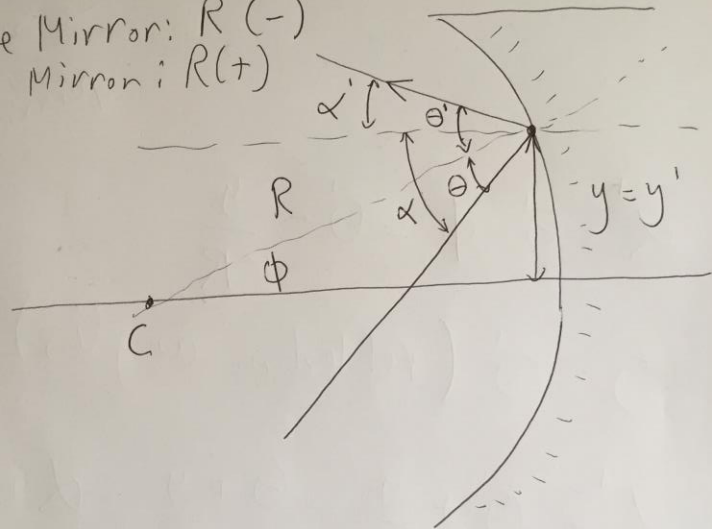


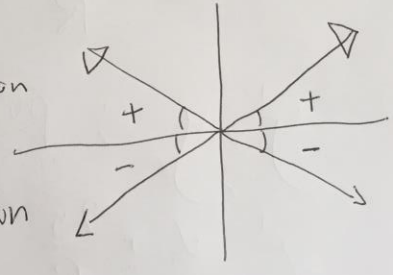
9.4 Reflection & Refraction at Curved Surfaces (1)

Concave Mirror: $R(-)$
 Convex Mirror: $R(+)$



Sign Convention

(+) for rays pointing up
 (-) for rays pointing down



$\alpha' \rightarrow \text{up} \rightarrow (+)$
 $\alpha \rightarrow \text{up} \rightarrow (+)$

$$\alpha = \theta + \phi = \theta + \frac{y}{-R} \quad (2)$$

$\rightarrow \sin \phi = \frac{y}{R}$
paraxial $\sin \phi = \phi$

$$\alpha' = \theta' - \phi = \theta' - \frac{y}{-R}$$

$$\theta = \theta' \quad (\text{reflection angles}).$$

$$\alpha' = \theta' + \frac{y}{R} = \theta + \frac{y}{R} = \alpha + \frac{y}{R} + \frac{y}{R}$$

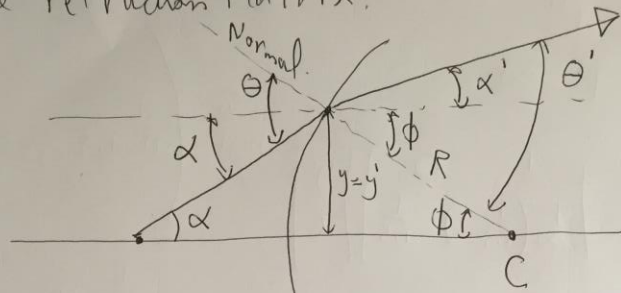
$y' = y$; point of reflection.

$$\alpha' = \alpha + \frac{2y}{R}$$

\rightarrow ABCD matrix:
$$\begin{bmatrix} y' \\ \alpha' \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ \frac{2}{R} & 1 \end{bmatrix} \begin{bmatrix} y \\ \alpha \end{bmatrix}$$

The refraction Matrix.

(3)



n n'

Convex Surface
 $R(+)$
 Concave Surface
 $R(-)$

$$\alpha' = \theta' - \phi = \theta' - \frac{y}{R}$$

$$\alpha = \theta - \phi = \theta - \frac{y}{R}$$

Using Snell's law: $n \sin \theta = n' \sin \theta'$
 $n \theta = n' \theta'$

$$\alpha' = \left(\frac{n}{n'}\right) \theta - \frac{y}{R} = \frac{n}{n'} \left(\alpha + \frac{y}{R}\right) - \frac{y}{R}$$

$$\therefore \begin{bmatrix} y' \\ \alpha' \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ \frac{1}{R} \left(\frac{n}{n'} - 1\right) & \frac{n}{n'} \end{bmatrix} \begin{bmatrix} y \\ \alpha \end{bmatrix}$$

Example:

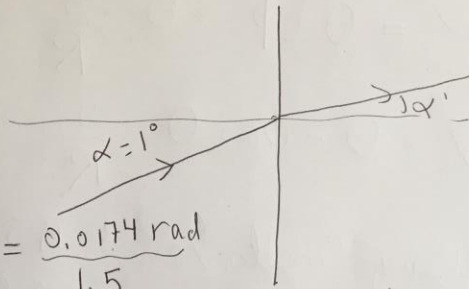
(4)

let $R = \infty$

$$\begin{bmatrix} y' \\ \alpha' \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ \frac{1}{\infty} \left(\frac{1}{1.5} - 1 \right) & \frac{1}{1.5} \end{bmatrix} \begin{bmatrix} y \\ \alpha \end{bmatrix}$$

air $n=1$ (glass $n=1.5$)

