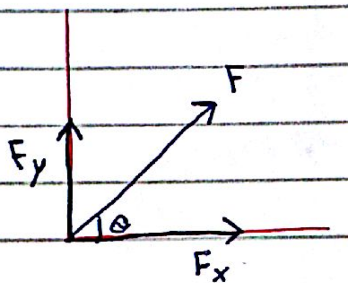
Chapter 2 (Force Vectors)

* Force characterized by:-
  1- point of application
  2- magnitude
  3- direction (line of action)

* Force can be expression by:-

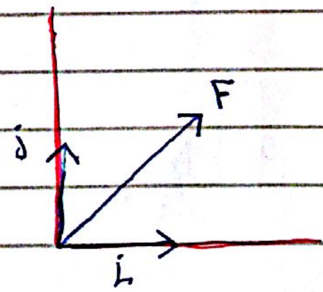
1- Components:-
any Force could be replaced by two Components F_x and F_y as shown



$F_x = F\cos\theta$
$F_y = F\sin\theta$

$$F = \sqrt{F_x^2 + F_y^2} \quad , \quad \theta = \tan^{-1}(F_y/F_x)$$

2- Vectors:
Vector expression of the Force:-



$F = F_x + F_y$
$F = i\,F_x + j\,F_y$

$$F = i\,F\cos\theta + j\,F\sin\theta$$

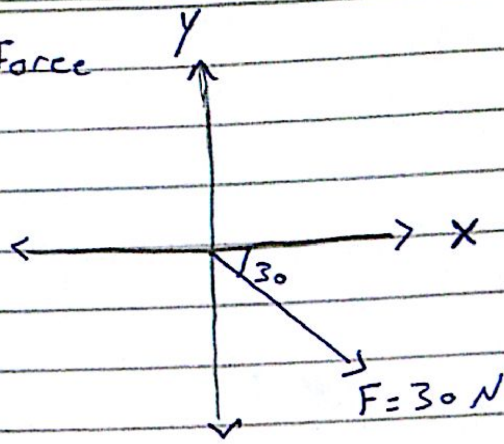
Ex: Find the vector expression of the Force

$F = iF\cos\theta + jF\sin\theta$

$F_x = 300 \times \cos 30 = 260$

$F_y = -300 \sin 30 = -150$

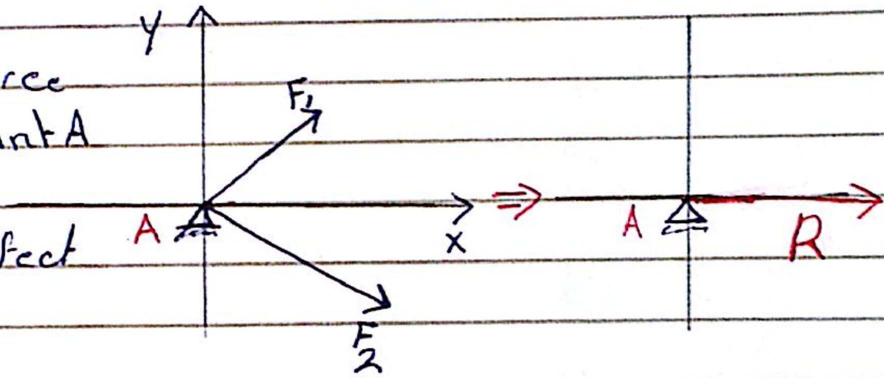
$F = (260i - 150j)\ N$



Resultant Force:

The resultant force is the single force that it has the same effect on the rigid body as the original system of the force.

If we replace the force system (F_1, F_2) at the joint A by a single force (R) we get the same effect



* Procedure to find R

1- Replace all forces by Fx and Fy then find ΣF_x and ΣF_y

2- $R = \sqrt{(F_x)^2 + (F_y)^2}$

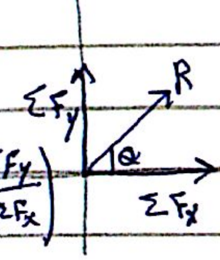
3- $\theta = \tan^{-1}\left(\frac{\Sigma F_y}{\Sigma F_x}\right)$

(1)

ΣF_x +

ΣF_y +

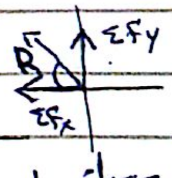
$\theta = \tan^{-1}\left(\frac{\Sigma F_y}{\Sigma F_x}\right)$



(2)

ΣF_x −

ΣF_y +

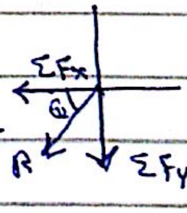


$\theta = \tan^{-1}\left(\frac{\Sigma F_y}{\Sigma F_x}\right)$

(3)

ΣF_x −

ΣF_y −

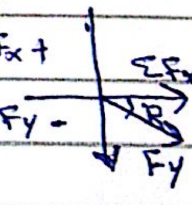


$\theta = \tan^{-1}\left(\frac{\Sigma F_y}{\Sigma F_x}\right)$

(4)

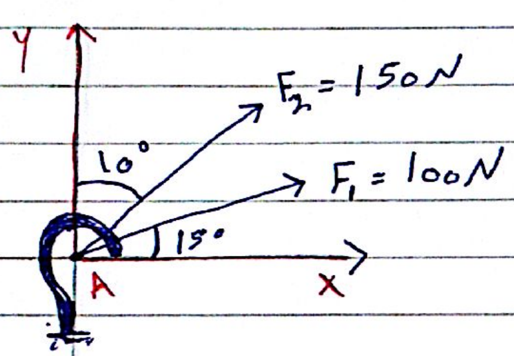
ΣF_x +

ΣF_y −

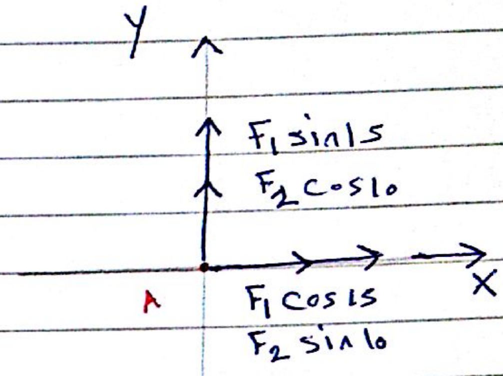


$\theta = \tan^{-1}\frac{\Sigma F_y}{\Sigma F_x}$

Ex



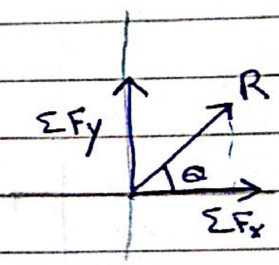
$\Rightarrow$



solu:-

$\Sigma F_x = F_1 \cos 15 + F_2 \sin 10$
$= 100 \times \cos 15 + 150 \times \sin 10 = 122.64\ N$

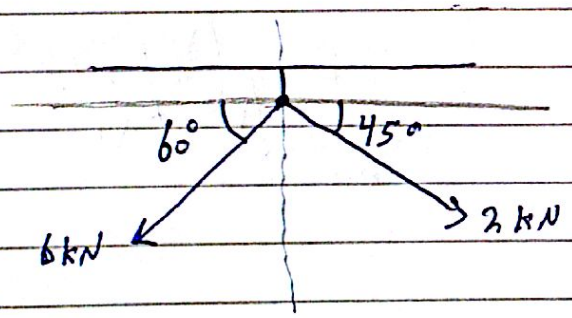
$\Sigma F_y = F_1 \sin 15 + F_2 \cos 10$
$= 100 \times \sin 15 + 150 \cos 10 = 173.6\ N$



$$R = \sqrt{(\Sigma F_x)^2 + (\Sigma F_y)^2} = 212.55\ N$$

$$\theta = \tan^{-1}\left(\frac{\Sigma F_y}{\Sigma F_x}\right) = 54.78°$$

Ex

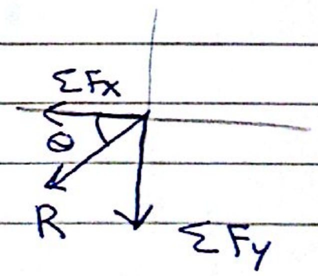


solu

$\Sigma F_x = 2 \times \cos 45 - 6 \times \cos 60 = -1.586\ kN$

$\Sigma F_y = -2 \sin 45 - 6 \times \sin 60 = -6.61\ kN$

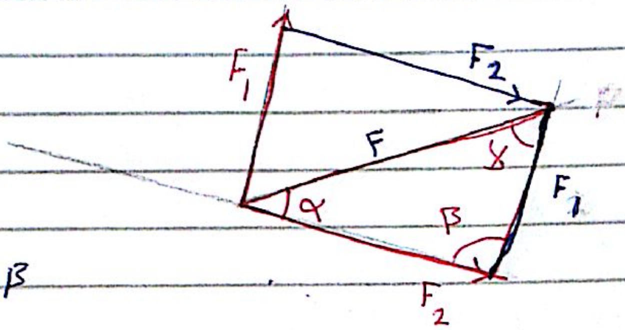
$R = \sqrt{(\Sigma F_x)^2 + (\Sigma F_y)^2} = 6.79\ kN$



$$\theta = \tan^{-1}\left(\frac{\Sigma F_y}{\Sigma F_x}\right) = 76.51°$$

$$R = (-1.586\,\hat{i} - 6.61\,\hat{j})\ N$$

Non-Rectangular components:-



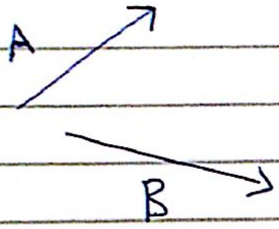
① Law of sines :-

$$\frac{F}{\sin\beta} = \frac{F_1}{\sin\alpha} = \frac{F_2}{\sin\gamma}$$

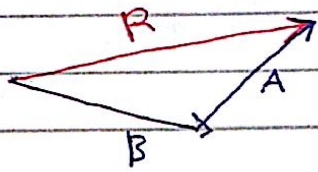
② Law of cosines

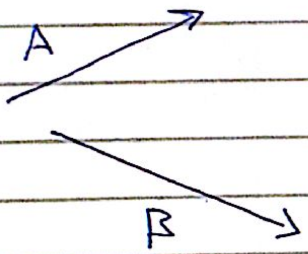
$$F^2 = F_1^2 + F_2^2 - 2F_1F_2\cos\beta$$

If we have 2-forces the R :-

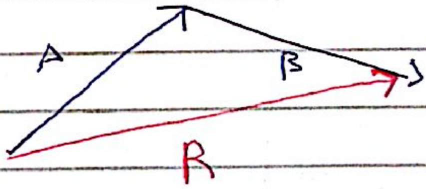


Add A to B

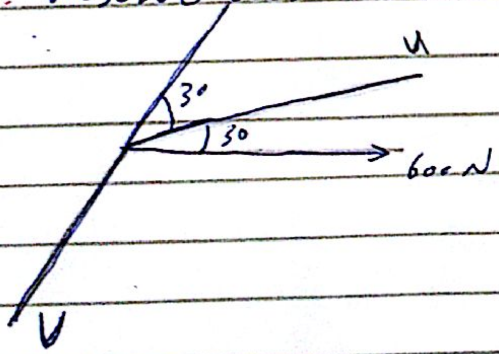




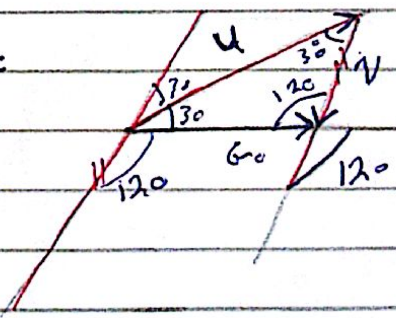
Add B to A



Ex: Resolve 600N Force into Forces acting on u and v



Solu:



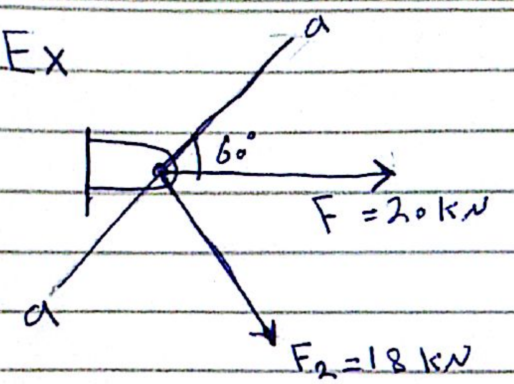
R = V + U

$$\frac{600}{\sin30} = \frac{U}{\sin120} = \frac{V}{\sin30}$$

$F_u = 1039\,N$

$F_v = 600\,N$

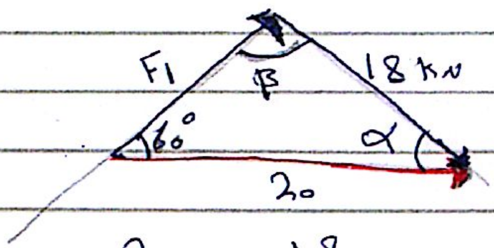
Ex



What is the F_1 = ?? along the axis a-a

solu:-

$F = F_2 + F_a$



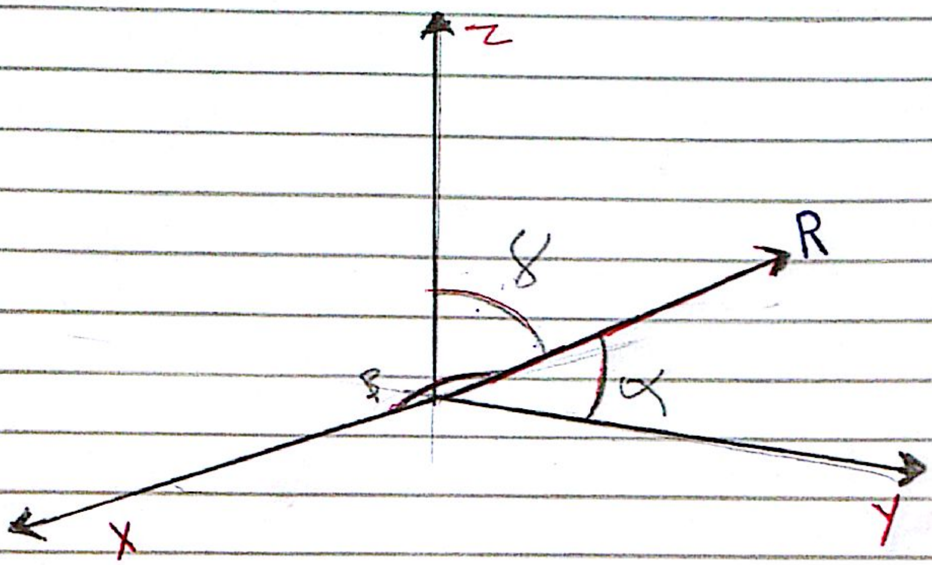
$\frac{20}{\sin\beta} = \frac{18}{\sin 60}$ $\beta = 74.2°$

$\beta + \alpha + 60° = 180°$ $\alpha = 45.7°$

$\frac{F_1}{\sin 45.7} = \frac{20}{\sin 74.2}$ $F_1 = 14.9$ KN

3D-system

Revision



1) $A_x = R\cos\beta$

$A_y = R\cos\alpha$

$A_z = R\cos\gamma$

2) The force as a Cartesian vector

$$R = A_x\hat{i} + A_y\hat{j} + A_z\hat{k}$$

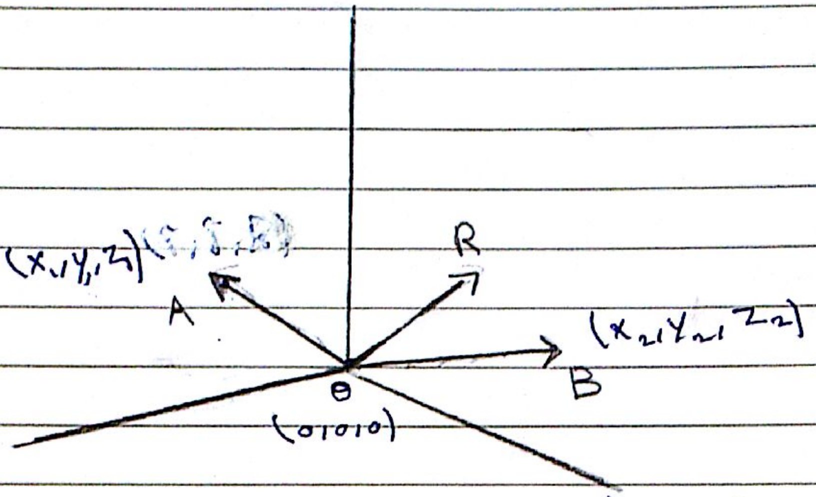
3) The magnitude of the force

$$|R| = \sqrt{(A_x)^2 + (A_y)^2 + (A_z)^2}$$

4) The force as a Unit vector

$$U_R = \frac{A_x}{|R|}\hat{i} + \frac{A_y}{|R|}\hat{j} + \frac{A_z}{|R|}\hat{k} \qquad \vec{R} = U_R \times |R|$$

5) $\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$



6) Addition of cartesin vector:-

$\vec{A} = A_x \hat{i} + A_y \hat{j} + A_z \hat{k}$

$\vec{B} = B_x \hat{i} + B_y \hat{j} + B_z \hat{k}$

$$R = A + B = (A_x + B_x)\hat{i} + (A_y + B_y)\hat{j} + (A_z + B_z)\hat{k}$$

7) Position vector:-

the position vector between A, O $\quad r_{AO}$

$$r_A = (x_1 - 0)\hat{i} + (y_1 - 0)\hat{j} + (z_1 - 0)\hat{k}$$

the position vector between A, B $\quad r_{AB}$

$$r_{AB} = (x_2 - x_1)\hat{i} + (y_2 - y_1)\hat{j} + (z_2 - z_1)\hat{k}$$

8) Dot product

A) $A \cdot B = B \cdot A$

B) $A \cdot (B + D) = A \cdot B + A \cdot D$

c) $A \cdot B = A_x B_x + A_y B_y + A_z B_z = C$

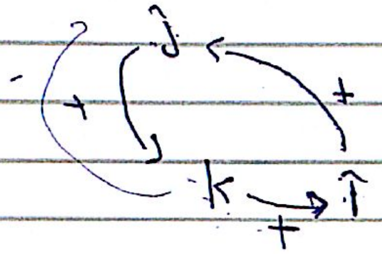
9) Cross product:

1) $A \times B = -B \times A$

2) $\alpha (A \times B) = \alpha A \times B = A \times \alpha B$

3) $A \times (B + D) = A \times B + A \times D$

4)



$\hat{i} \times \hat{i} = 0$

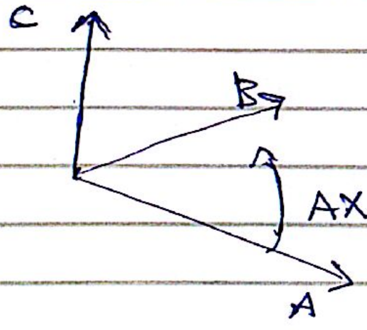
$\hat{k} \times \hat{k} = 0$

$\hat{j} \times \hat{j} = 0$

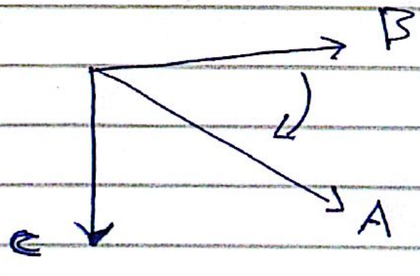
5) $A \times B = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$

$A \times B = (A_y B_z - A_z B_y)\hat{i} - (A_x B_z - A_z B_x)\hat{j} + (A_x B_y - A_y B_x)\hat{k}$

6)



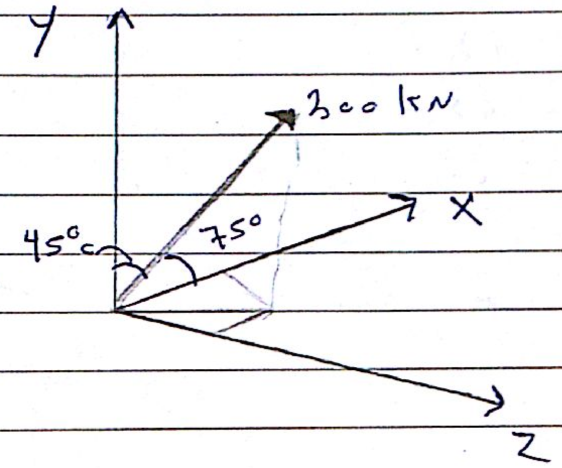
$A \times B = C$



$C = B \times A$

Ex:-

1) Determine the components of the 200kN force on x, y and z axis.
2) express the force as a cartesian vector
3) Determine the unit vector.



