Birzeit University-Department of Physics
Quantum Mechanics I Phys433
Fall 2020
Final Exam, Feb. 2^{nd} 2021

1. Calculate the following quantitites:
   (a) $< nlm|L_z L_+|n'l'm' >$
   (b) $[L_z, \phi]$
2. Consider a spin-1/2 particle which we shall describe in the basis of eigenstates for S_z. The basis for S_z are:

$$\chi_{+z} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \chi_{-z} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

   (a) What are the eigenvalues and eigenvectors of S_y. Write the eigenvectors of S_y (i.e χ_{+y}, χ_{-y}) in terms of those of S_z
   (b) If the particle is initially in the following state:

$$\chi = \frac{1}{\sqrt{13}}[3\chi_{+y} + 2\chi_{-y}]$$

   What is the probability of getting $\frac{\pm\hbar}{2}$ if we measure S_z, and what is the expectation value of S_z
   (c) What is the probability of getting $\frac{+\hbar}{2}$ if we measure S_y
3. Consider a spin-1/2 particle with magnetic moment $\mu = \gamma S$ in a uniform magnetic field that points in the z-direction. If at time t=0 the x-component of the spin as measured and were found to be $\frac{+\hbar}{2}$. At time t, y-component of the spin was measured and were found to be $\frac{+\hbar}{2}$, what is t?
4. An operator A has the following two properties
   - $A^2 = 0$
   - $\{A, A^\dagger\} = AA^\dagger + A^\dagger A = I$

   Show that $(A^\dagger A)^n = (A^\dagger A)$
5. At time t $= 0$ a particle in the potential $V(x) = \frac{1}{2}m\omega^2 x^2 is$ described by the wave function:

$$\Psi(x, 0) = A \sum_n (\frac{1}{\sqrt{2}})^n \phi_n(x)$$

   where $\phi_n(x)$ are eigenstates of the energy with eigenvalues $E_n = (n + \frac{1}{2})\hbar\omega$
   (a) Find the normalization constant A.
   (b) Write an expression for $\Psi(x, t)$ for t > 0.
   (c) Find the expectation value of the energy at t $= 0$.

6. A certain 3-level system can have three values of energy associated with 3 stationary states:

Energy	state
0	$\|1>$
$\hbar\omega$	$\|2>$
$2\hbar\omega$	$\|3>$

   (a) An operator $\hat{Q}$ is written in the following form:

$$\hat{Q} = a|1><1| + b(|2><3| + |3><2|)$$

   with $0 < a < b$. What will be the outcome if we measure the operator $\hat{Q}$

   (b) If at time t = 0, the operator $\hat{Q}$ was measured,and the largest eigenvalue was obtained, write the wavefunction at any later time t.

   (c) If $\hat{H}$ was measured at any later time t, what might we get and with what probability.

7. With your choice of basis, find the matrix element of $L \cdot S$. Where L is the orbital angular momentum, and S is the spin.

8. Let $V(x, y, z) = \frac{1}{2}m\omega_1^2(X^2 + Y^2) + \frac{1}{2}m\omega_2^2 Z^2$, where $\omega_1 \neq \omega_2$. Solve the corresponding Schrödinger equation in cylindrical coordinates

Good Luck