$$= \begin{bmatrix} 1 & 0 \\ 0 & \frac{1}{1.5} \end{bmatrix} \begin{bmatrix} y \\ \alpha \end{bmatrix} \Rightarrow \begin{aligned} y' &= y \\ \alpha' &= \frac{\alpha}{1.5} \end{aligned}$$



$$\alpha' = \frac{\alpha}{1.5} = \frac{0.0174 \text{ rad}}{1.5}$$

$$\alpha' = 0.012 \text{ rad} = 0.69 \text{ deg.}$$

let us use Snell's law

$$n_i \sin \theta_i = n_t \sin \theta_t$$

$$(1) (\sin 1) = 1.5 \sin \theta_t$$

$$\theta_t = 0.67 \text{ deg.}$$

Same problem, but let $\alpha = 30^\circ$ (5)

$$\alpha' = \frac{\alpha}{1.5} = \frac{30}{1.5} = 20^\circ$$

Using Snell's law

$$(1) \sin 30 = 1.5 \sin \theta_t$$

$$\theta_t = 19.5^\circ$$

let $\alpha = 60^\circ$

$$\alpha' = \frac{60^\circ}{1.5} = 40^\circ$$

Using Snell's law

$$(1) \sin 60 = 1.5 \sin \theta_t$$

$$\theta_t = 35.3^\circ$$

The closer to paraxial approximation range, the better.

Exemple:

air



$$|R| = 10 \text{ cm}$$

convex (convexe)

$$R(+) = +10 \text{ cm}$$

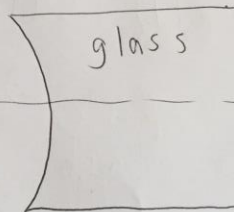
$$\begin{bmatrix} y' \\ \alpha' \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ \frac{1}{0.1} \left(\frac{1}{1.5} - 1 \right) & \frac{1}{1.5} \end{bmatrix} \begin{bmatrix} y \\ \alpha \end{bmatrix}$$

$$y' = y$$

$$\alpha' = 10 \left(\frac{-1}{3} \right) y + \frac{2}{3} \alpha = -\frac{10}{3} y + \frac{2}{3} \alpha$$

Exemple:

air



$$|R| = 10 \text{ cm}$$

concave (concave)

$$R(-) = -10$$

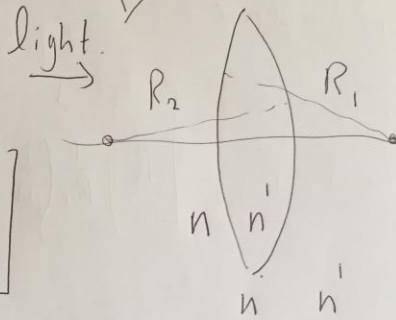
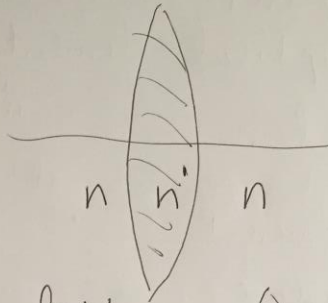
$$\begin{bmatrix} y' \\ \alpha' \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ \frac{1}{-0.1} \left(\frac{1}{1.5} - 1 \right) & \frac{1}{1.5} \end{bmatrix} \begin{bmatrix} y \\ \alpha \end{bmatrix} \Rightarrow \begin{cases} y' = y \\ \alpha' = +\frac{10}{3} y + \frac{2}{3} \alpha \end{cases}$$

Examples:

1) Thin lens

R_1 : Convex(+)

R_2 : Concave(-)



$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ \frac{1}{R_2} \left(\frac{n'}{1} - 1 \right) & \frac{n'}{1} \end{bmatrix}$$

$$\times \begin{bmatrix} 1 & 0 \\ \frac{1}{R_1} \left(\frac{1}{n'} - 1 \right) & \frac{1}{n'} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ \frac{1}{R_2} (n'-1) + \frac{n'}{R_1} \left(\frac{1}{n'} - 1 \right) & 1 \end{bmatrix}$$

$$\Rightarrow \left(\begin{array}{c} \text{---} \\ \text{---} \\ \text{---} \\ \text{---} \end{array} \right) + \left(\begin{array}{c} \text{---} \\ \text{---} \\ \text{---} \\ \text{---} \end{array} \right) = \begin{bmatrix} 1 & 0 \\ \frac{n'}{R_2} - \frac{1}{R_2} + \frac{1}{R_1} - \frac{n'}{R_1} & 1 \end{bmatrix}$$

(7)

(8)

$$= \begin{bmatrix} 1 & 0 \\ -n\left(\frac{1}{R_1} - \frac{1}{R_2}\right) + 1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 \\ -(n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) & +1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 \\ -\frac{1}{f} & 1 \end{bmatrix}$$

f = Focal length

= lens maker's formula

$$= (n-1) \left(\frac{1}{R_1} - \frac{1}{R_2} \right)$$

Exmpl:



(9)

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & n \end{bmatrix} \begin{bmatrix} 1 & d \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & \frac{1}{n} \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 \\ 0 & n \end{bmatrix} \begin{bmatrix} 1 & \frac{d}{n} \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & \frac{d}{n} \\ 0 & 1 \end{bmatrix}$$