Solu:-

1) We must find θ_z

$$\cos 45^2 + \cos 75^2 + \cos^2\theta_z = 1 \Rightarrow \theta_z = 48.85°$$

$F_x = 200 \times \cos 75 = 52$ kN $\quad F_y = 200 \times \cos 45 = 141$ kN

$F_z = 200 \times \cos 48.85 = 131.7$ kN

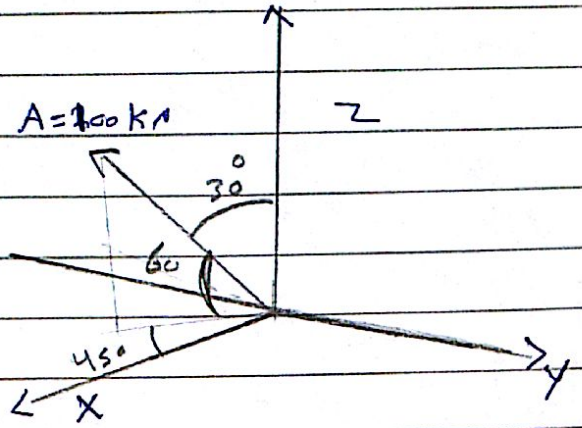
2) $R = 52\hat{i} + 141\hat{j} + 131.7\tilde{k} \quad |R| = 200$

3) $U_R = \frac{52}{200}\hat{i} + \frac{141}{200}\hat{j} + \frac{131.7}{200}\hat{k}$

Ex

Determine

A_x, A_y, A_z



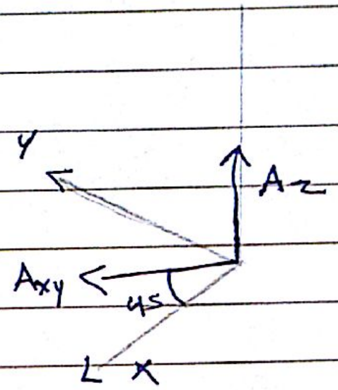
Solu

$A_z = 100 \times \cos 30 = 86.6$ kN

$A_{xy} = 100 \times \cos 60 = 50$ kN

$A_x = A_{xy} \times \cos 4 = 35.4$ kN

$A_y = A_{xy} \sin 45 = 35.4$ kN



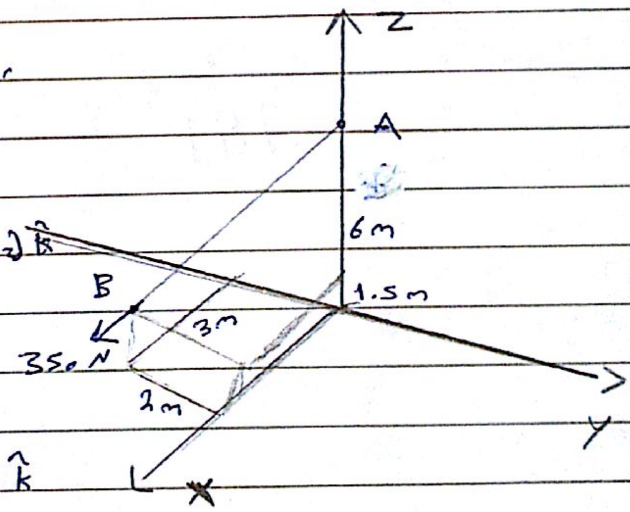
Ex

expeess the force as a cartesian vector

Solu

1) Position vector

$$\vec{r}_{AB} = (B_x - A_x)\,\hat{i} + (B_y - A_y)\,\hat{j} + (B_z - A_z)\,\hat{k}$$



A (0, 0, 7.5) B (3, -2, 1.5)

$$\vec{r}_{AB} = 3\hat{i} + -2\hat{j} + 6\hat{k} = 3\hat{i} - 2\hat{j} - 6\hat{k}$$

2) unit vector $$U_{AB} = \frac{\vec{r}_{AB}}{|\vec{r}_{AB}|} = \frac{3\hat{i} - 2\hat{j} + 6k}{\sqrt{3^2 + 2^2 + 6^2}} = 0.42\hat{i} - 0.285\hat{j} - 0.8\hat{k}$$

$\alpha = \cos^{-1}(3/7)$

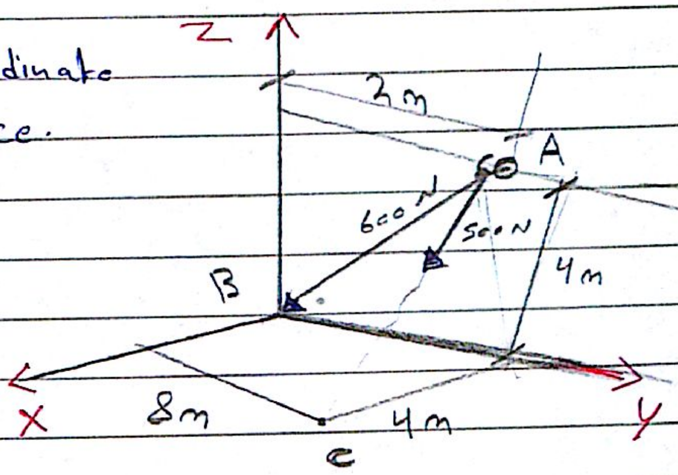
$\beta = \cos^{-1}(\frac{-2}{7})$

$\gamma = \cos^{-1}(\frac{-6}{7})$

$$\vec{F} = F \times U_{AB} = 150\hat{i} - 100\hat{j} - 300k$$

Ex:-

Determine the magnitude and coordinate direction angels of resultant force.



solu:-

A (0, 2, 4)

B (0, 0, 0)

C (4, 8, 0)

$r_{AB} = -2\hat{j} - 4\hat{k}$ $\qquad |r_{AB}| = \sqrt{(2)^2 + (4)^2} = 4.472$

$$U_{AB} = \frac{r_{AB}}{|r_{AB}|} = -0.447\hat{j} - 0.894\hat{k}$$

$$F_{AB} = U_{AB} \times F = 600 \times (-0.447\hat{j} - 0.894\hat{k})$$
$$= -268.3\hat{j} - 536.6\hat{k}$$

$r_{AC} = 4\hat{i} + 6\hat{j} - 4\hat{k}$ $\qquad |r_{AC}| = 8.25$

$$U_{AC} = \frac{r_{AC}}{|r_{AC}|} = 0.485\hat{i} + 0.728\hat{j} - 0.485\hat{k}$$

$$F_{AC} = U_{AC} \times F = 242.54\hat{i} + 363.8\hat{j} - 242.5\hat{k}$$

$$F_R = F_{AB} + F_{AC} = 242.54\hat{i} + 95.47\hat{j} - 779.2\hat{k}$$

$$|F_R| = \sqrt{(242.54)^2 + \cdots} = 822\ N$$

$$\cos\theta_x = \frac{F_x}{F} = \frac{242.54}{822} \qquad \theta = \cos^{-1}\left(\frac{242.5}{822}\right)$$

$$\cos\theta_y = \frac{F_y}{F}$$

$$\cos\theta_z = \frac{F_z}{F}$$

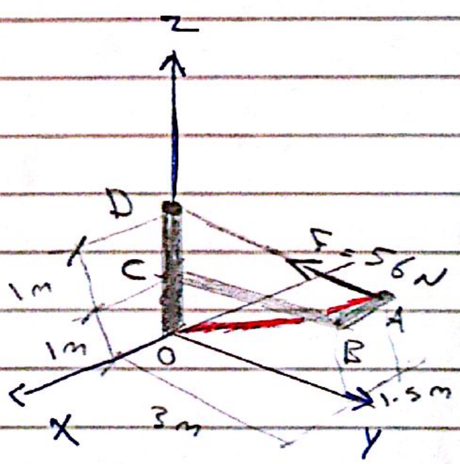
Dot Product

We use dot Product for
1- To find the angle formed between two vectors or intersecting lines.
2- To find the components of a vector parallel and perpendic to line

→ to find the angle $\theta = \cos^{-1}\left(\frac{A \cdot B}{|A| * |B|}\right)$

→ 1) We find the angle between the vector and the line (θ)
2) if we want the force that ~~part~~ parallel to the line
$F_{//} = F * \cos\theta$
if we want the force that perpendicular to the line
$F_{\perp} = F \sin\theta$

Ex:-
Determine the magnitudes of force $F = 56$ N acting along and perpendicular to line OA



Solu:-
A (1.5, 3, 1)
D (0, 0, 2)

$r_{AD} = 1.5\hat{i} - 3\hat{j} + 1\hat{k}$ $|r_{AD}| = 3.5$

$r_{AO} = -1.5\hat{i} + 3\hat{j} \mp 1\hat{k}$ $|r_{AO}| = 3.5$

$U_{AD} = \frac{r_{AD}}{|r_{AD}|} = \frac{3}{7}\hat{i} - \frac{6}{7}\hat{j} + \frac{2}{7}\hat{k}$

$\theta = \cos^{-1}\left(\frac{r_{AD} \cdot r_{AO}}{r_{AD} \times r_{AO}}\right) = 33.2$

$F_{AD} = F * U_D = 24\hat{i} - 48\hat{j} + 16\hat{k}$

$F_{//} = 56 \times \cos\theta = 46.8$ N

$F_{\perp} = F \times \sin\theta = 30.7$ N

Chapter 3: Equilibrium

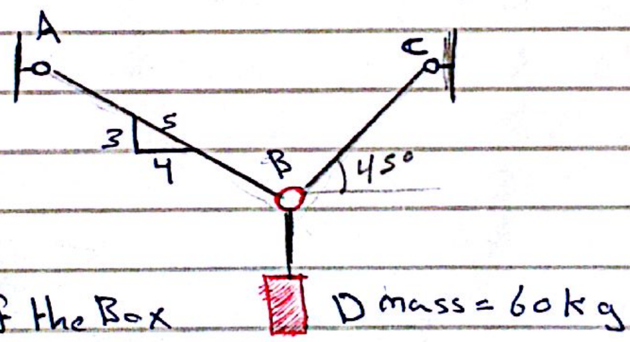
A rigid body is said to be in equilibrium when the external force acting on it form a system of forces equivalent to zero thus the equilibrium equations are

$$\boxed{\Sigma F = 0}$$

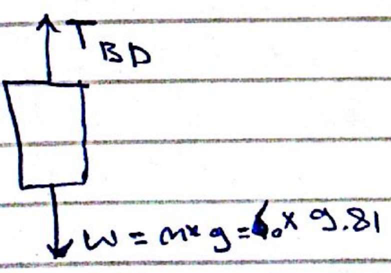
Free Body diagram (F.B.D):-
Diagramatic representation of a body or bodies under Considiration showing all forces applied to it by other bodies that are imagined to be removed

Procuder for soluing:-
1) Choice of the F.B.D to be isolated to give all unknowns.
2) indecating all external force act on the body
3) Use $\Sigma F = 0$ to give us the knowns

Ex:-
Determine the tension on cable AB and BD



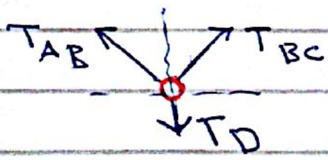
Solu:-
take a F.B.D of the Box

① 

$\Sigma F_y = 0$

$T_{AB} = 588.6\ N$

② take F.B.D of the ring



$\Sigma F_x = 0$

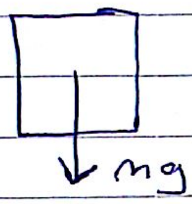
$T_{BC} \times \cos 45 - T_{AB} \times \frac{4}{5} = 0$

$\Sigma F_y = 0$

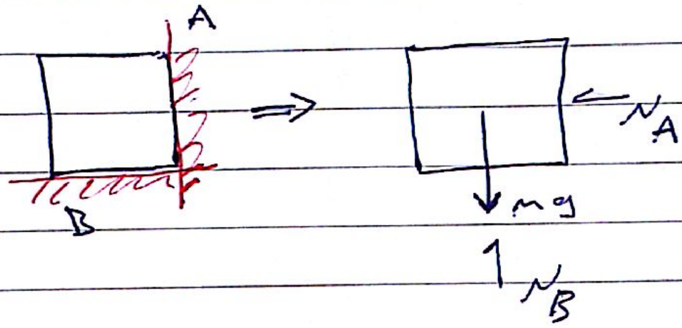
$T_{AB} = 420\ N$

Type of Forces

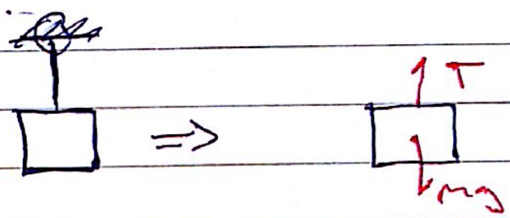
1- gravity Force



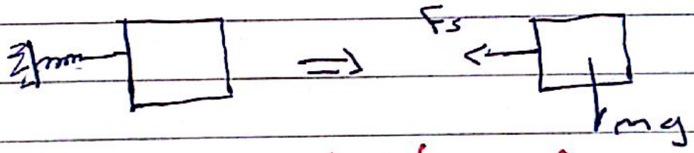
2- normal Forces



3- tension force



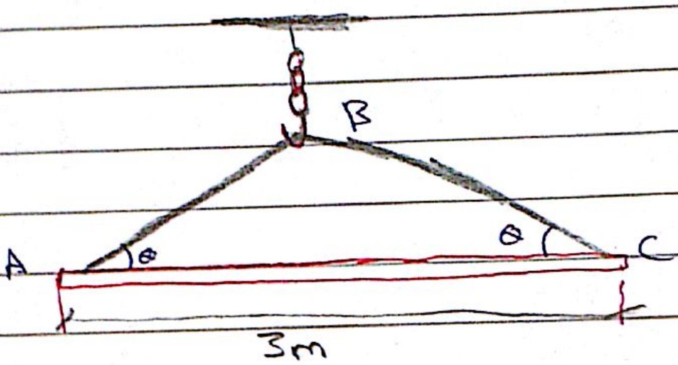
4- spring force



$$F_s = K * (x_2 - x_1)$$
$$= K * \delta$$

Ex:-

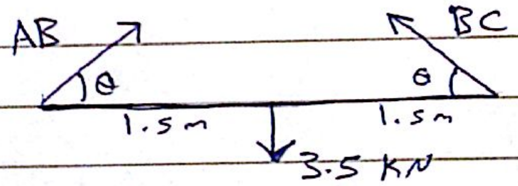
Determine the length of the cable that can be used to lifit the beam if the maximum force in the Cable is 7.5 KN and the weight of the beam is 3.5 KN

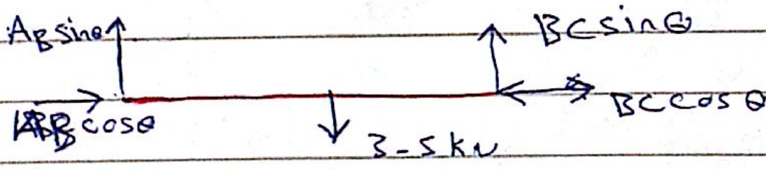


Solu:-

We must find θ then we find the length

1) F.B.D





$\Sigma F_x = 0$

$AB\cos\theta - BC\cos\theta = 0$ $\quad AB = BC$

$\Sigma F_y = 0$

$AB\sin\theta + \underset{AB}{\cancel{BC}}\sin\theta = 3.5\ kN$

$2AB\sin\theta = 3.5$

$2 \times 7.5 \sin\theta = 3.5$ $\quad \theta = 13.49$

Equilibrium on 3D

We use $\Sigma F_x = 0$

$\Sigma F_y = 0$

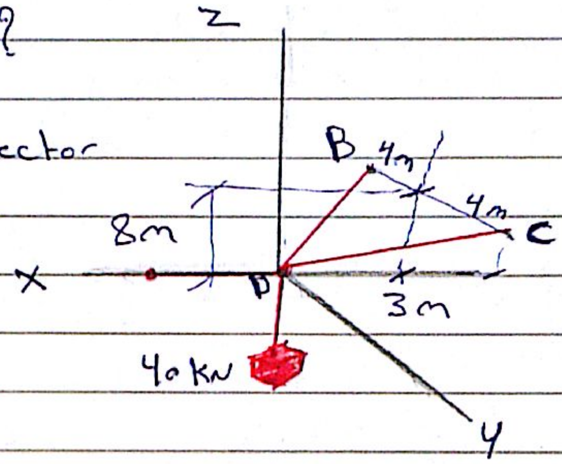
$\Sigma F_z = 0$

Ex:-

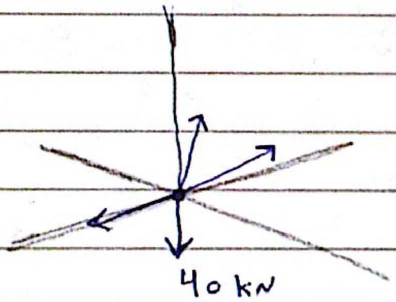
Determine the force on each cable?

We must express the tension on each cable as a cartesian vector

$F = F \times \left|\frac{r}{|r|}\right|$ the We use $\Sigma F = 0$



Soln:-



as a cartesian

$$F_B = F_B \times \left(\frac{-3\hat{i} - 4\hat{j} + 8\hat{k}}{\sqrt{(3)^2 + (4)^2 + (8)^2}} \right)$$

$$= -0.318 F_B \hat{i} - 0.424 F_B \hat{j} + 0.848 F_B \hat{k}$$

$$F_C = -0.318 F_C \hat{i} + 0.424 F_C \hat{j} + 0.848 F_C \hat{k}$$

$$F_D = F_D \hat{i}$$

$$W = -40 \hat{k}$$

$\Sigma F = 0$

$F_x = 0 \quad -0.318 F_B - 0.318 F_C + F_D = 0$

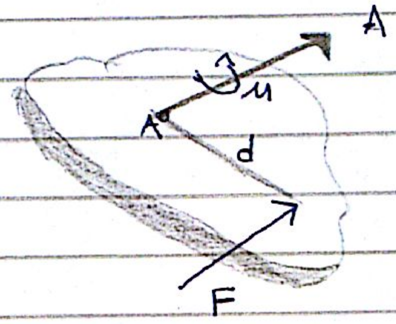
$\Sigma F_y = 0 \quad -0.424 F_B + 0.424 F_C = 0 \quad \Rightarrow \quad F_B = F_C = 23.6$ kN, $F_D = 15$ kN

$\Sigma F_z = 0 \quad 0.848 F_B + 0.848 F_C - 40 = 0$

Chapter 4:- Force system Resultants:-

Moment:- The tendency to rotate about a given axis caused by the force acting on a body provided that axis is not parallel to the force line of action and does not intersect it.

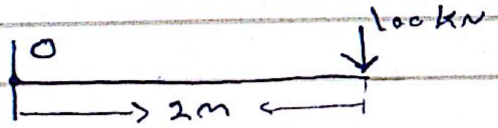
$M = F \cdot d$



notes:-
the moment of a force about a given point is equal to the moment of its component about the same point

Ex:-
Determine the moment at Point O

1)

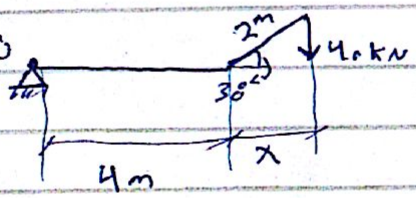


$M = 100 \times 2 = 200 \, kN.m$ C.W

2)

$M = 50 \times 0.75 = 37.5 \, kN.m$ C.W

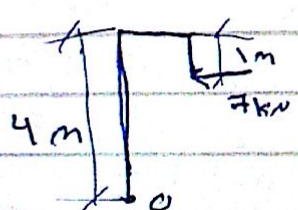
3)



$x = 2 \times \cos 30°$

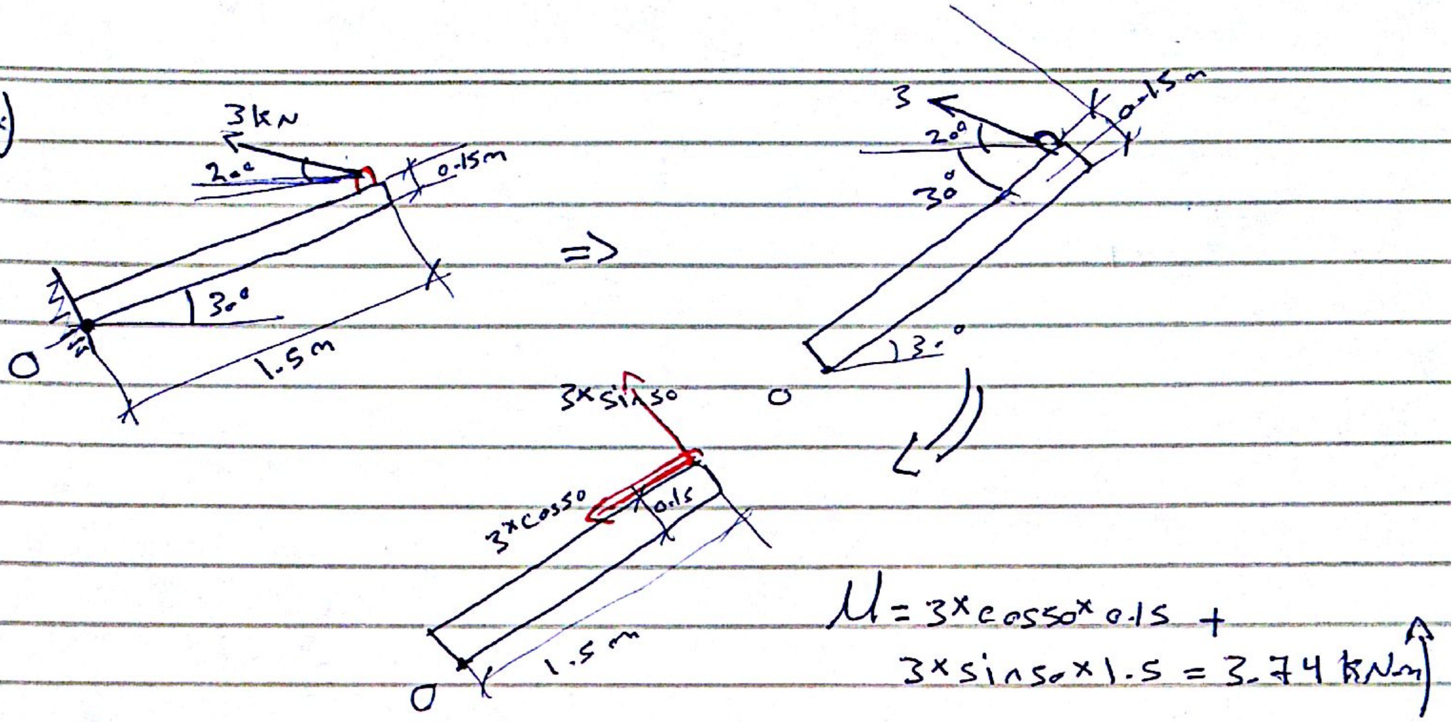
$M = 40 \times (4 + 2\cos 30°) = 229 \, kN.m$

4)



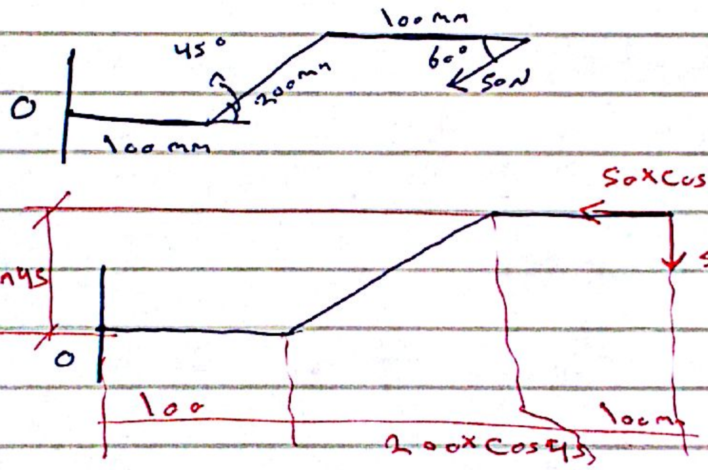
$M = 7 \times (4 - 1) = 21 \, kN.m$ C.C.W

5)



$M = 3 \times \cos 50 \times 0.15 + 3 \times \sin 50 \times 1.5 = 3.74\ kN.m$

6)



$M = 50 \times \sin 60 \times (100 + 200\cos 45 + 100) - 50 \times \cos 60 \times 0.2 \times \sin 45$
$= 11.24\ N.m$

if the answer is - thats mean the moment is

Moment on 3D

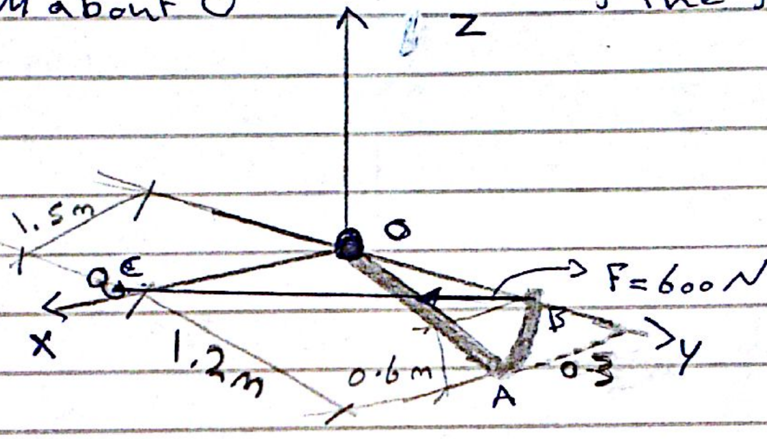
Moment about a point

$$M = r \times F$$

F as a cartesian vector of the force
r as a Position vector for a point that we want to find the moment on it to any point on the line of action of the force.

