Birzeit University
Mathematics Department
Second Semester 2021/2022
MATH339 – Midterm Exam
Instructor: Dr. Hani Kabajah

40

Name (بالعربية): ...Key........... **Student No.:** ....

Question 1 (10 points)

Let S be the following statement in the language of incidence geometry:

If l and m are any two distinct lines, then there exists a point P that does not lie on either l or m.

Show that S is not a theorem in incidence geometry, i.e., cannot be proved from the axioms of incidence geometry.

Model 1
- A, B, C are Points
- $\{A,B\}$, $\{B,C\}$, $\{A,C\}$ are lines
- incidence means set membership

In model 1: If we take lines $\{A,B\}$ and $\{B,C\}$, then we cannot find a point that doesn't lie on either $\{A,B\}$ or $\{B,C\}$ $\Rightarrow$ S is false in Model 1.

Model 2
- A, B, C, D, E are Points
- $\{A,B\}$, $\{A,C\}$, $\{A,D\}$, $\{A,E\}$, $\{B,C\}$, $\{B,D\}$, $\{B,E\}$, $\{C,D\}$, $\{C,E\}$, $\{D,E\}$ are lines
- incidence means set membership

In model 2: If we take any two distinct lines, then there is always a point that doesn't lie on either the first line or the second line $\Rightarrow$ S is true in Model 2.

conclusion: S is not a theorem in incidence geometry. S can't be proved from the axioms of incidence geometry.

10

Question 2 (10 points)

Justify each step of in the following proof of Proposition 3.14 (see Figure 3.39).

Proposition 3.14. Supplements of congruent angles are congruent.

Proof:

Given $\sphericalangle ABC \cong \sphericalangle DEF$. To prove $\sphericalangle CBG \cong \sphericalangle FEH$:

(1) The points A, C, and G being given arbitrarily on the sides of $\sphericalangle ABC$ and the supplement $\sphericalangle CBG$ of $\sphericalangle ABC$, we can choose the

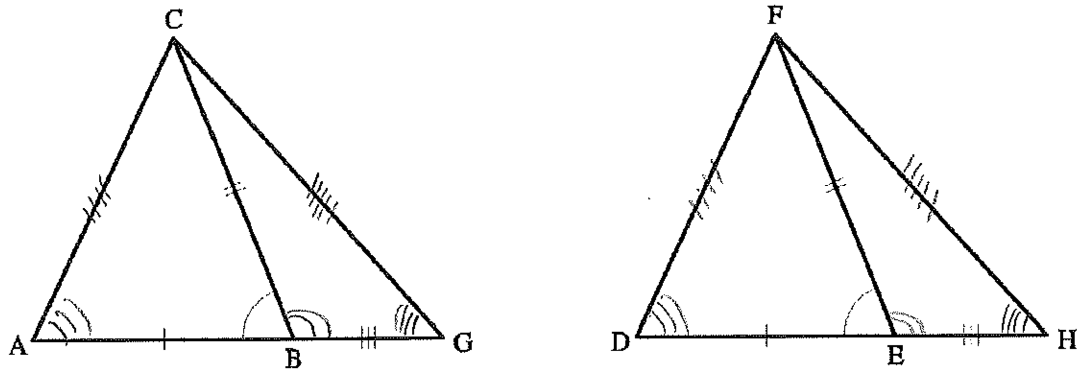


Figure 3.39

points D, F, and H on the sides of the other angle and its supplements so that $AB \cong DE$, $CB \cong FE$, and $BG \cong EH$.

(2) Then, $\triangle ABC \cong \triangle DEF$.

(3) Hence, $AC \cong DF$ and $\sphericalangle A \cong \sphericalangle D$.

(4) Also, $AG \cong DH$.

(5) Hence, $\triangle ACG \cong \triangle DFH$.

(6) Therefore, $CG \cong FH$ and $\sphericalangle G \cong \sphericalangle H$.

(7) Hence, $\triangle CBG \cong \triangle FEH$.

(8) It follows that $\sphericalangle CBG \cong \sphericalangle FEH$, as desired. ◀

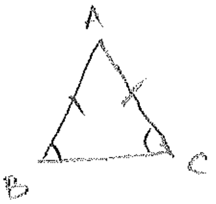
(1)	CA1	(5)	Step 3 + step4 + CA6
(2)	Step1 + Given + CA6	(6)	step 5 + Def. of congruent triangles
(3)	Step2 + Def. of congruent triangles	(7)	Step1 + Step 6 + CA6
(4)	step 1 + CA3	(8)	step 7 + Def of congruent triangles

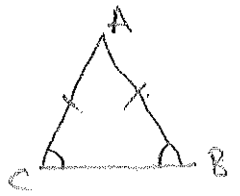
10

Question 3 (10 points)

Write a formal proof for Proposition 3.18.

Proposition 3.18. If in ΔABC we have $\sphericalangle B \cong \sphericalangle C$, then $AB \cong AC$ and ΔABC is isosceles.





proof

(1) Given ΔABC with $\sphericalangle B \cong \sphericalangle C$ (hypothesis)

(2) Given ΔACB with $\sphericalangle C \cong \sphericalangle B$ (hypothesis)

(3) $BC \cong CB$ (CA2)

(4) $\sphericalangle B \cong \sphericalangle C$ (step 1)

(5) $\sphericalangle C \cong \sphericalangle B$ (step 2)

(6) $\Delta ABC \cong \Delta ACB$ (steps 3, 4, 5, prop. 3.17 (ASA))

(7) $AB \cong AC$ and $AC \cong AB$ (step 6, def of congruent triangles)

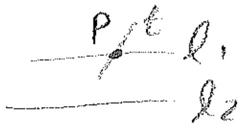
(8) ΔABC isosceles (step 7, def of isosceles triangles).

10

Question 4 (10 points)

Write a formal proof for the following statement, which is part of Proposition 4.7.

In any Hilbert plane, Hilbert's Euclidean parallel postulate ⇒ if a line intersects one of two parallel lines, then it also intersects the other.



proof

(1) Assume a Hilbert plane (Hypothesis)

(2) Assume Hilbert's Euclidean parallel postulate (Hypothesis)

(3) Assume we have two parallel lines l_1, l_2 (Hypothesis)

(4) Assume a line t intersects one of the two parallel lines l_1 in a point P (hypothesis)

(5) Assume the line t does not intersect the other line l_2 (RAA hypothesis)

(6) l_2 and t are parallel (step 5, def of parallel lines)

(7) There is a point P not on l_2 such that l_1 and t are distinct and both pass through P and both are parallel to l_2 (step 4, step 3, step 6)

(8) step 7 contradicts step 2.

(9) the line t must intersect l_2 (RAA conclusion)

10