Excellent

$\left(\frac{9.5}{10}\right.$ $\boxed{+0.5}$

BIRZEIT UNIVERSITY

MATHEMATICS DEPARTMENT

MATH 1321 - Quiz (1)

First Semester 2021/2022

Name [redacted] ID Number [redacted] Sec- 15

tion..........

Show ALL your work CAREFULLY

(a) Evaluate the improper integral

$$\int_{-\infty}^{\infty} 2xe^{-x^2}\,dx$$

$\cancel{\int_{-\infty}^{\infty} 2x e^{-x^2} dx = \int_{-\infty}^{1} 2x e^{-x^2} dx + \int_{1}^{\infty} 2}$ $\int_{-\infty}^{\infty} 2x e^{-x^2} dx = \int_{-\infty}^{0} 2x e^{-x^2} dx + \int_{0}^{\infty} 2x e^{-x^2} dx$

$= \lim_{a\to-\infty} \int_a^0 2xe^{-x^2} dx + \lim_{b\to\infty} \int_0^b 2xe^{-x^2} dx$ by substitution

$u = -x^2$

$du = -2x\,dx$

$dx = \frac{du}{-2x}$

$= \lim_{a\to\infty} \int \cancel{2x} \frac{e^u\,du}{-\cancel{2x}} + \lim_{b\to\infty} \int \cancel{2x} \frac{e^u\,du}{-\cancel{2x}}$

when $x = a \Rightarrow u = \boxed{-a^2}$

$x = 1 \Rightarrow \boxed{u = -1}$

$x = b \Rightarrow \boxed{-b^2}$

$= \lim_{a\to\infty} -e^u \Big|_{-a^2}^{0} + \lim_{b\to\infty} -e^u \Big|_{0}^{-b^2}$

$= \lim_{a\to\infty} \left(-e^{0} + e^{-a^2}\right) + \lim_{b\to\infty} \left(-e^{-b^2} + e^{0}\right)$

-0.5

$= \lim_{a\to\infty} \left(\frac{-1}{e}\right) + \frac{1}{e^{a^2}} + \lim_{b\to\infty} \left(\frac{-1}{e^{b^2}} + \frac{1}{e}\right)$ if conv. to zero

$-1 + 1 = 0$

$= \left(\frac{-1}{e} + \cancel{\frac{1}{\infty}}\right) + \left(\cancel{\frac{-1}{\infty}} + \frac{1}{e}\right) = \frac{-1}{e} + \frac{1}{e} = \boxed{0}$

(b) Use comparison test to determine whether the improper integral converges or diverges.

$f(x) = \frac{1}{\sqrt{x}+1}$ $g(x) = \frac{1}{\sqrt{x}}$

$$\int_1^{\infty} \frac{1}{\sqrt{x}+1} dx$$

by the L.C.T

$$\lim_{x \to \infty} \frac{\frac{1}{\sqrt{x}}}{\frac{1}{\sqrt{x}+1}} = \frac{\sqrt{x}+1}{\sqrt{x}} = \boxed{1}$$ finite

the both diverge or both converge

so $g(x) = \int_1^{\infty} \frac{1}{\sqrt{x}}$ diverges

by the L.C.T $f(x) = \int_1^{\infty} \frac{1}{\sqrt{x}+1}$ diverges

$\int_1^{\infty} \frac{1}{\sqrt{x}}$ diverges

because it's P-function and $P < 1$

D.C.T. Fails

$$0 < \int_1^{\infty} \frac{1}{\sqrt{x}+1} < \int_1^{\infty} \frac{1}{\sqrt{x}}$$

no info. diverges

D.C.T Fails

MATH1321 - Quiz (2)
Second Semester 2021/2022

Excellent $\frac{10}{10}$

Name. [redacted] ID Number. [redacted] Section.........