Ex

Determin the M about O



Solu

B (0.3, 1.2, 0.6)

C (1.5, 0, 0)

O (0, 0, 0)

$$r_{BC} = 1.2\hat{i} + 1.2\hat{j} - 0.6k \qquad |r_{BC}| = 1.8$$

$$F = F \times u_{BC} = 600 \times \left(\frac{1.2\hat{i} + 1.2\hat{j} - 0.6\hat{k}}{1.8}\right)$$

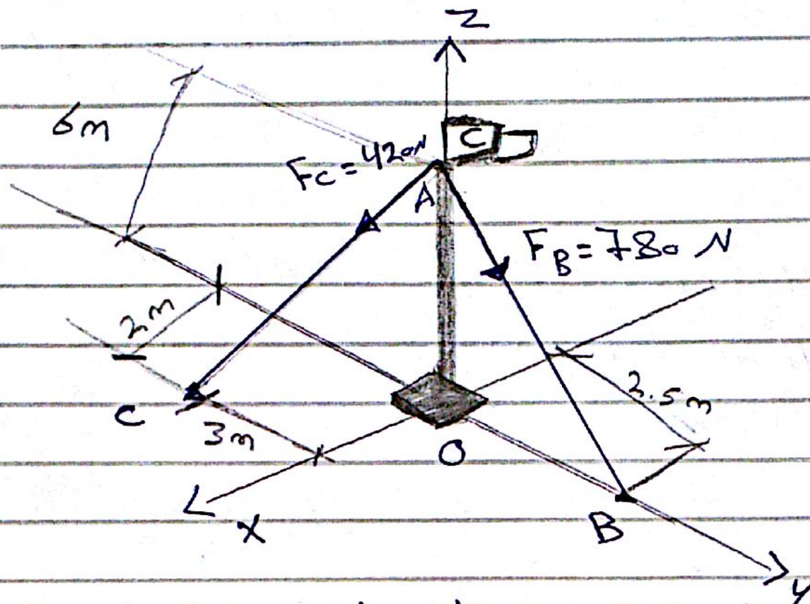
$$= 400\hat{i} - 400\hat{j} - 200\hat{k}$$

$$r_{OC} = 1.5\,\hat{i} \qquad \text{from } O \rightarrow C$$

$$M = r_C \times F = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1.5 & 0 & 0 \\ 400 & -400 & -200 \end{vmatrix}$$

$$= (0 \times -200 - 0 \times -400)\hat{i} - (1.5 \times -200 - 0 \times 400)\hat{j} + (1.5 \times -400 - 0 \times 400) = \underline{\underline{(300\hat{j} - 600\hat{k})}}$$

Ex
Determine the moment about O



solu

O (0,0,0)   A (0,0,6)   B (0, 2.5, 6)   C (2, -3, 0)

$r_{AC} = +2\hat{i} - 3\hat{j} - 6\hat{k} \qquad |r_{AC}| = 7$

$F_{AC} = F_{AC} \times u_{AC} = 420 \times \left(\frac{2\hat{i} - 3\hat{j} - 6k}{7}\right) = 120\hat{i} - 18\hat{j} - 360k$

$F_{AB} = 780 \times \left(\frac{(0-0)\hat{i} + (2.5-0)\hat{j} + (0-6)k}{\sqrt{0^2 + (2.5)^2 + (-6)^2}}\right) = 300\hat{j} - 720\hat{k}$

$r_{OA} = 6\hat{k}$

~~M = rOA x FAc~~ $M = r_{OA} \times F_{AC} + r_{OC} \times F_{BA}$

$= (-720\hat{i} + 720\hat{j})\ N.m$

Moment about a axis :-

$$M = U \cdot (r \times F)$$

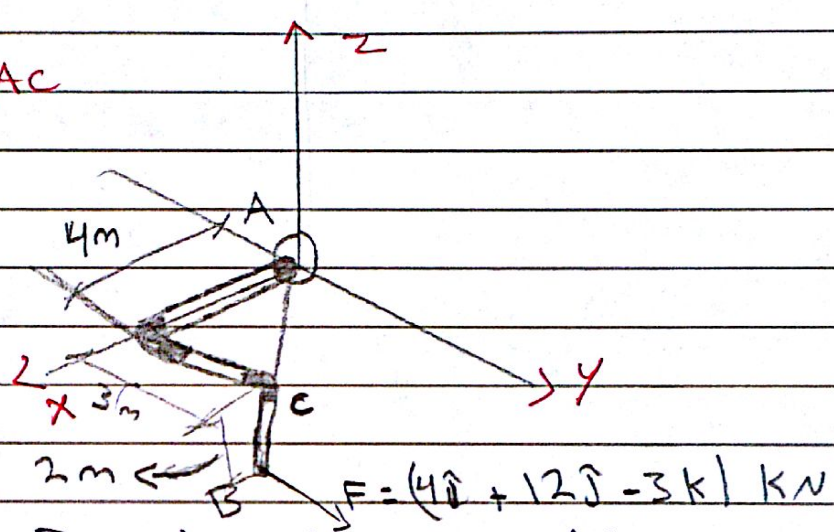
U = unit vector of the axis

r = is a Position vector between a point on the axis and a point on a line of action of the force

F = Force as a cartesian vector.

Ex

Determine the moment about line AC



Solu B (4, 3, -2) C (4, 3, 0) A (0, 0, 0)

$r_{CB} = -2\hat{k}$

$$U_{AC} = \frac{(4-0)\hat{i} + (3-0)\hat{j} - (0-0)k}{\sqrt{4^2+9}} = 0.8\hat{i} + 0.6\hat{j}$$

$$M = U_{AC} \cdot (r_{CB} \times F)$$

$$= (0.8\hat{i} + 0.6\hat{j}) \cdot \begin{vmatrix} 0 & 0 & -2 \\ 4 & 12 & -3 \end{vmatrix}$$

$$= (0.8\hat{i} + 0.6\hat{j}) \left((0 \times -3 - 12 \times -2)\hat{i} - (0 \times -3 - -2 \times 4)\hat{j} + 0\hat{k} \right)$$

$$= 14.4 \text{ kN.m}$$

Couple :-

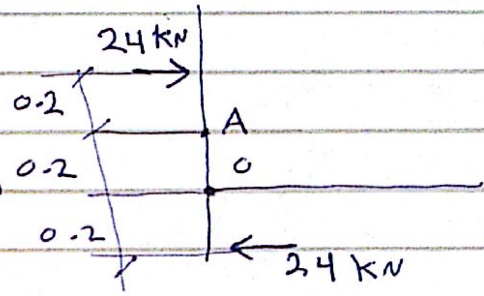
Two forces having the same magnitude prallel lines of action and opposite sense.

* the moment of the couple is given by $M = F \cdot d$ about any point

* A force may be replace by an equal force at same point and couple.

Ex

find M_o, M_A

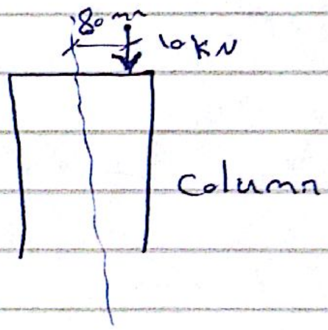


Solu :-

$M_o = M_A = 24 \times 0.6 = 14.4 \text{ kN.m}$

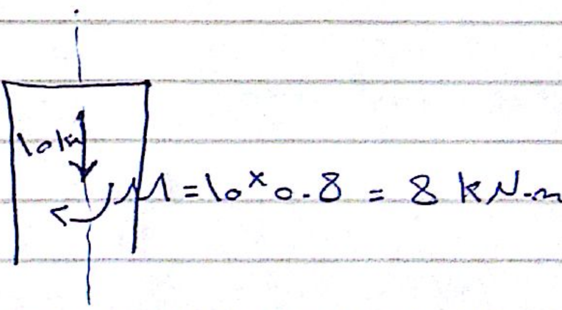
Ex :-

replace the 10 kN force by a force acting at the center of the col and a couple



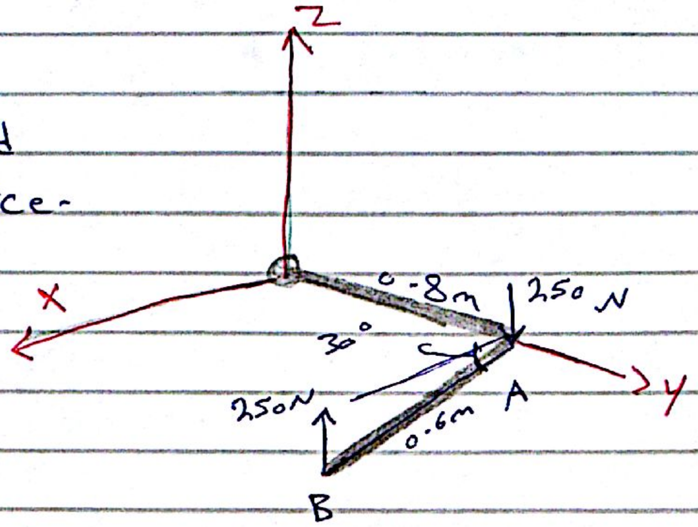
Solu:

$M = 10 \times 0.8 = 8 \text{ kN.m}$



Ex:-

Determine the moment produced by the couple force.



Solu

A (0, 0.8, 0)

B = (cos30 × 0.6, 0.8, − sin3 × 0.6)

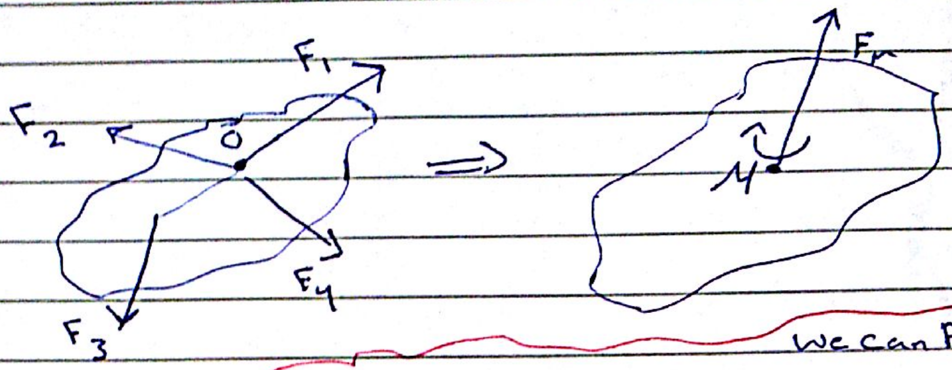
$r_{AB} = (0.52\hat{i} - 0.3k)$ $\qquad$ $F = 250\, k$

$$M = r_{AB} \times F$$

$$= \begin{vmatrix} 0.52 & 0 & -0.3 \\ 0 & 0 & 250 \end{vmatrix} = -130\hat{j}\ N.m$$

Simplification of force and couple system :-

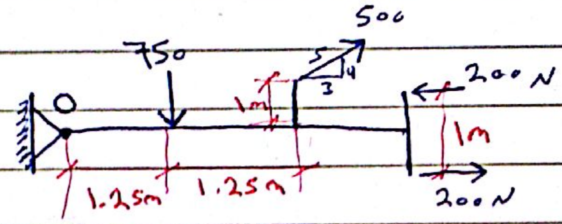
Case 1



$$F_R = \Sigma F$$
$$M_{R_o} = \Sigma M_o + \Sigma M$$

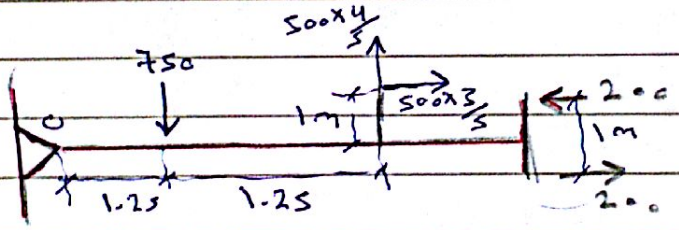
we can Replace the force system by a F_R and M

Ex



Replace the force and couple system acting on the member in Fig by an equivilant resultant force and couple moment acting on point O ??

Solu :-



$$\Sigma F_x = 500 \times \frac{3}{5} = 300\ N$$

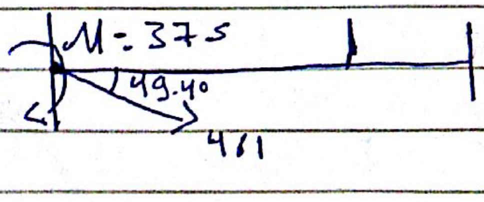
$$\Sigma F_y = 500 \times \frac{4}{5} - 750 = -350\ N$$

$$\circlearrowleft + \Sigma M_o = -750 \times 1.25 - 300 \times 1 + 400 \times 2.5 + 200 = -37.5\ N.m$$

$$= 37.5\ N.m \circlearrowright$$

$$F_R = \sqrt{(F_x)^2 + (F_y)^2} = 461\ N$$

$$\theta = \tan^{-1}\left(\frac{F_y}{F_x}\right) = 49.4°$$



Case 2

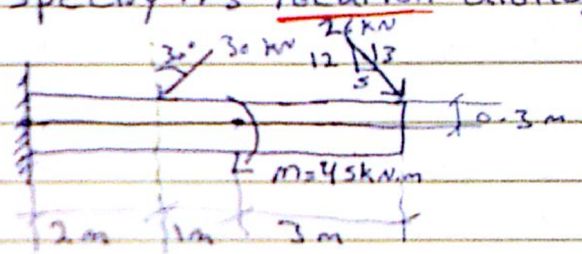
On case two we Replace the Force system by a F_R and we determine the location of F_R

$$\Sigma F = F_R$$

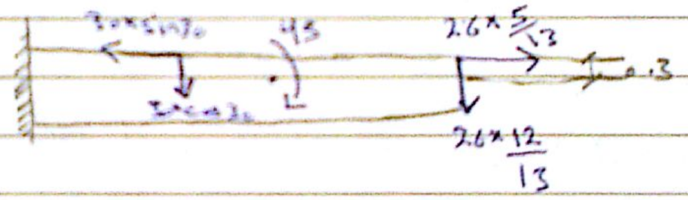
$$F_R \hat{X} = \Sigma M$$

Ex:-

Replace the force and couple moment system acting on the beam by ~~an~~ resultant force and specify it's location along AB measured from point A:-



Solu

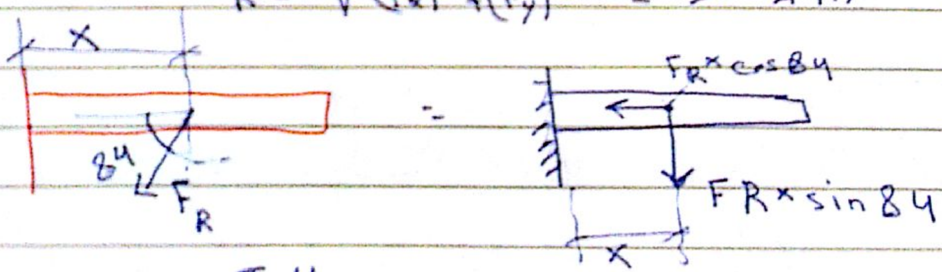


$$\Sigma F = \Sigma F_x + \Sigma F_y$$

$$\Sigma F_x = 26\times\frac{5}{13} - 30\times\sin 30 = -5 \text{ kN}$$

$$\Sigma F_y = -30\cos 30 - 26\times\frac{12}{13} = -49.98 \text{ kN}$$

$$F_R = \sqrt{(F_x)^2+(F_y)^2} = 50.2 \text{ kN} \qquad \theta = \tan\left(\frac{F_y}{F_x}\right) = 84^\circ$$



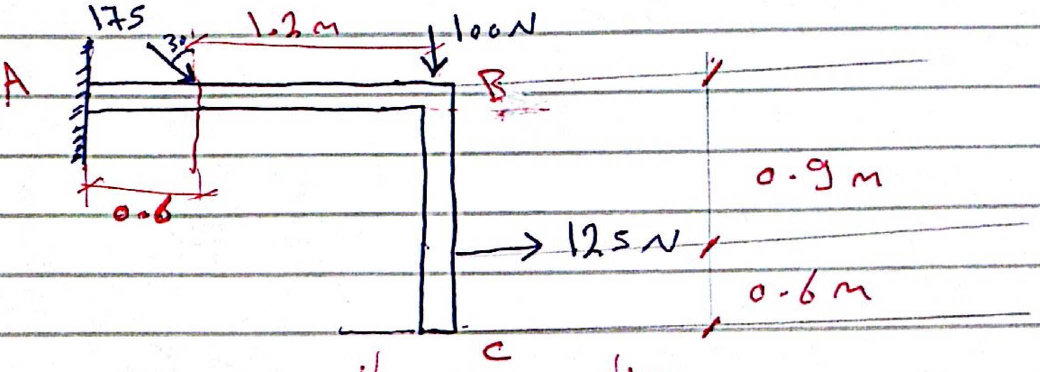
$$\Sigma M_{F_R} = \Sigma M$$

$$FR\times\sin 84\times X = -30\times\cos 30\times 2 + 30\times\sin 30\times 0.3 - 45 - 26\times\frac{5}{13}\times 0.3 - 26\times\frac{12}{13}$$

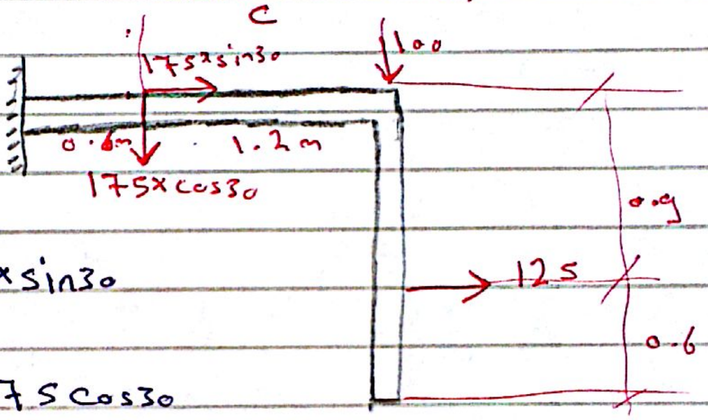
$$X = 4.79 \text{ m}$$

Ex:

Replace the force system by a F_R and Determine the location of aplication at member Bc



Solu

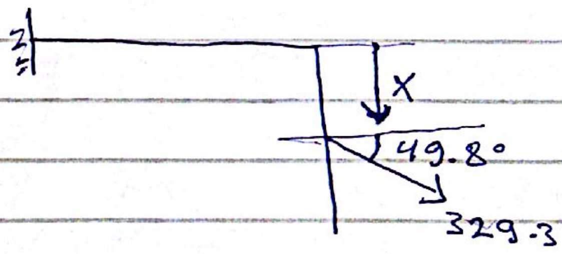


$$\Sigma F_x = 125 + 175 \times \sin 30 = 212.5\ N$$

$$\Sigma F_y = -100 - 175 \cos 30 = -251.55\ N$$

$$F_R = \sqrt{(F_x)^2 + (F_y)^2} = 329.3\ N$$

$$\theta = \tan^{-1}\left(\frac{F_y}{F_x}\right) = 49.8°$$



$$\circlearrowright \Sigma M_F = \circlearrowright \Sigma M$$

$$-251.55 \times 1.8 + 212.5 \times x = -175 \times \cos 30 \times 0.6 - 100 \times 1.8 + 125 \times 0.9$$

$$x = 1.38\ m$$

Chapter 5 Equilibruim of a rigid body

table 5-1 P202

name	shape	reaction
roller		F_y
pin		F_x, F_y
fixed		F_x, F_y, M
cable	cable	F

Procuder for find the reactions

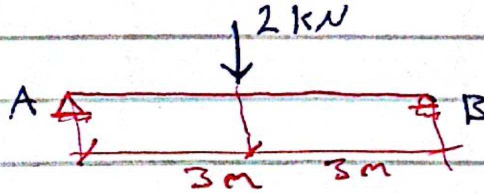
1) Draw the F.B.D
2) Convert the Distributed load to concentrated load
3) Use $\Sigma F_x = 0$ to find the Reactions.

   - $\Sigma F_y = 0$

   $\Sigma M = 0$ about any point

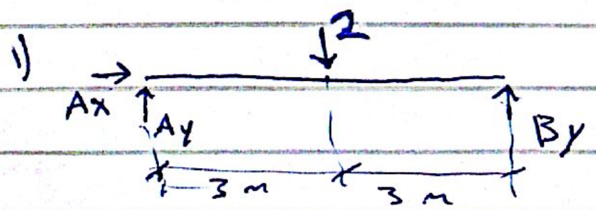
Ex:

find the reations



solu

1)



$\Sigma F_x = 0$ $A_x = 0$

$\Sigma F_y = 0$ $A_y + B_y - 2 = 0$

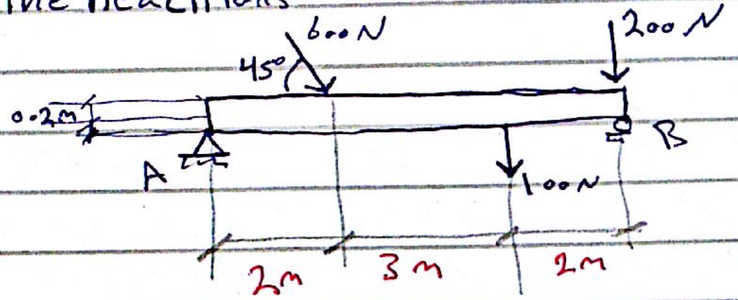
$\Sigma M_A = 0$

$B_y \times 6 - 2 \times 3 = 0$ $B_y = 1$ kN

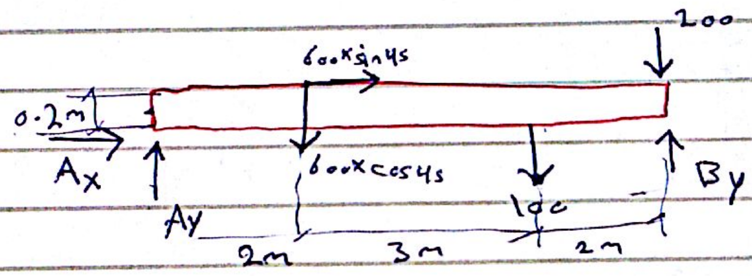
$A_y = 1$ kN

Ex

Determine the Reacitions



Solu



$\Sigma F_x = 0 \qquad A_x + 600\sin 45 = 0 \qquad A_x = -424.3\ N = 424.3 \leftarrow$

$\circlearrowleft + \Sigma M_A = 0$

$0.2 \times -600 \times \sin 45 - 600 \times \cos 45 \times 2 - 100 \times 5 + B_y \times 7 - 200 \times 7 = 0$

$B_y = 405\ N$

$\Sigma F_y = 0$

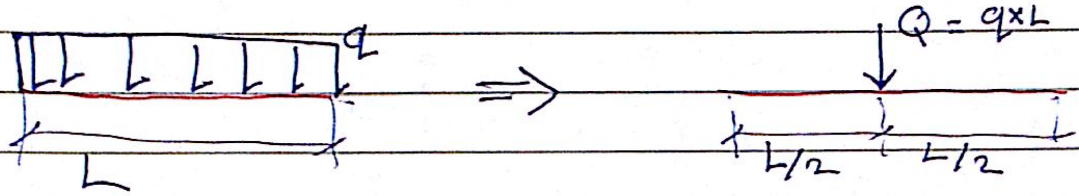
$A_y - 600\cos 45 - 100 - 200 + B_y = 0$

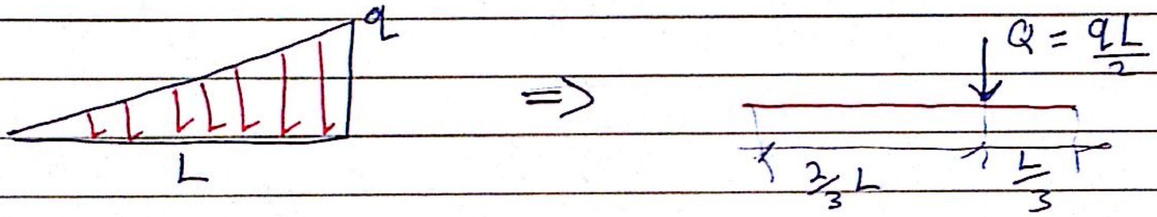
$A_y = 319\ N$

How to convert ~~for~~ from distributed load to consentrated

1) the value of the concentrated load is equal to the area of distributed load.

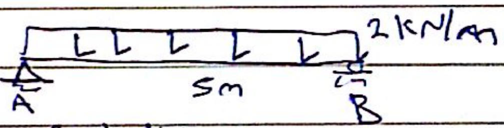
2) the location of the consentrated load ... the centroid of the dis. load.



