Birzeit University-Department of Physics
Quantum Mechanics I Phys433
Fall 2020
Midterm Exam, Nov. 30^{th} 2020

1. For a particle of mass m, placed in infinite square potential of width a.
   (a) (10 points) Find the solution of the wavefunction in the momentum space for any state n
   (b) (10 points) Compute the expectation value for P^4 both using wavefuvtion in momentum space and position space, compare your results. Is it what you expected why or why not

2. Consider a particle of mass m and charge q in one dimensional harmonic oscillator, we can define the following operator:

$$
\begin{aligned}
U(\lambda) = e^{\lambda(a-a^\dagger)} &= e^{-\lambda a} e^{-\lambda a^\dagger} e^{\lambda^2/2} \\
\tilde{H} &= U(\lambda) H U^\dagger(\lambda) \\
\tilde{a} &= U(\lambda) a U^\dagger(\lambda) \\
\tilde{a^\dagger} &= U(\lambda) a^\dagger U^\dagger(\lambda)
\end{aligned}
$$

   (a) (6 points) Show that $\tilde{a} = a - \lambda$
   (b) (6 points) Show that $\tilde{a^\dagger} = a^\dagger - \lambda$
   (c) (9 points) Write $\tilde{H}$ in terms of a, $a^\dagger$ and λ
   (d) (9 points) If the harmonic oscillator is subjected to an external E uniform electric field, Show that $\lambda = \frac{qE}{\omega}\sqrt{\frac{1}{2m\hbar\omega}}$

3. In Ch2 we learned about the raising and lowering operators for Harmonic oscillator. This approach can be generalized for any potential by making the following definition:

$$
\begin{aligned}
\hat{A} &= i\frac{\hat{P}}{2m} + \hat{W}(x) \\
\hat{A}^\dagger &= -i\frac{\hat{P}}{2m} + \hat{W}(x) \\
\hat{H}_1 = \hat{A}\hat{A}^\dagger &= \frac{P^2}{2m} + V_1(x) \\
\hat{H}_2 = \hat{A}^\dagger \hat{A} &= \frac{P^2}{2m} + V_2(x)
\end{aligned}
$$

   Now the trick is as follows:

   - Let $\psi_n^{(1)}$ is a solution to $\hat{H}_1$ with eigenvalue $E_n^{(1)}$, then $\hat{A}\psi_n^{(1)}$ is a solution to $\hat{H}_2$ with the same energy.
   - Let $\psi_n^{(2)}$ is a solution to $\hat{H}_2$ with eigenvalue $E_n^{(2)}$, then $\hat{A}^\dagger\psi_n^{(2)}$ is a solution to $\hat{H}_1$ with the same energy.

   (a) (10 points) Apply this method to the case of Harmonic oscillator taught in the class, define $\hat{W}(x)$, $V_1(x)$ and $V_2(x)$
   (b) (25 points) Apply it to the one dimensional Hydrogen atom:

$$
\hat{H} = -\frac{d^2}{dx^2} + \frac{L(L+1)}{x^2} - \frac{1}{x}
$$

4. (20 points) Consider the following potential

$$
V(x) = \begin{cases} 0 & \text{if x} < 0 \\ V_1 & \text{if a} > \text{x} > 0 \\ V_2 & \text{if a} < \text{x} \end{cases}
$$

   where $0 < V_1 < V_2$, and a particle of total energy E> V_2 approaching x=0 in the direction of increasing x. show that the probability of continuing into the region x>a is a unity if a equals an integral or half-integral number of deBroglie wavelengths in the region $0 < x < a$.

5. (15 points) Prove that there is no degeneracy in 1-D quantum mechanics

6. A Hamiltonian H has two orthonormal eigenstates $|1>$ and $|2>$ such that:

$$\hat{H}|1>= E_1|1> \qquad \hat{H}|2>= E_2|2> \qquad E_1 \neq E_2$$

Two states $|A>$ and $|B>$ are defined as follows:

$$< 1|A>= \frac{1}{\sqrt{2}} \qquad < 2|A>= \frac{i}{\sqrt{2}}$$
$$< 1|B>= \frac{1}{\sqrt{2}} \qquad < 2|B>= \frac{-i}{\sqrt{2}}$$

(a) (6 points) Calculate $< A|B>$ and $< B|A>$

(b) (6 points) If the system initially in the state $|\psi(t=0)>= |A>$, what is the time dependent state $|\psi(t)>$?

(c) (8 points) What is the probability of finding the particle at state $|A>$ at any time t?

Question:	1	2	3	4	5	6	Total
Points:	20	30	35	20	15	20	140
Score:							

Good Luck