Question (1) Choose the correct answer

1) For the **sequence** defined by $a_n = n^2 - 5n + 2$, what is the smallest value of n for which a_n is positive?

a) 2

b) 3

c) 5

$f(n) = n^2 - 5n + 2$

$f'(n) = 2n - 5$

$2n - 5 = 0$

$n = \frac{5}{2}$

$- - - \; + + + +$

$\frac{5}{2}$

2) The sum of the **series** $\sum_{n=1}^{\infty} \frac{2^{3n}}{3^{2n}}$

a) The series diverges by nth term test

b) is 0

c) is 8

$\left(\frac{2^3}{3^2}\right)^n$ $\left(\frac{8}{9}\right)^n$ $\left|\frac{8}{9}\right| < 1$ conv

3) The sum of the **series** $\sum_{n=0}^{\infty} 2(\frac{3}{2})^n$

a) Is 16

b) is 4

c) this series diverges

$2 \times \left(\frac{1}{1-\frac{3}{2}}\right)$ $|r| > 1$ $\frac{\frac{8}{9}}{1-\frac{8}{9}}$

4) Does the **sequence** $a_n = \ln(n+1) - \ln(n)$ converge or diverge?

a) Diverge

b) Converge and the limit is 0

c) Converge and the limit is e

$S_n = \ln(n+1)$

$\lim_{n\to\infty} a_n = \ln\left(\frac{n+1}{n}\right)$

$e^0 = \boxed{1}$

∞

5) Every monotonic increasing sequence which is bounded above is

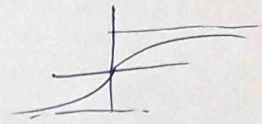
a) Convergent

b) Divergent

$a_n = \ln\left(\frac{n+1}{n}\right)$

6) Which statement is true about the **sequence** $\{\tan^{-1}(\ln n)\}$?

lim lan^{-1}



a) It diverges.

b) it converges to 0.

c) it converges to $\frac{\pi}{2}$.

7) Which of the following **sequences** diverges?

a) $a_n = \frac{3^{2n}}{4^{3n}}$

$\left(\frac{3^2}{4^3}\right)$ $\frac{9}{64}$

b) $b_n = \frac{2^{2n}}{3^n}$

$\frac{4}{3}$

c) $c_n = \frac{n}{n!}$

$\cancel{\frac{n}{n \times (n-1)}} = 1$

8) If $\sum_{n=0}^{\infty} r^n = \frac{4}{3}$, then $r =$

$\frac{4}{3} = \frac{1}{1-r}$

$3 = 4 - 4r$

-4

$-\frac{1}{4} = -\frac{4r}{4}$

a) $\frac{1}{4}$

b) 3

c) 4

9) The nth partial sum of $\sum_{n=1}^{\infty} \frac{5}{n(n+1)}$ is

$\frac{5}{n(n+1)} = \frac{A}{n} + \frac{B}{n+1}$

$= \frac{5}{n} + \frac{-5}{n+1}$

a) $S_n = \frac{n-4}{n+1}$ and its sum equals 1

b) $S_n = \frac{5n}{n+1}$ and its sum equals 5

c) $S_n = \frac{5n+10}{n+1}$ and its sum equals 5

$\left(5 - \frac{5}{2}\right) + \left(\frac{5}{2} - \frac{5}{3}\right)$

$\left(\frac{5}{n-1} - \frac{5}{n}\right) + \left(\frac{5}{n} - \frac{5}{n+1}\right)$

10) Series? How do you feel about them?

a) I'm confused

b) way better than integration

c) I hate this so much

d) I have got this so far

$5 - \frac{5}{n+1}$

$\frac{5n + 5 - 5}{n+1}$

MATH1321 - Quiz (3)
Second Semester 2021/2022

Excellent $\frac{10}{10}$

Name..   ID Number   Section..12...

Question (1) Choose the correct answer

1) The series $\sum_{n=1}^{\infty} \frac{(\ln n)^2}{n^3}$

a) converges by direct comparison with $\sum_{n=1}^{\infty} \frac{1}{n^3}$

b) converges by limit comparison with $\sum_{n=1}^{\infty} \frac{1}{n^3}$

c) converges by limit comparison with $\sum_{n=1}^{\infty} \frac{1}{n^2}$

$\lim \frac{\frac{(\ln n)^2}{n^3}}{\frac{1}{n^3}} = \infty$ $\frac{1}{n^2}$

$\frac{(\ln n)^2}{n^3} > \frac{1}{n^3}$

$\frac{\frac{(\ln n)^2}{n^3}}{\frac{1}{n^2}} = \frac{\infty}{\infty}$ $\frac{2\ln n \frac{1}{n}}{}$ $\frac{2\ln n}{n}$ $2\frac{1}{n}$ $\frac{1}{\infty}$

$\frac{(\ln n)^2}{n^3} < \frac{1}{n^2}$ $\Rightarrow$ conv.