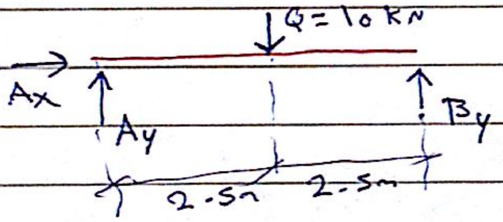


Ex.

Determine the reaction



Solu:



$\Sigma F_x = 0$

$A_x = 0$

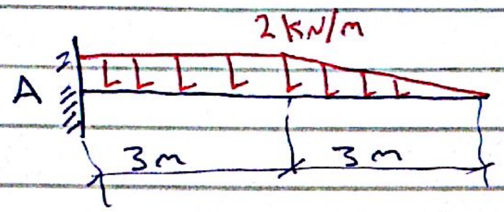
$\Sigma M_A = 0$

$-10 \times 2.5 + B_y \times 5 = 0 \qquad B_y = 5\ kN$

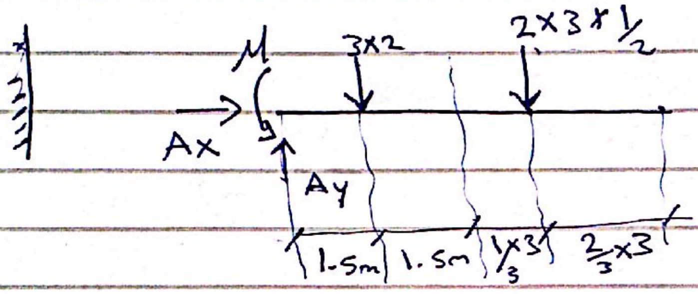
$\Sigma F_y = 0$

$A_y + B_y - 10 = 0 \qquad A_y = 5\ kN$

Ex



Solu:-



$\Sigma F_x = 0 \quad A_x = 0$

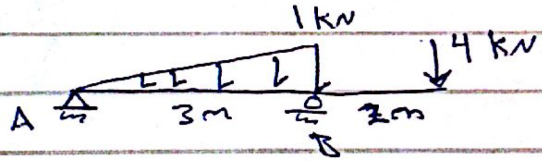
$\Sigma F_y = 0 \quad A_y = 9\ kN$

$\Sigma M_A = 0$

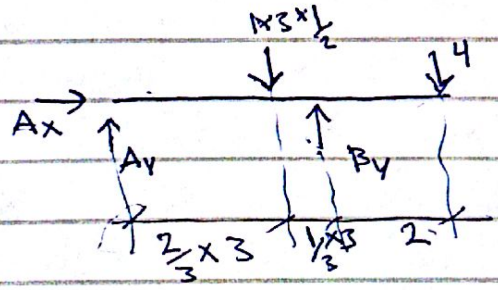
$M - 6 \times 1.5 - 3 \times 4 = 0$

$M = 21\ kN.m$

Ex:-



Solu



$\Sigma F_x = 0 \quad A_x = 0$

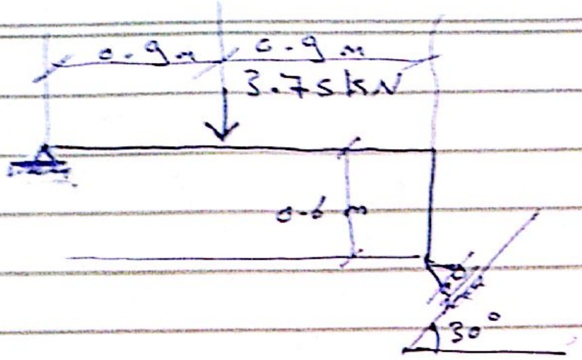
$\Sigma M_A = 0$

$-\frac{3}{2} \times 2 + B_y \times 3 - 4 \times 5 = 0 \qquad B_y = 7.67\ kN$

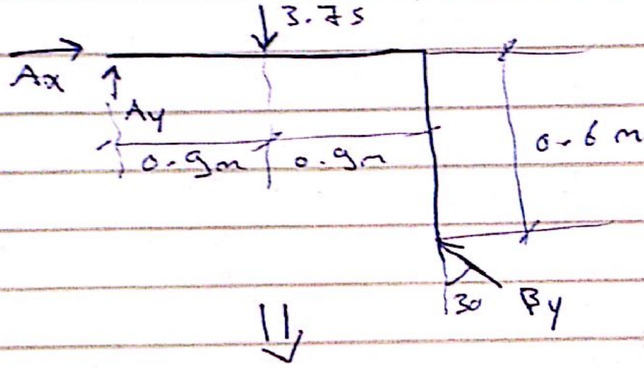
$\Sigma F_y = 0$

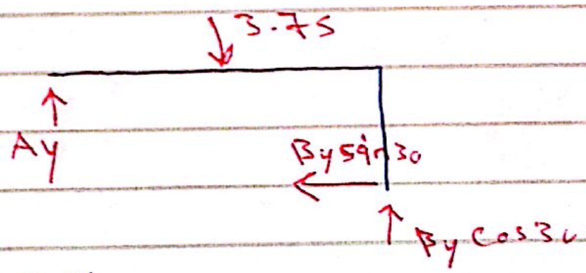
$A_y + B_y - \frac{3}{2} - 4 = 0 \qquad A_y = 2.17\ kN$

Ex



Solu





$\Sigma M_A = 0$

$B_y \times \cos 30 \times 1.8 - B_y \times \sin 30 \times 0.6 - 3.75 \times 0.9 = 0$

$B_y = 2.681 \text{ kN}$

$\Sigma F_y = 0 \Rightarrow A_y = 1.428 \text{ kN}$

$\Sigma F_x = 0 \Rightarrow A_x = 1.3405 \text{ kN}$

Ch6:- Trusses:

محمد مازن

A truss is a structrue composed of selnder members jointed together at their end points, the truses member only suport tension or compresion force.

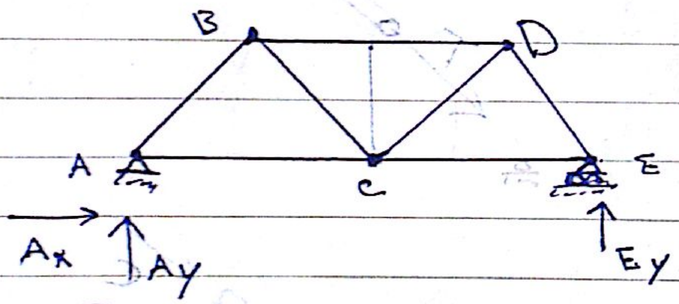
- stability and determinacy:-

$$m + s = 2J$$

m = # of members.
s = # number of external reaction.
J = # of Joints.

if ⟹ 1) $m+s = 2j$ Determinate and stable
2) $m+s < 2j$ Un stable
3) $m+s > 2J$ Indeterminate, Extra fixity.

ex ⟹

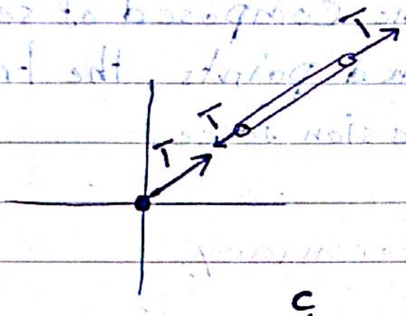


$m = 7$, $s = 3$, $J = 5$

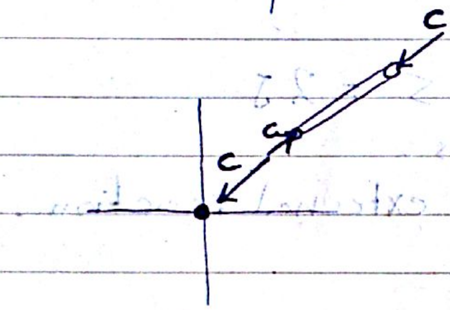
$m + s = 7 + 3 = 10$
$2J = 10$ ⟹ $m+s = 2J$
$10 = 10$ ⟹ the truss Determinate and stable.

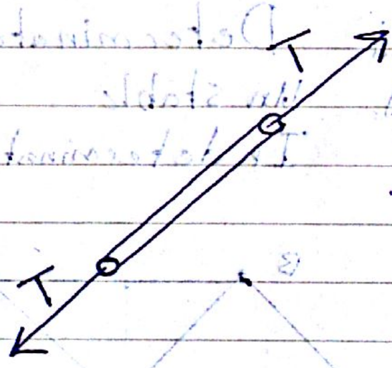
- Members are either in :-

1- Tension :-

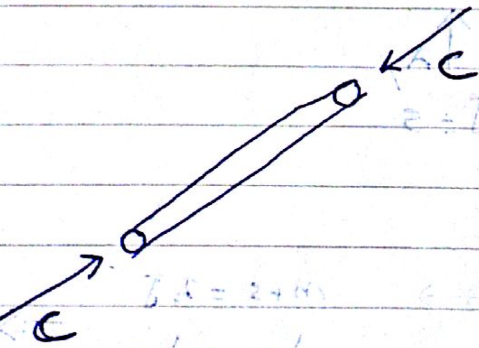


2- Compression :-





Tension



Compression

- There is Two methods of solving trusses :-
  1- method of joints
  2- method of section

1- Method of Joints :-

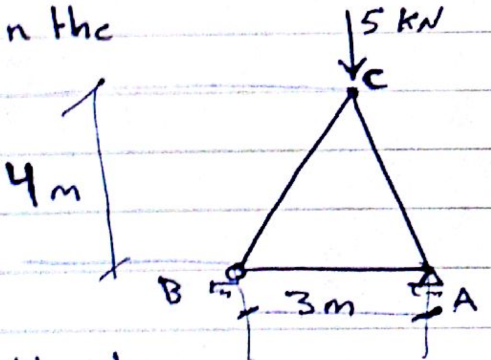
- Procedure for analysis :-

1- Draw the free body diagram of the truss and then find the Reactions.

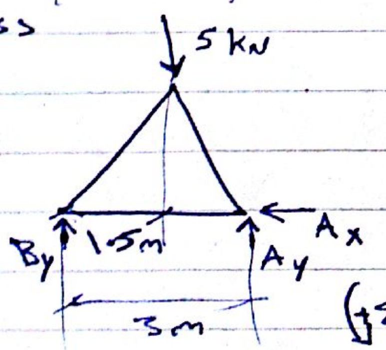
2.- Draw the free body diagram of the Joint that have at least one Known force and at most tow unknown forces. then we use $\Sigma F_x = 0$, $\Sigma F_y = 0$ to find the unknown forces in the members (We Continue in the same way Until we reach the desired member).

Examples :-

Ex (1) :-
find the force in the member AB.



Solu :-
the F.B.D of the truss



$\Sigma F_x = 0$

$A_x = 0$

$\Sigma F_y = 0$

$A_y + B_y - 5 = 0$

$\circlearrowleft + \Sigma M_B = 0$

$A_y \times 3 - 5 \times 1.5 = 0$

$A_y = 2.5 \text{ KN}$

$B_y = 5 - A_y$

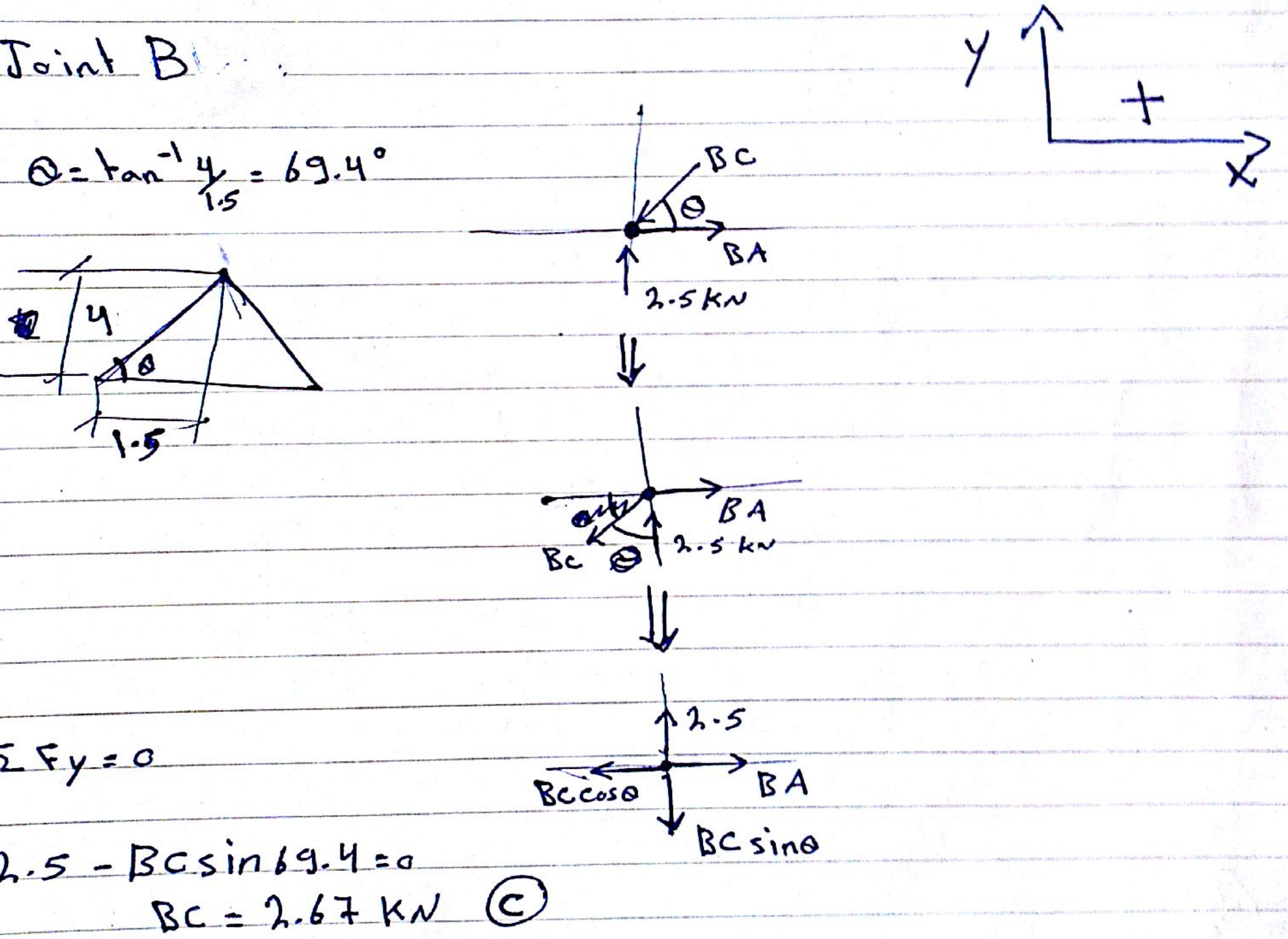
$B_y = 2.5\ KN$

the next step we take the F.B.D of Joint B or A

Joint B have one known and 2- unknwon so we can find the forces in the members (Also Joint A)

Joint B

$\theta = \tan^{-1} \frac{4}{1.5} = 69.4°$



$\Sigma F_y = 0$

$2.5 - BC \sin 69.4 = 0$

$BC = 2.67\ KN$ (C)

$\Sigma F_x = 0$

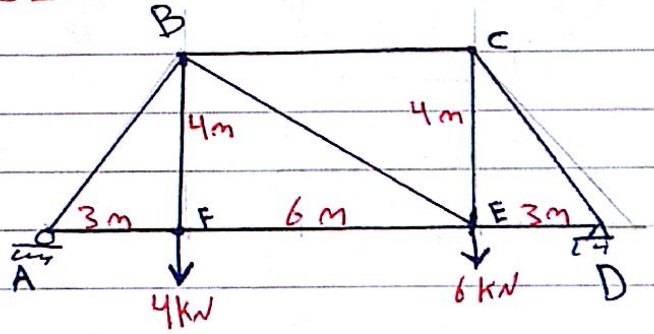
$BA - BC \cos 69.4 = 0$

$BA = 2.67 \times \cos 69.4$

$BA = 0.93\ KN$ (T)

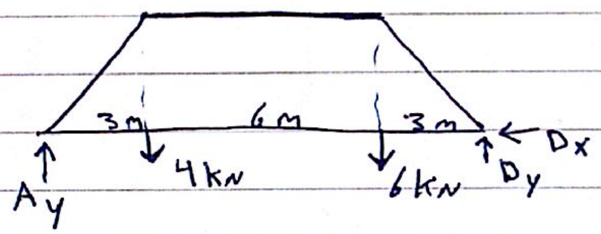
Ex 2:-

Find the force in the member BE



Solu:-

– F.B.D of the truss



$\Sigma F_x = 0$

$D_x = 0$

$\circlearrowright + \Sigma M_A = 0$

$-4 \times 3 - 6 \times (3+6) + D_y \times (3+6+3) = 0$

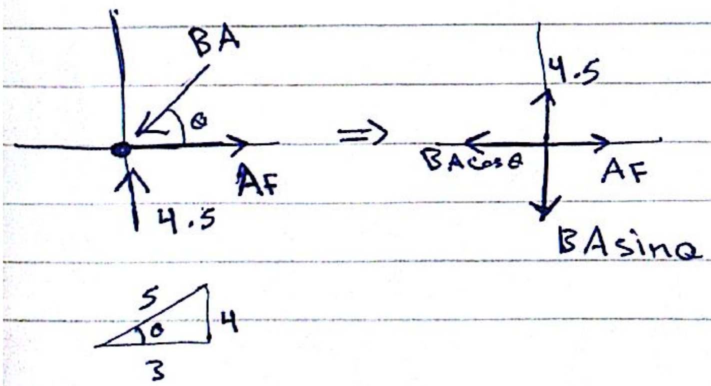
$D_y = 5.5 \text{ kN}.$

$\Sigma F_y = 0$

$A_y - 4 - 6 + D_y = 0$

$A_y = 4.5 \text{ kN}.$

– take Joint A



$\Sigma F_y = 0$

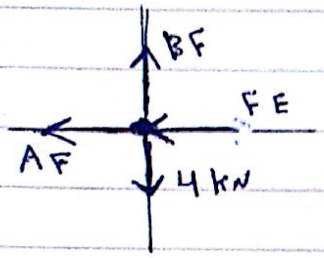
$-BA \times \frac{4}{5} + 4.5 = 0 \Rightarrow BA = 5.6 \text{ kN}$ (C)

$\Sigma F_x = 0$

$AF - BA \times \frac{3}{5} = 0$

$AF = 3.375 \text{ kN}$ (T)

take Joint F



$\Sigma F_x = 0$

$-AF - FE = 0$

$-FE = AF$ AF = 3.375

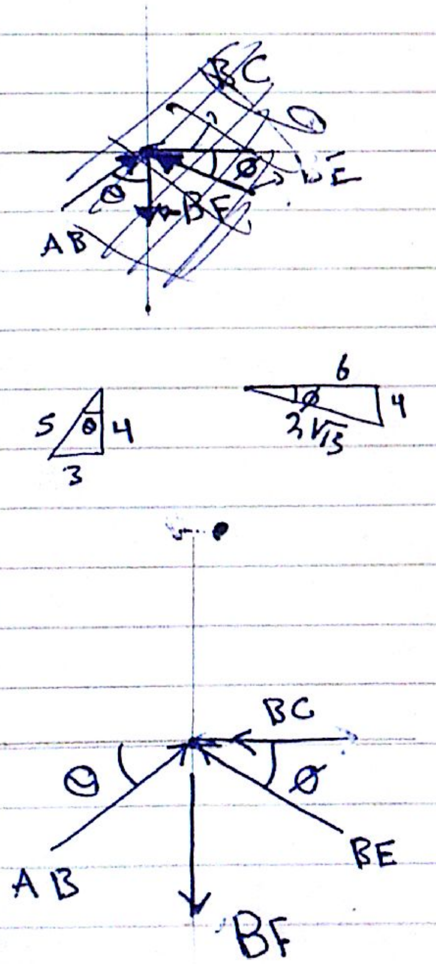
$FE = -3.375 = 3.375 \rightarrow = 3.375$ T

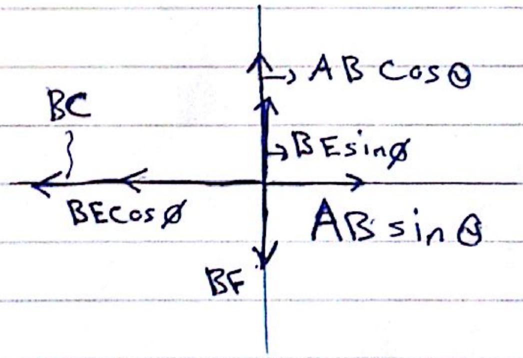
$\Sigma F_y = 0$

$BF - 4 = 0$

$BF = 4\,kN$ T

take Joint B





BF = 4 kN T

AB = 5.6 kN T

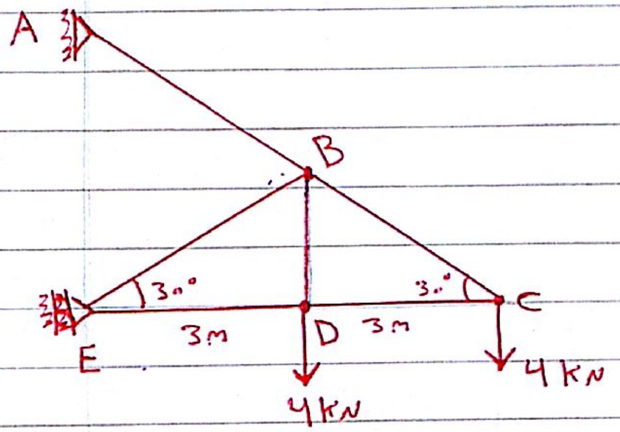
$\Sigma F_y = 0$

$-BF + BE \cdot \frac{4}{2\sqrt{13}} + AB \times \frac{4}{5} = 0$

$BE = -0.901 = 0.901 \searrow = 0.901$ T

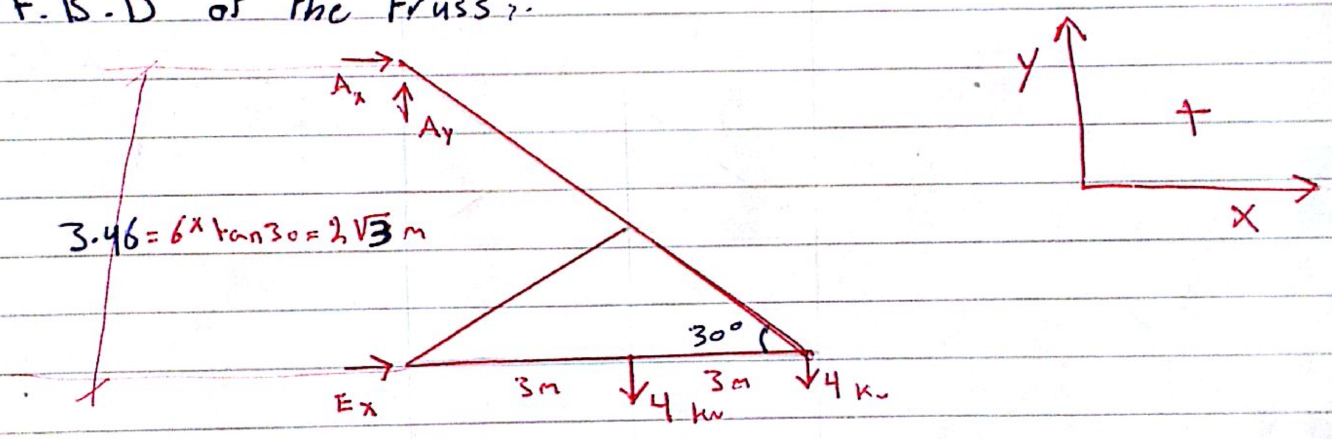
Ex 3 (P 6-7)

