

Instructions:

1. You are allowed to use three books, namely:
 - (a) Griffiths, David J, Introduction to Quantum Mechanics, 3rd edition
 - (b) J. J. Sakurai, Modern Quantum Mechanics, Revised Edition (required)
 - (c) Quantum Mechanics (2 vol. set) by Claude Cohen-Tannoudji, Bernard Diu and Frank Laloe
2. You are not allowed to communicate with each others.
3. you are not allowed to communicate with anybody regarding the exam.
4. You can communicate with me through Ritaj

1. We define the standard components of a vector operator V as the three operators:

$$\begin{aligned}
 V_1^{(1)} &= -\frac{1}{\sqrt{2}}(V_x + iV_y) \\
 V_0^{(1)} &= V_z \\
 V_{-1}^{(1)} &= \frac{1}{\sqrt{2}}(V_x - iV_y)
 \end{aligned}$$

Using the standard components $V_p^{(1)}$ and $W_q^{(1)}$ of the two vector operators V and W, we construct the operators:

$$[V^{(1)} \otimes W^{(1)}]_M^{(K)} = \sum_p \sum_q \langle 11; pq | KM \rangle V_p^{(1)} W_q^{(1)}$$

where the $\langle 1, 1; p, q | K, M \rangle$ are the Clebsch-Gordan coefficients entering into the addition of two angular momenta 1

- (a) Show that $[V^{(1)} \otimes W^{(1)}]_0^{(0)}$ is proportional to the scalar product $V \cdot W$ of the two vector operators.
 - (b) Show that the three operators $[V^{(1)} \otimes W^{(1)}]_M^{(1)}$ are proportional to the three standard components of the vector operator $V \times W$
 - (c) Express the five components $[V^{(1)} \otimes W^{(1)}]_M^{(2)}$ in terms of $V_z, V_{\pm} = V_x \pm iV_y, W_z, W_{\pm} = W_x \pm iW_y$
2. In the Hydrogen atom each state is labeled by $|nlm\rangle$ Let :

$$g = \langle 322 | xy | 300 \rangle$$

Find the values of the following in term of g:

$$\begin{aligned}
 &\langle 32m | G | 300 \rangle \\
 G &= xy, xz, yz, xx, yy, zz
 \end{aligned}$$

3. Consider an isotropic harmonic oscillator in two dimensions. The Hamiltonian is given by:

$$\hat{H}_0 = \frac{\hat{P}_x^2 + \hat{P}_y^2}{2m} + \frac{1}{2}m\omega^2(X^2 + Y^2)$$

- (a) What are the energies of the three lowest-lying states? Is there any degeneracy?
- (b) Apply a perturbation $V = \delta m\omega^2 XY$, where δ is a dimensionless real number much smaller than unity. Find the zeroth-order energy eigenket and the corresponding energy to first order [that is, the unperturbed energy obtained in the previous part plus the first-order energy shift] for each of the three lowest-lying states.
- (c) Solve the $H_0 + V$ problem exactly. Compare with the perturbation results obtained in the second part.

4. A particle with Spin 1 is subjected to a spin dependent potential:

$$V = A\hat{S}_x^2 + B\hat{S}_z^2$$

Find the first order correction for the energy and the corresponding eigenvectors.

5. What is the effect of the relativistic and spin-orbit correction on the ground state energy of 3-D Harmonic oscillator.
6. Find the configuration for Ge atom and write it as $^{2S+1}L_J$

Question:	1	2	3	4	5	6	Total
Points:	0	0	0	0	0	0	0
Score:							

Good Luck