

Ch8 Coherence Theory

(1)

Correlation: for example: $A \uparrow B \uparrow$

Temporal Coherence: $\vec{E}(\vec{r}, t)$ & $\vec{E}(\vec{r}, t - \tau)$

Spatial Coherence: $\vec{E}(\vec{r}, t)$ & $\vec{E}(\vec{r} + \Delta\vec{r}, t)$

High coherence \rightarrow consistently
constructive, or destructive

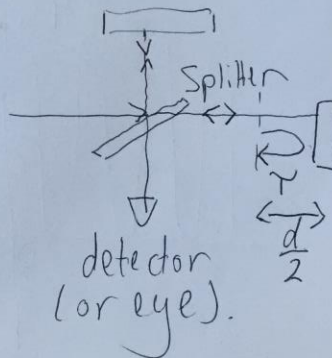
Low Coherence \rightarrow time-averaged signal
shows no interference.

8.1 Michelson Interferometer:

- path difference d leads
to time delay τ .

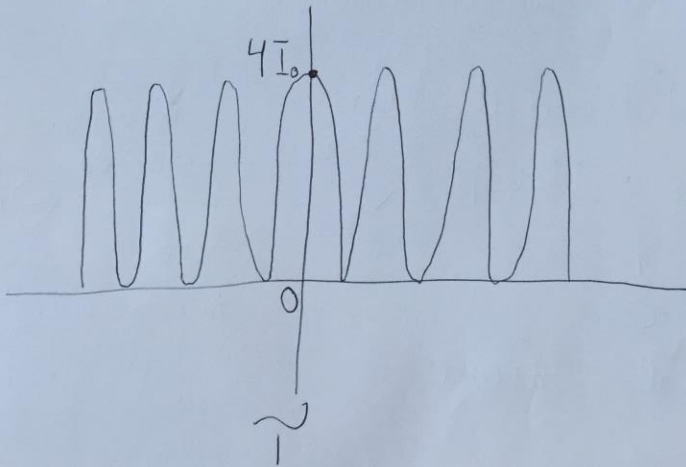
$$\tau = \frac{d}{c}$$

Temporal coherence

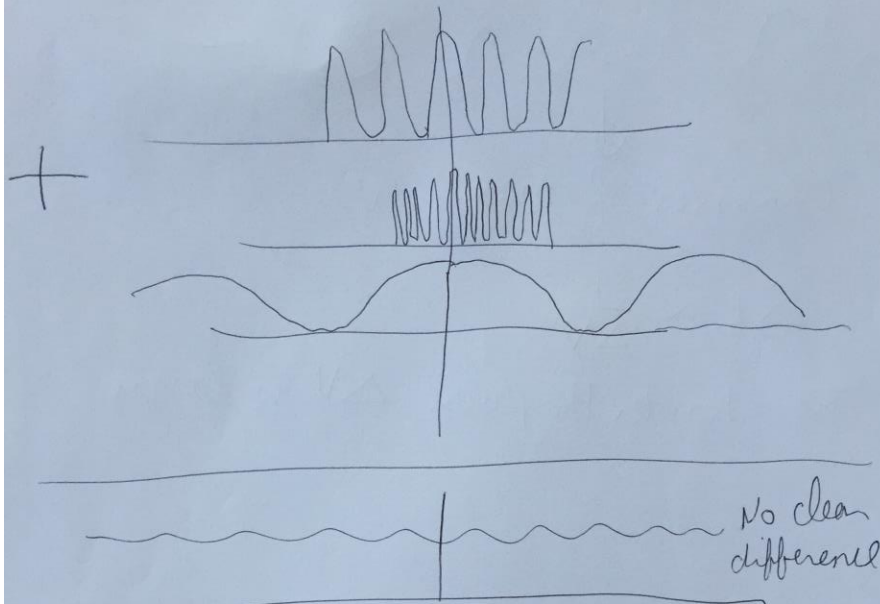


Assume input light is a plane wave (2)
 : We have two signals at detector
 $\vec{E}_0 e^{i(kz - \omega t)}$ and $\vec{E}_0 e^{i(kz - \omega(t - \gamma))}$

$$\begin{aligned} \bar{I}_{\text{tot}}(\gamma) &= \frac{c\epsilon_0}{2} \left[\vec{E}_{\text{tot}} \cdot \vec{E}_{\text{tot}}^* \right] \\ &= \frac{c\epsilon_0}{2} \left[\vec{E}_0 e^{i(kz - \omega t)} + \vec{E}_0 e^{i(kz - \omega(t - \gamma))} \right] \left[\right]^* \\ &= 2 \bar{I}_0 [1 + \cos(\omega\gamma)] \quad ; \quad \bar{I}_0 = \frac{c\epsilon_0}{2} \vec{E}_0 \cdot \vec{E}_0^* \end{aligned}$$

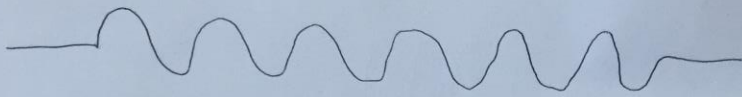


What if we have multiple frequencies? (3)



No constructive or destructive interference seen at detector.

