Quantum Statistics

10/12/2019

- System can exchange particles & energy with reservoir

[illegible] E , N

$$\frac{P(S_2)}{P(S_1)} = \frac{\Omega_R(S_2)}{\Omega_R(S_1)} = e^{\frac{1}{k}\left[S_R(S_2) - S_R(S_1)\right]}$$

[illegible]

system [illegible] in equilibrium [illegible] reservoir [illegible]

[illegible] E [illegible]

[illegible] (U) Energy

$$\Rightarrow dS_R = \frac{1}{T}\left[dU_R - \underbrace{P\,dV_R}_{\text{negligble}} - \mu\, dN_R\right]$$

$$\Delta S_R = S_R(S_2) - S_R(S_1) = \frac{-1}{T}\left[E(S_2) - E(S_1) - \mu N(S_2) + \mu N(S_1)\right]$$

[illegible] exchange في E&N

$$\boxed{U_{R_{S_2}} + E_{S_2} = U_{R_{S_1}} + E_{S_1}}$$

$$\frac{P(S_2)}{P(S_1)} = \frac{e^{\frac{-1}{kT}\left[E(S_2) - \mu N(S_2)\right]}}{e^{-\frac{1}{kT}\left(E(S_1) - \mu N(S_1)\right)}}$$

$\frac{N}{\mu} - \frac{N}{\mu}$

Gibbs factor : $e^{\frac{-1}{kT}\left[E(S) - \mu N(S)\right]}$

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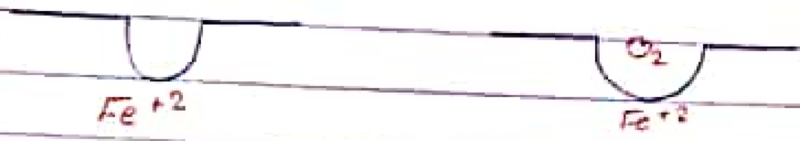
Grand partition function (Z) "Gibbs Sum"

$$Z = \sum_s e^{-\frac{1}{kT}\left(E(s) - \mu N(s)\right)}$$

Exp) CO poisoning

Case 1)



unoccupied $(E=0)$ occupied $\left[\begin{array}{l} E = -0.7\,eV \\ \mu = -0.6\,eV \end{array}\right]$

$$Z = 1 + e^{-\frac{1}{kT}\left[-0.7 - -0.6(1)\right]}$$

$$= 1 + e^4$$

$$P(\text{occupied by } O_2) = \frac{e^4}{1+e^4} \approx 98\%$$

Case 2

$E = 0$ $E = -0.7\,eV$ $E = -0.85\,eV$

$\mu = 0$ $\mu = -0.6\,eV$ $\mu = -0.72\,eV$

$$Z = 1 + e^4 + e^{\frac{0.13}{0.025}}$$

$$= 1 + e^4 + e^{5.2}$$

$$P(O_2) = \frac{e^4}{1+e^4+e^{5.2}} \approx 25\%$$

7.6) $Z = \sum_s e^{-\frac{1}{kT}(E(s) - \mu N(s))}$ ← اذا كانت Z معلومة مباشرة

$$\bar{N} = \frac{kT}{Z}\frac{\partial Z}{\partial \mu}$$

$$\frac{\partial Z}{\partial \mu} = \sum \frac{1}{kT} N(s) \, e^{-\frac{1}{kT}(E(s) - \mu N(s))}$$

$$\frac{kT}{Z}\left(\frac{\partial Z}{\partial \mu}\right) = \frac{\sum N(s) \, e^{-\frac{1}{kT}(E(s) - \mu N(s))}}{Z}$$

$$\frac{kT}{Z}\frac{\partial Z}{\partial \mu} = \frac{\sum N P}{Z} = \bar{N}$$

Bosons & Fermions

- We derived $Z = \frac{1}{N!} Z_1^N$ for a system of N indistinguishable ; non ~~iter~~ interacting particles

This is correct only if the particles are always in different States

Exp.) We have 2 noninteracting particles, 5 states
All states have zero energy, therefore
Boltzmann factor $= e^0 = 1$, $\boxed{Z = \Omega}$

عدد الـ States

Case 1) Particles are distinguishable

$$Z = 5 \times 5 = 25 \text{ states}$$

Case 2) Particles are indistinguishable

$$Z = \frac{25}{2} = 12.5$$?!!

وين صار الغلط ؟!

لما استثنينا انه يكونوا الـ 2 ضمن State

خلينا نشوف شو هم الـ States

11000	01010	20000
10100	01001	02000
10010	00110	00200
10001	00101	00020
01100	00011	00002

هنا عملنا الاشي الجديد
نقصنا عدد الـ (States)

الغلط هون
انه قسمناهم ع 2
بس هدول بكونوا
Fermion لو كانوا موجودات

- Bosons : No limit on Particles in a State
- Fermions : Only one particle per state (Pauli Exclusion principle)
- Bosons have integer spin
- Fermions have ½ integer spin
- If $Z_1 \gg N$; # of Available Single particle states is more than # of particles then there's a Small chance of Two particles sharing the same state.

- $Z_1 = \frac{V}{V_Q} Z_{int}$ (Ideal gas)

- $\frac{V}{V_Q} Z_{int} \gg N \Rightarrow \frac{V}{N} \gg V_Q$ "Z_{int} : Small Number"

low density → classical limit

- For air, Avg. distance between particles is ≈ 3 nm & $\lambda_{avg} \approx 0.02$ nm

So $\frac{3}{0.02} = 150$

We need Quantum Statistics } Low Temp, & high densities

Pb 7.8) 10 Single-particle States with Zero energy

a) Z for one particle $= \Omega e^{0} = 10$

b) Z for 2 distinguishable particles $= 10 \times 10 = 100$

c) Two Identical bosons $Z = 10 + 45 = 55$

d) Two Identical Fermions $\dfrac{100 - \overset{\text{مكرر}}{10}}{2} = 45$

e) Z according to $\frac{1}{N!}(Z_1)^N = \frac{1}{2!}(10)^2 = 50$

f) Probability of finding both particles in same state

2 Distinguishable particles $= \frac{10}{100}$

2 Identical bosons $= \frac{10}{55}$

2 Identical Fermions $= 0$

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single particle states with zero ...

Z for one particle $= \Omega e^{0} = 10$

Z for 2 distinguishable particles $= 10 \times 10 = 100$

Z for 2 identical bosons $Z = 10 + 45 = 55$

Z for 2 identical Fermions $\frac{100 - 10}{2} = 45$

Z according to $\frac{1}{N!}(Z_1)^N = \frac{1}{2!}(10)^2 = 50$

Probability of finding both particles in same state

Distinguishable particles $= \frac{10}{100}$

Identical bosons $= \frac{10}{55}$

Identical Fermions $= 0$