Birzeit University
Faculty of Science-Department of Physics
Quantum Mechanics II, Phys4332
Spring 2021
Homework 1: Due date Apr. 18th 2022

1. Find the energy levels of a spin S =3/2 particle whose Hamiltonian is given by:

$$\hat{H} = \frac{\alpha}{\hbar^2}(\hat{S}_x^2 + \hat{S}_y^2 - 2\hat{S}_z^2) - \frac{\beta}{\hbar}\hat{S}_z$$

   α and β are constants. Are these level degenerate.

2. Particle 1 which is spin $\frac{1}{2}$, and particle 2 which is spin 2 are combined into a single particle with a total spin $\vec{S} = \vec{S_1} + \vec{S_2}$
   (a) If the system is at state $|Sm_S>=|\frac{5}{2}\frac{3}{2}>$, write it in terms of $|S_1S_2;m_1m_2>$
   (b) If S_{1z} and S_{2z} were measured and found to be $\frac{\hbar}{2}$ and $\hbar$ respectively. What values might we get if we measure S^2, and with what probability.

3. Consider a spin-1/2 particle which we shall describe in the basis of eigenstates for S_z. The basis for S_z are:

$$|+>_z = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad |->_z = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

   (a) What are the eigenvalues and eigenvectors of S_y. Write the eigenvectors of S_y (i.e $|+>_y$, $|->_y$) in terms of those of S_z
   (b) If the particle is initially in the following state:

$$\chi = \frac{1}{\sqrt{13}}[3|+>_y + 2|->_y]$$

   What is the probability of getting $\frac{\pm\hbar}{2}$ if we measure S_z, and what is the expectation value of S_z
   (c) What is the probability of getting $\frac{+\hbar}{2}$ if we measure S_y

4. Consider a spin-1/2 particle described by the Hamiltonian:

$$H = \omega_1 S_x + \omega_2 S_z \tag{1}$$

   where $\omega_1 = 3, \omega_2 = 4$
   (a) What is the matrix representation of H in the basis where S_z is diagonal.
   (b) Find the eigenvalues and eigenvectors of H
   (c) Suppose at t=0, the particle was in a state in which $S_z = +\hbar/2$, what is the probability of getting $S_z = -\hbar/2$ at a later time t.

5. Consider a spin-1/2 particle with magnetic moment $\mu = \gamma S$ in a uniform magnetic field that points in the z-direction. If at time t=0 the x-component of the spin as measured and were found to be $\frac{+\hbar}{2}$. At time t, y-component of the spin was measured and were found to be $\frac{+\hbar}{2}$, what is t?

6. Particle 1 which has a spin $\frac{1}{2}$, and particle 2 which has spin 2, are combined to form a particle with spin $\vec{S} = \vec{S_1} + \vec{S_2}$. The combined particle is in state $|\frac{5}{2}\frac{3}{2}>$. Write it in terms of $|S_1S_2m_1m_2>$. Don't use the table.