2) The series $\sum_{n=1}^{\infty}(1+\frac{2}{n})^n$

a) converges by the root test

b) diverges by the root test

c) diverges by the nth term test

$\lim_{n\to\infty} 1+\frac{2}{n} = \boxed{1}$ X

$\lim_{n\to\infty} (1+\frac{2}{n})^n = e^2$ div

3) The series $\sum_{n=1}^{\infty} \frac{1}{n^2+1}$

a) converges by the integral test

b) diverges by the integral test

c) diverges by the nth term test

$f(x) = \frac{1}{x^2+1}$

$\int_1^{\infty} \frac{1}{x^2+1}dx = \lim_{b\to\infty} \tan^{-1}x \Big|_1^b$

$= \lim_{b\to\infty} \tan^{-1}b - \tan^{-1}1$

$\frac{\pi}{2} - \frac{\pi}{4}$

4) The series $\sum_{n=0}^{\infty} a_n$ where $a_1 = 0$ and $a_{n+1} = \frac{1+tan^{-1}(n)}{n} a_n$

a) converges by ratio test

b) diverges by ratio test

c) the ratio test fails

$\frac{a_{n+1}}{a_n} = \frac{1+\tan^{-1} n}{n} = \frac{1+\frac{\pi}{2}}{\infty} = 0 < 1$

5) Which of the following statements must be true about the series $\sum_{n=0}^{\infty} a_n$ with positive terms if $\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = L$

pos.

$\frac{a_{n+1}}{a} = L$

a) the series converges if $L = \infty$ div.

b) the series converges if $L = 1$ fail.

c) the series converges if $L = \frac{1}{2}$

~~15.5~~

$\frac{4.5}{5}$

Birzeit University

Mathematics Department

Second semester 2021-2022

MATH1321- Quiz 4

Name: sec:....15........ Student Number:.

1) $\sum_{n=2}^{\infty}(-1)^{n+1}\frac{1}{n\ln n}$

Is the series convergent absolutely, conditionally or divergent? Give reasons for your answers.

by the A.S.T : $\sum_{n=2}^{\infty}\frac{(-1)^{n+1}}{n\ln n}$

① $U_n = \frac{1}{n\ln n} > 0$ ✓

② $U_n = \frac{1}{n\ln n}$ decreasing ✓

③ $\lim_{n\to\infty} U_n = \lim_{n\to\infty}\frac{1}{n\ln n} = 0$ ✓

By the A.S.T

$\sum_{n=2}^{\infty}(-1)^{n+1}\frac{1}{n\ln n}$ converges.

let's take $\sum_{n=2}^{\infty}\frac{1}{n\ln n} \Longrightarrow$ By the I.T $f(x) = \frac{1}{x\ln x}$ ~~d~~/con/>0

$\Downarrow$

$\sum_{n=1}^{\infty}|a_n|$

$u = \ln x$

$du = \frac{1}{x}dx$

$dx = x\,du$

$\int_2^{\infty} f(x) = \int_2^{\infty}\frac{1}{x\ln x}dx =$

$= \int_2^{\infty}\frac{1}{\cancel{x}u}\cancel{x}\,du = \ln u\Big|_2^{\infty} = \ln(\ln x)\Big|_2^{\infty} = \lim_{b\to\infty}\ln(\ln x)\Big|_2^{b}$

$= \lim_{b\to\infty}(\ln(\ln b)) - \ln(\ln 2) = \boxed{\infty}$ diverges by Integral test

so $\sum_{n=2}^{\infty}\frac{1}{n\ln n}$ diverges by I.T

so not abs

So $\sum_{n=2}^{\infty}(-1)^{n+1}\frac{1}{n\ln n}$ conv. cond

converge conditionally