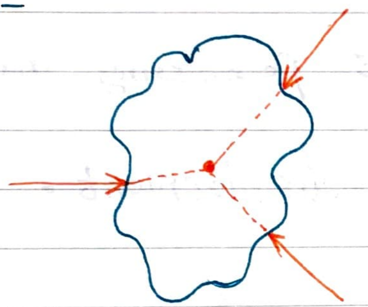
CH 2 Statics of Particles

* the size and shapes of bodies under considration don't significantly affect the solution of problems.

تلتقي بنقطة معينة

⇒ All forces are $\boxed{\text{Concurrent}}$ forces.

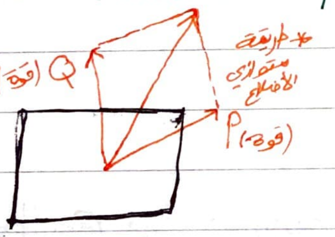
Exp:-



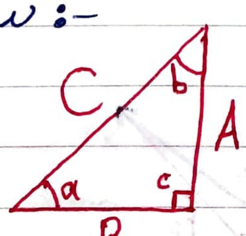
تقع في نفس الـ plan

2.1 addition of $\boxed{\text{planar}}$ forces

2.1 A :- force on a particle : resultant of two forces.



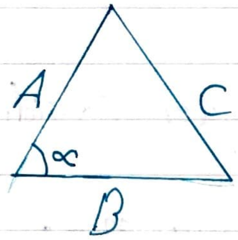
* Sine Law:-



$$\frac{C}{\sin c} = \frac{B}{\sin b} = \frac{A}{\sin a}$$

* Cosine Law :-

* هذه الحالة تستخدمها عندما يكون ضلعين الزاوية بينهم معروفة

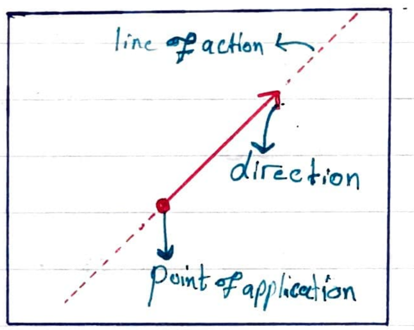


$$C^2 = A^2 + B^2 - 2AB\cos\alpha$$

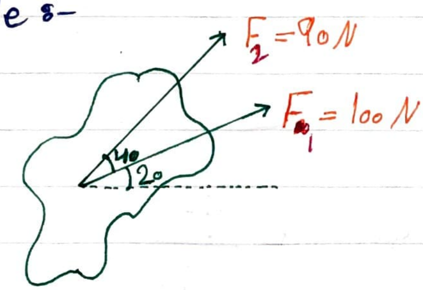
* ملاحظة: هذا الضلع هو الضلع الذي يقابل الزاوية المعروفة "∝".

2.1 B :-

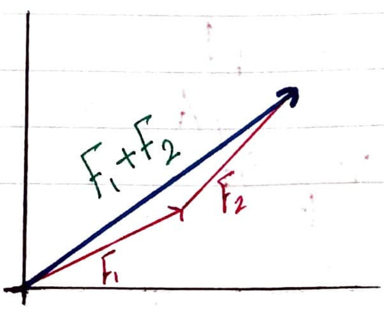
* vectors are mathmetical expressions possessing magnitude and direction, like : force, velocity and dis placment.



Example :-

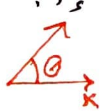


* determine the resultant force of the forces F_1 and F_2 ?

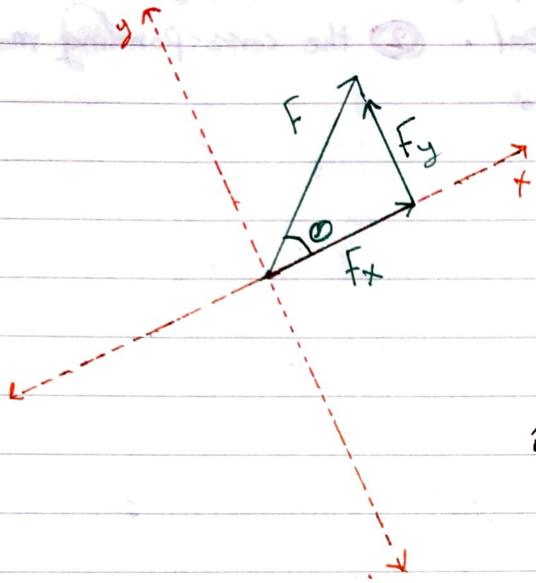


* ملاحظة: عند الإجابة يجب التوضيح والرسم وذلك مثلا :-

$\theta = 38.18$



2.1 C :- Resolution of a force into Components :-

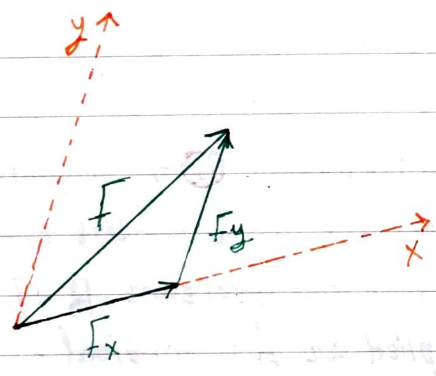


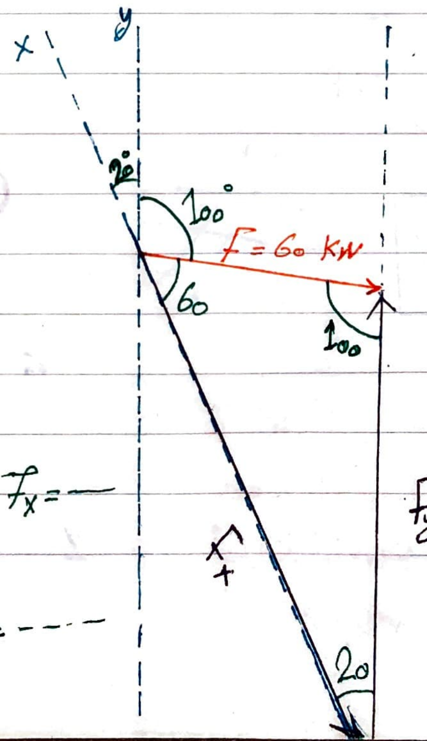
$$F_x^2 + F_y^2 = F^2$$

$$\theta = \tan^{-1}\frac{F_y}{F_x}$$

* هذه الحالة فقط عندما تكون المحاور متعامدة
"$y \perp x$" ....

Case "2" :- X-y non Perpendicular :-



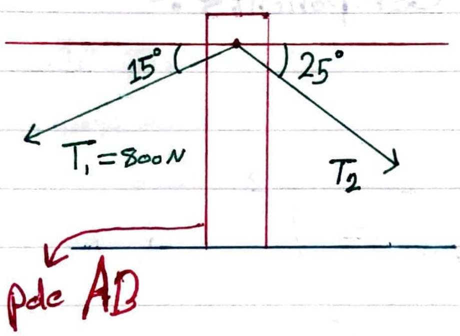


exp :- * نرسم محور عمودي على x ليس من رأس المتجه.

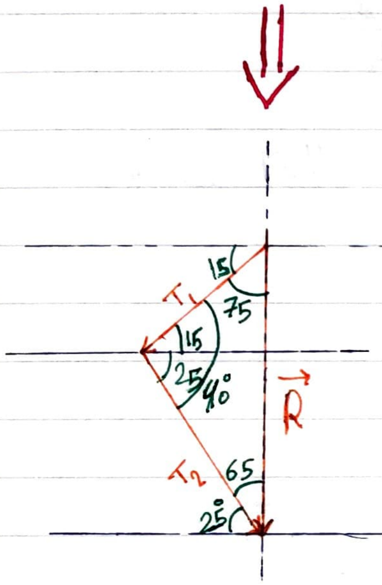
* $\frac{F_x}{\sin 100} = \frac{60}{\sin 20} \Rightarrow F_x = ---$

$\frac{F_y}{\sin 60} = \frac{60}{\sin 20} \Rightarrow F_y = ----$

example :-



* Determine ① the required T_2 in the right hand portion of the cable if the resultant force R of exerted by cable A is vertical. ② the corresponding magnitude of R.

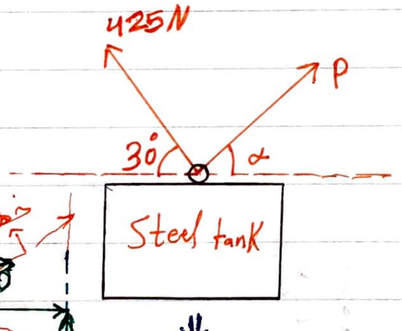


∴ Using Sine Law :-

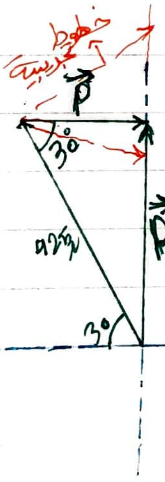
$$\frac{800}{\sin 65} = \frac{T_2}{\sin 40} \Rightarrow \boxed{T_2 = 853N}$$

$$\Rightarrow \frac{800}{\sin 65} = \frac{R}{\sin 40} \Rightarrow \boxed{R = 567N}$$

example :-



* Determine ⓐ the magnitude and direction of the smallest force p for which the resultant R of the two forces applied at A is vertical.
ⓑ Corresponding magnitude of R.



