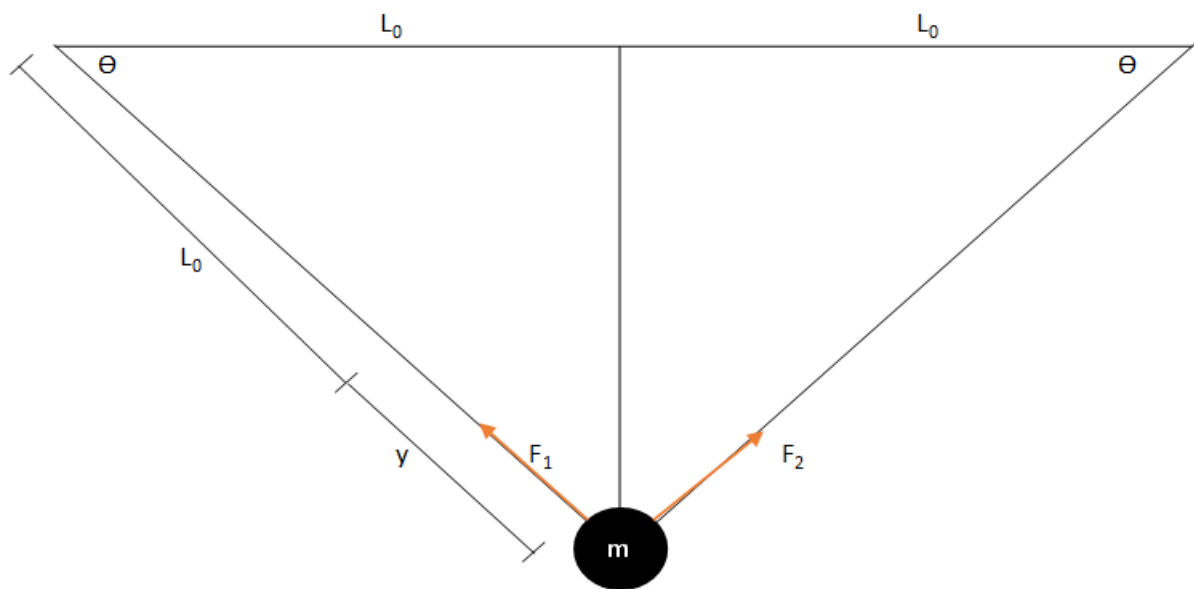


PHYS338:Computational Physics

HW6

Student Name: Hamza AlHasan-1181636

Submission Date: Nov 11, 2020



In this system, i want to find θ when the system is at equilibrium, that is, the acceleration is zero.

$$mg - 2ky \sin(\theta) = ma = 0 \quad (1)$$

Writing y as function of θ :

$$\cos(\theta) = \frac{L_0}{L_0 + y}$$
$$y = \frac{L_0}{\cos(\theta)} - L_0$$

Hence, equation(??) becomes:

$$2kL_0[\tan(\theta) - \sin(\theta)] - mg = 0 \tag{2}$$

This is a non linear equation that is very hard to solve, i tried to use Wolfram Mathematica code to solve it and get the analytical solution. It shows many slides for its solution formula. It is enough to show the first slide in this figure:

```

In[5]= Solve[mg == 2 kL (Tan[theta] - Sin[theta]), theta]
In[6]= TraditionalForm[%5]
Out[6]/TraditionalForm=

```

$$\left\{ \left\{ \theta \rightarrow \text{ConditionalExpression} \left[\tan^{-1} \left(\frac{\frac{\frac{mg^4}{\sqrt{kL^2 \sqrt[3]{mg^6 + 864 kL^4 mg^2 + 24 \sqrt{3} \sqrt{432 mg^4 kL^8 + mg^8 kL^4}} - \frac{2 mg^2}{kL^2} + \frac{\sqrt[3]{mg^6 + 864 kL^4 mg^2 + 24 \sqrt{3} \sqrt{432 mg^4 kL^8 + mg^8 kL^4}}}{kL^2} + 12}{4 \sqrt{3}}}{12 kL^2 \sqrt[3]{mg^6 + 864 kL^4 mg^2 + 24 \sqrt{3} \sqrt{432 mg^4 kL^8 + mg^8 kL^4}} - \frac{1}{2} \left(\frac{mg^4}{12 kL^2 \sqrt[3]{mg^6 + 864 kL^4 mg^2 + 24 \sqrt{3} \sqrt{432 mg^4 kL^8 + mg^8 kL^4}} - \frac{mg^2}{3 kL^2} \right)} \right. \right. \right.$$

