

Phys338/Homework #6

Due on Monday 2/11/2020

In 2012 Felix Baumgartner jumped from an altitude of $h_0 = 39,045 \text{ m}$ towards the earth. We would like to model his fall trajectory, velocity and duration. To achieve that we consider having two forces affecting him during the fall. The first force is the gravitational force $F_g = G \frac{mM_{\text{earth}}}{(R_{\text{earth}}+h)^2} = \frac{mg}{\left(1+\frac{h}{R_{\text{earth}}}\right)^2}$. Where $m = 70 \text{ kg}$

is the mass of Felix and h is his distance from the surface of the earth. The second force is the drag force which we will model with $F_D = \frac{1}{2}C\rho Av^2$. Where C is the drag coefficient; ρ is the density of air; $A = 0.120 \text{ m}^2$ and v are the effective cross section area and velocity of the falling object, respectively. We need to further consider that the air density over earth surface is not constant. It decreases with height. As an approximation, consider that $\rho(h) = \rho_0 \exp\left(-\frac{h}{H}\right) \text{ kg/m}^3$. Where $\rho_0 = 1.225 \text{ kg/m}^3$ and $H = 10.4 \text{ km}$. In addition, the drag coefficient usually depends on the shape and velocity of the moving object relative to the speed of sound ($v_{\text{sound}} = 343 \text{ m/s}$). Assume for our case, it is given by

$$C(v) = \begin{cases} 0.65 & \frac{v}{v_{\text{sound}}} < 0.6 \\ 0.65 + 0.55 \left(\frac{v}{v_{\text{sound}}} - 0.6\right)^2 & 0.6 < \frac{v}{v_{\text{sound}}} < 1.1 \\ 0.7875 - 0.32 \left(\frac{v}{v_{\text{sound}}} - 1.1\right) & \frac{v}{v_{\text{sound}}} > 1.1 \end{cases}$$

Therefore, Felix trajectory can be modeled using

$$\frac{dv}{dt} = \frac{g}{\left(1 + \frac{h}{R_{\text{earth}}}\right)^2} - \frac{1}{2} \frac{A}{m} C(v) \rho_0 \exp\left(-\frac{h}{H}\right) v^2$$

$$\frac{dh}{dt} = -v$$

Write your own code or modify the codes provided to you to solve these equations and subsequently

- Plot Felix velocity as a function of time.
- Plot Felix position as a function of time.
- Plot Felix velocity as a function of height.
- From last plot, how long will take him to reach the ground?