Find the Force in the member ED



Solu :-

F.B.D of the truss :-



$\curvearrowright + \Sigma M_E = 0$

$-A_x \times 2\sqrt{3} - 4 \times 3 - 4 \times 6 = 0$

$A_x = -10.39 = 10.39 \leftarrow$

$\Sigma F_x = 0$

$A_x + E_x = 0$

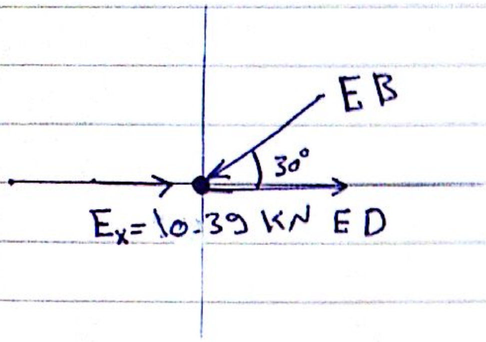
$-10.39 + E_x = 0$ $\quad E_x = 10.39$ kN

$\Sigma F_y = 0$

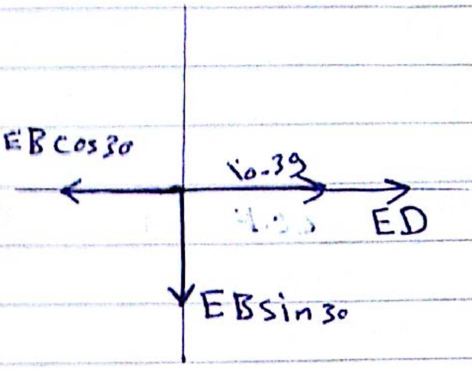
$A_y - 4 - 4 = 0$

$A_y = 8$ kN.

take Joint E



$\Longrightarrow$

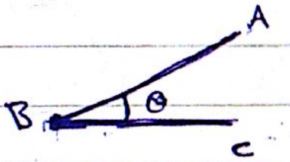


$\sum F_y = 0$

$EB \sin 30 = 0$ $\quad \boxed{EB = 0}$ zero force member

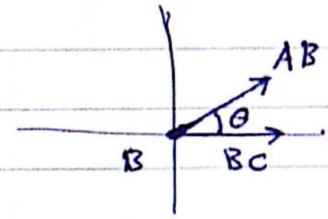
Zero force member

1-

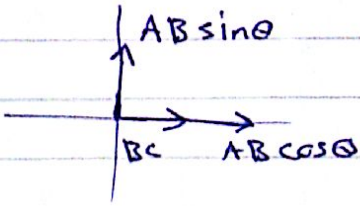


if there is no load on Joint B

Joint B



$\Rightarrow$



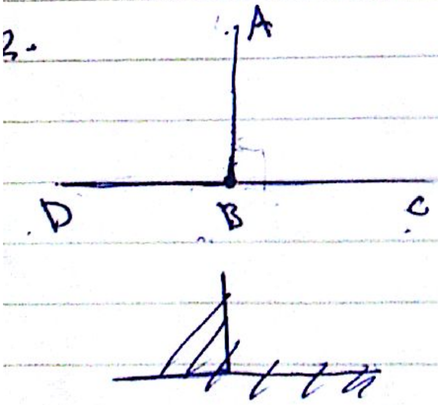
$\sum F_y = 0$

$\boxed{AB = 0}$

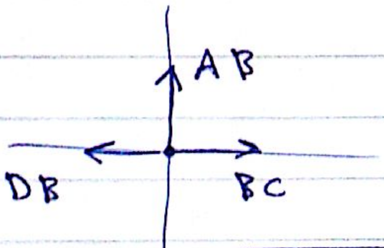
$\sum F_x = 0$

$BC + AB\cos\theta = 0 \Rightarrow \boxed{BC = 0}$

2-



$\Rightarrow$



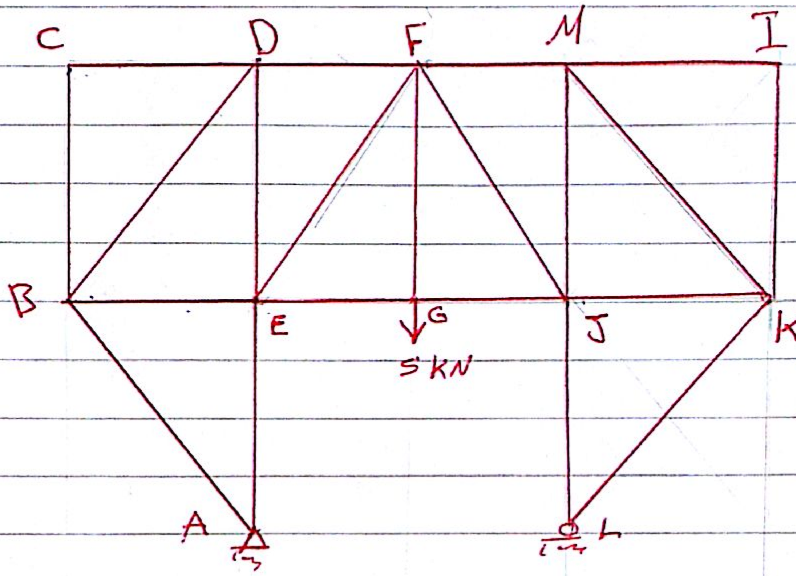
$\sum F_x = 0$ $\quad \boxed{BC = BD}$

$\sum F_y = 0$

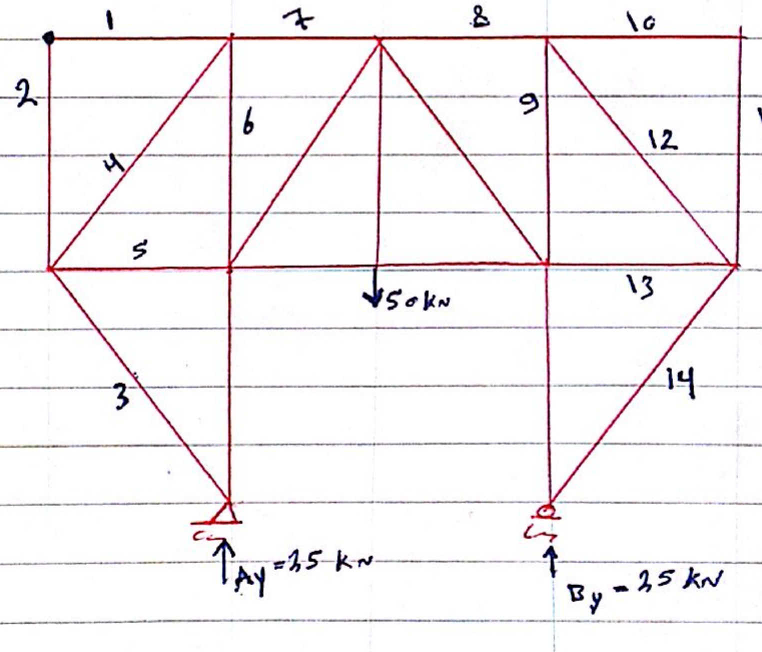
$\boxed{AB = 0}$

Example

Find the Zero force members



Solu



Solve

Ex6.4 , P (6-1 , 6-9 , 6-15 , 6-19) and all Fundamental Problems.

2- Method of section:-

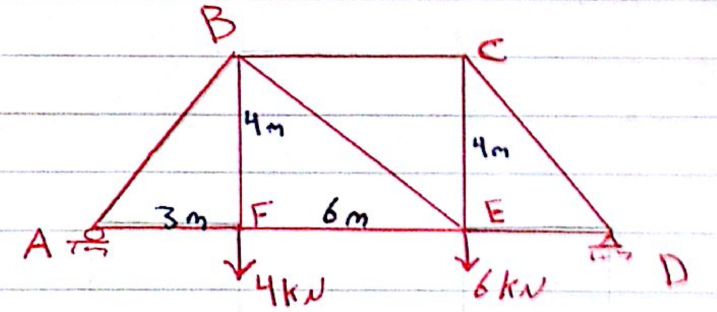
Procedure for analysis:-

1- Draw the free body diagram of the truss and then find the reactions. (some times we Dont have to find the Reactions)
2- Choose the section which will give you the required force (required member) the section must have at least one Unknown and maximum 3 knowns (because we have only 3-equations of equilibruim).
3- Isolating the section then draw the F.B.D of the section.
4- Use the equations of equilibrium to find the UKnowns. $\Sigma F_x = 0$ $\Sigma F_y = 0$ $\Sigma M = 0$

Examples:-

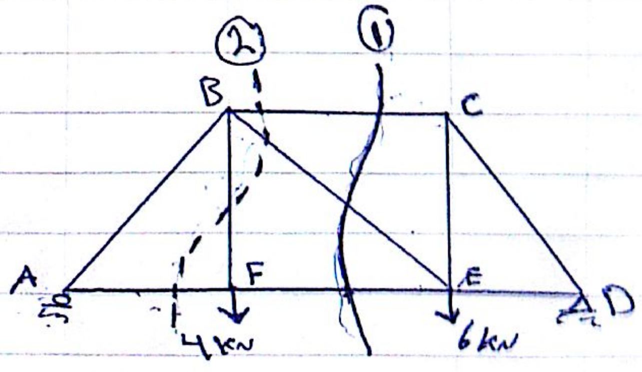
Ex 1:-

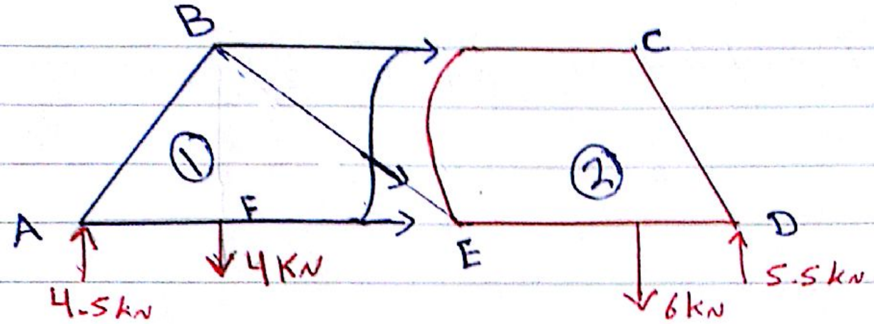
Find the forces in BC, BE, FE



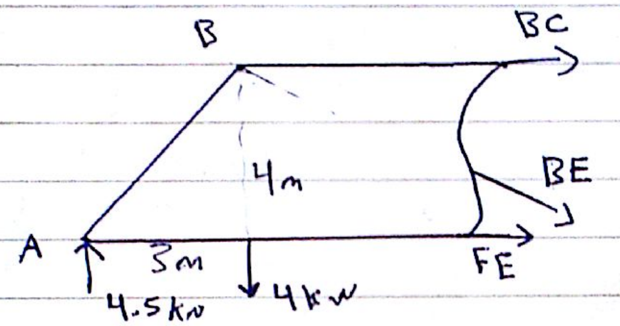
Solu:-

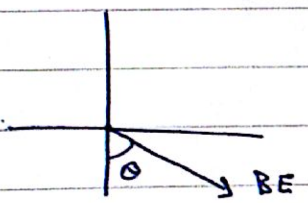
Draw the F.B.D then Find the Reaction ⇒ $A_y = 4.5\ kN$ $D_y = 5.5\ kN$

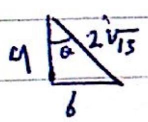




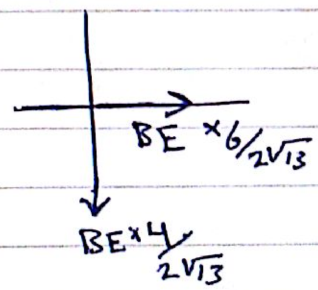
if we take section ①



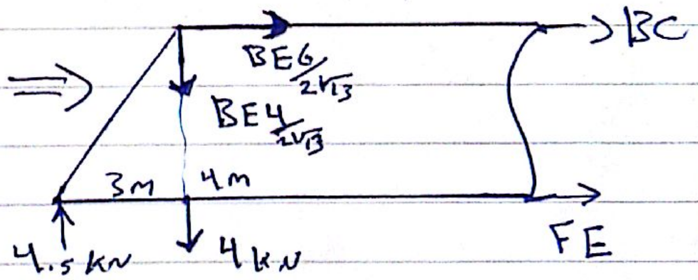




$\Rightarrow$



$\Rightarrow$



if we take $\sum F_y = 0$

$4.5 - 4 - BE \times \frac{4}{2\sqrt{13}} = 0 \qquad BE = 0.901\,kN$ (T)

$\sum F_x = 0$

$BE\frac{6}{2\sqrt{13}} + FE + BC = 0$

$\circlearrowright + \sum M_A = 0$

$-4 \times 3 - BE \times \frac{6}{2\sqrt{13}} \times 4 - BE \times 3 \times \frac{4}{2\sqrt{13}} - BC \times 4 = 0$

$BC = -4.125\,kN = 4.125\,kN$ (C)

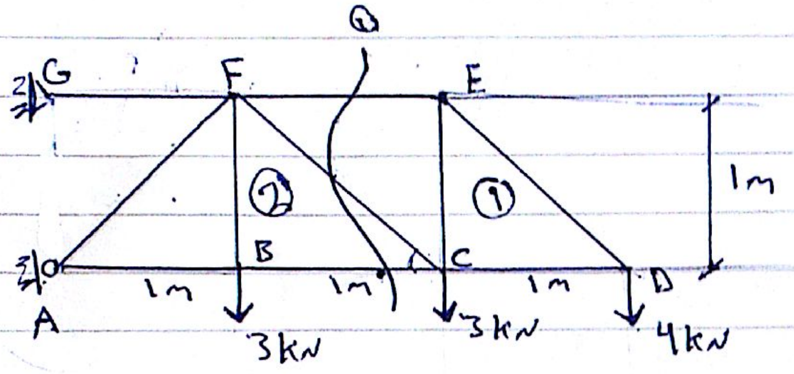
$\sum F_x = 0$

$0.901 \times \frac{6}{2\sqrt{13}} + FE - 4.125 = 0 \qquad FE = -3.375 = 3.375\,kN$ (C)

(10)

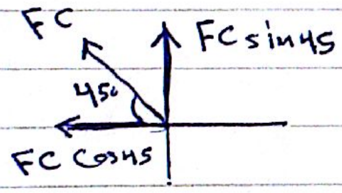
Ex(3) :-

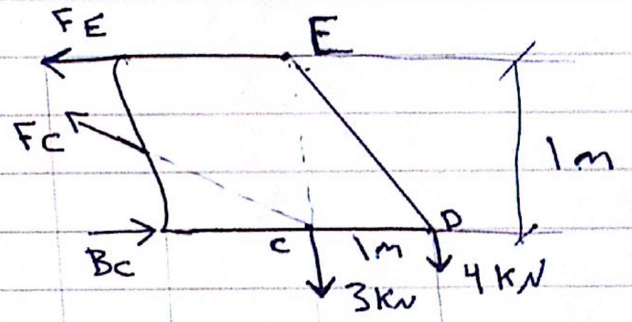
Find the Forces in member FC, FE

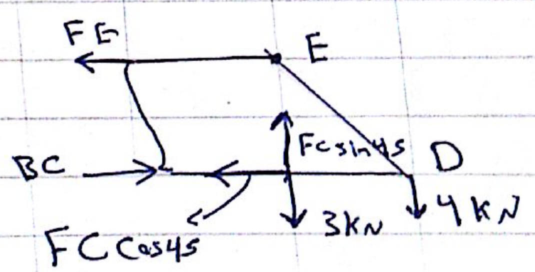


Soln :-

We can take section ① so that we Dont have to Find the reactions







$\Sigma F_y = 0$

$Fc \sin 45 - 3 - 4 = 0 \Rightarrow Fc = 9.9\ kN$ (T)

$+\Sigma M_D = 0$

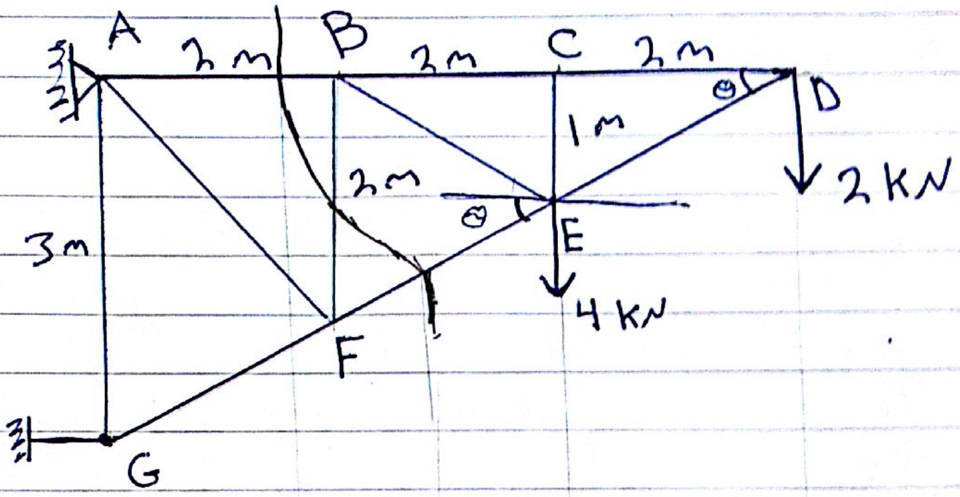
$-Fc \sin 45 \times 1 + 3 \times 1 + FE \times 1 = 0$ $\quad FE = 4\ kN$ (T)

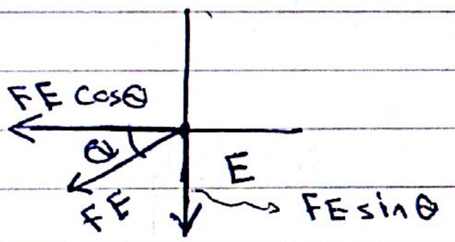
$\Sigma F_x = 0$

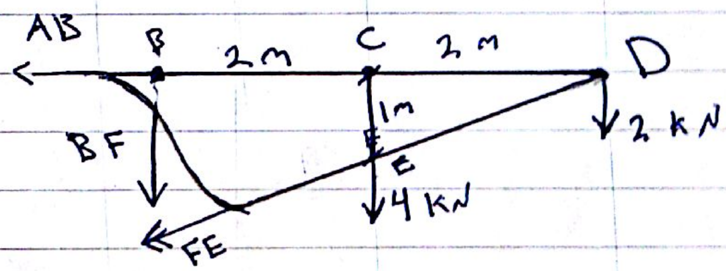
$-FE + BC - FC \cos 45 = 0$ $\quad BC = 11\ kN$ (T)

EX 3 :-

Determine the forces in members AB, BF and FE :







$\theta = \tan^{-1}(1/2) = 26.6°$

$\circlearrowright + \Sigma M_D = 0$ (note Keep FE ↙)

$4 \times 2 + BF \times 4 = 0 \Rightarrow BF = -2\,kN = 2\,kN$ (C)

$\Sigma F_y = 0$

$-BF - 4 - 2 - FE \sin\theta = 0$

$-(-2) - 4 - 2 = FE \sin 26.6$

$FE = -8.9\,kN = 8.9\,kN$ (C)

$\Sigma F_x = 0$

$-AB - FE \times \cos\theta = 0$

$-AB = FE \cos\theta$

$\cancel{-}AB = \cancel{-}8.9 \times \cos 26.6$ $AB = 8\,kN$ (T)

Subject Statics mid term. Date No.

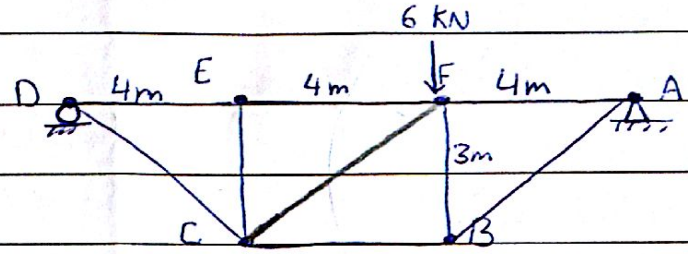
1. Calculate the force in member CF of the truss

a. 5 KN (T)

→ b. 3.33 KN (C).

c. 6 KN (T)

d. 7 KN (C)



For the truss shown below

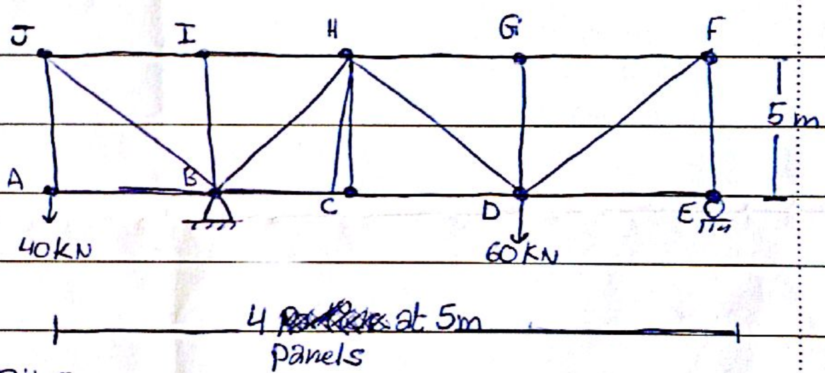
2. The force in member JB is

→ a. 56.6 KN (C)

b. 60 KN (C).

c. 70 KN (T)

d. 80 KN (T)



3. The Force in member BH is

→ a. 47.1 KN. (C).

b. 50 KN. (C)

c. 60 KN (T)

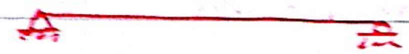
d) 55 KN (T)

محمد مازن

Ch:7 Beams

Beams:- it's a perimistic member that subjucted to M and V. (P=0)

types of beam:-

 → simply supported beam

 → over hanging beam

 → fixed beam (cantilever)

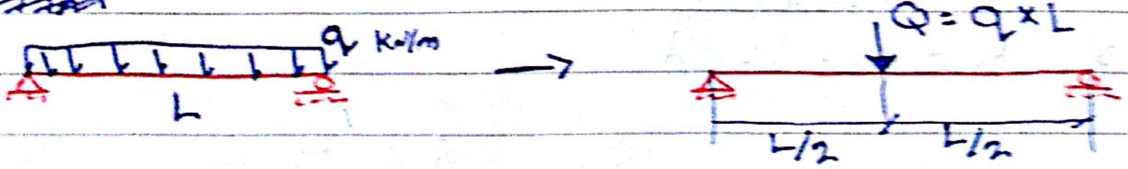
* Objectives:-

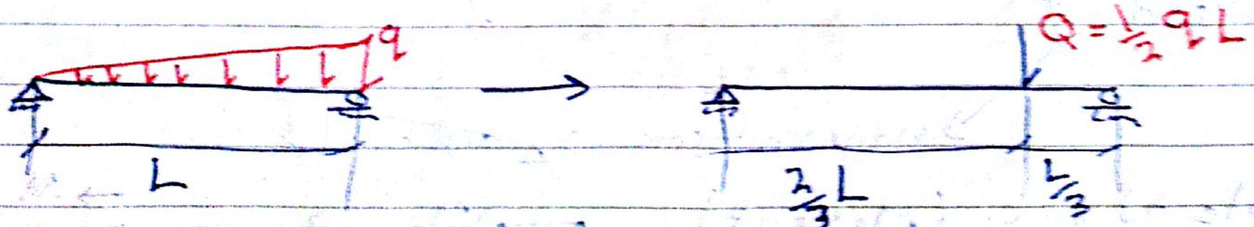
1- Find the shearforce and moment at any point on the beam.

2- Draw the moment Diagram and shear Diagram.

