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Phys338/Final Exam
Sunday 24/1/2021

In all problems below: 1) work on them alone. 2) Mention the computational method you used 3) Attached your solution code (no direct solutions from other computational packages is allowed)

1) (20%) Consider the differential equation

$$\frac{dy}{dx} = \sqrt{y + x^2}$$

If $y(0) = 1$,

a) Plot y for x between 0 and 3.

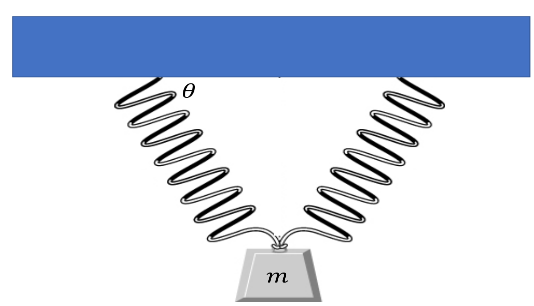
b) Find the value of y(3) to at least 8 decimal places? How did you estimate the accuracy?

2) (15%) Evaluate to at least 12 decimal places

$$I = \int_{-\pi}^{\pi} e^{-sin(x)} dx$$

What method did you use? and how did estimate accuracy?

3) (20%) Consider two springs of equilibrium length $L_0 = 10.0\ cm$ and spring constant $k = 640\ N/m$ that are fixed to the ceiling at two points $2L_0$ apart. If an object of mass m is linked to the other sides of the two springs as shown in the figure. Find the angle θ that a 5 kg object makes.



4) (15%) Consider a very long rectangular capillary tube of edge length 10nm. This tube is filled with electrolyte solution. If three sides of the tube where held at a potential of 1 V, while the fourth side is held at zero potential. Find the electrostatic potential of the system by solving

$$\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} - k^2 V = 0$$

With the boundary conditions

$$V(0,x) = 1.0, V(x,10) = 1.0, V(0,y) = 1.0 \; and \; V(10,y) = 0$$

Solve the equation above for different values of k to plot $V(5,5)$ as a function of k for k between 0 and 10.

5) (30%) In 2012, Felix Baumgartner jumped from an altitude of $h_0 = 39{,}045\ m$ towards the earth. We would like to model his fall trajectory, velocity and duration. To achieve that we consider having two forces affecting him during the fall. The first force is the gravitational force $F_g = G\frac{mM_{earth}}{(R_{earth}+h)^2} = \frac{mg}{\left(1+\frac{h}{R_{earth}}\right)^2}$. Where $m = 70\ kg$ is the mass of Felix and h is his distance from the surface of the earth.
   The second force is the drag force which we will model with $F_D = \frac{1}{2}C\rho A v^2$. Where C is the drag coefficient; ρ is the density of air and v are the effective cross section area and velocity of the falling object, respectively. We need to further consider that the air density over earth surface is not constant. It decreases with height. As an approximation, consider that $\rho(h) = \rho_0 \exp\left(-\frac{h}{H}\right)\ kg/m^3$. Where $\rho_0 = 1.225\ kg/m^3$ and $H = 10.4\ km$. In addition, the drag coefficient usually depends on the shape and velocity of the moving object relative to the speed of sound ($v_{sound} = 343\ m/s$). Assume for our case, it is given by

$$C(v) = \begin{cases} 0.65 & \frac{v}{v_{sound}} < 0.6 \\ 0.65 + 0.55\left(\frac{v}{v_{sound}} - 0.6\right)^2 & 0.6 < \frac{v}{v_{Sound}} < 1.1 \\ 0.7875 - 0.32\left(\frac{v}{v_{sound}} - 1.1\right) & \frac{v}{v_{sound}} > 1.1 \end{cases}$$

Therefore, Felix trajectory can be modeled using

$$\frac{dv}{dt} = \frac{g}{\left(1+\frac{h}{R_{earth}}\right)^2} - \frac{1}{2}\frac{A}{m}C(v)\rho_0 \exp\left(-\frac{h}{H}\right)v^2$$

$$\frac{dh}{dt} = -v$$

Write your own code or modify the codes provided to you to solve these equations and subsequently

a) Plot Felix velocity as a function of distance from earth surface of the case of skydiving head down (; $A = 0.120\ m^2$) and skydiving with belly down ($A = 0.450\ m^2$).

b) Compare the flight time of both cases mentioned in (a).