Birzeit University
Faculty of Science-Department of Physics
Quantum Mechanics II
Spring 2021
Final Exam, Due June. 10^{th} 2021

Instructions:

1. You are allowed to use one books, namely: Griffiths, David J, Introduction to Quantum Mechanics, 3^{rd} edition
2. You are not allowed to communicate with each others.
3. you are not allowed to communicate with anybody regarding the exam.
4. You can communicate with me through Ritaj

1. Consider a one-dimensional harmonic oscillator of frequency ω_0- Denote the energy eigenstates by n, starting with n = 0 for the lowest. To the original harmonic oscillator potential a time-independent perturbation V(x) is added. The matrix representation of $V(x)$ of the unperturbed eigenstates. A portion of the matrix is given below, where ϵ is a small dimensionless constant. [Note that the indices on this matrix run from n = 0 to 4.]

$$\epsilon\hbar\omega_0 \begin{pmatrix} 1 & 0 & -\sqrt{\frac{1}{2}} & 0 & \sqrt{\frac{3}{8}} \\ 0 & 0 & 0 & 0 & 0 \\ -\sqrt{\frac{1}{2}} & 0 & \frac{1}{2} & 0 & -\sqrt{\frac{3}{16}} \\ 0 & 0 & 0 & 0 & 0 \\ \sqrt{\frac{3}{8}} & 0 & -\sqrt{\frac{3}{16}} & 0 & \frac{3}{8} \end{pmatrix}$$

   (a) (8 points) Find the new energies for the first five energy levels to second order in perturbation theory.

   (b) (12 points) Find the new energies for n = 0 to second order in perturbation theory.

2. (15 points) A quantum mechanical rigid rotor constrained to rotate in one plane. It has moment of inertia I about its rotational axis, and electric dipole moment μ.
This rotor is placed in a weak uniform electric field E, which is in the plane of rotation. Treating the electric field as a perturbation, find the first non-vanishing corrections to the energy levels of the rotor.

3. (15 points) A particle of mass m is placed in half harmonic oscillator, that is for $x < 0$ $V(x) = 0$ and for $x > 0$ $V(x) = \frac{1}{2}m\omega^2 x^2$. Use WKB approximation to find the allowed energies.

4. (15 points) Consider a particle of mass m in an one dimensional infinite square well: V(x) = 0 (for $-a \leq x \leq a$), V(x) = ∞(otherwise). Let the wave function of the ground state to be given:

$$\psi(x) = (a^2 - x^2)$$

(for $-a \leq x \leq a$) Calculate the mean value of the Hamiltonian in this state. Compare the result obtained with the true value.

5. (10 points) What should be the condition on α so that the potential

$$V(x) = -\frac{V_0}{(x^2 + a^2)^\alpha}$$

may have infinite number of levels?

6. (15 points) Consider a one-dimensional simple harmonic oscillator whose classical angular frequency is ω_0. For t < 0 it is known to be in the ground state. For t > 0 there is also a time-dependent potential $V(t) = V_0 x e^{-t/\tau}$, where V_0 is constant in both space and time. What is the probability of transition to the first excited state.

Question:	1	2	3	4	5	6	Total
Points:	20	15	15	15	10	15	90
Score:							

Good Luck