PHYS338:Computational Physics
Final Exam

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Q1:

Consider the differential equation:

$$\frac{dy}{dx} = \sqrt{y + x^2} \tag{1}$$

$$y(0) = 1 \tag{2}$$

Using Forward Euler Method to solve this differential equation with $dx = 10^{-5}$. Then the solution of $y(x)$ is plotted:

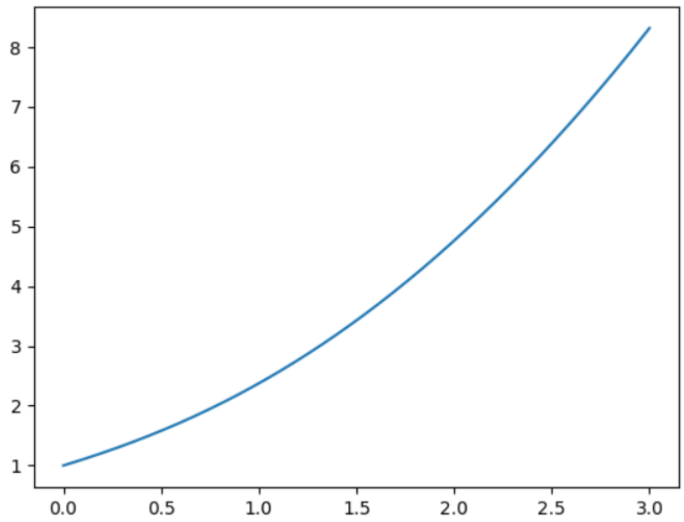


Figure 1: y VS x; $0 \leq x \leq 3$ - Q1

The code used is:

`ExamQ1.py`

From the code, y(3) is calculated:

$$y(3.00000000011186) = 8.319602661671594$$

The error is: $O(h^2) = O(dx^2) = O(10^{-10})$

Q2:

Consider this integral:

$$I = \int_{-\pi}^{\pi} e^{-\sin(x)} dx \tag{3}$$

I will use Simpson's Rule to find the value of I, with $n = 100000$. The code used is:

`ExamQ2.py`

And the result is:

$$I = 7.954926521012942$$

To measure the error uncertainty in Simpson's Rule, i used this formula:

$$E \leq \frac{(b-a)^5}{180n^4}(max(|f^{(4)}(x)|)) \tag{4}$$

In Fig(2), i used Wolfram Mathematica code to find the error.

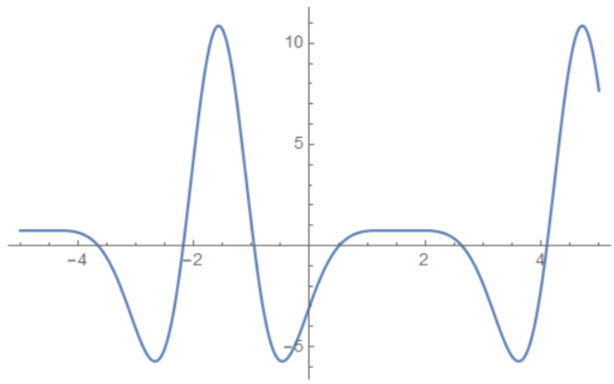
```
In[1]:= F[x_] := e^-Sin[x]

In[4]:= F''''[x]

        -4 e^-Sin[x] Cos[x]^2 + e^-Sin[x] Cos[x]^4 - e^-Sin[x] Sin[x]
        +6 e^-Sin[x] Cos[x]^2 Sin[x] + 3 e^-Sin[x] Sin[x]^2

In[8]:= Plot[F''''[x], {x, -5, 5}]
```

Out[8]=

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In[9]:= FindMaximum[F''''[x], {x, -2}]

