

Name (بالعربية): *Key*

Student No.:

Question (10 points)

Write a formal proof of Proposition 2.2.

proposition 2.2 There exist three ^{distinct} lines that are not concurrent.

proof:

- (1) There exist three distinct points A, B, C with the property that no line is incident with all three of them (IA3)
- (2) There is a unique line \overleftrightarrow{AB} incident with A, B (step 1, IA1)
- (3) There is a unique line \overleftrightarrow{BC} incident with B, C (step 1, IA1)
- (4) There is a unique line \overleftrightarrow{AC} incident with A, C (step 1, IA1)
- (5) $\overleftrightarrow{AB} \neq \overleftrightarrow{BC}$ (Assuming $\overleftrightarrow{AB} = \overleftrightarrow{BC}$ contradicts step 1)
- (6) $\overleftrightarrow{AB} \neq \overleftrightarrow{AC}$ (Assuming $\overleftrightarrow{AB} = \overleftrightarrow{AC}$ contradicts step 1)
- (7) $\overleftrightarrow{BC} \neq \overleftrightarrow{AC}$ (Assuming $\overleftrightarrow{BC} = \overleftrightarrow{AC}$ contradicts step 1)
- (8) There exist three lines $\overleftrightarrow{AB}, \overleftrightarrow{AC}, \overleftrightarrow{BC}$ that are distinct (steps 2-7)
- (9) Assume that $\overleftrightarrow{AB}, \overleftrightarrow{AC}, \overleftrightarrow{BC}$ are concurrent lines (RAA hypothesis)
- (10) There is a point P lying on all lines $\overleftrightarrow{AB}, \overleftrightarrow{AC}, \overleftrightarrow{BC}$ (step 9, def of concurrent lines)
- (11) $\overleftrightarrow{AB} = \overleftrightarrow{AP}$ (step 10)
- (12) $\overleftrightarrow{AC} = \overleftrightarrow{AP}$ (step 10)
- (13) $\overleftrightarrow{AB} = \overleftrightarrow{AC}$ (steps 11, 12, logic rule 12)
- (14) step 13 contradicts step 6.
- (15) The lines $\overleftrightarrow{AB}, \overleftrightarrow{AC}, \overleftrightarrow{BC}$ are not concurrent (RAA conclusion)
- (16) There exist three lines $\overleftrightarrow{AB}, \overleftrightarrow{AC}, \overleftrightarrow{BC}$ that are distinct and not concurrent (steps 8, 15)