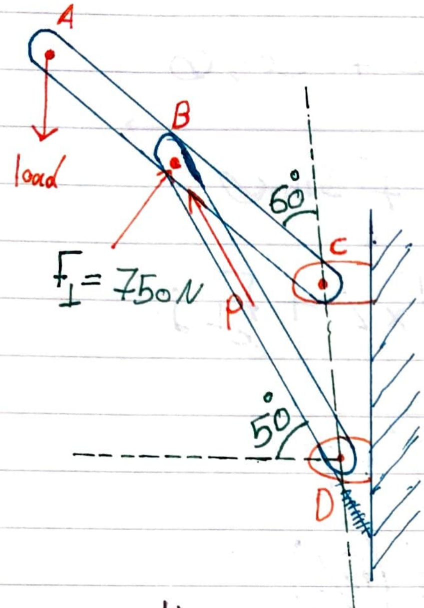
* p تكون أفقية حسب المطلوب اصغر قوة (أصغر مسافة) أو (أصغر طول متجه) لهذا هي أفقية

$$\therefore \frac{P}{\sin 60} = \frac{425}{\sin 90} \Rightarrow p = 425 \sin 60 \ N \ (\rightarrow)$$

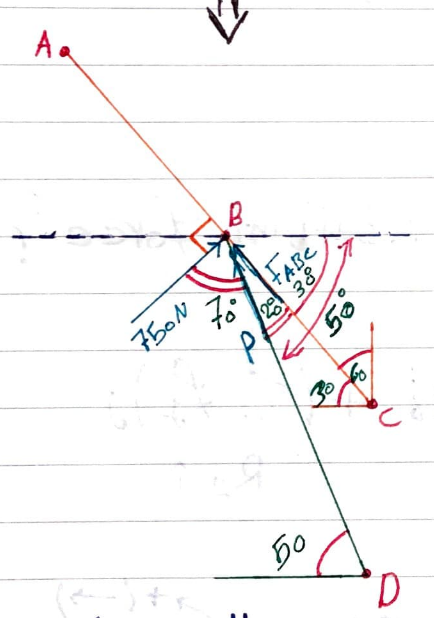
$$\therefore R = 425 \sin 30 \ N \ (\uparrow)$$

example:-

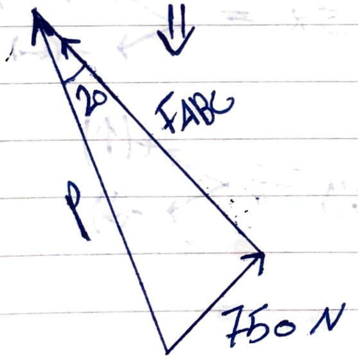
* The hydraulic cylinder BD exert on member ABC a force p directed a long line BD. Knowing that p must have a 750-N component perpendicular to member ABC, determine: ① the mgnitude of the force p. ② the component parallel to ABC.



⇓



⇓



1) $P \cos 70 = 750 \Rightarrow P = \frac{750}{\cos 70}\,N$

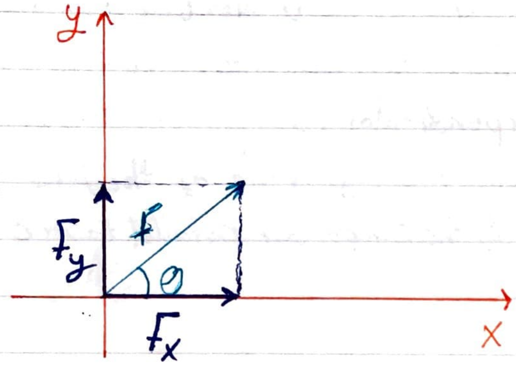
$= 2192\,N$

2) $F_{ABC} = P \sin 70$

$= 2192 \sin 70\,N$

#

2.2 Adding forces by Components

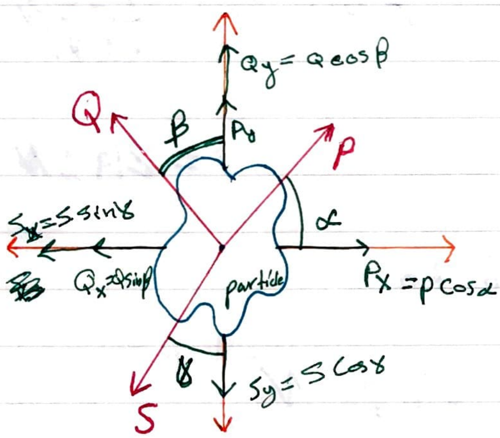


$F_x = F\cos\theta$

$F_y = F\sin\theta$

$\vec{F} = F_x i + F_y j$

$$\begin{cases} F = \sqrt{F_x^2 + F_y^2} \\ \theta = \tan^{-1}(F_y/F_x) \end{cases}$$



, find the resultant force?

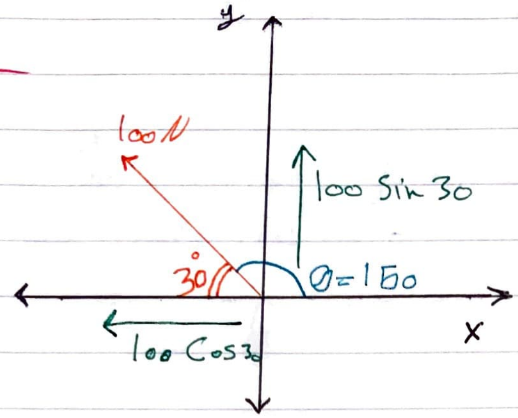
$$\vec{R} = \underbrace{(\Sigma F_x)}_{R_x} i + \underbrace{(\Sigma F_y)}_{R_y \uparrow} j$$

$\xrightarrow{+}$ $R_x = \Sigma F_x = +P_x - Q_x - S_x =$ $+(\rightarrow)$ / $-(\leftarrow)$

$+\uparrow$ $R_y = \Sigma F_y = P_y + Q_y - S_y =$ $+(\uparrow)$ / $-(\downarrow)$

$R = \sqrt{(\Sigma F_x)^2 + (\Sigma F_y)^2}$

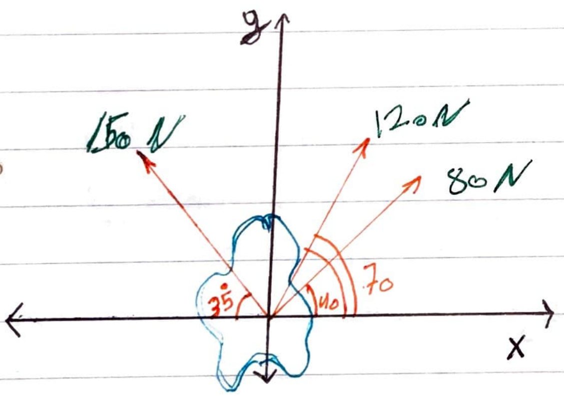
exp :-



$F = -100 \cos 30 \, i + 100 \sin 30 \, j$

or $F = 100 \cos 150 \, \underline{i} + 100 \sin 150 \, \underline{j}$

examples:-



* determine the resultant force?

$\overset{+}{\rightarrow} R_x = \Sigma F_x = +80 \cos 40 + 120 \cos 70 - 150 \cos 35$

$= -20.55 \, N \; (\leftarrow)$

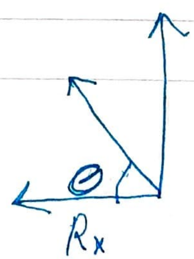
$+\uparrow R_y = \Sigma F_y = +80 \sin 40 + 120 \sin 70 + 150 \sin 35$

$= +250.22 \, N \; (\uparrow)$

$\Rightarrow R = \sqrt{(20.55)^2 + (250.22)^2}$

$= 251.1 \, N$

$\theta = \tan^{-1}\left(\frac{250.22}{20.55}\right) = 85.3$

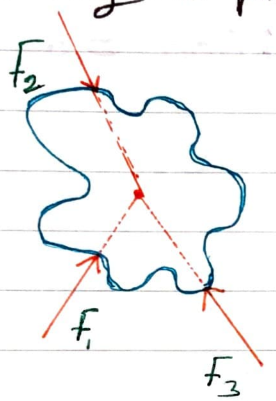


2.3 Forces and equilibrium in a plane.

when the resultant of all forces acting on particle is zero, the particle is in equilibrium.

$$R = \sum F = 0$$

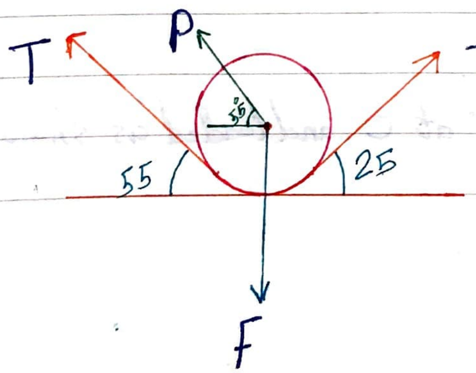
* choose a significant particle and draw a separate diagram showing this particle and all the forces acting on it.

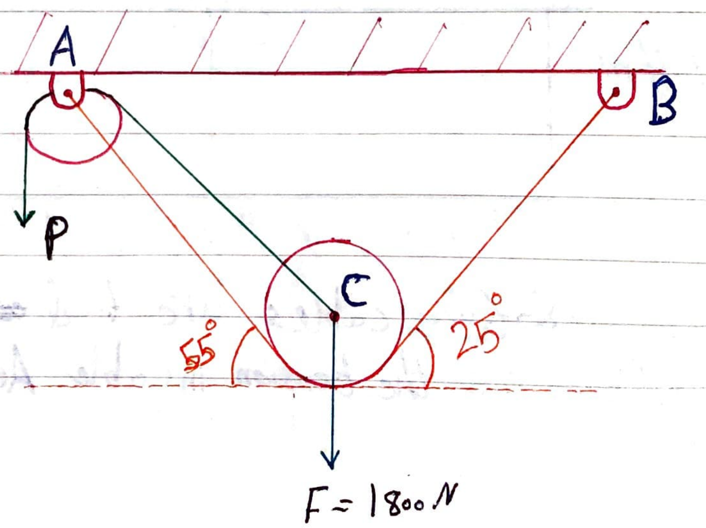


Exp:- an 1800 N load F is applied to pulley C, when roll on the cable ACB The pulley is held in the position shown by a second cable [redacted] CAP, which passes over the pulley A and support a load P.