Out[9]= {10.8731, {x -> -1.5708}}

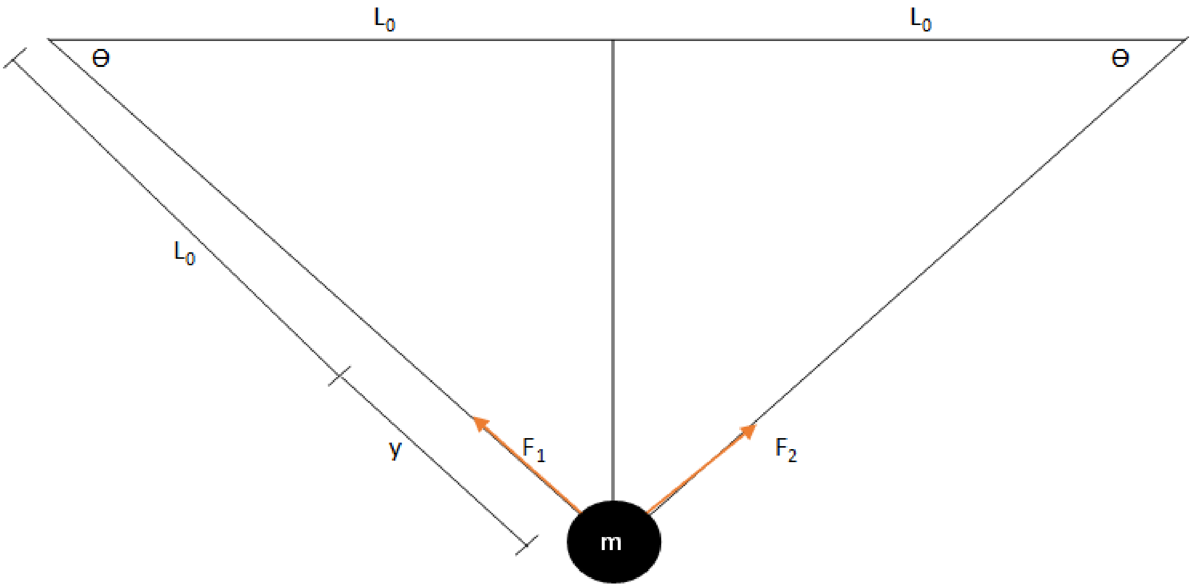
         (2 π)^5 * 10.87312731383618
In[10]:= ---------------------------
            180 * (100000)^4

