Birzeit University-Department of Physics
Quantum mechanics Phys635
Final Exam Jan. $23^{rd}$ 2021

1. The Hamiltonian operator for a two-level system is given by:

$$H = a(|1><1| - |2><2| + |1><2| + |2><1|)$$

where a is a number with the dimension of energy. Find the energy eigenvalues and the corresponding energy eigenkets

2. Suppose that a hydrogen atom is exposed to a uniform electric field, $\vec{\varepsilon}$, and a parallel, uniform magnetic field, $\vec{B}$. Consider the first excited energy level, corresponding to n = 2. Ignore spin
   (a) Show that in general the level is split into four nondegenerate energy levels.
   (b) For what values of $\varepsilon$ and B are there instead only three levels, and what are the degeneracies of these levels?
   (c) For what values of $\varepsilon$ and B are there only two levels, and what are the degeneracies of these levels?

3. A three dimensional harmonic oscillator hamiltonian can be written in the following way:

$$\hat{H} = \frac{-\hbar^2}{2m}\nabla^2 + \frac{1}{2}m(\omega_x^2 X^2 + \omega_y^2 Y^2 + \omega_z^2 Z^2)$$

The Hamiltonian is in cartesian coordinates, its basis is characterized by 3 quantum numbers $|n_x n_y n_z>$ if $\omega_x = \omega_y = \omega_z$, then it will become an isotropic oscillator and can be solved in spherical coordinates, and its basis can be written as $|Nljm_j>$, where $N = n_x + n_y + n_z$, $L = N, N-2, N-4, ...., 0 or 1$

a spin-1/2 particle is placed in this 3D-oscillator.Now lets do the following:
   (a) If the particle is subjected to a perturbation $H_1 = \mu\vec{\sigma}\cdot\vec{r}$ ,where $\sigma_x$,$\sigma_y$, and $\sigma_z$ are the Pauli spin matrices. Find the expectation value of $x\sigma_x$ in first order perturbation theory for the ground state.
   (b) If $\omega_x = \omega_y = \omega_0(1+\frac{1}{3}\epsilon)$ and $\omega_z = \omega_0(1-\frac{2}{3}\epsilon)$. Find the energy and wave-function correction up to first order for the first excited state. Comment on the linear combination obtained for the wave-function correction. Do not use cartesian representation
   (c) If $\omega_x = \omega_0(1-\frac{2}{3}\epsilon cos(\gamma+120^o))$, $\omega_y = \omega_0(1-\frac{2}{3}\epsilon cos(\gamma-120^o))$ and $\omega_z = \omega_0(1-\frac{2}{3}\epsilon cos(\gamma))$. Find the energy and wave-function correction up to first order for the first excited state. Comment on the linear combination obtained for the wave-function correction. Do not use cartesian representation

4. For a spin-1/2 particle, is it possible to have a state $\chi$ such that the expectation value of the three component of the spin operator to be zero. If yes, then find the state $\chi$. If no, justify your answer.

5. Let $J = J_1 + J_2$, If $J_1 = 1$ and $J_2$ can be either an integer or half-integer. Then the vector $|JM>$ can be written as:

$$|JM>= A|11J_2M-1> + B|10J_2M> + C|1-1J_2M+1>$$

Find a general expression for A, B and C

Note: You can ignore any integrals and just rename them, if their value does not effect the results.