Wavetrain



Consists of many ω 's.

$$\Delta t \propto \frac{1}{\Delta \omega}$$

The shorter the pulse, $\Delta \omega$ is larger.

What happens if the light incident consists of a continuous band of frequencies? (3a)

Let $\vec{E}(t)$ be an arbitrary waveform, e.g. a pulse with many frequencies.

$$\vec{E}_{\text{tot}}(t, \tau) = \vec{E}(t) + \vec{E}(t - \tau)$$

$$\bar{I}_{\text{tot}}(t, \tau) = \frac{n c \epsilon_0}{2} \vec{E}_{\text{tot}}(t, \tau) \cdot \vec{E}_{\text{tot}}^*(t, \tau)$$

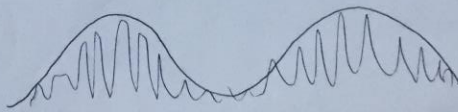
$$= \frac{c \epsilon_0}{2} \left[\vec{E}(t) \cdot \vec{E}^*(t) + \vec{E}(t) \cdot \vec{E}^*(t - \tau) \right. \\ \left. + \vec{E}(t - \tau) \cdot \vec{E}^*(t) + \vec{E}(t - \tau) \cdot \vec{E}^*(t - \tau) \right]$$

$$= \bar{I}(t) + \bar{I}(t - \tau) + \frac{c \epsilon_0}{2} \text{Re} \left\{ \vec{E}(t) \cdot \vec{E}^*(t - \tau) \right\}$$

↳ rapid oscillations averaged out.

We only keep the slowly changing

envelope.



The detected signal is $Sig(\tau)$ (4)

$$Sig(\tau) \propto \int_{-\infty}^{\infty} I_{tot}(t, \tau) dt$$

$Sig(\tau)$ is time-integrated intensity

↳ called the "fluence"

$$\text{let } \mathcal{E} = \int_{-\infty}^{\infty} I(t) dt = \int_{-\infty}^{\infty} I(t-\tau) dt \quad \begin{matrix} \text{Why} \\ ?? \\ \end{matrix}$$

$$(8.9) \quad Sig(\tau) \propto 2\mathcal{E} [1 + \text{Re}\{\gamma(\tau)\}]$$

(derivation in book; not important)

$\gamma(\tau)$ = degree of coherence function

$$\equiv \int_{-\infty}^{\infty} I(\omega) e^{-i\omega\tau} d\omega$$

$$\frac{\int_{-\infty}^{\infty} I(\omega) d\omega}{\int_{-\infty}^{\infty} I(\omega) d\omega}$$

$$\mathcal{E} = \int_{-\infty}^{\infty} I(t) dt = \int_{-\infty}^{\infty} I(\omega) d\omega$$

Example 8.1

(5)

Compute output signal when a gaussian pulse with spectrum (7.25) is sent into a Michelson Interferometer.

$$\text{given: } \underline{I}(\omega) = \frac{\epsilon_0 c}{2} \vec{E}_0 \cdot \vec{E}_0^* T e^{-T^2(\omega - \omega_0)^2}$$

T = pulse duration

$$\int_{-\infty}^{\infty} \underline{I}(\omega) d\omega = \frac{\epsilon_0 c}{2} \vec{E}_0 \cdot \vec{E}_0^* T \sqrt{\pi} \quad (\text{Example 7.3})$$

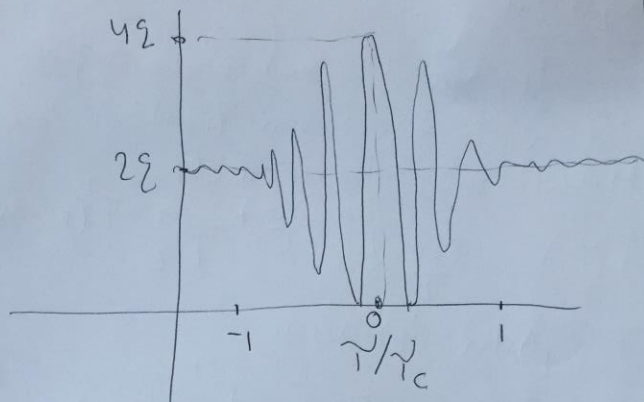
$$\chi(\tau) = \frac{T}{\sqrt{\pi}} \int_{-\infty}^{\infty} e^{-T^2(\omega - \omega_0)^2} e^{-i\omega\tau} d\omega$$

$$= e^{-\frac{\tau^2}{4T^2}} e^{-i\omega_0\tau}$$

$$\text{Sig}(\tau) \propto 2\mathcal{E} [1 + \text{Re}\{\chi(\tau)\}]$$

$$= 2\mathcal{E} \left[1 + e^{-\frac{\tau^2}{4T^2}} \cos(\omega_0\tau) \right]$$





$$\tilde{T}_c \equiv \int_{-\infty}^{\infty} |\chi(\tau)|^2 d\tau$$

\tilde{T}_c = coherence time
 = delay necessary for $\chi(\tau)$ to become steady (no oscillation).

l_c = coherence length
 = $c \tilde{T}_c$