Out[10]= 5.91536 × 10^-18
```

Figure 2: Finding Error Uncertainty in Simpson's Rule - Q2

From the Fig(2), we find that the error is $E = 5.91536 \times 10^{-18}$, which is a very small error.

Q3:



In this system, i want to find θ when the system is at equilibrium, that is, the acceleration is zero.

$$mg - 2ky\sin(\theta) = ma = 0 \tag{5}$$

Where $k = 640N/m$ and $L_0 = 0.1m$.

Writing y as function of θ:

$$\cos(\theta) = \frac{L_0}{L_0 + y} \tag{6}$$

$$y = \frac{L_0}{\cos(\theta)} - L_0 \tag{7}$$

Hence, equation(5) becomes:

$$2kL_0[\tan(\theta) - \sin(\theta)] - mg = 0 \tag{8}$$

This is a non linear equation that is analytically very hard to solve. So i used Bisection Method to solve it by making $f(\theta)$ equal the left hand side of the equation(8). Then i got θ as function of mass m. Finally i found the value of angle θ corresponding to mass $m = 5kg$.

To implement the method, i used this C language code:

`ExamQ3.c`

However, it is necessary to find θ_0 and θ_1 such that: $f(\theta_0)f(\theta_1) < 0$. So i used Desmos to find them: From this i found that: $\theta_0 = 0.5$ and $\theta_1 = 0.85$.

