Bir Zeit University
Faculty of Science Department of Physics
Quantum Mechanics I Phys431
Fall 2015
1st Exam, Nov. 5th 2015

1. At t = 0, a particle of mass m trapped in an infinite square well of width L is in a superposition of the first excited state ($n=2$) and the 6th excited state ($n=6$)

$$\Psi(x,0) = A[3\psi_2(x) - 2\psi_6(x)]$$
$$\psi_n(x) = \sqrt{\frac{2}{L}}\sin\left(\frac{n\pi x}{L}\right)$$
$$E_n = \frac{n^2\pi^2\hbar^2}{2mL^2}$$

ground $\rightarrow n = 0$
first $\rightarrow n = 2$

(a) (5 points) Find the Normalization constant A $1/\sqrt{13}$
(b) (5 points) Write the wavefunction at any later time t
(c) (10 points) If the Hamiltonian is measured, at time t, what might you get and with what probability
(d) (10 points) Calculate σ_{p}

$\langle x \rangle = \langle p \rangle = \frac{d\langle x \rangle}{dt} m \Rightarrow \hat{H} = \frac{\hat{p}^2}{2m}$

$2m\langle \hat{H} \rangle = \langle p^2 \rangle$

$\sigma_p = \sqrt{\langle p^2 \rangle - \langle p \rangle}$

$\langle x \rangle = \int x |\psi|^2.$

2. Consider a particle of mass m in the following potential

$$V(x) = \begin{cases} \infty & \text{if } x < 0 \\ 0 & \text{if } 0 < x < L \\ U & \text{if } L < x \end{cases}$$

If the particle energy E<U
(a) (9 points) Write the wavefunction in each region
(b) (6 points) Setup the equations needed to find the particle energy

3. The wavefunction for a particle of mass m moving in a potential V(x) is given by: ~~free particle~~

$$\Psi(x,t) = \begin{cases} xe^{-Bx^2 - iCt/\hbar} & \text{if } x > 0 \\ 0 & \text{elsewhere} \end{cases}$$

where B and C are real constants such that $\Psi(x,t)$ is a properly normalized wave function that obeys the Schrodinger time-evolution equation for a potential V(x).
(a) (5 points) What is the energy of the particle
(b) (10 points) Find V(x)

$\boxed{c}$ $\langle \hat{H} \rangle = \frac{9}{13}\left(\frac{4\pi^2\hbar^2}{2mL^2}\right) + \frac{4}{13}\left(\frac{36\pi^2\hbar^2}{2mL^2}\right)$

$= \frac{(18) + (4 \times 18)}{13}\left(\frac{\pi^2\hbar^2}{mL^2}\right)$

Good Luck

$\psi^2 = \frac{9}{13}\psi_2^2 + \frac{4}{13}\psi_6^2 + \frac{12}{13}\psi_2\psi_6 \sin\omega t.$

BirZeit University
Faculty of Science Department of Physics
Quantum Mechanics I Phys432
1st 2016
Final Exam, Jan. 20th 2017

1. Consider a three-level system where the Hamiltonian and observable A are given by the matrix

$$\hat{A} = \mu \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

$$\hat{H} = \hbar\omega \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

(a) (6 points) What are the possible values obtained in a measurement of A
(b) (3 points) Does a state exist in which both the results of a measurement of energy H and observable A can be predicted with certainty? Why or why not?
(c) (6 points) Two measurements of A are carried out, separated in time by t. If the result of the first measurement is the largest possible value, determine the expectation value $<\psi(t)|A|\psi(t)>$ for the second measurement.

2. (20 points) For spin 1 particle, construct S_x, S_y, S_z and S^2 matrix

3. (15 points) If the state $|l, m_l>$ is an eigenvector of L^2 and L_z with eigenvalues of $l(l+1)\hbar^2$ and $m_l\hbar$, respectively. Find $<L_x>$ and $<L_x^2>$

4. An electron is placed in a uniform magnetic field $B = B_0\hat{k}$. At time $t = 0$ S_x was measured and was found to be $\hbar/2$
(a) (5 points) Write its spin wavefunction at any later time t.
(b) (5 points) Calculate $<S_x>$
(c) (5 points) At what time t if you measure the y component of the spin you will get $\hbar/2$

5. (15 points) Particle 1 which is spin $S_1 = 1$, and particle 2 which is spin $S_2 = 1$ are combined into a single particle with a total spin $S = S_1 + S_2$. Find the angle between S_1 and S_2 such that the value of S is 2.