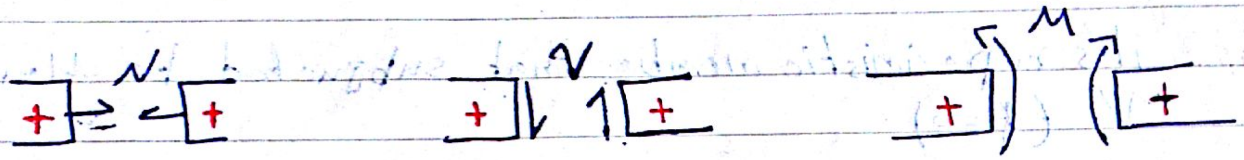
* Convert from uniform load to Concentrated load:-

~~F Find~~

1- 

2- 

* sign Convention:-



1- Find V and M at any point on the beam

1) Find the reactions:-
   - → Draw F.B.D for the beam
   - → Convert the distributed load to a Concentrated load
   - → $\Sigma F_x = 0$

     $\Sigma F_y = 0$ → almost we use this two equ.

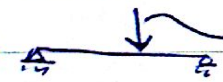
     $\Sigma M = 0$
   - → Calculate the reactions.

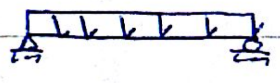
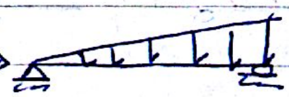
2) take a cut (section) at the point that we want to find V and M on it

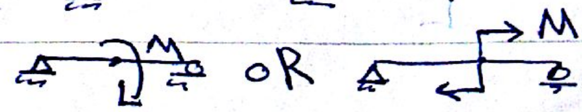
3) Put V, M, and P (P = 0 always on beams)

4) $\Sigma F_x = 0$ $\Sigma F_y = 0$ $\Sigma M = 0$ then find M and V

Loads

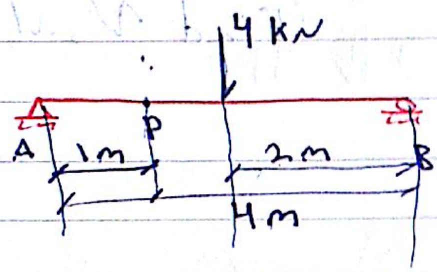
1)  → Concentrated

2) Distributed
   - → Uniform → 
   - → non-uniform → 

3) couple of force (Moment) 

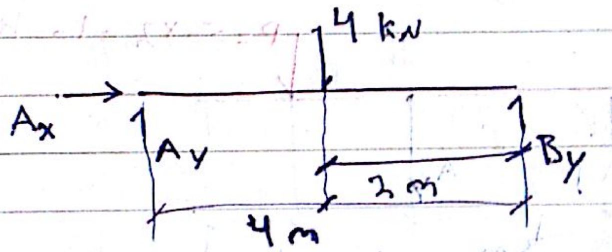
Ex (1)

Find V and M at Point P



Solution:-

1- Find the reaction:-



F.B.D

$\sum F_x = 0$

① $\boxed{Ax = 0}$

$\sum F_y = 0$

$By + Ay = 4$

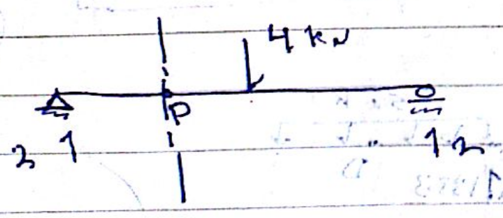
$\circlearrowleft + \sum M_B = 0$

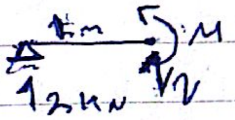
$-Ay * 4 + 4 * 2 = 0$

② $\boxed{Ay = 2\ kN \uparrow}$

$By + 2 = 4$

③ $\boxed{By = 2\ kN \uparrow}$





$\Rightarrow$ $\sum F_y = 0$

$-V + 2 = 0$

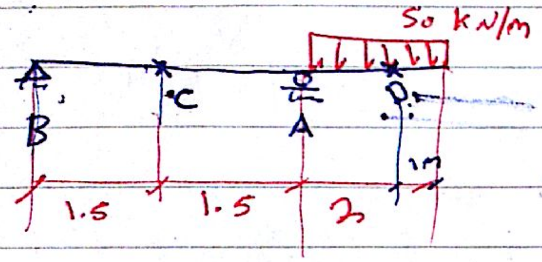
$V = +2\ kN = 2\ kN \downarrow$

$\circlearrowleft + \sum M_P = 0$

$M - 2 \times 1 = 0$

$M = 2\ kN \cdot m \circlearrowright$

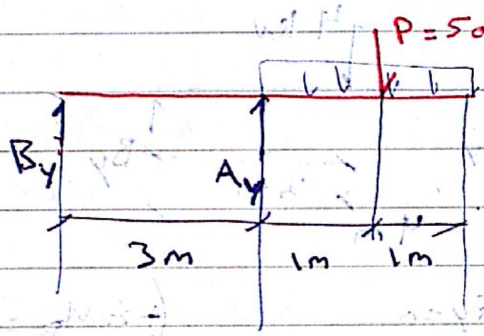
Ex (2) (F 7-3)



Find V, M at C and D

Solu:-

1- Find the Reactions:-



$\Sigma F_y = 0$ ↑

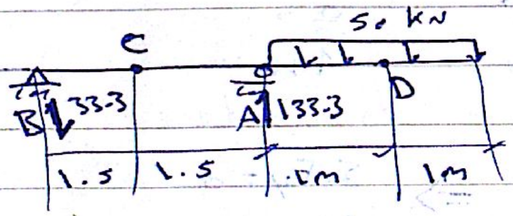
$B_y + A_y - 100 = 0 \Rightarrow B_y + A_y = 100\ kN$

$\circlearrowright + \Sigma M_B = 0$

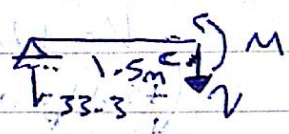
$A_y \times 3 - 100 \times (3+1) = 0$

$\boxed{A_y = 133.3\ kN \uparrow}$ $\quad B_y = 100 - 133.3$

$\boxed{B_y = 33.3\ kN \downarrow}$



take section BC



$-V - 33.3 = 0$

$\Sigma F_y = 0$ ↑ $+ \quad V = -33.3\ kN$

$V = 33.3\ kN \uparrow$

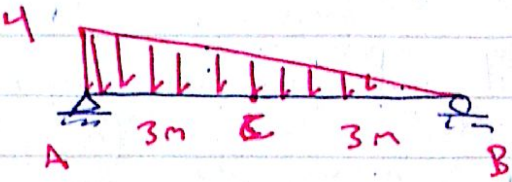
$\circlearrowright + \Sigma M_C = 0$

$33.3 \times 1.5 + M = 0$

$M = -50\ kN.m = 50\ kN.m \circlearrowright$

4

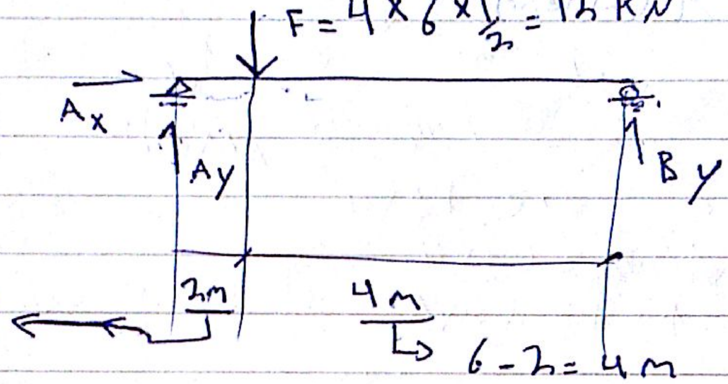
Ex(3) P (7-8)



Determine the internal Forces at C

Solu:-

1) F.B.D



$6 \times \frac{1}{3} = 2m$

$6 - 2 = 4m$

$\sum F_x = 0$ | $\sum F_y = 0$ | $\circlearrowleft + \sum M_A = 0$

$A_x = 0$ | $A_y + B_y - 12 = 0$ | $-12 \times 2 + B_y \times 6 = 0$

$B_y = 4 kN \uparrow$

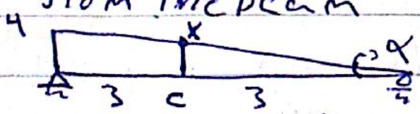
$A_y + 4 - 12 = 0$

$A_y = 8 kN \uparrow$

2) Determine the forces:-

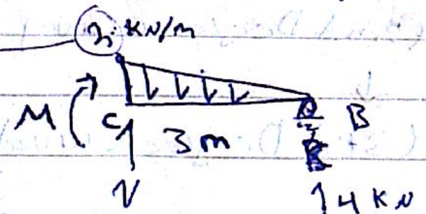
we take section BC for simplisity:-

from the Beam

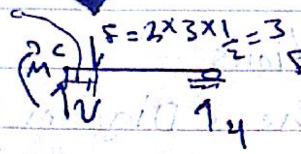


من تشابه المثلثات

$\frac{x}{3} = \frac{4}{6}$ $x = 2 kN/m$



1m



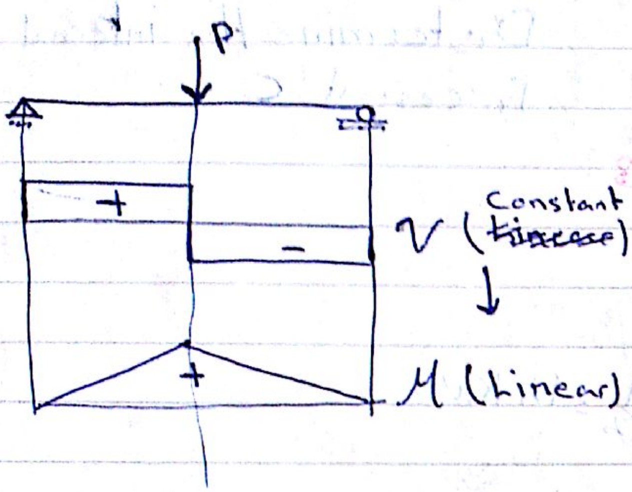
$+\uparrow \sum F_y = 0$ $V + 4 - 3 = 0$

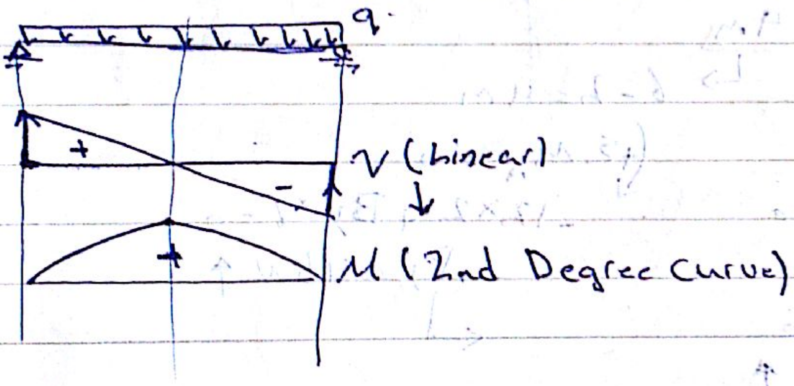
$V = -1 kN = 1 kN \downarrow$

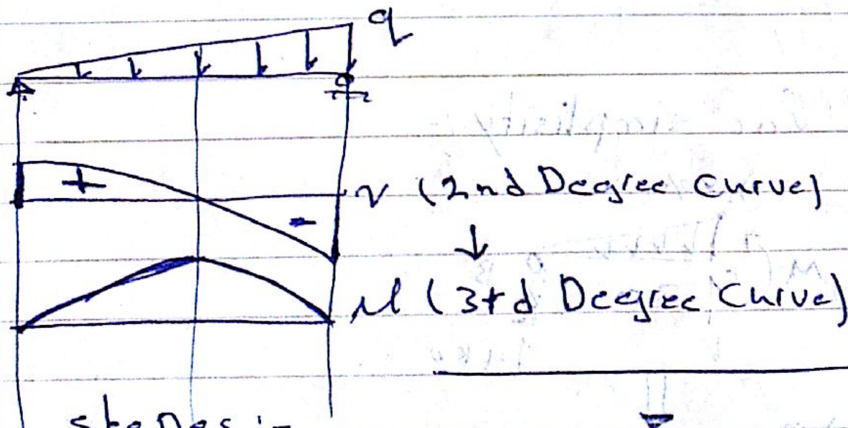
$\circlearrowleft + \sum M_c = 0$

$-M - 3 \times 1 + 4 \times 3 = 0$ $M = 9 kN.m$

Moment and shear Diagram:-







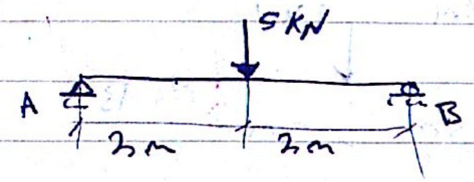
stepes:-

1) Find the reactions

2) Draw the shear Digram then moment Diagram.

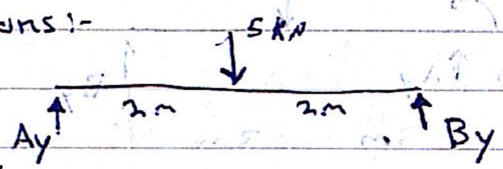
Ex(4):-

Draw the V and M diagrams:-



Solu:-

1- Find the reactions:-



$\uparrow + \Sigma F_y = 0$

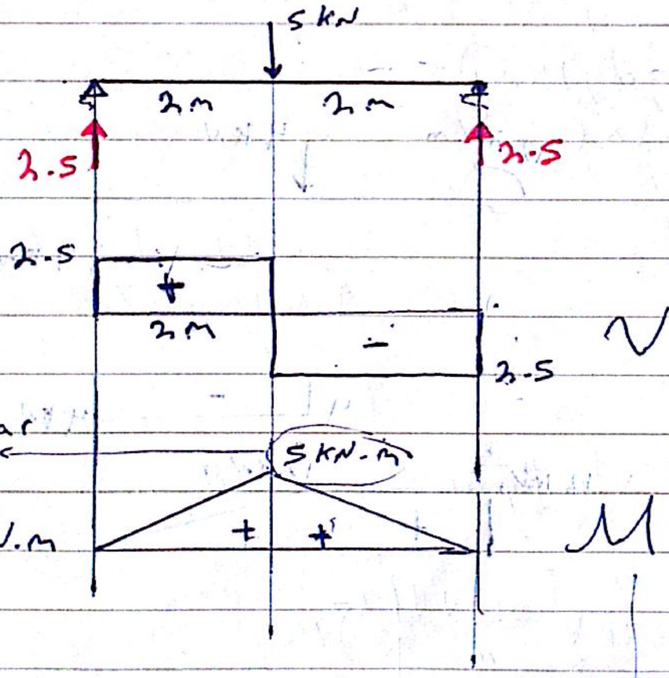
$A_y + B_y - 5 = 0$

$\circlearrowleft + \Sigma M_A = 0$

$B_y \times 4 - 5 \times 2 = 0$

$B_y = 2.5\ kN \uparrow$

$A_y = 2.5\ kN \uparrow$

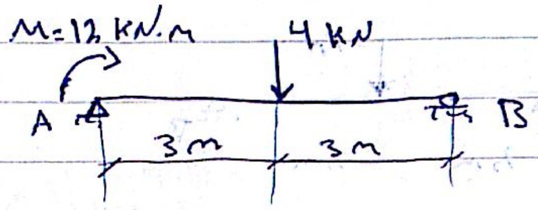


Area of the shear Diagram

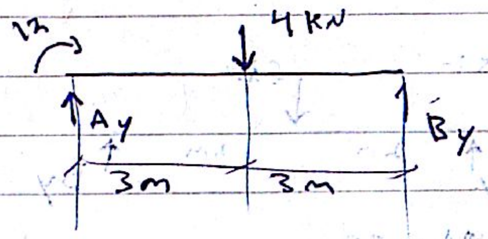
$A = 2.5 \times 2 = 5\ kN.m$

Ex (5) :-

Draw the V and M



Solu:-



$\circlearrowleft + \sum M_A = 0$

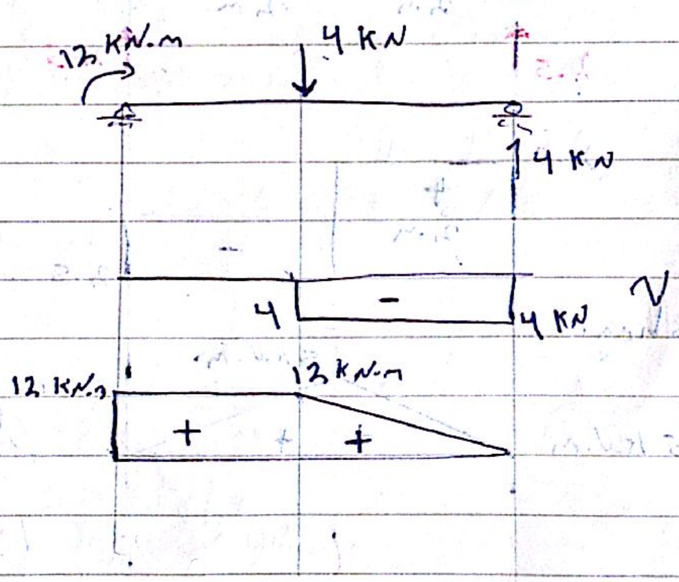
$-12 - 4 \times 3 + B_y \times 6 = 0$

$B_y = 4\ kN \uparrow$

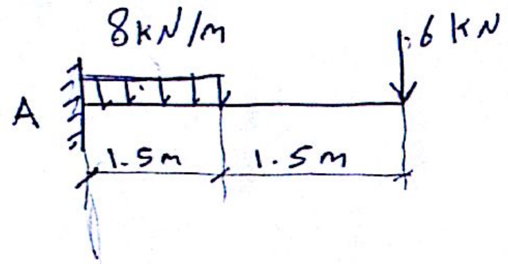
$+\uparrow \sum F_y = 0$

$A_y + B_y - 4 = 0$

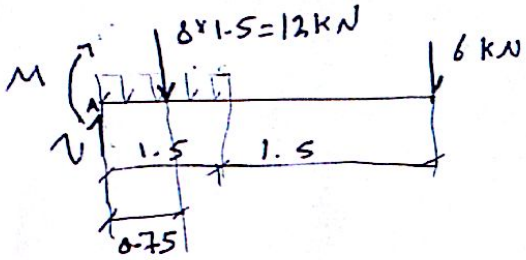
$A_y = 0$



Ex6 Draw the V and M :-



Solu:-



$\curvearrowleft + \sum M_A = 0$

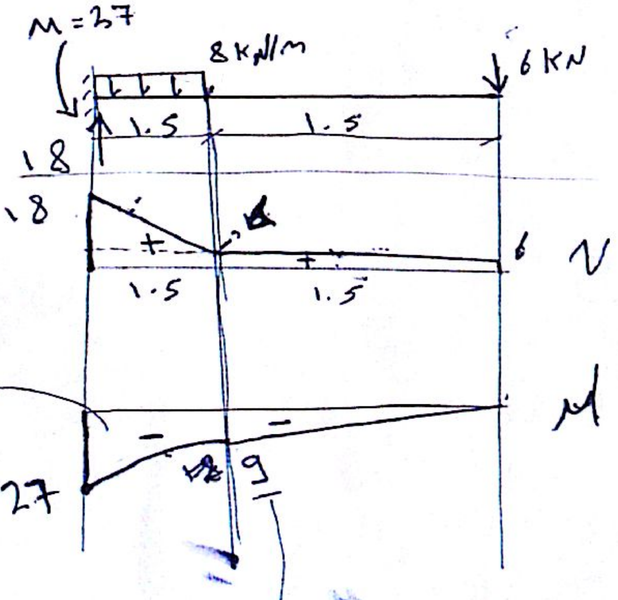
$-12 \times 0.75 - M - 6 \times 3 = 0$

$M = -27 \text{ kN/m} = 27 \text{ kN.m}$

$+\uparrow \sum F_y = 0$

$V - 12 - 6 = 0$

$V = 18 \text{ kN}$

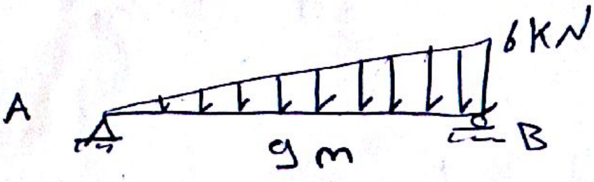


$A = (18-6) \times 1.5 \times 0.5$
$+$
$1.5 \times 6 = 18$

$27 - 18$

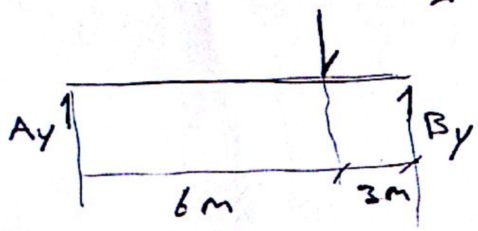
Ex 4

Draw the V and M



Solu:-

$6 \times 9 \times \frac{1}{2} = 27\ KN$



$\circlearrowright + \sum M_A = 0$

$B_y \times 9 - 27 \times 6 = 0$

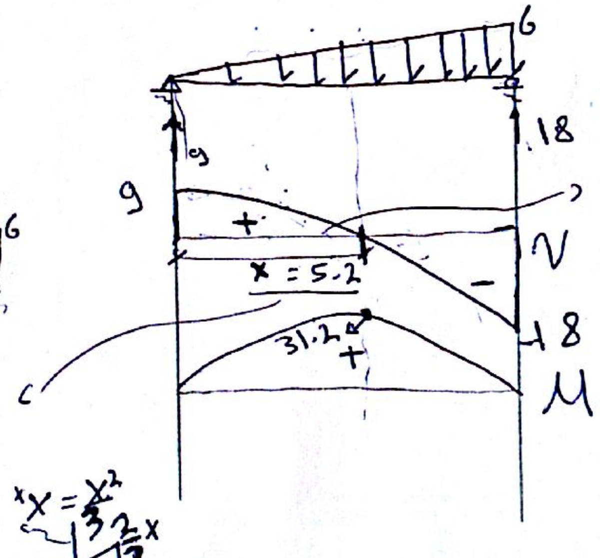
$B_y = 18\ KN \uparrow$

$\sum F_y = 0$

$A_y + B_y - 27 = 0$

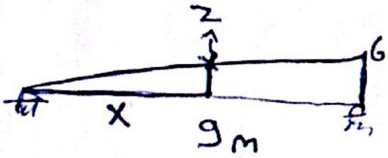
$A_y = 9\ KN \uparrow$

maximum moment at $V = 0$



$A = \frac{2}{3} \times 5.2 \times 9$

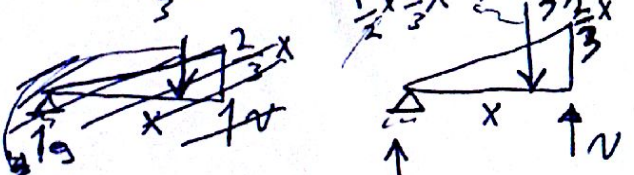
$= 31.2$



$\frac{X}{9} = \frac{Z}{6}$

$Z = \frac{2}{3} X$

$\frac{1}{2} \times \frac{2}{3} X \times X = \frac{X^2}{3}$



$V + 9 - \frac{X^2}{3} = 0$

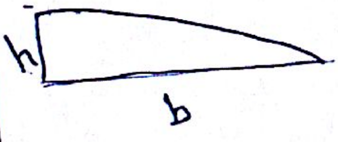
$V = \frac{X^2}{3} - 9$

when $V = 0$

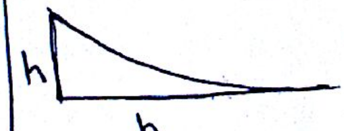
$0 = \frac{X^2}{3} - 9$

$X = 5.2$

note:



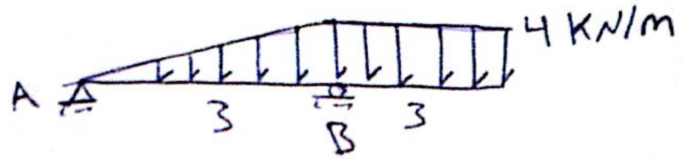
$A = \frac{2}{3} \times b \times h$



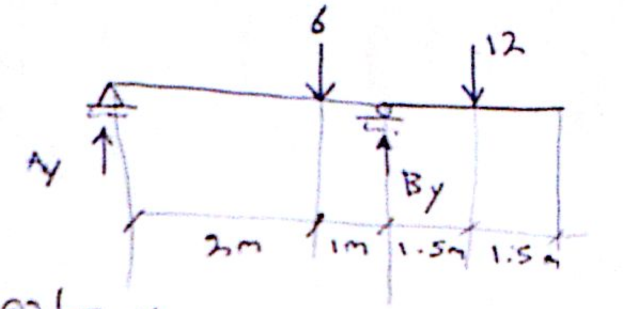
$A = \frac{1}{3} \times b \times h$

Ex 8

Draw V and M :-



Solu :-



① $\Sigma F_y = 0$

$A_y + B_y - 6 - 12 = 0$

$A_y + B_y = 18$

② $+\Sigma M_A = 0$

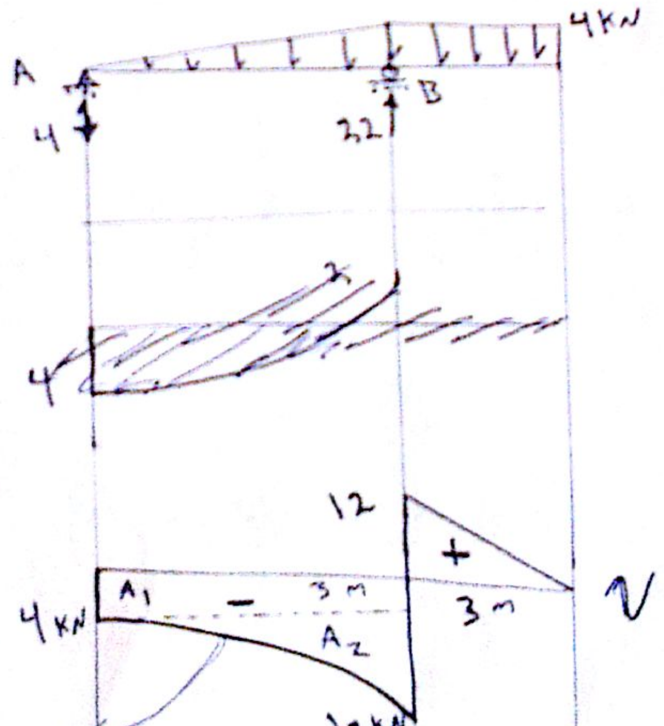
$-6 \times 2 + B_y \times 3 - 12 \times 4.5 = 0$

$B_y = 22 \text{ KN} \uparrow$

③

$A_y + 22 = 18$

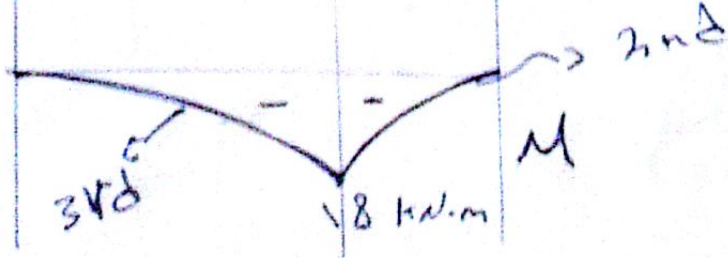
$A_y = -4 \text{ KN} = 4 \text{ KN} \downarrow$

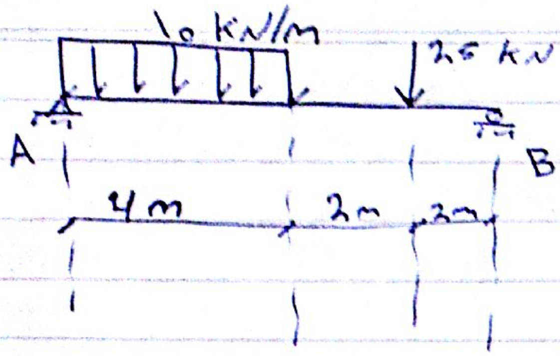


$A = A_1 + A_2$

$A = 4 \times 3 + \frac{1}{2} \times (10 - 4) \times 3$

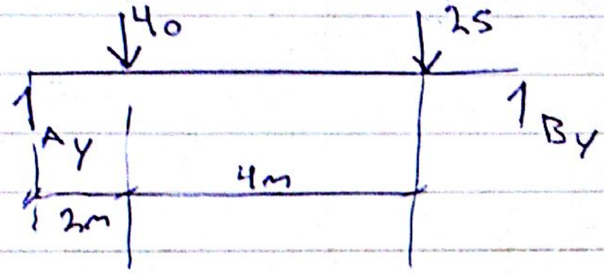
$= 18$





Draw V and M

1- Find the reactions:-



$(+\Sigma M_A = 0$

$-40 \times 2 - 25 \times 6 + B_y \times 8 = 0$

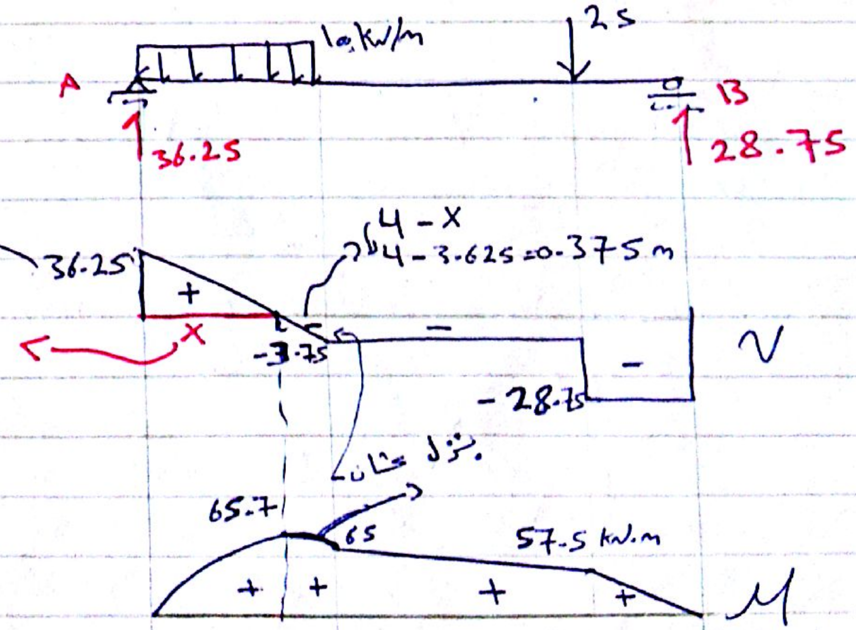
$B_y = 28.75 \text{ kN}$

$\Sigma F_y = 0$

$A_y + B_y - 40 - 25 = 0 \Rightarrow A_y = 36.25$

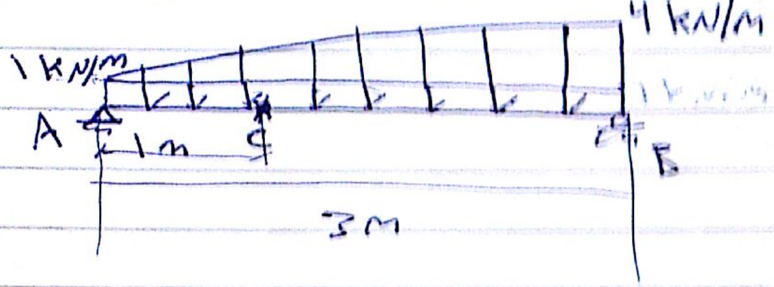
$x = \frac{36.25}{10} = 3.625$

intensity of the load



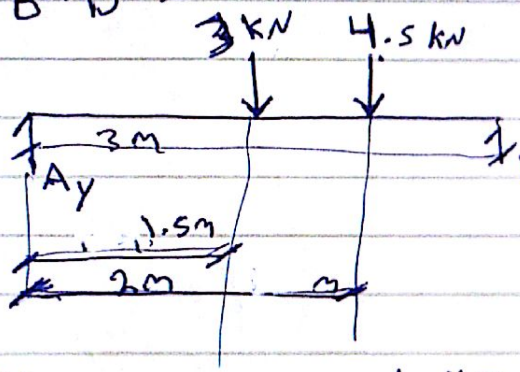
Ex (4):-

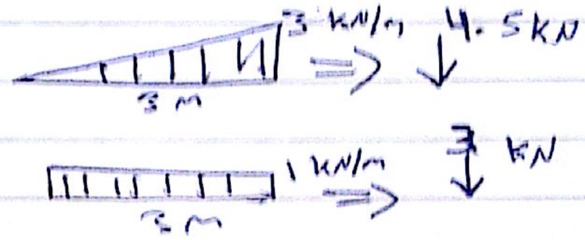
Determin the internal forces at C



Solu:-

1) F.B.D:





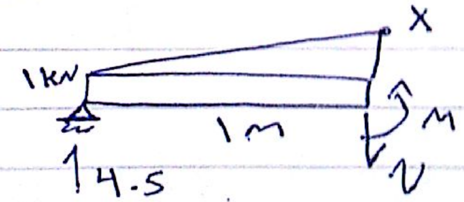
$\circlearrowleft + \Sigma M_B = 0 \qquad -A_y \times 3 + 4.5 \times 1 + 3 \times 1.5 = 0$

$\boxed{A_y = 3\ kN}$

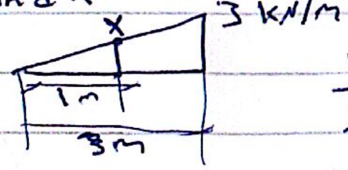
$\Sigma F_y = 0$

$B_y + A_y - 7.5 = 0 \qquad \boxed{B_y = 4.5\ kN}$

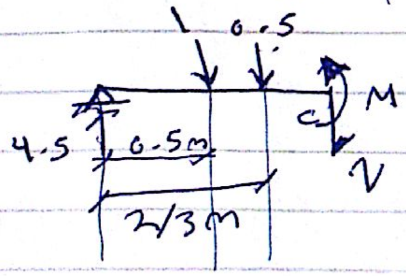
2) Determine the internal forces

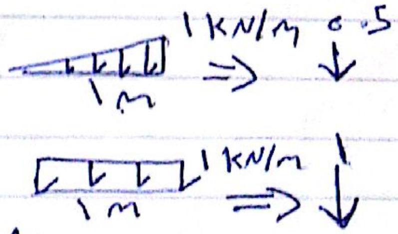


to find x



$\frac{x}{3} = \frac{1}{3}$ $\qquad x = 1\ kN/m$





$\Sigma F_y = 0$

$-V - 1.5 = 0$

$V = -1.5\ kN = 1.5\ kN \uparrow$

$\circlearrowleft + \Sigma M_c = 0$

$M + 0.5 \times \frac{1}{3} + 1 \times 0.5 - 4.5 \times 1 = 0$

$M = 2.33\ kN.m \circlearrowright$

Ch 9 + 10

Definitions:-

* **Centroid**:- it's the center of mass of ~~two~~-Dimentional figure or three-Dimentional solid, and it's represents the point at which it could be balanced if it were ~~out~~ cut out of.

* **Moment of inertia**:- it's the mass property of a rigd body that defines the torque needed for a desired change in the angular velocity about an axis of rotation.

Steps to find the Centroid and moment of inertia:-

1- divide the general area into subareas so that you can calculate the Centroid, I of each one of them.

2- Select a reference point.

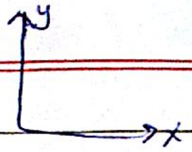
3- Construct the table shown and fill it.

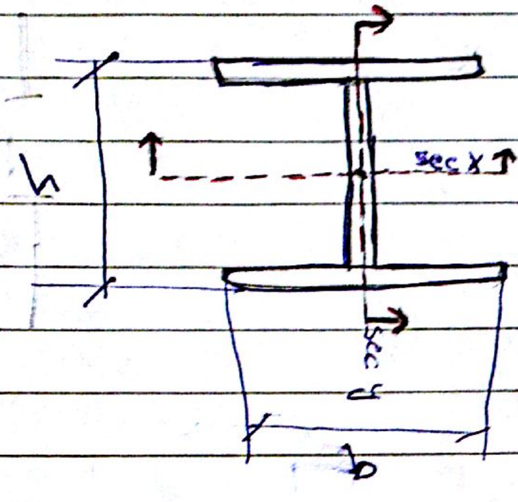
Sub. area	Area	$\bar{x}$	$\bar{y}$	$A.\bar{x}$	$A\bar{y}$
	ΣA			$\Sigma A\bar{x}$	$\Sigma A\bar{y}$

4- $\bar{x} = \frac{\Sigma A\bar{x}}{\Sigma A}$, $\bar{y} = \frac{\Sigma A\bar{y}}{\Sigma A}$

5- $I_x = \bar{I}_x + Ay^2$

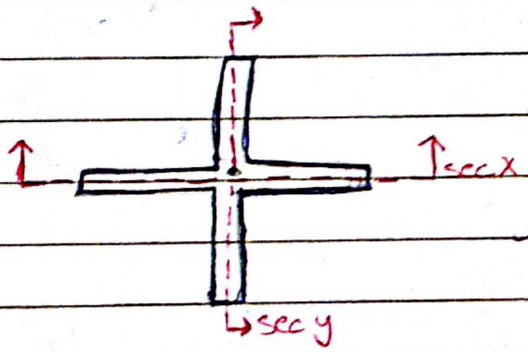
$I_y = \bar{I}_y + Ax^2$





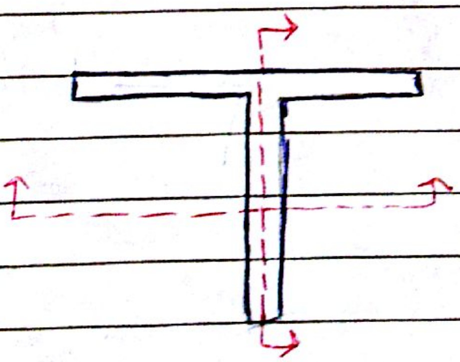
→ symmetric on x and y

→ centroid $(\frac{b}{2}, \frac{h}{2})$



→ symmetric on x, y

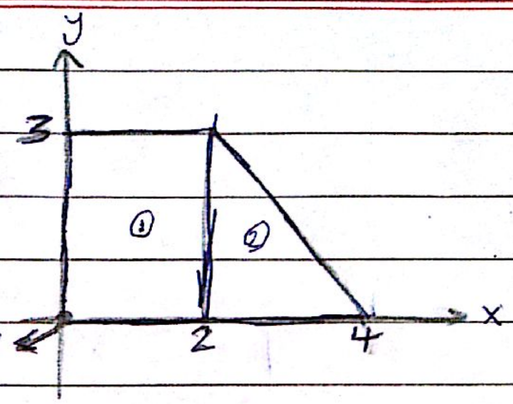
→ centroid $(\frac{b}{2}, \frac{h}{2})$



→ symmetric about x, but not symmetric about y.

→ centroid $(\frac{b}{2}, y)$ $\quad y \neq h/2$

ex1 Calculate the Centroid, I



Solu:

①

	S.A	A	$\bar{x}$	$\bar{y}$	$\bar{x}A$	$\bar{y}A$
□	1	$2*3=6$	1	1.5	6	9
◺	2	$\frac{1}{2}*2*3=3$	$2+\frac{2}{3}=2.\bar{6}$	1	8	3
$\Sigma(\ )$		9			14	12

② $\bar{X} = \frac{\Sigma \bar{x}A}{\Sigma A} = \frac{14}{9} = 1.56$ / $\bar{y} = \frac{\Sigma \bar{y}A}{\Sigma A} = \frac{12}{9} = 1.33$

③ $I_{x,y} = I_{(\text{for rectangular})} + I_{(\text{triangular})}$

$\rightarrow I_x = \bar{I} + Ay^2$

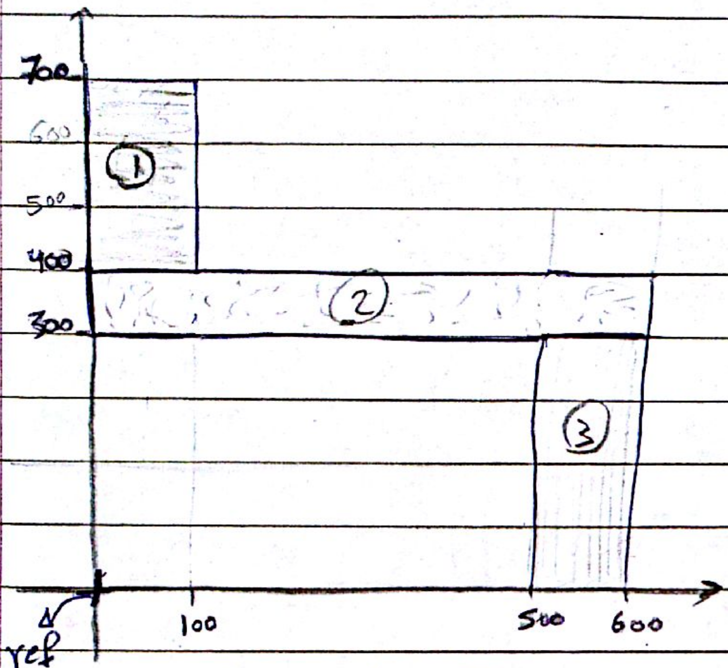
$$= \underbrace{\frac{bh^3}{12} + b*h*y^2}_{Rec} + \underbrace{\frac{bh^3}{36} + \frac{1}{2}*b*h*y^2}_{Tri}$$

$$= \frac{2*3^3}{12} + 6*(1.5-1.33)^2 + \frac{2*3^3}{36} + \frac{1}{2}*2*3\ (1-1.33)^2 = 6.44$$

$\rightarrow I_y = \bar{I} + Ax^2$

$$\frac{b^3h}{12} + 6*(1-1.56)^2 + \frac{3*2^3}{36} + 3*(2.\bar{6}-1.56) = 8.22$$

ex 2



S.A	$\bar{x}$	$\bar{y}$
□1	50	550
▭2	300	350
□3	550	150

Centroid $= \bar{x} = \frac{600}{2} = 300$ / $y = \frac{700}{2} = 350$
because the shape is symmetric on x, y

$$I_x = \bar{I} + Ay^2 = \frac{100(300)^3}{12} + 100(300)*(550-350)^2 +$$

$$\frac{600(100)^3}{12} + 100(600)*(350-350)^2 +$$

$$\frac{100(300)^2}{12} + 100(300)*(150-350)^2 = 2.9*10^9$$

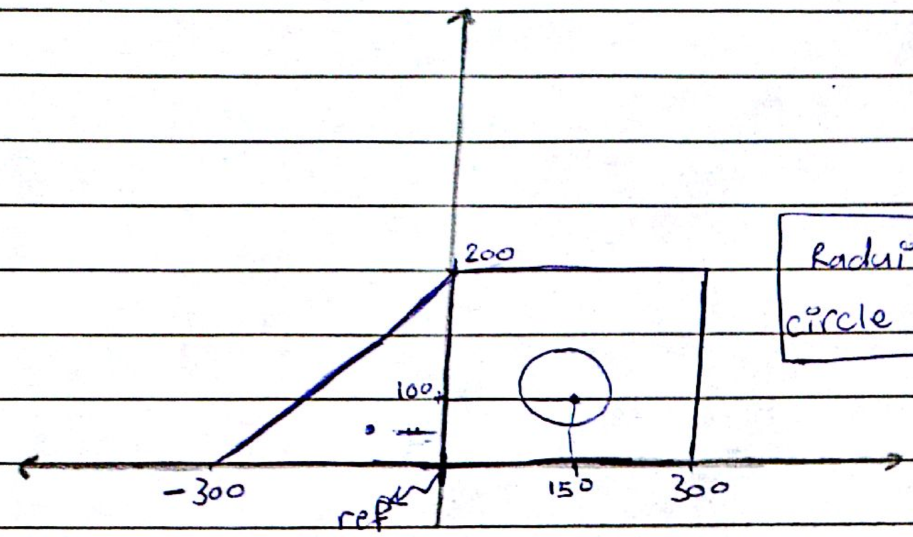
$$I_y = \bar{I} + A\cancel{y}x^2 = \frac{300(100)^3}{12} + 100(300)*(50-300)^2 +$$

$$\frac{100(600)^3}{12} + 100(600)*(300-300)^2 +$$

$$\frac{300(100)^3}{12} + 100(300)*(550-300)^2 = 5.6*10^9$$

(4)

ex 3



Radius of the circle = 75 mm

S.A	A	$\bar{x}$	$\bar{y}$	$\bar{x}A$	$\bar{y}A$
1 triangle	3×10^4	$\frac{-300}{3} = -100$	200/3	-3×10^6	2×10^6
2 rectangle	6×10^4	150	100	9×10^6	6×10^6
3 circle	-1.76625×10^4	150	100	-2.6494×10^6	-1.76625×10^6
$\Sigma()$	72337.5			3350625	6233750

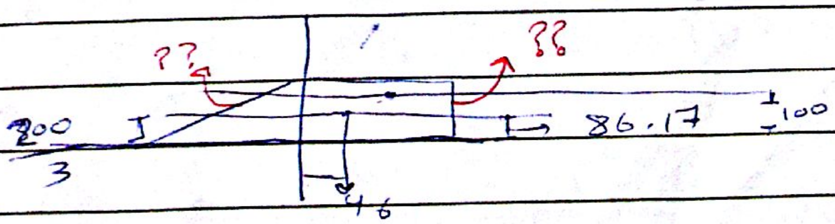
$$\bar{X} = \frac{\Sigma \bar{x}A}{\Sigma A} = 46.32\text{mm} \quad / \quad \bar{y} = \frac{\Sigma \bar{y}A}{\Sigma A} = 86.17\text{mm}$$

$$I_x = \frac{300(200)^3}{36} + 3\times10^4 \times \left(86.17 - \frac{200}{3}\right)^2 + \frac{300(200)^3}{12}$$

$$+ 6\times10^4(100 - 86.17)^2 - \left[\frac{\pi \times (75)^4}{4} + \pi(75)^2(100 - 86.17)^2\right]$$