Determine a) the tension in cable ACB.

b) the magnitude of load P.





$\overset{+}{\rightarrow} \sum F_x = 0$

$-T\cos 55 + T\cos 25 - P\cos 55 = 0$

$P = 0.5801\,T$

$+\uparrow \sum F_y = 0$

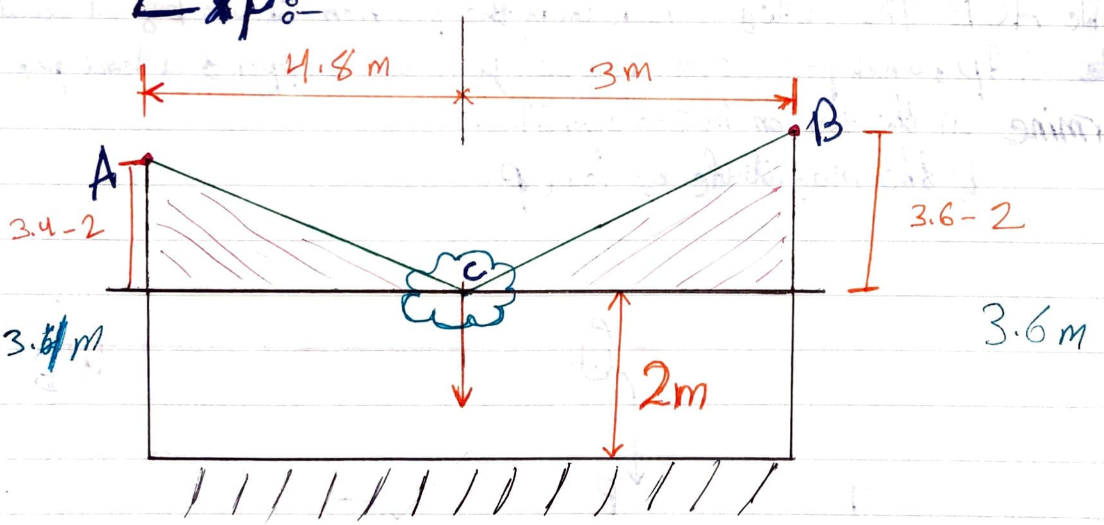
$+T\sin 55 + T\sin 25 + P\sin 55 - 1800 = 0$

$1.71696\,T - 1800 = 0$

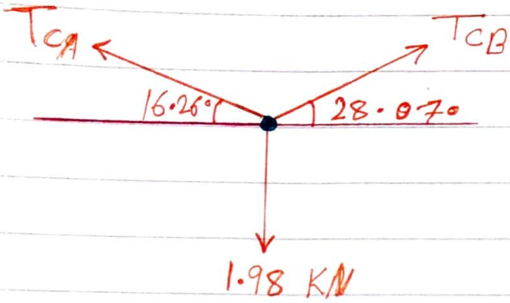
$\boxed{T = 1048\,N}$

$\therefore P = 0.5801(1048) = 608\,N$

Exp:-



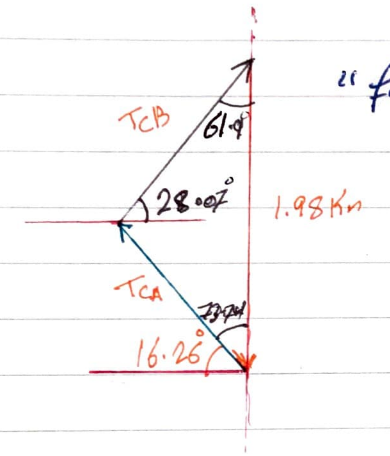
Two cables are tied Together at C and loaded as shown. Determine the tension in Cable AC and BC.



$$\frac{T_{CB}}{\sin 73.74} = \frac{T_{CA}}{\sin 61.93} = \frac{1.98}{\sin 44.33}$$

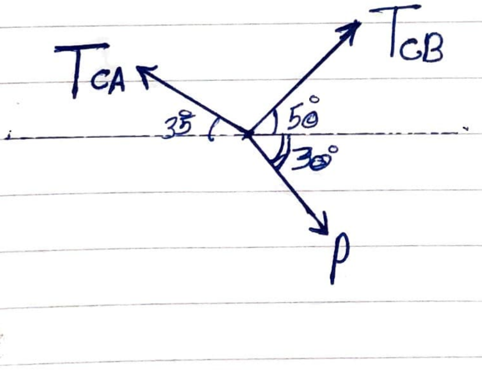
$\Rightarrow T_{CA} = 2.5\ KN$

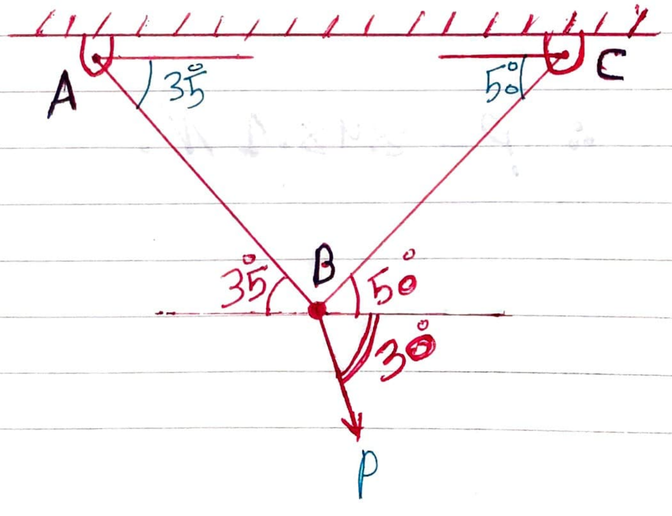
$\Rightarrow T_{CB} = 2.72\ Kn$



"force Triangle"

exp :- Two cables tied together at C are loaded as shown Knowing that the maximum allowable tension in each cable is 800N, Determine : The magnitude of the largest force P that can be applied at C.





Applying equilibrium equations :-

$(\xrightarrow{+})\ \sum F_x = 0$:-

$-T_{CA}\cos 35 + T_{CB}\cos 50 + P\cos 60 = 0$

$(\uparrow +)\ \sum F_y = 0$:-

$+T_{CA}\sin 35 + T_{CB}\sin 50 - P\sin 60 = 0$

Assume $T_{CA} = 800$ Newton :-

$-800\cos 35 + T_{CB}\cos 50 + P\cos 60 = 0$ ---- (1)

$800\sin 35 + T_{CB}\sin 50 + P\sin 60 = 0$ ---- (2)

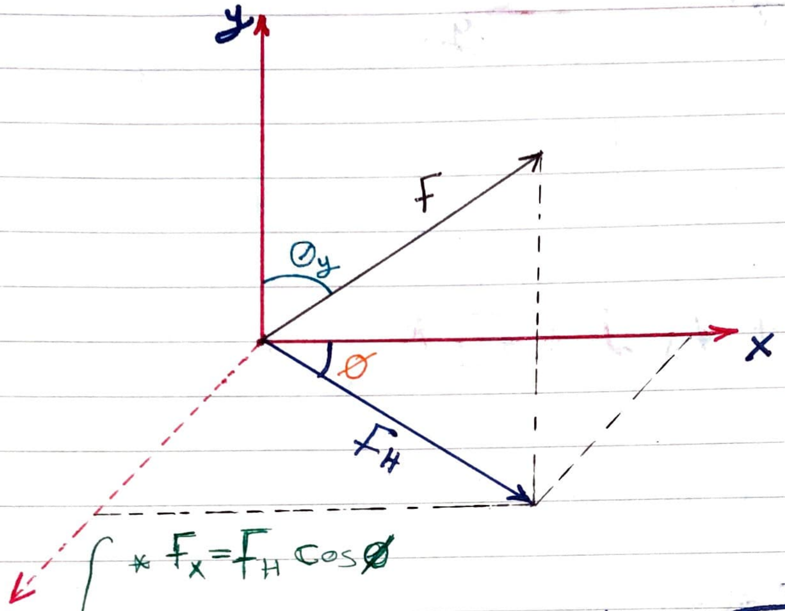
after solving the equations :-

$T_{CB} = 359.8\ N \le 800\ N \Rightarrow$ The assumption is correct.

$\therefore P = 848.1\ N.$

2.4 Adding Forces in space.

2.4 A recantegular components of a force in space:-

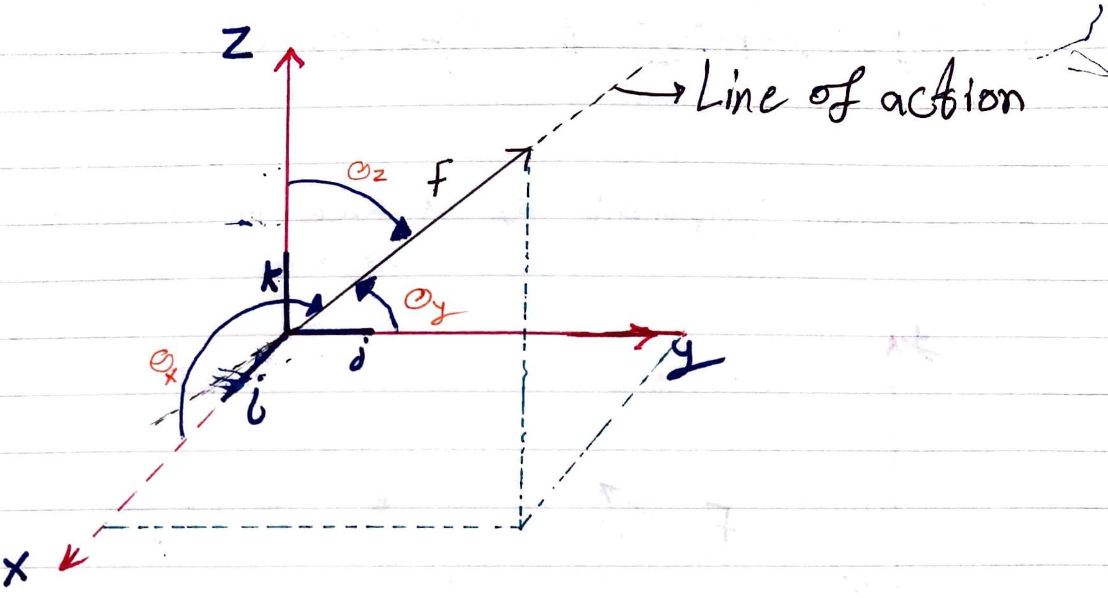


Z
- $F_x = F_H \cos\phi$
- $F_y = F \cos\theta_y$
- $F_z = F_H \sin\phi$

but $F_H = F \sin\theta_y$

⇓

$$F_x = F \sin\theta_y * \cos\phi$$
$$F_y = F \cos\theta_y$$
$$F_z = F \sin\theta_y * \sin\phi$$



