Birzeit University
Department of Physics
Quantum Mechanics Phys635
Fall 2018
Final Exam, Jan. 2nd 2019

1. Consider a particle of mass m and charge q that moves in the two dimensional x-y plane with the Hamiltonian:

$$\hat{H} = \frac{(\hat{P}_x + \hat{Y}qB/2)^2}{2m} + \frac{(\hat{P}_y + -\hat{X}qB/2)^2}{2m}$$
$$\omega = \frac{qB}{m}$$

   (a) (10 points) Show that $\hat{L}_z$ commutes with the Hamiltonian.

   (b) (10 points) Define the following operators:

$$\hat{V}_x = \frac{(\hat{P}_x + \hat{Y}qB/2)}{m}$$
$$\hat{V}_y = \frac{(\hat{P}_y + -\hat{X}qB/2)}{m}$$

   Show that $[\hat{V}_x, \hat{V}_y] = \frac{i\hbar\omega}{m}$

   (c) (15 points) Show that using the results of the previous part, the Hamiltonian can reduce to the one of a harmonic oscillator.

2. Consider an isotropic harmonic oscillator in two dimensions. The Hamiltonian is given by:

$$\hat{H}_0 = \frac{\hat{P}_x^2 + \hat{P}_y^2}{2m} + \frac{1}{2}m\omega^2(X^2 + Y^2)$$

   (a) (10 points) What are the energies of the three lowest-lying states? Is there any degeneracy?

   (b) (15 points) Apply a perturbation $V = \delta m\omega^2 XY$, where δ is a dimensionless real number much smaller than unity. Find the zeroth-order energy eigenket and the corresponding energy to first order [that is, the unperturbed energy obtained in the previous part plus the first-order energy shift] for each of the three lowest-lying states.

   (c) (15 points) Solve the $H_0 + V$ problem exactly. Compare with the perturbation results obtained in the second part.

3. (25 points) Phase space is defined by the position and momentum. Show that:

$$T(x_0, p_0) = e^{\frac{i(Xp_0 - Px_0)}{\hbar}}$$

4. (20 points) Three matrices M_x, M_y, M_z, each with 256 rows and columns, are known to obey the commutation rules
$[M_x, M_y] = iM_Z$ (with cyclic permutations of x, y and z). The eigenvalues of the matrix M_x are ± 2, each once; $\pm 3/2$, each 8 times; ± 1, each 28 times; $\pm 1/2$, each 56 times; and 0, 70 times. State the 256 eigenvalues of the matrix $M^2 = M_x^2 + M_y^2 + M_z^2$.

Good Luck

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Question:	1	2	3	4	Total
Points:	35	40	25	20	120
Score:					