$$\left. \left. \left. \frac{\sqrt[3]{mg^6 + 864 kL^4 mg^2 + 24 \sqrt{3} \sqrt{432 mg^4 kL^8 + mg^8 kL^4}}}{12 kL^2} - \frac{\sqrt{3} \left(-\frac{2 mg^2}{kL^2} - 8 \right)}{2 \sqrt{\frac{\frac{mg^4}{\sqrt{kL^2 \sqrt[3]{mg^6 + 864 kL^4 mg^2 + 24 \sqrt{3} \sqrt{432 mg^4 kL^8 + mg^8 kL^4}} - \frac{2 mg^2}{kL^2} + \frac{\sqrt[3]{mg^6 + 864 kL^4 mg^2 + 24 \sqrt{3} \sqrt{432 mg^4 kL^8 + mg^8 kL^4}}}{kL^2} + 12}{kL^2} + 12}}}{2} + 2 \right. \right. \right.$$

$$\left. \left. \left. \frac{1}{2 kL mg} \left(\frac{\frac{\frac{mg^4}{\sqrt{kL^2 \sqrt[3]{mg^6 + 864 kL^4 mg^2 + 24 \sqrt{3} \sqrt{432 mg^4 kL^8 + mg^8 kL^4}} - \frac{2 mg^2}{kL^2} + \frac{\sqrt[3]{mg^6 + 864 kL^4 mg^2 + 24 \sqrt{3} \sqrt{432 mg^4 kL^8 + mg^8 kL^4}}}{kL^2} + 12}{16 \sqrt{3} \sqrt[3]{mg^6 + 864 kL^4 mg^2 + 24 \sqrt{3} \sqrt{432 mg^4 kL^8 + mg^8 kL^4}}} + \right. \right. \right.$$

$$\left. \left. \left. \right) \right) \right\} \right\}$$

Figure 1: The analytical answer using Wolfram -first slide!-

So, it is necessary to use another methods. I tried to use two numerical methods to solve it:

- Secant method
- Bisection method

The function that i want to find its zero is the left side of the equation(??).
Let:

$$f(\theta) = 2kL_0[\tan(\theta) - \sin(\theta)] - mg$$

However, secant method showed wrong values and became inconsistent, because $f(x)$ especially at high masses shows approximately vertical line patterns, that makes the secant of two points approaches infinity. I used Desmos to demonstrate these patterns:

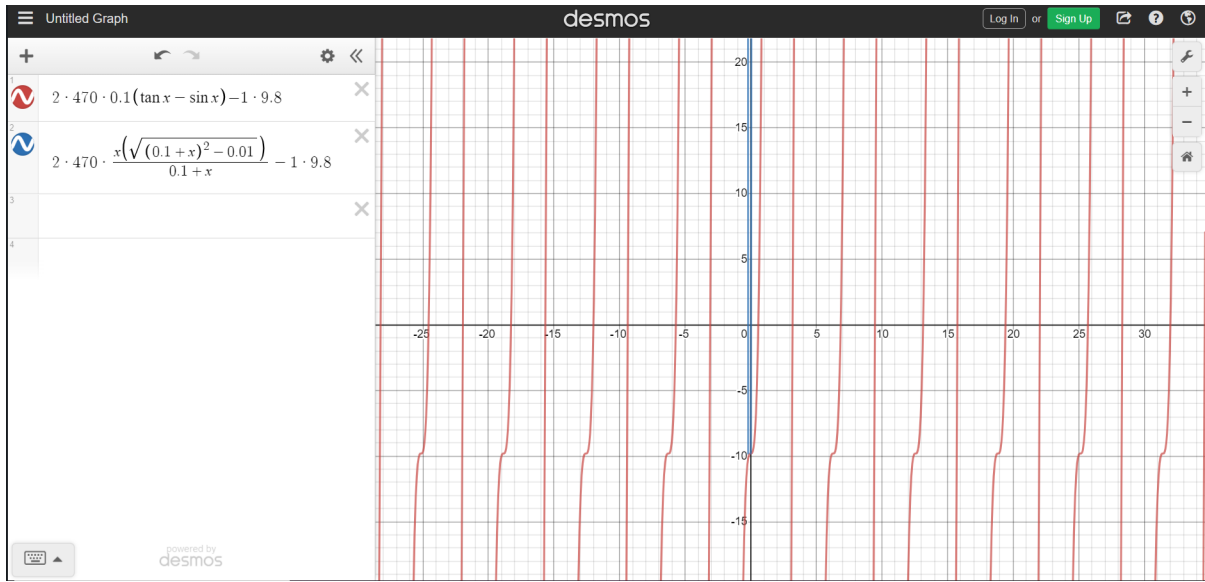


Figure 2: $f(x)$ with mass=1 kg

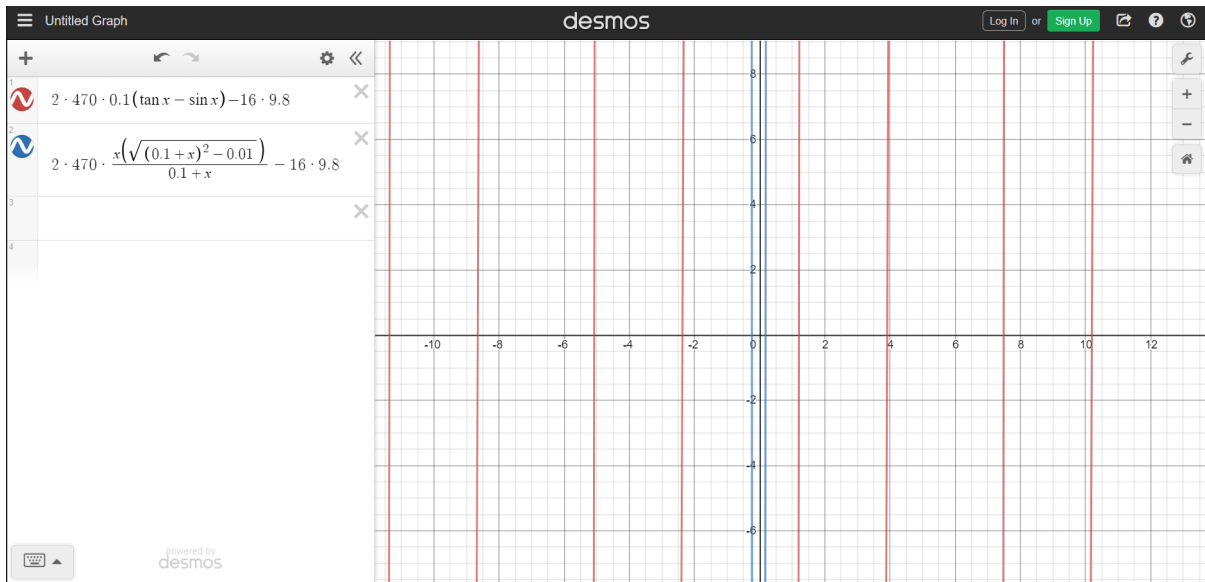


Figure 3: $f(x)$ with mass=16 kg

Anyway, using Bisection method, the problem is solved and i plot (θ VS m), with angles in radians and degrees.

