Birzeit University
Faculty of Science-Department of Physics
Quantum Mechanics II, Phys4332
Spring 2021
Homework 3:

1. Let $\vec{s}_1$ and $\vec{s}_2$ be the spin operators of two spin 1/2-particles. Then $\vec{S} = \vec{s}_1 + \vec{s}_2$ is the spin operator for this two-particle system.
   (a) Consider the Hamiltonian $H_0 = \frac{1}{\hbar^2}(S_x^2 + S_y^2 - S_z^2)$. Determine the eigenvalues and eigenvectors of this Hamiltonian.
   (b) Consider the perturbation $H_1 = s_{1x} - s_{2x}$. Calculate the eigenvalues of $H_0 + \lambda H_1$ in first-order perturbation theory.
2. Suppose that a hydrogen atom is exposed to a uniform electric field, $\vec{\varepsilon}$, and a parallel, uniform magnetic field, $\vec{B}$, both pointing in z-direction Consider the first excited energy level, corresponding to n = 2, neglect the spin .
   (a) Show that in general the level is split into four nondegenerate energy levels.
   (b) For what values of ε and B are there instead only three levels, and what are the degeneracies of these levels?
   (c) For what values of ε and B are there only two levels, and what are the degeneracies of these levels?
   (d) Re do the previous parts if the electric field and magnetic field where in 3 dimensions. Show that you can get the results of the three previous parts if the an appropriate choice of E and B.
3. A beam of excited hydrogen atoms in the 2s state passes between the plates of a capacitor in which a uniform electric field E exists over a distance L. The hydrogen atoms have velocity v along the x-axis and the E field is directed along the z-axis, as shown . All the n = 2 states of hydrogen are degenerate in the absence of the E field, but certain of them mix when the field is present.
   (a) Which of the n = 2 states are connected in first order via the perturbation?
   (b) Find the linear combination of n = 2 states which removes the degeneracy as much as possible.
   (c) For a system which starts out in the 2s state at t = 0, express the wave function at time t $\leq \frac{L}{v}$.
   (d) Find the probability that the emergent beam contains hydrogen in the various n = 2 states.
4. Consider an isotropic two-dimensional harmonic oscillator with mass m and frequency ω_0 i.e., a particle with Hamiltonian

$$H_{osc} = \frac{1}{2m}(P_x^2 + P_y^2) + \frac{1}{2}m\omega_0^2(X^2 + Y^2) - \hbar\omega_0$$
$$a_x = \sqrt{\frac{m\omega}{2\hbar}}(X + i\frac{P_X}{m\omega})$$

   Similarly, one can define the annihilation operator for the y-coordinate. Creation operator is the hermitian conjugate of the annihilation operator.

   1. Define $a_\pm = \frac{1}{\sqrt{2}}(a_x \mp i a_y)$

   Write L_z and the Hamiltonian in terms of $a_\pm$ and $a_\pm^\dagger$. Make a conclusion about the basis of both operators. Deduce a relation between the basis of the 2-D Harmonic oscillator and the basis of the orbital angular momentum.
5. A three dimensional harmonic oscillator hamiltonian can be written in the following way:

$$\hat{H} = \frac{-\hbar^2}{2m}\nabla^2 + \frac{1}{2}m(\omega_x^2 X^2 + \omega_y^2 Y^2 + \omega_z^2 Z^2)$$

   The Hamiltonian is in cartesian coordinates, its basis is characterized by 3 quantum numbers $|n_x n_y n_z >$ if $\omega_x = \omega_y = \omega_z$, then it will become an isotropic oscillator and can be solved in spherical coordinates, and its basis can be written as $|Nljm_j>$, where $N = n_x + n_y + n_z$, $L = N, N-2, N-4, ..., 0 or 1$

   a spin-1/2 particle is placed in this 3D-oscillator.Now lets do the following:

(a) If the particle is subjected to a perturbation $H_1 = \mu\vec{\sigma}\cdot\vec{r}$,where σ_x,σ_y, and σ_z are the Pauli spin matrices. Find the expectation value of $x\sigma_x$ in first order perturbation theory for the ground state.

(b) If $\omega_x = \omega_y = \omega_0(1+\frac{1}{3}\epsilon)$ and $\omega_z = \omega_0(1-\frac{2}{3}\epsilon)$. Find the energy and wave-function correction up to first order for the first excited state. Comment on the linear combination obtained for the wave-function correction. Do not use cartesian representation

(c) If $\omega_x = \omega_0(1-\frac{2}{3}\epsilon cos(\gamma+120^o))$, $\omega_y = \omega_0(1-\frac{2}{3}\epsilon cos(\gamma-120^o))$ and $\omega_z = \omega_0(1-\frac{2}{3}\epsilon cos(\gamma))$. Find the energy and wave-function correction up to first order for the first excited state. Comment on the linear combination obtained for the wave-function correction. Do not use cartesian representation