$$F = F_x\, i + F_y\, j + F_z\, K$$

→ Unit vectors

$$\begin{cases} F_x = F \cos\theta_x \\ F_y = F \cos\theta_y \\ F_z = F \cos\theta_z \end{cases}$$

$$\vec{F} = F_x\, i + F_y\, j + F_z\, K$$

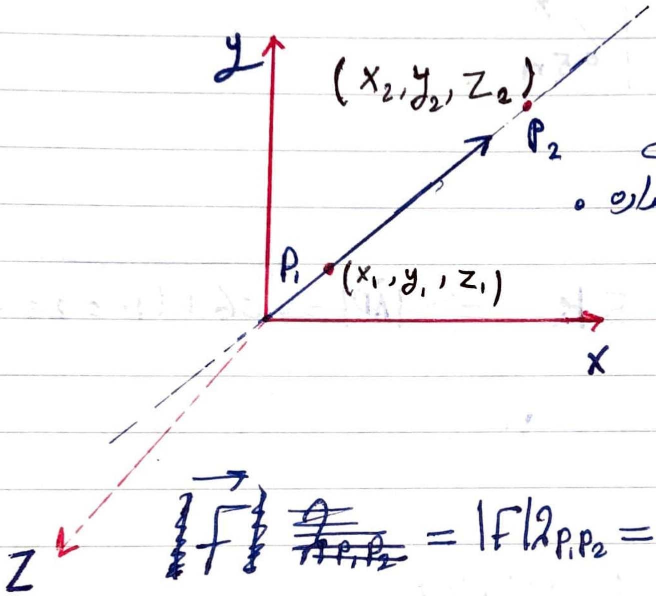
$$= (F\cos\theta_x)\, i + (F\cos\theta_y)\, j + (F\cos\theta_z)\, K$$

$$\Rightarrow \cancel{\vec{F}} = \cancel{F}\, \underbrace{\left[\cos\theta_x\, i + \cos\theta_y\, j + \cos\theta_z\, K\right]}_{\lambda}$$

but $|\lambda| = 1$

$$|\lambda| = \cos^2\theta_x + \cos^2\theta_y + \cos^2\theta_z = 1^2 = 1$$

2.4 B :-



* لايجاد مركبات متجه من خلال نقطتين معطيتين بحيث يكونين على المتجه أو على امتداده .

$$\vec{F} \cancel{\lambda_{P_1P_2}} = |F|\lambda_{P_1P_2} = |F| \left[\cos\theta_x + \cos\theta_y + \cos\theta_z\right]$$

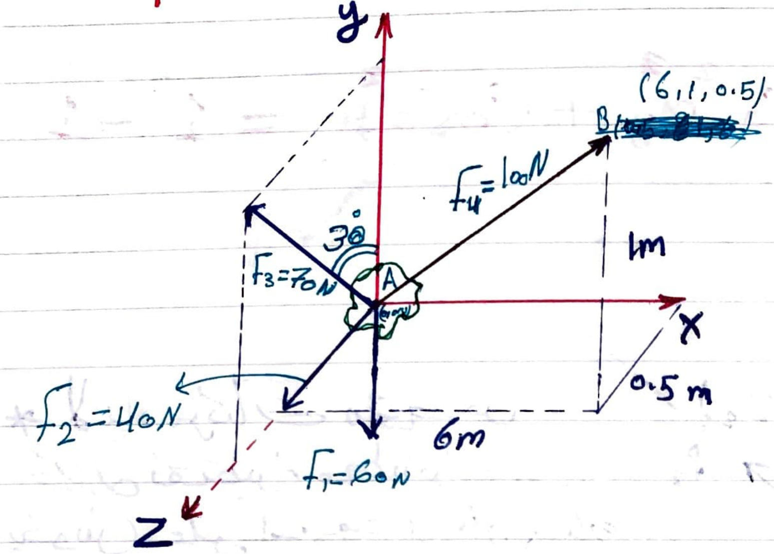
$$\text{but } \lambda_{P_1P_2} = \frac{\overrightarrow{P_1P_2}}{|P_1P_2|} = \frac{(x_2-x_1)\,i + (y_2-y_1)\,j + (z_2-z_1)\,k}{\sqrt{(x_2-x_1)^2 + (y_2-y_1)^2 + (z_2-z_1)^2}}$$

$$\cos\theta_x = \frac{x_2 - x_1}{\sqrt{(x_2-x_1)^2 + (y_2-y_1)^2 + (z_2-z_1)^2}}$$

$$\cos\theta_y = \frac{y_2 - y_1}{\sqrt{(x_2-x_1)^2 + (y_2-y_1)^2 + (z_2-z_1)^2}}$$

$$\cos\theta_z = \frac{z_2 - z_1}{\sqrt{(x_2-x_1)^2 + (y_2-y_1)^2 + (z_2-z_1)^2}}$$

Exp:-



* Determine the resultant force of the four forces actin on the particle at point A.

$$\vec{AB} = 6i + 1j + 0.5K \Rightarrow |\vec{AB}| = \sqrt{36 + 1 + 0.25} = 6.1m$$

$$\therefore \lambda_{AB} = \frac{\vec{AB}}{|AB|} = \frac{6i + 1j + 0.5k}{6.1} \Rightarrow F_4 = |F_4| \lambda_{AB}$$

$$\Rightarrow F_4 = 98.4i + 16.4j + 8.2K$$

$$F_3 = 70\cos 30\, j + 70 \sin 30\, K$$

$$F_2 = 40K$$

$$F_1 = -60j$$

$$\vec{R} = \sum F = \Sigma(F_x)i + \Sigma(F_y)j + \Sigma(F_z)K$$

$$\vec{R} = 98.4i + 17.02j + 83.2K$$

$$|\vec{R}| = 129.9N$$

Direction angles : (θ_x , θ_y , θ_z) :

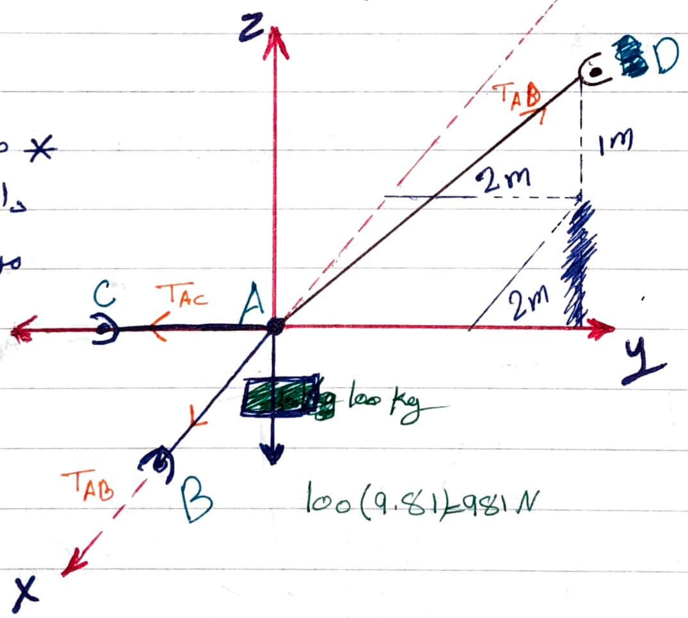
$$\cos\theta_x = \frac{|\Sigma F_x|}{|R|} = \frac{98.4}{129} \Rightarrow \theta_x = 40.7°$$

$$\cos\theta_y = \frac{|\Sigma F_y|}{|R|} = \frac{17.02}{129.9} \Rightarrow \theta_y = 82.5°$$

$$\cos\theta_z = \frac{|\Sigma F_z|}{|R|} = \frac{83.2}{129.9} \Rightarrow \theta_z = 50.2°$$

Exp :- forces and Equilibrium in space (2.5)

* ملاحظة : قوى الشد T دائماً ما تكون خارجة من الجسم .



* Determine the tension in the cables in order to support the 100-Kg crate in the equilibrium position shown.