6. Consider a simple harmonic oscillator with mass m and frequency ω. The two lowest energy eigenstates have energies $\frac{1}{2}\hbar\omega$ and $\frac{3}{2}\hbar\omega$ respectively
(a) (5 points) Calculate the expectation value of kinetic energy for the state with total energy $\frac{1}{2}\hbar\omega$
(b) (5 points) Calculate the expectation value of potential energy for the state with total energy $\frac{3}{2}\hbar\omega$

7. A particle of mass m is placed in an infinite square potential of width a, has the initial wavefunction :

$$\psi(x, 0) = \sqrt{\frac{8}{5a}}[1 + \cos(\frac{\pi x}{a})]\sin(\frac{\pi x}{a})$$

(a) (5 points) Write the wavefunction at any later time t
(b) (5 points) Find the expectation value of the Hamiltonian

Question	1	2	3	4	5	6	7	Total
Points	10	20	15	15	15	10	10	100
Score								

Good Luck

Quantum mechanics

BirZeit University
Faculty of Science-Department of Physics
Quantum Mechanics I Phys433
Fall 2016
2nd Exam, Jan. 10th 2017

1. Suppose we have a quantum mechanical system with an observable called color, and we have built a device which can measure the color of the system. We find that there are only three color states: $|red>$, $|green>$, and $|blue>$

   (a) (7 points) We measure the color of the system, and then we wait a little while and measure it again. We find that the second color measurement is not always the same as the first. Based on this observation, we guess the following Hamiltonian for our system:

   $$\hat{H} = E_1|red><red| + E_2|green><green| + E_3|blue><blue|$$

   Note: E_1, E_2, and E_3 are the eigenvalues of the Hamiltonian of the system. Is this a reasonable guess? In other words, is this Hamiltonian consistent or inconsistent with our observations? (Explain your answer.)

   (b) (8 points) We decide to measure the color of the system over and over again, once per second. When the color is red, we immediately follow the color measurement with an energy measurement. We find that 90% of the time we get energy E_1, 10% of the time we get E_2, and we never get E_3. Write down a plausible guess for how $|red>$ appears when written in terms of the energy eigenstates: $|1>$, $|2>$, and $|3>$

   (c) (5 points) What, if anything, can we say about the commutator of the color operator and the Hamiltonian for this system

2. The state of some system can be expressed using three orthonormal basis states $|1>$, $|2>$, and $|3>$. The action of the Hamiltonian $\hat{H}$ on each basis state is given is:

$$\hat{H}|1> = \frac{3i}{\sqrt{2}}\hbar\omega|2>$$

$$\hat{H}|2> = \frac{-3i}{\sqrt{2}}\hbar\omega|1> - \frac{3i}{\sqrt{2}}\hbar\omega|3>$$

$$\hat{H}|3> = \frac{3i}{\sqrt{2}}\hbar\omega|2>$$

   (a) (5 points) Find the matrix representation of $\hat{H}$ in this basis

   (b) (8 points) What are the possible energies of this system? The ground state $|E_0>$ is the state which corresponds to the lowest possible energy. What is the representation of $|E_0>$ in this basis

   (c) (7 points) If this system is in the state

   $$|\psi> = \frac{1}{\sqrt{8}}\begin{pmatrix}\sqrt{3}\\ -\sqrt{2}\\ \sqrt{3}\end{pmatrix}$$

   written in the original basis. What is the probability that the system is in the ground state.

3. In the basis of states $|1>$ and $|2>$, the matrix representations of two operators $\hat{A}$ and $\hat{B}$ are:

$$\hat{A} = \begin{pmatrix}1 & 2\\ 2 & 1\end{pmatrix}$$

$$\hat{B} = \begin{pmatrix}1 & i\\ -i & -1\end{pmatrix}$$

$$\begin{pmatrix}1 & -i\\ i & -1\end{pmatrix} = \begin{pmatrix}1 & i\\ -i & -1\end{pmatrix}$$

   (a) (5 points) For each operator determine whether it can be associated with a physical observable.

   (b) (5 points) Do quantum states exist for which the outcome of a measurement of both physical observables can be predicted with certainty.

   (c) (10 points) Determine the mean value and the variance in a measurement of $\hat{A}$ on a particle in the basis state $|1>$

$-\beta e^{-lk} =$

Quantum Mechanics (course 433)

First Hour Exam 2013/2014

1. A particle of mass m is in the state

$$\Psi(x,t) = A\exp\left[-a\left(\frac{mx^2}{\hbar} + it\right)\right],$$

where A and a are positive real constants.

(a) Find A.

(b) For what potential energy $V(x)$ does $\Psi(x,t)$ satisfy the Schrodinger equation? What is the name of this potential?

(c) Calculate the expectation values of x, x^2, p, and p^2.

(d) Find the product $\sigma_x \sigma_p$.

Two useful integrals are: $\int_{-\infty}^{\infty} e^{-\lambda x^2} dx = \sqrt{\frac{\pi}{\lambda}}$ and $\int_{-\infty}^{\infty} x^2 e^{-\lambda x^2} dx = \frac{1}{2\lambda}\sqrt{\frac{\pi}{\lambda}}$.

