Birzeit University
Mathematics Department
Second Semester 2021/2022
MATH339 – Quiz 2
Instructor: Dr. Hani Kabajah

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Name (بالعربية): ...Key..... **Student No.:** .....

Question (10 points)

Write a formal proof of Proposition 3.1 (ii).

Proposition 3.1(ii) For any two points A and B:

$\overrightarrow{AB} \cup \overrightarrow{BA} = \{\overleftrightarrow{AB}\}$

proof

(1) $\overrightarrow{AB} \subset \{\overleftrightarrow{AB}\}$ (Def. of ray)

(2) $\overrightarrow{BA} \subset \{\overleftrightarrow{AB}\}$ (Def. of ray)

(3) $\overrightarrow{AB} \cup \overrightarrow{BA} \subset \{\overleftrightarrow{AB}\}$ (step 1, step 2)

(4) $P \in \{\overleftrightarrow{AB}\}$ (hypothesis)

(5) $P = A$ or $P = B$ or ($P \neq A$ and $P \neq B$) (step 4, proof by cases)

(6) $P = A \Rightarrow P \in \overrightarrow{AB} \Rightarrow P \in \overrightarrow{AB} \cup \overrightarrow{BA}$ (step 5, def. of ray)

(7) $P = B \Rightarrow P \in \overrightarrow{BA} \Rightarrow P \in \overrightarrow{AB} \cup \overrightarrow{BA}$ (step 5, def. of ray)

(8) $P \neq A$ and $P \neq B \Rightarrow P * A * B$ or $A * P * B$ or $A * B * P$ (steps 4, 5, BA3 proof by cases)

(9) $A * P * B \Rightarrow P \in \overrightarrow{AB} \Rightarrow P \in \overrightarrow{AB} \cup \overrightarrow{BA}$ (step 8, def. of ray, proof by cases)

(10) $P * A * B \Rightarrow P \in \overrightarrow{BA} \Rightarrow P \in \overrightarrow{AB} \cup \overrightarrow{BA}$ (step 8, def. of ray, proof by cases)

(11) $A * B * P \Rightarrow P \in \overrightarrow{AB} \Rightarrow P \in \overrightarrow{AB} \cup \overrightarrow{BA}$ (step 8, def. of ray, proof by cases)

(12) $\{\overleftrightarrow{AB}\} \subset \overrightarrow{AB} \cup \overrightarrow{BA}$ (steps 6, 7, 9, 10, 11)

(13) $\overrightarrow{AB} \cup \overrightarrow{BA} = \{\overleftrightarrow{AB}\}$ (steps 3, 12)