$A(0,0,0)$, $D(-2,+2,+1)$ $\Rightarrow \vec{AD} = -2i + 2j + 1K$ (m)

$$|\vec{AD}| = \sqrt{(2)^2 + (2)^2 + (1)^2} = \sqrt{9} = 3$$

$$\lambda_{AD} = \frac{\vec{AD}}{|\vec{AD}|} = \frac{-2i + 2j + K}{3} = \frac{-2}{3} i + \frac{2}{3} j + \frac{1}{3} K$$

$$\vec{T}_{AD} = T_{AD}\, \lambda_{AD} = \frac{-2T_{AD}}{3} i + \frac{2T_{AD}}{3} j + \frac{T_{AD}}{3} K$$

from Equilibrium : $\Sigma F = 0$

$\Sigma F_x = 0$ $(i) \Rightarrow + T_{AB} - \frac{2}{3} T_{AD} = 0$

$\Sigma F_y = 0$: (j) : $- T_{AC} + \frac{2}{3} T_{AD} = 0$

$\Sigma F_z = 0$: (K) : $-981 + \frac{T_{AD}}{3} = 0$

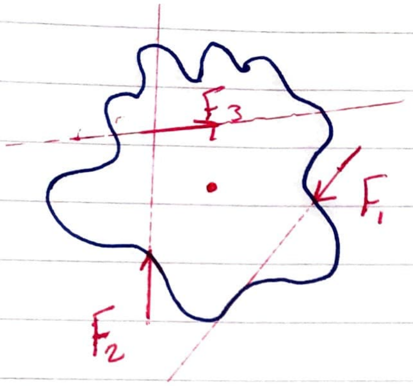
$\therefore T_{AD} = 2943\ N$

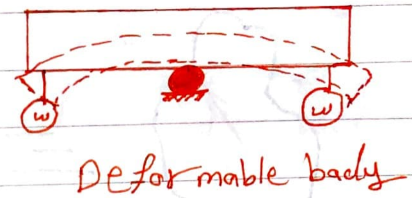
$T_{AC} = \frac{2}{3}(2943)$

$= T_{AB}$ (تصادفاً نفس القيمة)

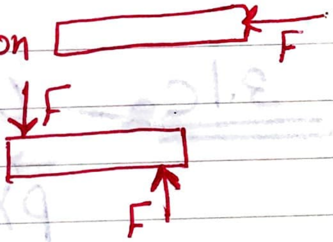
The END of chapter "2"

Ch3 Rigid Bodies : Equivalent system of forces.



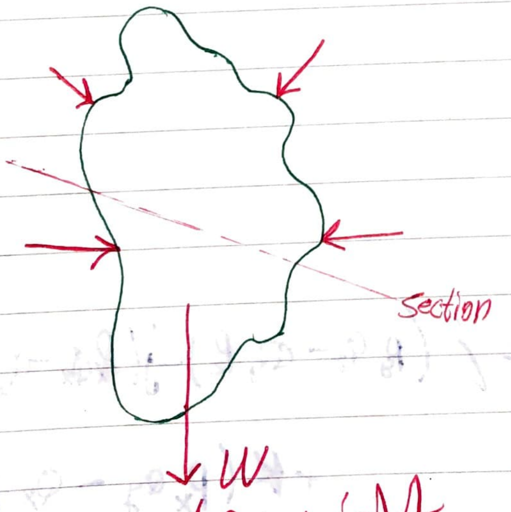


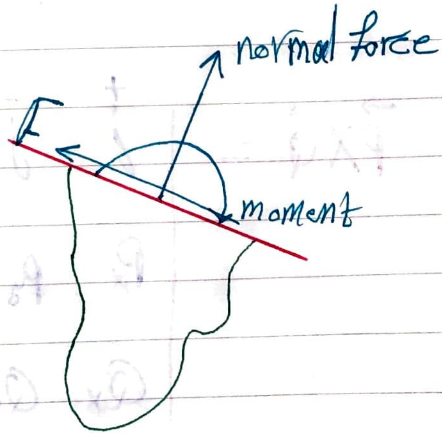
Rigid body motion ⟶ Translation
⟶ Rotation



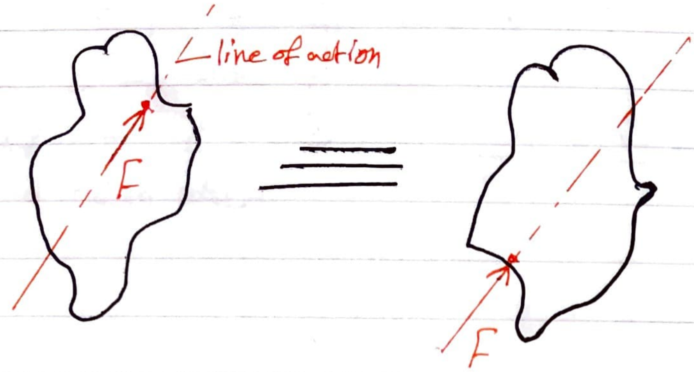
3.1 :- Forces and moments.

3.1 A : External and Internal forces.





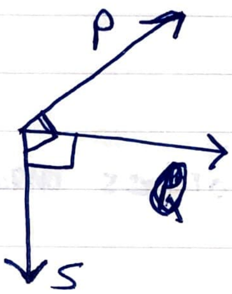
3.1 B: Principles of Transmissibility



3.1 C: Vector Product

$$\vec{P} \times \vec{Q} = \vec{S}$$

direction: right-hand rule



$$\vec{P} = P_x i + P_y j + P_z k$$

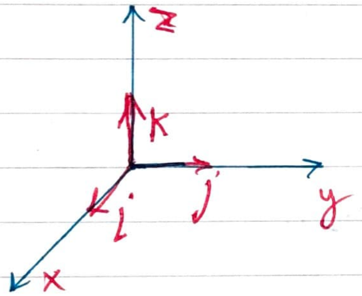
$$\vec{Q} = Q_x i + Q_y j + Q_z k$$

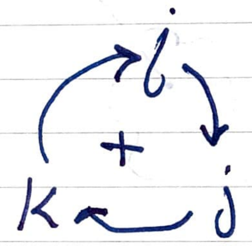
$$\vec{P} \times \vec{Q} = \begin{array}{ccc} + & - & + \\ \end{array}$$
$$\vec{P} \times \vec{Q} = \begin{vmatrix} i & j & K \\ P_x & P_y & P_K \\ Q_x & Q_y & Q_K \end{vmatrix} = i\,(P_y Q_z - Q_y P_z) - j\,(P_x Q_z - Q_x P_z)$$
$$+ K\,(P_x Q_y - Q_x P_y)$$

3.1

* Not Commutative : $(\vec{P} \times \vec{Q}) = -(\vec{Q} \times \vec{P})$
* ~~Distion~~ Distibutive : $\vec{P} \times (\vec{Q} + \vec{S}) = \vec{P} \times \vec{Q} + \vec{P} \times \vec{S}$
* Not associative : $(\vec{P} \times \vec{Q}) \times \vec{S} \neq \vec{P} \times (\vec{Q} \times \vec{S})$

unit Vectors :-

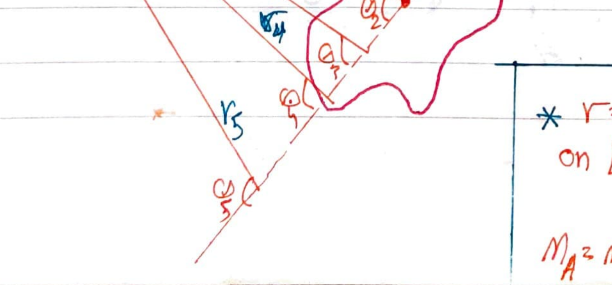




$i \times j = +k$

$k \times j = -i$

3.1E :-



$\vec{M} = \vec{r} \times \vec{F}$

$= r F \sin\theta$

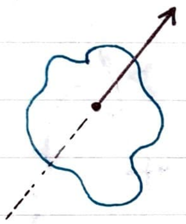
* r = is the position vector from A to any point on Line of action of the force F

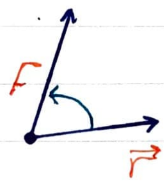
M_A = Moment of force F about point A

The moment of a force is a measure of the forces ability to produce twisting or rotation of a body about a point.

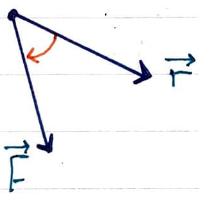
« Moment »

$M_A = \vec{r} \times \vec{F} = Fd$





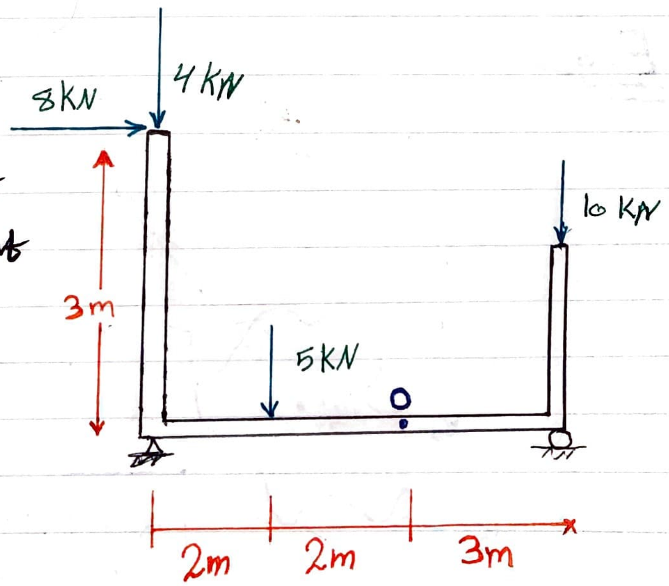
Counter Clockwise
«CCW»



clock wise
« CW »

example :-

— Determine the resultant moment of the forces about point O.