2. Derive the stationary states and the energy levels of the infinite square well,

$$V(x) = \begin{cases} 0, & \text{for } 0 \le x \le a, \\ \infty, & \text{otherwise} . \end{cases}$$

3. A particle of mass m is moving in the finite parabolic well potential:

$p = \hbar \frac{\partial \langle x \rangle}{\partial t}$

$$V(x) = \begin{cases} -V_0(1 - x^2/a^2), & \text{for } -a \le x \le a, \\ 0, & \text{otherwise} , \end{cases}$$

$= \frac{\hbar}{i}\frac{\partial}{\partial x}$

where V_0 and a are positive real constants. For a bound state do the following.

(a) Find the general (and physical) solutions of the time-independent Schrodinger equation in the regions $x \ge a$ and $x \le -a$.

(b) In the region $-a \le x \le a$ write down the time-independent Schrodinger equation (do not solve it).

(c) Assume now that you know the solution to the ordinary differential equation

$$\psi(x)'' + (c_1 + c_2 x^2)\psi(x) = 0,$$

where c_1 and c_2 are constants. Let us denote the even and odd solutions by $F_+(x, c_1, c_2)$ and $F_-(x, c_1, c_2)$, respectively. Write down the even and odd solutions of $\psi(x)$ in the whole range $x \in (-\infty, \infty)$.

$\frac{p^2}{2m}$

(d) Reach an equation that determines the allowed energies of the even states and then reach an equation that determines the allowed energies of the odd states.

4. (10 points) In Hydrogen atom, the recursion relation to obtain the radial wavefunction is given by

$$R_{nl}(r) = \frac{1}{r}\left(\frac{r}{an}\right)^{l+1} e^{-r/na}\, v\left(\frac{r}{an}\right)$$

$$v\left(\frac{r}{an}\right) = \sum_{j=0}^{\infty} c_j \left(\frac{r}{an}\right)^{j}$$

$$c_{j+1} = \frac{2(j+l+1-n)}{(j+1)(j+2l+2)} c_j$$

Find the radial wavefunctions that correspond to the n=2 states.

5. Consider an electron in the Hydrogen-atom. The wavefunction of the electron is, at time t=0, written as

$$\psi(r, t=0) = A(\psi_{211} + 2\psi_{100} + \psi_{321})$$

(a) (2 points) Find the normalization constant A.

(b) (2 points) Write the wavefunction at any later time t.

(c) (2 points) What is the expectation value of E_n

(d) (4 points) If the electron jump into the ground state, what wavelengths one might get and with what probability

Question	1	2	3	4	5	Total
Points	20	20	20	10	10	80
Score						

Good Luck

[illegible]

[illegible] University
Faculty of Science Department of Physics
Quantum Mechanics I Phys[illegible]
Fall 2018
1st Exam Nov. [illegible] 2018

1. Consider a particle with mass m in the one dimensional infinite square well potential

$$V(x) = \begin{cases} 0 & \text{if } |x| \le a \\ \infty & \text{if } |x| > a \end{cases}$$

[illegible]

(a) (5 points) Show that the eigenfunction is given by

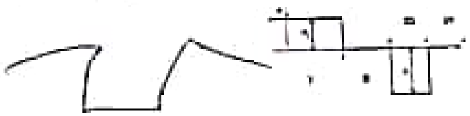
$$\psi(x) = \begin{cases} (-1)^{[illegible]}\sqrt{\frac{1}{a}}\sin\left(\frac{[illegible]}{[illegible]}\right) & \text{if } n \text{ is even} \\ (-1)^{[illegible]}\sqrt{\frac{1}{a}}\cos\left(\frac{[illegible]}{[illegible]}\right) & \text{if } n \text{ is odd} \end{cases}$$

(b) At time t = 0, the wave function for this particle is

$$\psi(x, t = 0) = A(a - |x|)$$

i. (2 points) Find the coefficient A so that the wave function is properly normalized.
ii. (4 points) Determine the time dependent wave function $\psi(x, t)$
iii. (3 points) What is the wave function after time $t = [illegible]$ has elapsed?
iv. (3 points) Calculate the expectation value of the energy

2. Consider the potential sketched below. Consider particles only coming from the right



(a) (2 points) What are the values of E that will create a bound state or a scattering state
(b) (3 points) Set up the solutions in the different regions I-IV, for $0 < E < V_0$. Relate the coefficients appearing in your expressions for the wavefunctions in the different regions to E, m, [illegible], V_0, and V_1
(c) (3 points) Write down the equations that specify the boundary conditions at [illegible]
(d) (3 points) If E < 0, in which region would you expect to have a bound state

3. (10 points) For a particle in one dimension harmonic oscillator potential show that for the nth state ψ_n: $E_n = \hbar\omega(n + \frac{1}{2})$

Question	1	2	3	Total
Points	13	22	10	[illegible]
Score				

Good Luck