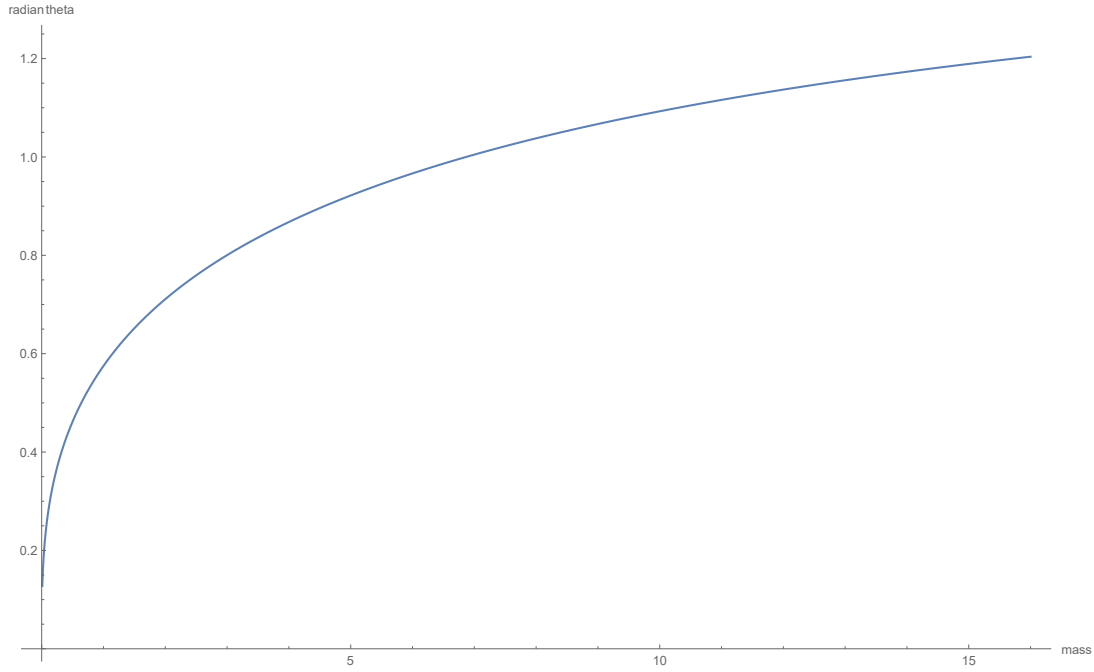


Figure 4: $\theta(\text{radian})$ VS m

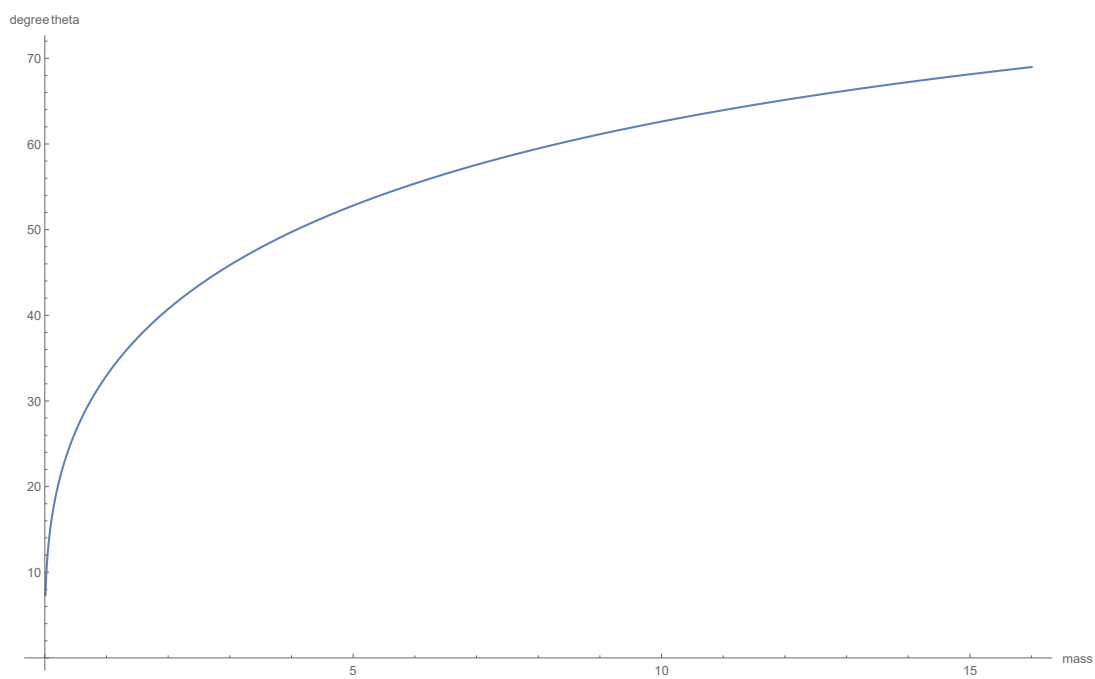


Figure 5: $\theta(\text{degree})$ VS m