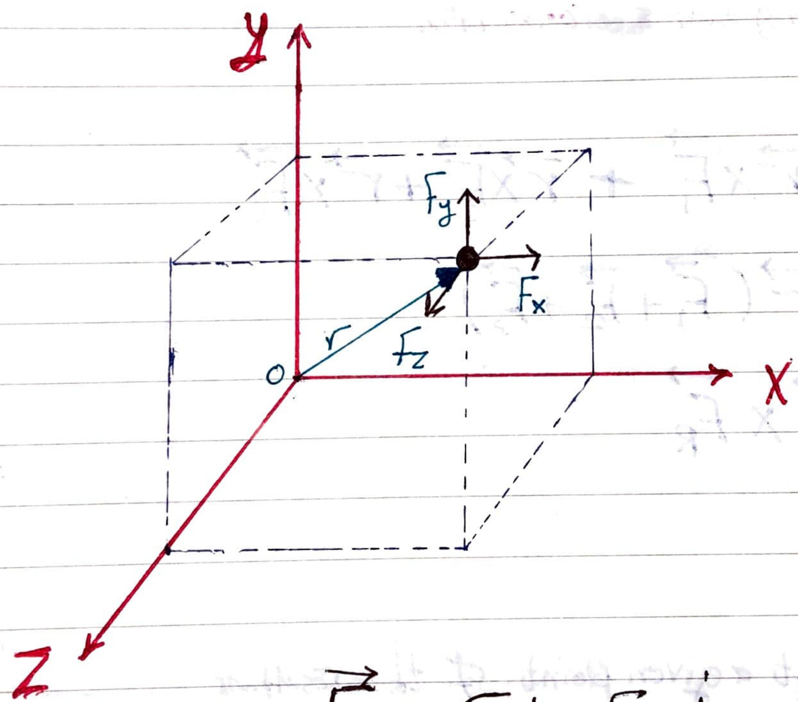


$\circlearrowleft M_o = -(8)(3) + (4)(4) + (5)(2) - (10)(3)$

$= -28 \text{ KN.m}$

or $M_o = 28 \text{ KN.m}$ ↻

3.1 F :- Rectangular Components of the moment of force.



$$\vec{F} = F_x i + F_y j + F_z K$$

$$M_o = \vec{r} \times \vec{F} = M_x i + M_y j + M_z K$$

$$r = x i + y j + z K$$

$$M_o = \begin{vmatrix} \oplus i & \ominus j & \oplus k \\ x & y & z \\ F_x & F_y & F_z \end{vmatrix}$$

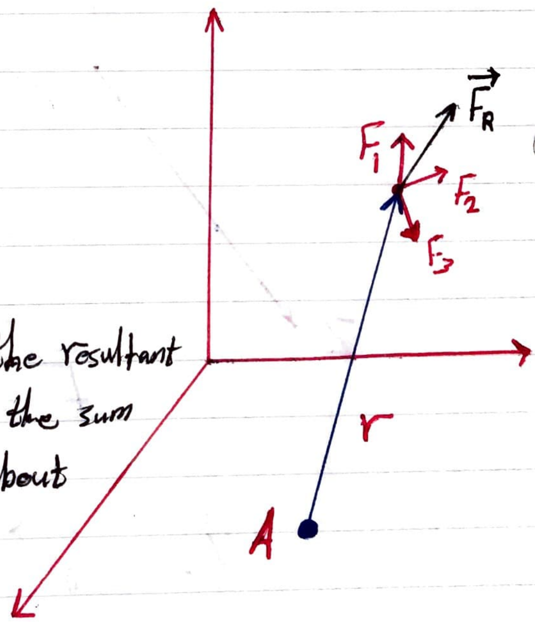
$$= (y \cdot F_z - F_y \cdot z)(i) + (x \cdot F_z - z \cdot F_x)(-j) + (x \cdot F_y - y \cdot F_x)\,k$$

3.1F:- Varignon's theorem

$$M_A = \vec{r} \times \vec{F_1} + \vec{r} \times \vec{F_2} + \vec{r} \times \vec{F_3}$$

$$= \vec{r}\,(\vec{F_1} + \vec{F_2} + \vec{F_3})$$

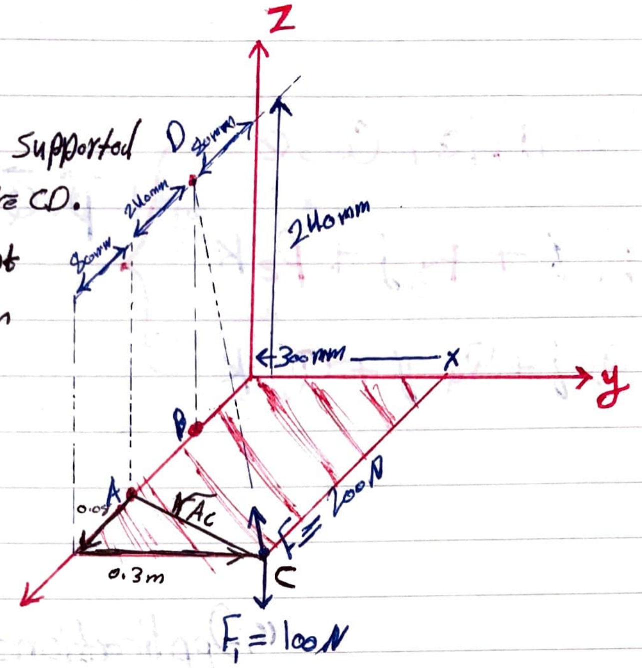
$$= \vec{r} \times \vec{F_R}$$



* The moment about a given point of the resultant of several concurrent forces is equal to the sum of the moments of the various forces about point O.

exp 8-

- Rectangular plate is supported by brackets at A and B and wire CD. Determine the moment about point A of the forces exerted on point C



$$f_1 = -100\,K$$

$$\vec{CD} = D - C = -320\,i - 300\,j + 240\,K \text{ (mm)}$$

$$|\vec{CD}| = \sqrt{(320)^2 + (300)^2 + (240)^2} = 500 \text{ m}$$

$$\lambda_{CD} = \frac{\vec{CD}}{|\vec{CD}|} = -\frac{320}{500}\,i - \frac{300}{500}\,j + \frac{240}{500}\,K$$

$$= -0.64\,i - 0.6\,j + 0.48\,K$$

$$F_2 = |F_2|\,\lambda_{CD} = 200(-0.64\,i - 0.6\,j + 0.48\,K)$$

$$= -128\,i - 120\,j + 96\,K \text{ (N)}$$

$$\vec{F_R} = \vec{F_1} + \vec{F_2} = -128\,i - 120\,j - 4\,K \text{ (N)}$$

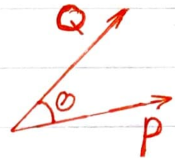
3.2:- Scalar Product (Dot product)

$$\vec{P}\cdot\vec{Q} = |\vec{P}||\vec{Q}| \cos\theta$$

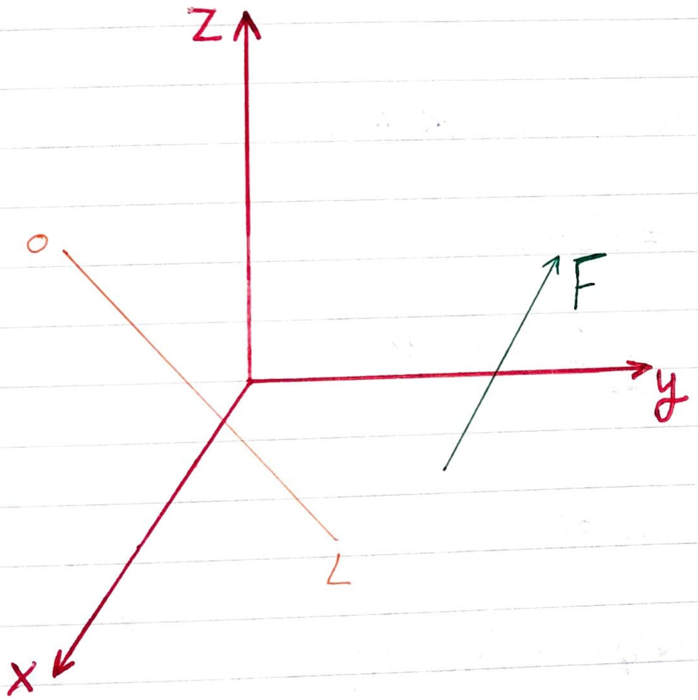
$$\vec{P} = P_x i + P_y j + P_z K$$

$$\vec{Q} = Q_x i + Q_y j + Q_z K$$

$$\vec{P}\cdot\vec{Q} = (P_x)(Q_x) + (P_y)(Q_y) + (P_z)(Q_z)$$



"Applications"

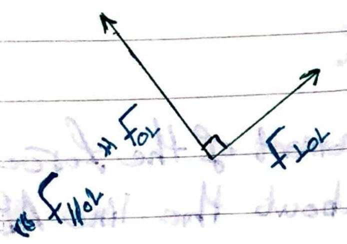


F_{OL} = Component of the force F along line direction OL.

$$F_{OL} = \vec{F}\cdot\vec{U}_{OL}$$

* unit vector باتجاه OL (أو على امتداده)

$$|\vec{F}|^2 = |F_{oL}|^2 + |F_{\perp oL}|^2$$

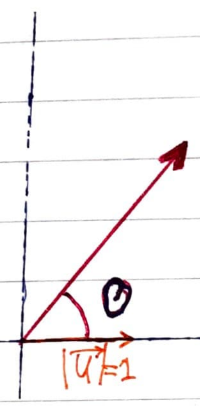


$$\therefore |F_{\perp oL}| = \sqrt{(|F|^2) + (|F_{oL}|^2)}$$

*حفظ/ مقدار "scalar"

$$\vec{F}_{\perp oL} + \vec{F}_{oL} = \vec{F}$$

جمع إتجاهي .