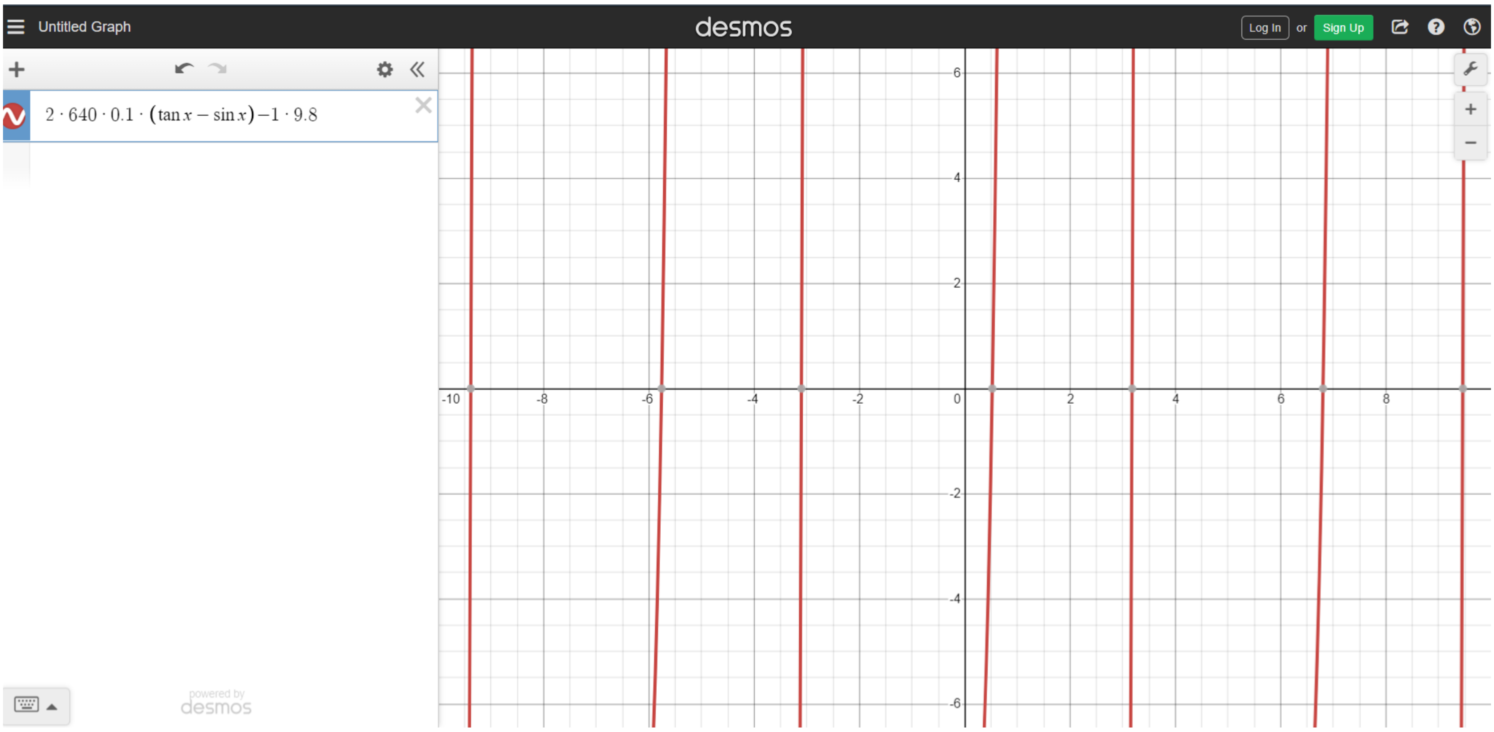


Figure 3: Using Desmos to plot $f(\theta)$ - Q3

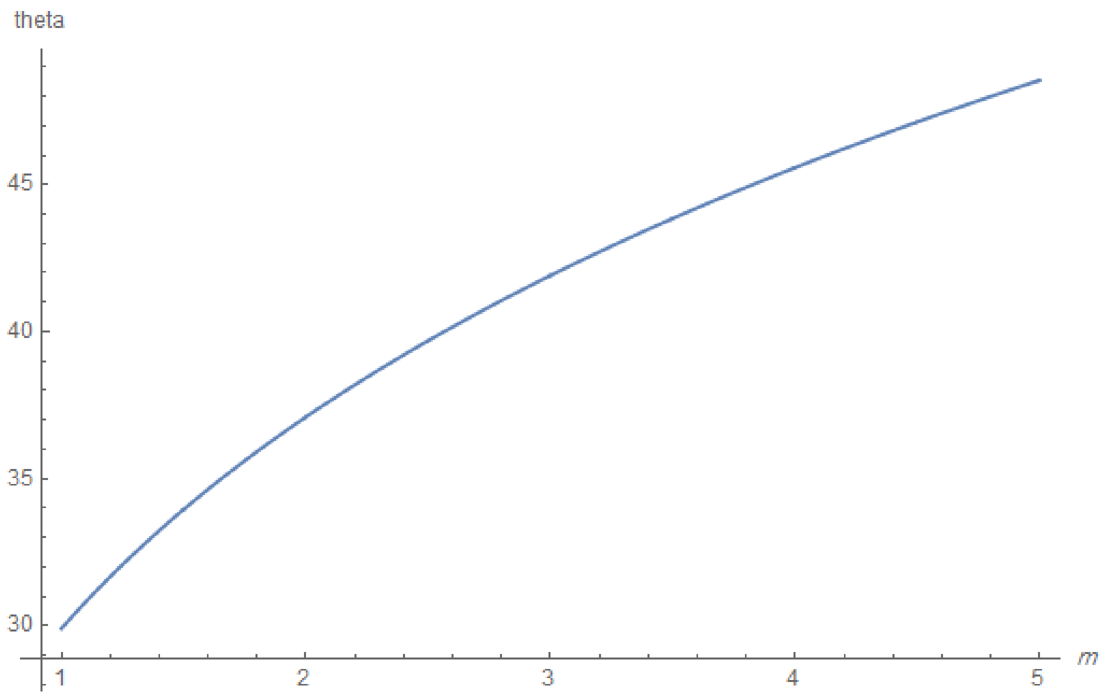


Figure 4: θ(degree) VS mass(kg) - Q3

After solving the equation with different values of m, i got the plot of θ as a function of mass used. Fig(4).

From the calculated data, i found:

$$\theta = 48.552202^o \text{ at mass} = 5.000000 \text{ kg} \tag{9}$$

Q4:

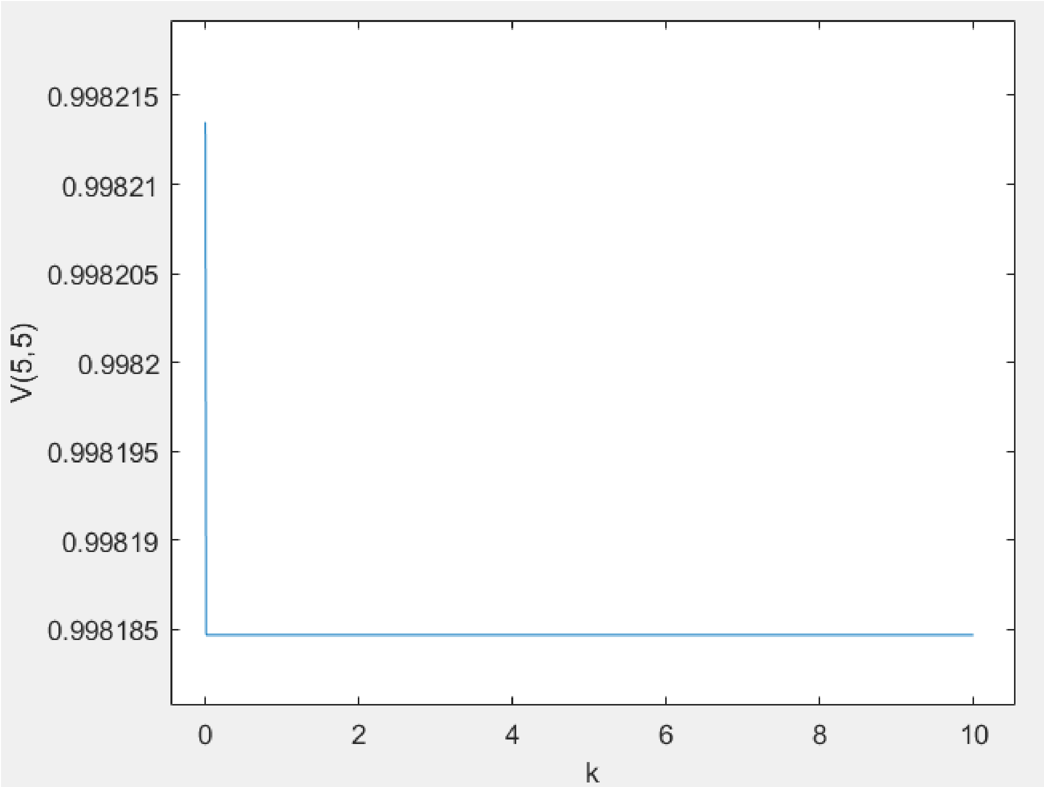


Figure 5: V(5,5) VS k - Q4

Consider this physical partial differential equation with boundary conditions:

$$\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} - k^2 V = 0 \tag{10}$$

$$V(x,0) = 1 \tag{11}$$

$$V(x,10) = 1 \tag{12}$$

$$V(0,y) = 1 \tag{13}$$

$$V(10,y) = 0 \tag{14}$$

Where $V(x,y)$ is 2D-voltage function, and k is inverse screening length. $k = \frac{1}{\lambda}$ where λ is screening length.

We want to solve this partial equation for different values of k, such that $0 \leq k \leq 10$. Then i plot V(5,5) as a function of k. In Fig(5) shows this plot.

To solve the equation(10), i used matlab code:

`ExamQ4m.m`

Q5:

The two differential equations that describe the Felix model are:

$$\frac{dv}{dt} = F_g - F_d \tag{15}$$

$$= \frac{g}{(1+\frac{h}{R_e})^2} - \frac{A}{2m}C(v)\rho(h)v^2 \tag{16}$$

$$\frac{dh}{dt} = -v \tag{17}$$

where F_g is the gravitational force, and F_d is the drag force in opposite direction, and $C(v)$ and $\rho(h)$ as shown in the question.

I used <u>Forward Euler</u> method to solve these equations with $dt = 0.001$. I used C language code:

`ExamQ5.c`

And i used this python code to plot the results:

`ExamQ5Plot.py`

The results are in the following plots:

Notice that at $A = 0.120m^2$, the velocities are higher. Also, the flight time(can be found in h VS t txt) of $A = 0.120m^2$ is less that of $A = 0.450m^2$:

$$A = 0.120m^2 : \text{ Flight time} = 1.695020 \times 10^2 s \tag{18}$$

$$A = 0.450m^2 : \text{ Flight time} = 2.962780 \times 10^2 s \tag{19}$$

We notice that as the area is being smaller, the drag force is smaller. Hence, the velocity gets higher.

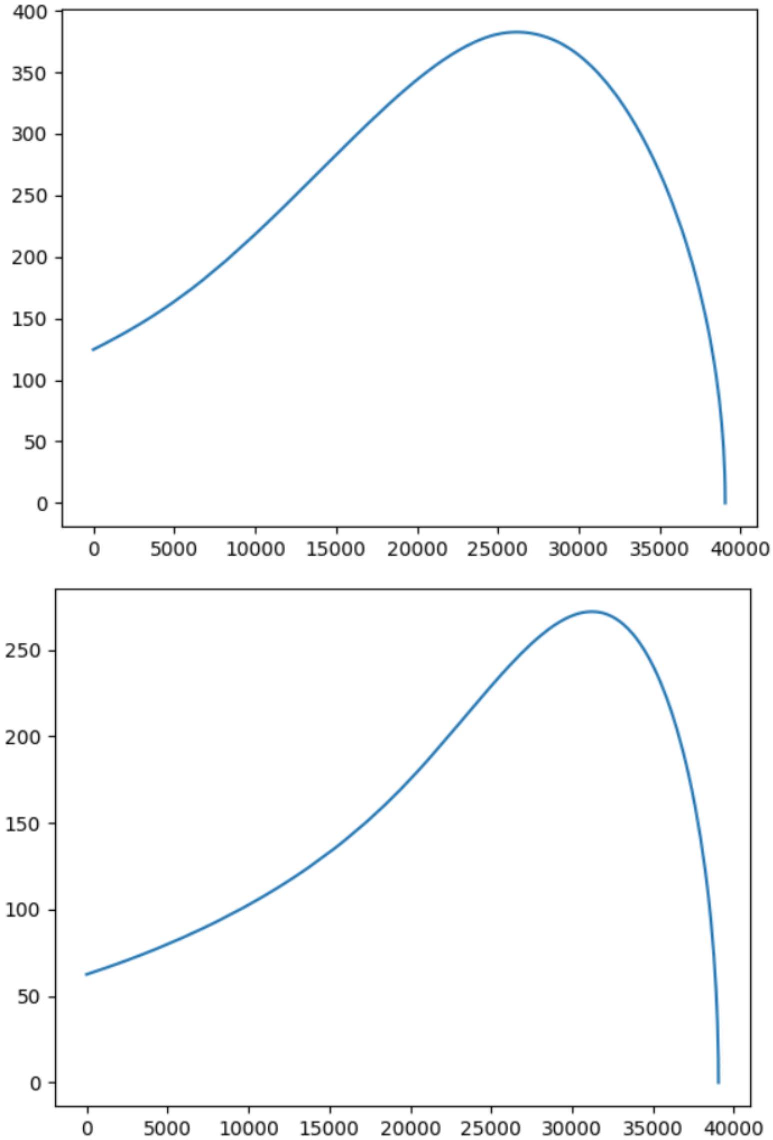


Figure 6: v VS h; above Fig: $A = 0.120m^2$, below Fig: $A = 0.450m^2$ - Q5