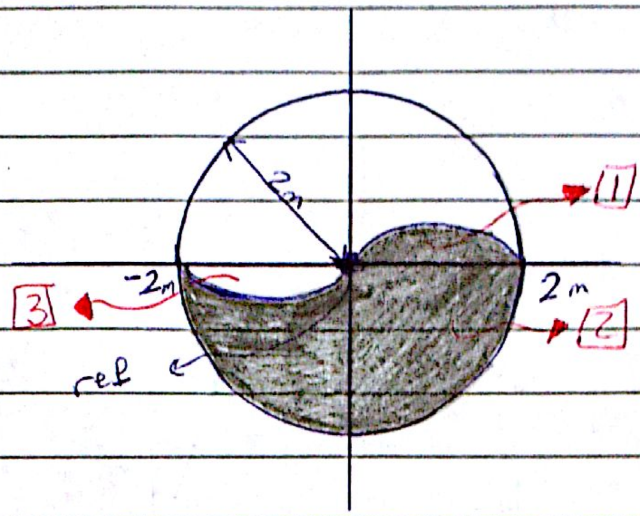
$$= 261.34\times10^6$$

$I_y = ??!$



(5)

ex 4



S.A	A	$\bar{X}$	$\bar{y}$	$\bar{X}A$	$\bar{y}A$
1	$\frac{1}{2}\pi\times1^2=1.57$	1	$\frac{4R}{3\pi}=0.42$	1.57	0.66
2	$\frac{1}{2}\pi 2^2=6.28$	0	-0.85	0	-5.34
3	-1.57	1	-0.42	1.57	0.66
$\Sigma()$	6.28			3.14	-4

$$\bar{X}=\frac{\Sigma\bar{X}A}{\Sigma A}=\frac{3.14}{6.28}=0.5 \quad / \quad \bar{y}=\frac{\Sigma\bar{y}A}{\Sigma A}=\frac{-4}{6.28}=-0.64.$$

$$I_x=\left(\frac{\pi}{8}*1^4-\frac{8*1^4}{9\pi}\right)+\frac{1}{2}*\pi*1^2*(-0.64-0.42)^2+$$

$$\left(\frac{\pi}{8}*2^4-\frac{8}{9\pi}\,2^4\right)+\frac{1}{2}*\pi*2^2*(-0.85+0.64)-$$

$$\left(\left(\frac{\pi}{8}*1^4-\frac{8*1^4}{9\pi}\right)+\frac{\pi}{2}*1^2\,(-0.64+0.42)^2\right)$$

$$=2.027\ m^4$$

$$I_y = \frac{\pi}{8} * 1^4 + 1.57 * (1-0.5)^2 + \frac{\pi}{2} * 2^4 + 6.28 * (0.5)^2$$

$$-\left(\left(\frac{\pi}{8} * 1^4 + 1.57 * (1+0.5)^2\right)\right) = 7.85 \ m^4$$

bookat : F (10-6)

p 10-35

p 10-37 p.g = 54

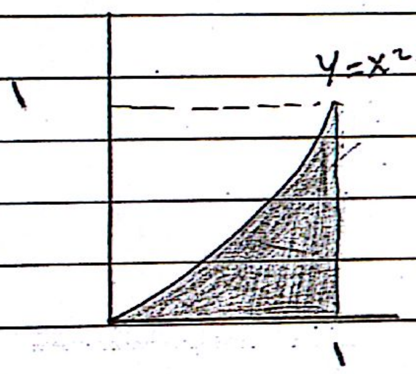
p 10-48 p.g - 60

إمثاله صفحه ٨٩/٩٠ من دفتر الدكتوره حنان

Calculate the Centroid using $\left(\int\right)$ integration :-

$$\bar{X} = \frac{\int_A x \cdot dA}{\int_A dA} \qquad \bar{Y} = \frac{\int_A y \cdot dA}{\int_A dA}$$

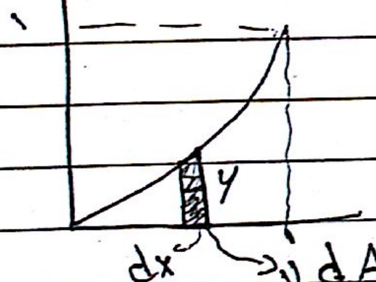
Ex (5) ;



1) we take Differential element
2) find dA (   )
3) find the integration $\int$
4) Calculate the integration

note :-

solu :-



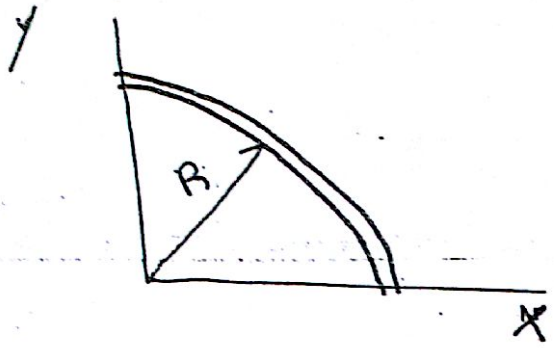
1) $dA = y \cdot dx$ (مساحة مستطيل)

2) $\boxed{y = x^2}$ → م

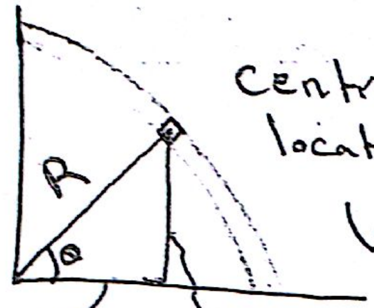
$$\bar{X} = \frac{\int_A x \cdot dA}{\int_A dA} = \frac{\int_0^1 x \cdot y \cdot dx}{\int_0^1 y \cdot dx} = \frac{\int_0^1 x \cdot x^2 \cdot dx}{\int_0^1 x^2 \cdot dx} = \frac{\int_0^1 x^3 \cdot dx}{\int_0^1 x^2 \cdot dx} = 0.75$$

$$\bar{Y} = \frac{\int_A y \cdot dA}{\int_A dA} = \frac{\int_0^1 y \cdot y \cdot dx}{\int_0^1 y \cdot dx} = \frac{\int_0^1 x^4 \cdot dx}{\int_0^1 x^2 \cdot dx} = 0.3$$

Ex 9.2



Solu :-

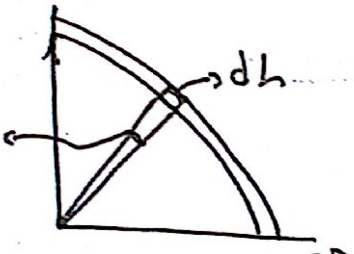


centroid of the diff. element located from the center. (orgin point)

$R\cos\theta$ $R\sin\theta$

$$\bar{C} = (\tilde{x}, \tilde{y})$$

$$C = (R\cos\theta , R\sin\theta)$$

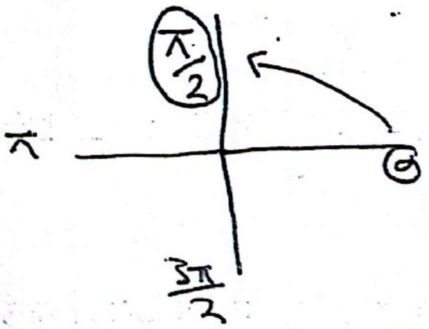


$$L = R\,\theta$$

$$dL = R\,d\theta$$ اشتقاق

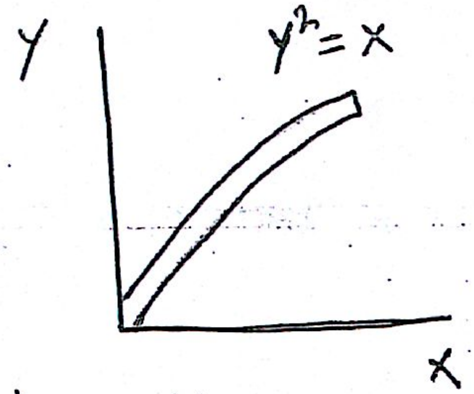
$$\bar{X} = \frac{\int_L \tilde{x} \cdot dL}{\int_L dL} = \frac{\int_0^{\pi/2} (R\cos\theta) \cdot R\,d\theta}{\int_0^{\pi/2} R\,d\theta}$$

تكون بدلالة θ إي (Rad)

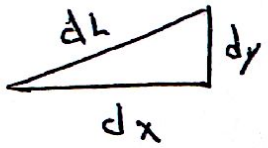


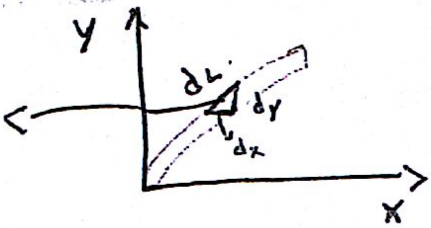
Ex 9.11

find the $\bar{c}$



1) We take diff. element





$dL \Rightarrow$ وحده طول
عشان هيك بستعمل
$dL = \sqrt{dx^2 + dy^2}$

$$dL = \sqrt{dx^2 + dy^2} \quad \text{(Pythagrean theorem)}$$

$$dL = \left(\sqrt{\left(\frac{dx}{dy}\right)^2 + 1}\right) \cdot dy \Rightarrow$$ شرح كيف صارت هيك
بـ صفحه $\underline{451}$

$y^2 = x \qquad \boxed{\frac{dx}{dy} = 2y}$ إشتقاق

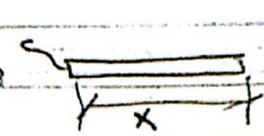
$$dL = \left(\sqrt{(2y)^2 + 1}\right) \cdot dy \Rightarrow$$ إصبحت معادله
بدلاله y

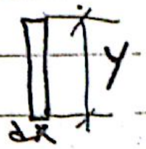
$\tilde{x} = x = y^2$

$$\bar{X} = \frac{\int_L \tilde{x} \cdot dL}{\int_L dL} = \frac{\int_L y^2 \sqrt{(2y)^2 + 1} \cdot dy}{\int_L \sqrt{(2y)^2 + 1} \cdot dy} = \frac{\int_0^1 y^2 \sqrt{(2y)^2 + 1} \cdot dy}{\int_0^1 \sqrt{(2y)^2 + 1} \cdot dy}$$

تكون الابعاد
في y مش x !!
عشان تكامل
بالنسبه لـ y فبأخذ
الابعاد في y

Find I using $\int f(x) \cdot dx$

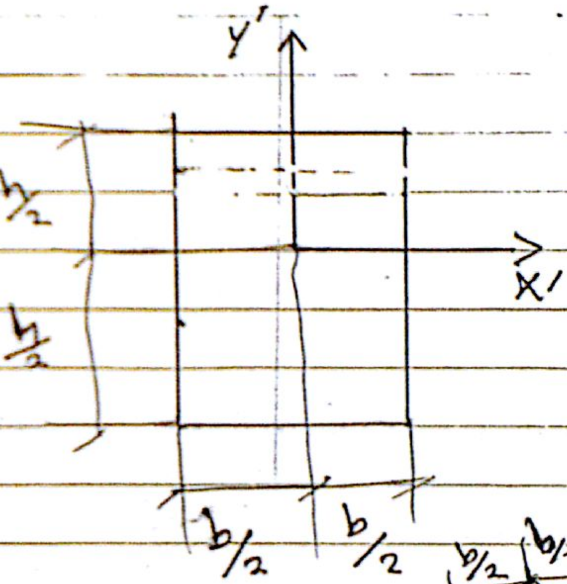
If we want to find $I_x = \int y^2 \cdot dA$ take 

$I_y = \int x^2 \cdot dA$ take 

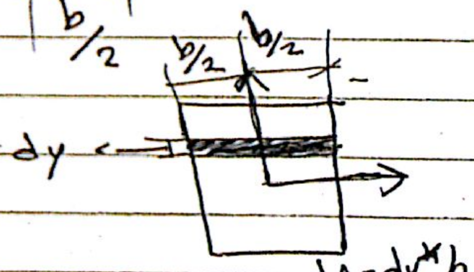
يجب الالتزام بالقوانين

Ex (1)

Find I_x and I_y about x', y'



$$I_x = \int y^2 dA = \int_{-h/2}^{h/2} y^2 \cdot b \cdot dy$$

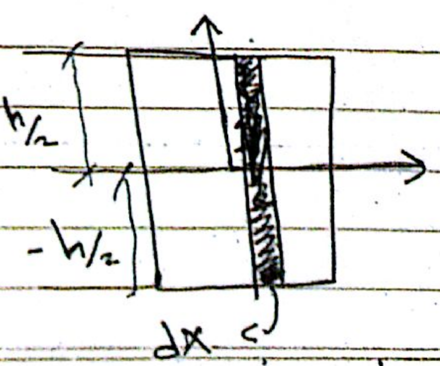


$dA = dy * b$

$$= \frac{y^3}{3} \times b \Big|_{-h/2}^{h/2} = \frac{b}{3} * \left(\frac{h^3}{8} - \left(-\frac{h^3}{8}\right)\right) = \frac{bh^3}{12}$$

$$I_y = \int x^2 \cdot dA = \int_{-b/2}^{b/2} x^2 \; h \, dx$$

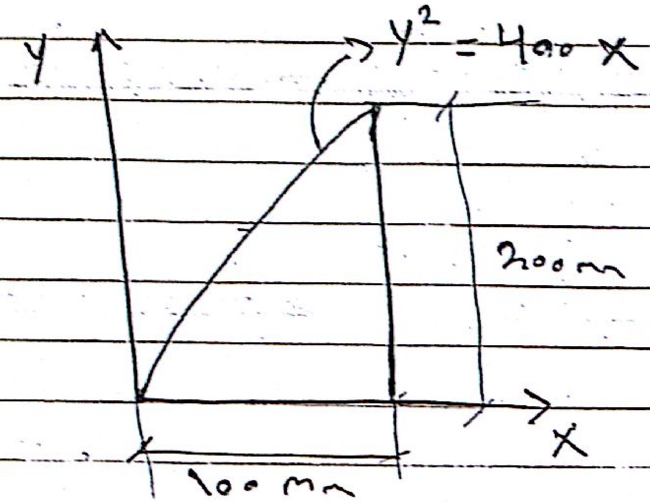
$$= h\frac{x^3}{3} \Big|_{-b/2}^{b/2} = \frac{hb^3}{12}$$



$dA = dx * h$

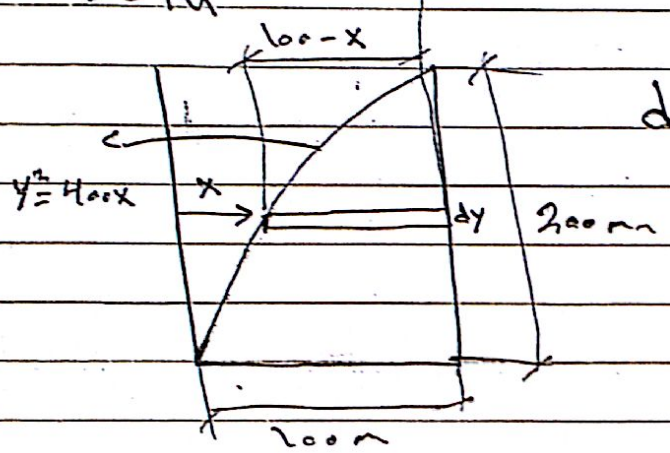
(١٠)

Ex2



Find I_x about x-axis

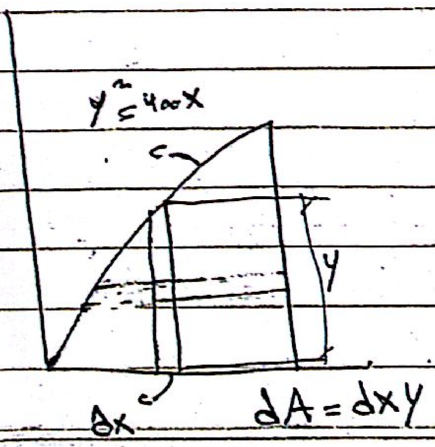
Solu



$dA = dy(100-x)$

$$I_x = \int y^2 \cdot dA = \int_0^{200} y^2 (100-x) \cdot dy$$

$$= \int_0^{200} y^2 \left(100 - \frac{y^2}{400}\right) dy = 1.07 \times 10^6 \ mm^4$$



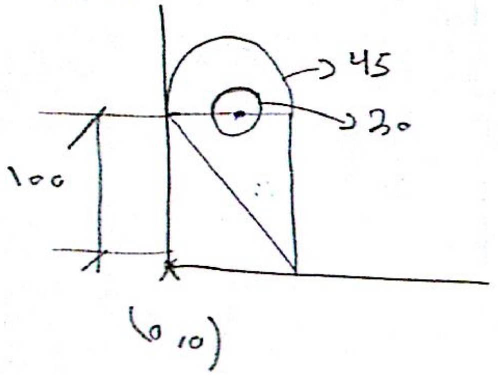
$dA = dx\,y$

$$I_y = \int x^2 \cdot dA = \int_0^{100} x^2 y \, dx$$

$$= \int_0^{100} \cancel{x^3 \times 400} \, dx = \cancel{1.6} \ mm^4$$

$x^2 \sqrt{400x}$

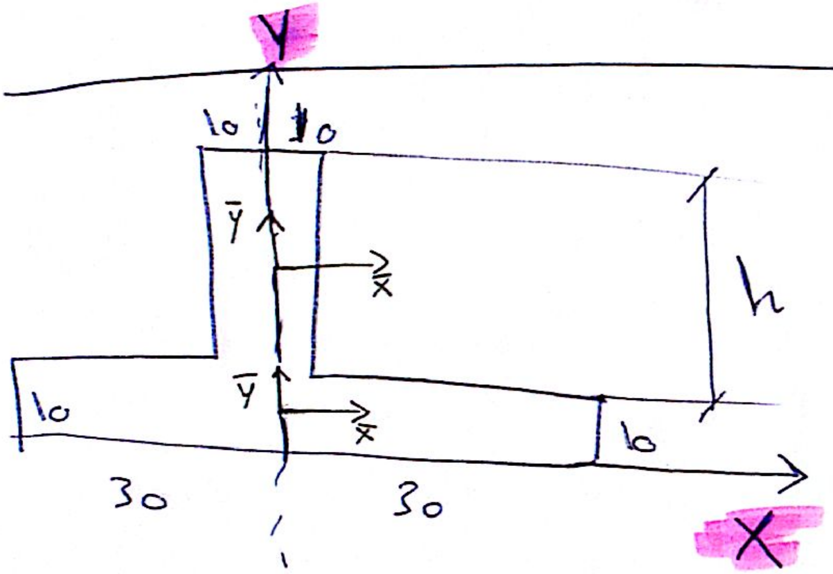
Find $(\bar{x}, \bar{y})$



S.A	A	$\bar{X}$	$\bar{Y}$	$\bar{X}A$	$\bar{Y}A$
90, 100	4500	60	66.6		
45	3181.2	45	119.		
30	-1257	45	100		

$\bar{X} = 55.5$ mm

$\bar{Y} = 86.01$ mm



Find value of h for where

$I_x = I_y$

$$I_x = \frac{bh^3}{12} + Ay^2$$

$$= \frac{60 \times 10^3}{12} + 6 \times 10 \times (5)^2 + \frac{30 \times h^3}{12} + 30 \times h \times (h/2 + 10)^2$$

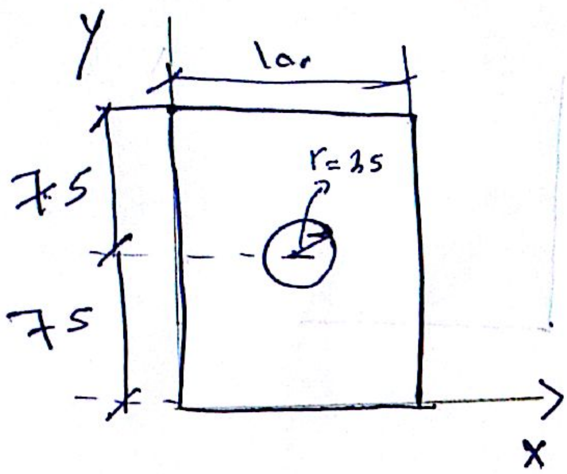
$$I_y = \frac{10 \times (60)^3}{12} + \cancel{60 \times 10 \times 0} + \frac{h \times (30)^3}{12} + \cancel{h \times 30 \times 0}$$

$$I_x = I_y$$

$$\boxed{h = 20.75}$$

$$6500 + \frac{5}{3}h^3 + 30h \times (h/2 + 10)^2 = 18000 + \frac{3000h}{3}$$

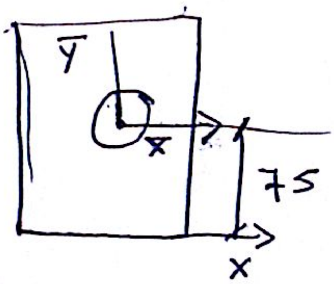
Find I about x-axis



$$I_x = \underbrace{\frac{bh^3}{12} + Ay^2}_{\text{Rec}} + \underbrace{\frac{\pi R^4}{4} + Ay^2}_{\text{Cir}}$$

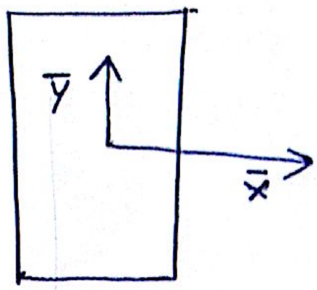
$$= \frac{100\times(150)^3}{12} + 100\times150\times(75)^2 + \frac{\pi\times(35)^4}{4} + \pi(35)^2\times(75)^2$$

بناخدها 75 عشان
السؤال نفسو طالب
I على محور x مش
حول (centroidal axis)
فعشان هيك بناخدها
بين centroid تاعها (مستطيل
و دائرة) و x-axis



$$I_x = \frac{bh^3}{12}$$

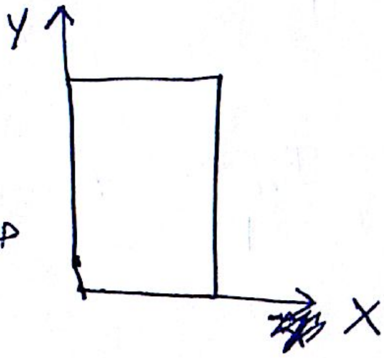
Centroidal حوالين axis



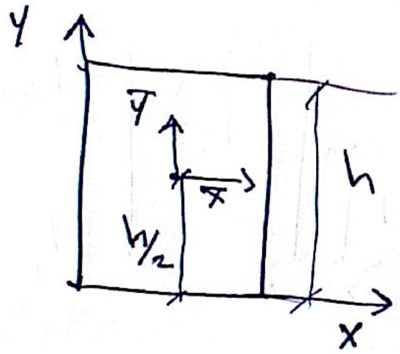
فرق

$$I_x = \frac{bh^3}{3}$$

x-axis حوالين

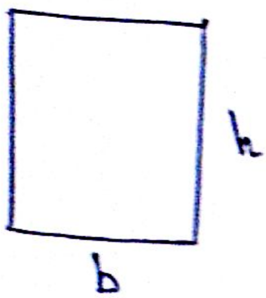


Find the I_x about x-axis



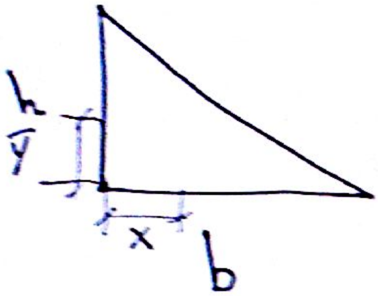
$$I_x = \frac{bh^3}{12} + Ay^2 = \frac{bh^3}{12} + bh \times \left(\frac{h}{2}\right)^2 = \frac{bh^3}{12} + \frac{bh^3}{4}$$

$$= \frac{bh^3}{3}$$

1) 

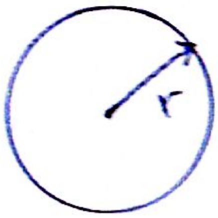
$$\bar{x} = \frac{b}{2} \qquad I_x = \frac{bh^3}{12}$$

$$\bar{y} = \frac{h}{2} \qquad I_y = \frac{hb^3}{12}$$

2) 

$$\bar{x} = b/3 \qquad I_x = \frac{bh^3}{36}$$

$$\bar{y} = h/3 \qquad I_y = \frac{hb^3}{36}$$

3) 

$$\bar{x}, \bar{y} = \text{Center} \qquad I_x = \frac{\pi R^4}{4}$$

$$I_y = \frac{\pi R^4}{4}$$

4) 

$$\bar{x} = \text{Center} \qquad I_x = \frac{\pi R^4}{8} - \frac{8R^4}{9\pi}$$

$$\bar{y} = \frac{4R}{3\pi} \qquad I_y = \frac{\pi R^4}{8}$$

5) 

$$\bar{x} = \bar{y} = \frac{4R}{3\pi}$$

$$I_x = I_y = \frac{\pi R^4}{16} - \frac{4R^4}{9\pi}$$