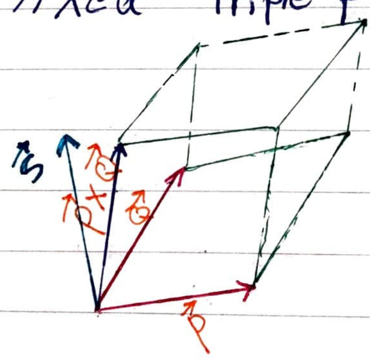


$$\Rightarrow \vec{F}\cdot\vec{u} = (F)(1)\cos\theta$$

$$= F\cos\theta$$

* مركبة موازية للمحور الموجود عليه u

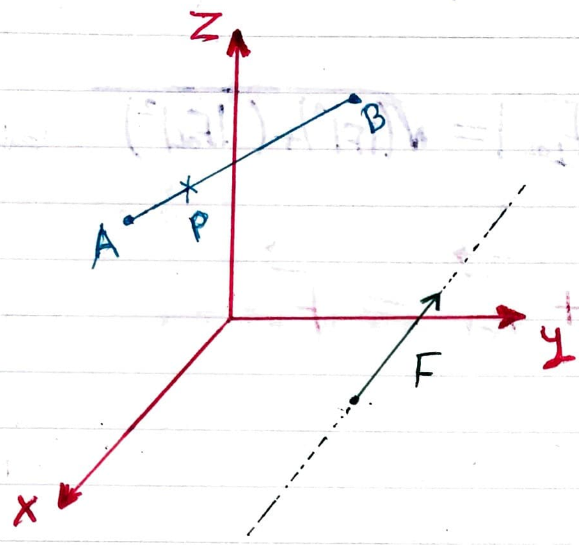
3.2 B :- Mixed Triple product of Three vectors.



$$\vec{S}\cdot(\vec{P}\times\vec{Q})$$

$$= \begin{vmatrix} S_x & S_y & S_z \\ P_x & P_y & P_z \\ Q_x & Q_y & Q_z \end{vmatrix}$$

3.2 "C" :- Moment of a force about a given axis



* M_{AB} = moment of the force (F) about the line AB
* point "p" : select any point lies on the line AB.

$$M_p = \vec{r} \times \vec{F}$$

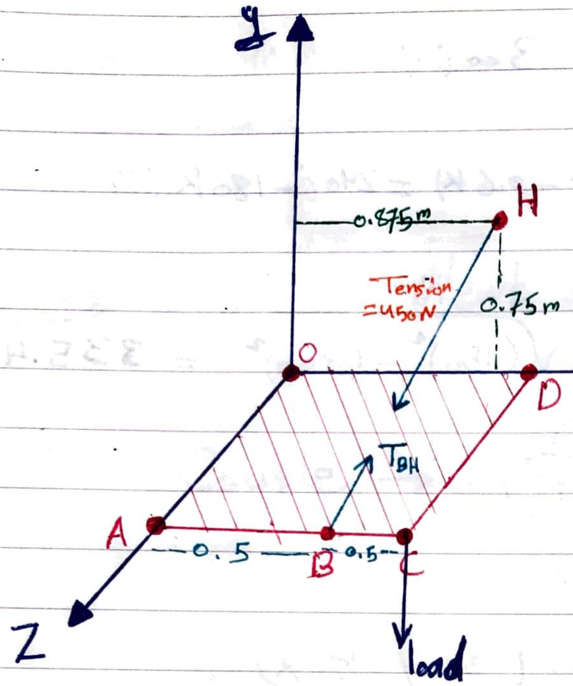
λ_{AB} = unit vector along line AB

$$|M_{AB}| = \lambda_{AB} \cdot M_p$$

$$= \lambda_{AB} \cdot |M_{AB}|$$

$\vec{r}$: is the position vector from any point on line AB to any point on the line of action of the force $\vec{F}$.

Ex.1:-



① Determine the component of the tension in cable BH exerted at point B parallel and perpendicular to the line AD.

② Determine the moment about the line AD of the force exerted on the force by cable BH

③ Determine the perpindicular distance between cable BH and line AD.

1) $\overrightarrow{BH} = 0.375i + 0.75j - 0.75K \ (m)$

$|\overrightarrow{BH}| = \sqrt{(0.375)^2 + (0.75)^2 + (0.75)^2} = 1.125 \ m$

$\lambda_{BH} = \dfrac{\overrightarrow{BH}}{|\overrightarrow{BH}|} = \frac{1}{3} i + \frac{2}{3} j - \frac{2}{3} K$

$\overrightarrow{T_{BH}} = 450 \ \lambda_{BH} = 150 i + 300 j + -300 K \ (N)$

$\overrightarrow{AD} = i - 0.75 K \ (m)$

$|\overrightarrow{AD}| = \sqrt{(1)^2 + (0.75)^2} = 1.25 \ m$

$\lambda_{AD} = \dfrac{\overrightarrow{AD}}{|\overrightarrow{AD}|} = 0.8i - 0.6K$

$$T_{\parallel AD} = \lambda_{AD} \cdot T_{BH} = (0.8 i - 0.6 k) \cdot (150 i + 300 j - 300 k)$$

$$= (0.8)(150) + (-0.6)(-300) = 300 \ (N)$$

$$T_{\parallel AD} = |T_{\parallel AD}| \ \lambda_{AD} = (300)(0.8 i - 0.6 k) = 240 i - 180 k \ (N)$$

$$\Rightarrow F_{\perp AD} = \sqrt{(T_{BH})^2 - (T_{\parallel AD})^2} = \sqrt{(450)^2 - (300)^2} = 335.41 \ (N)$$

or

$$T_{\perp AD} = (150 i + 300 j - 300 k) - (240 i - 180 k)$$

$$= -90 i + 300 j - 120 k$$

② $\vec{r}_{AB} = 0.5 i \ (m)$ * $\vec{AB}$ هو أنسب وأسهل وأقرب متجه بين القوة والخط.

$$|M_{AD}| = \lambda_{AD} \cdot (\vec{r}_{AB} \times \vec{T}_{BH})$$

$$= \begin{vmatrix} 0.8 & 0 & -0.6 \\ \overset{\ominus}{0.5} & \overset{\oplus}{0} & \overset{\ominus}{0} \\ 150 & 300 & -300 \end{vmatrix} = -0.5 [0 + 180] = -90 \ N.m$$

* عندما نركز في صف أسفل مباشرة نغير الإشارات.

③ $M_{AD} = (T_{IAD})\, d$

$\therefore d = \frac{90 \text{ N.m}}{335.41 \text{ N}} = \text{—} \text{ m}$

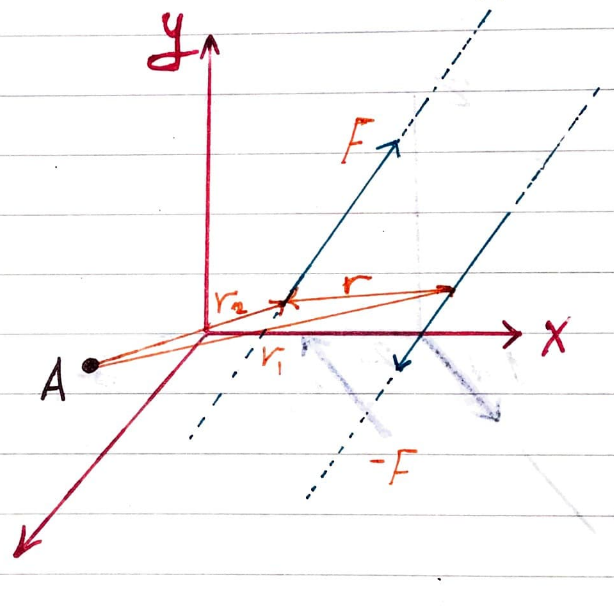
3.3:- A:- Moment of acouple

Couple → two forces → Same magnitude
→ parrallet line of action
→ opposite sense

Ⓐ ⟶ ⓘ

$\vec{r_1} = \vec{r_2} + \vec{r}$

$\vec{r} = \vec{r_1} - \vec{r_2}$



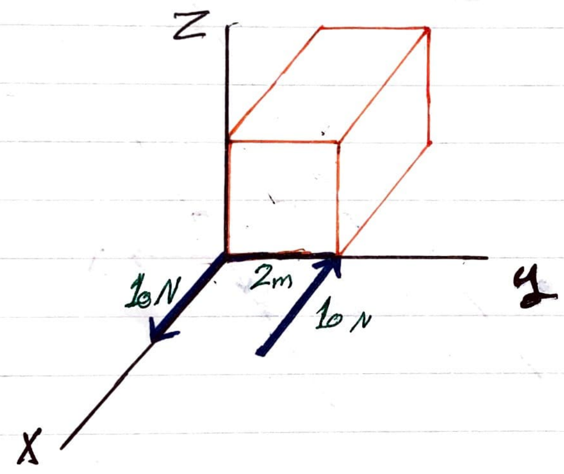
$$M_A = \vec{r_1} \times \vec{F} + \vec{r_2} \times (-F) = (\vec{r_1} - \vec{r_2}) \times \vec{F}$$
$$= \vec{r} \times \vec{F}$$

$F_R = 0$

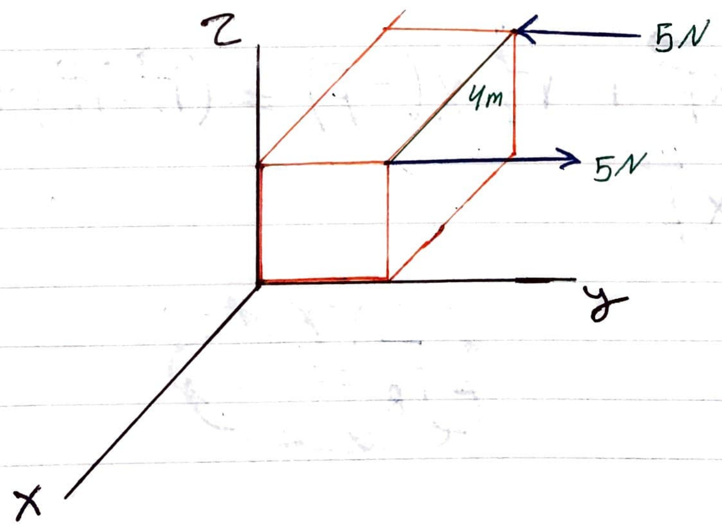
* The moment at a couple is independent at the point a bout which it's Computed.

* The moment vector is free Vector.

equivallent Couples

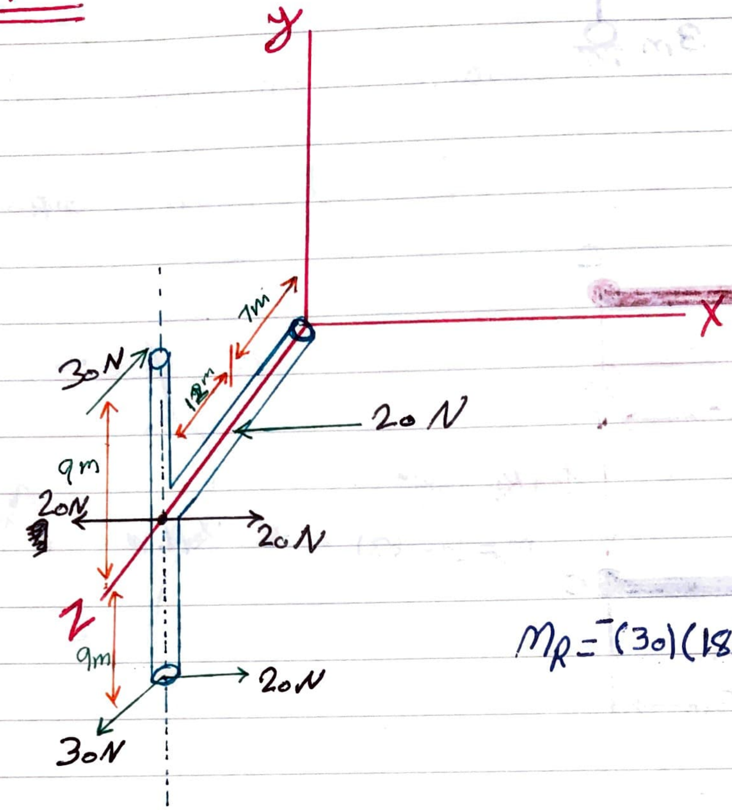


$$M = (10)(2)$$
$$= 20\ N.m \quad +Z$$



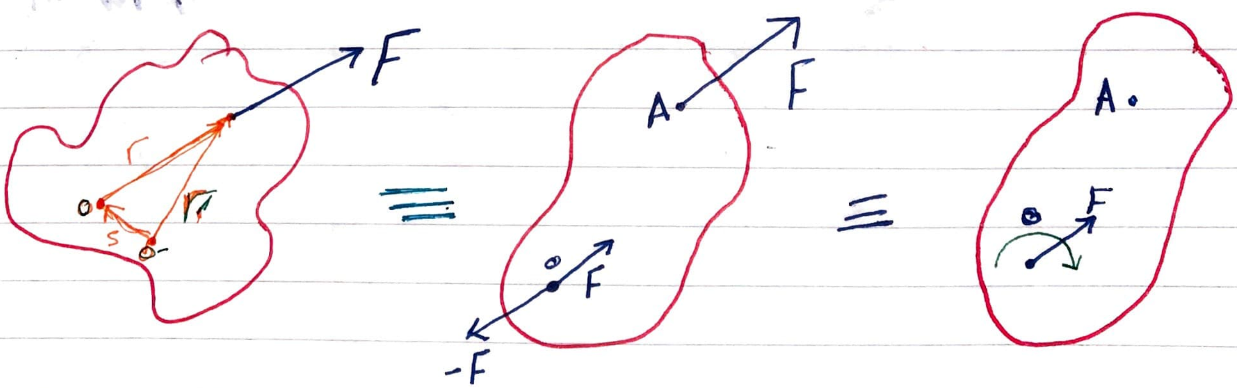
$$M = (5)(4)$$
$$= 20\ N.m$$
$$+Z$$

EX. Determine the equivalent resultant couple to the couples shown.



$M_R = -(30)(18)j + 20(12)j + 20(9)k$

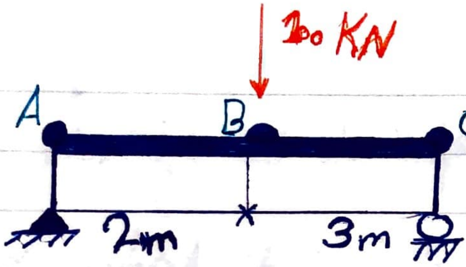
3.3 e :- Resolution of a given force into a force at O and a couple



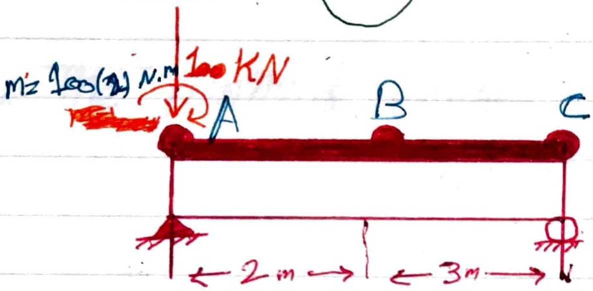
$A \to \bar{o}$:- $M_{\acute{o}} = \acute{r} \times \vec{F} = (\vec{s} + \vec{r}) \times \vec{F}$

$M_{\acute{o}} = \vec{s} \times \vec{F} + \vec{r} \times \vec{F}$

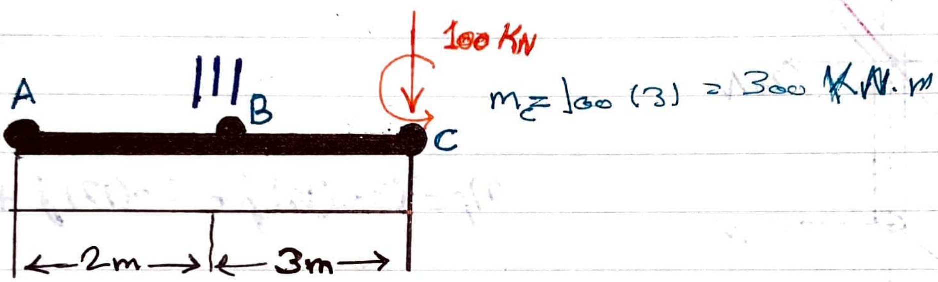
$= M_o + \vec{s} \times \vec{F}$



III



B → C



* B → A → C :-

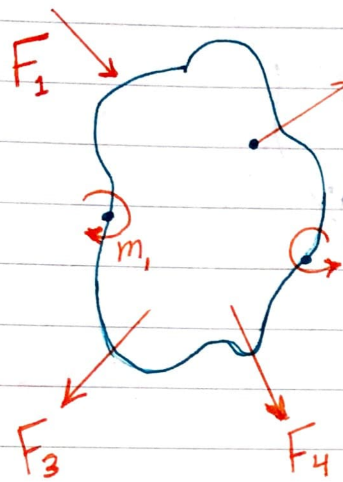
$$\circlearrowleft^{+} M_c = -200 + 100(5) = -200 + 500$$

$$= 300 \ KN \cdot m$$

3.4 Simplifying systems of forces

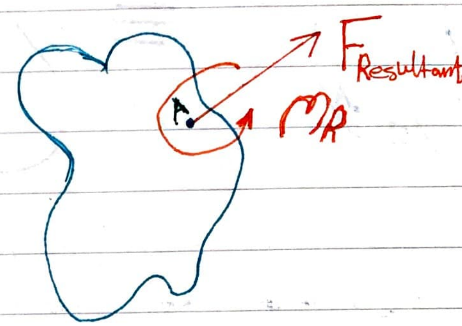
3.4 A :- Reducing a system of forces to a single Forces

- (single Couple system).



*same body

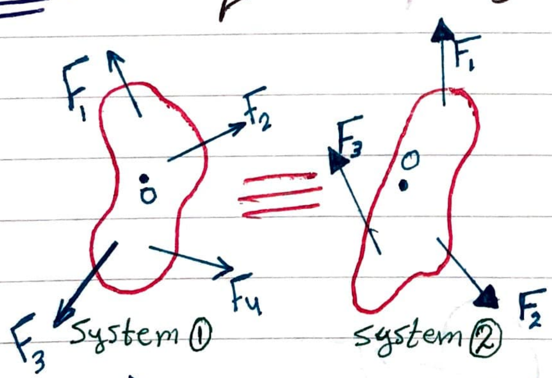
$\equiv$



$$\vec{F}_R = \sum \vec{F}$$

$$\vec{M}_A^R = \sum \vec{r} \times \vec{F}$$

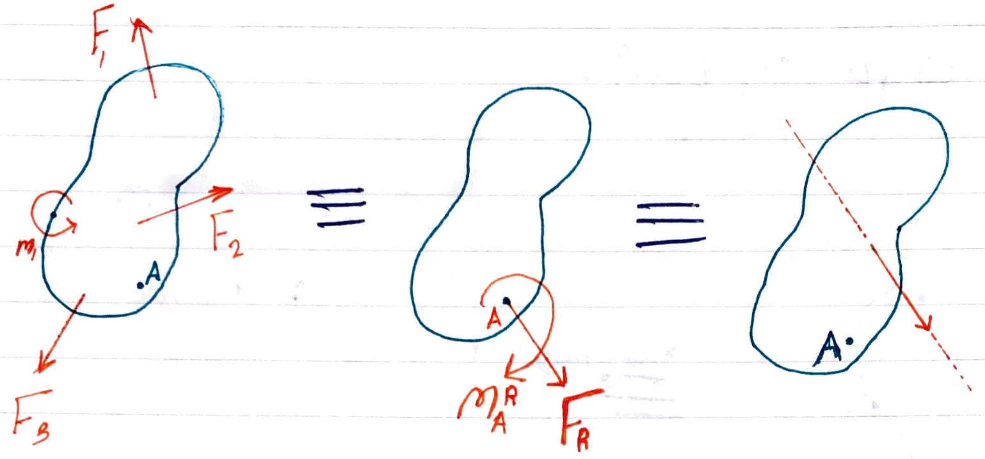
3.4B:- Equivalent systems of forces



$$(\vec{F}_R)_{system ①} = (\vec{F}_R)_{system ②}$$

$$(M_O^R)_{system} = (M_O^R)_{system ②}$$

3.4 c . Farther Reduction of a system of forces.



Force Couple

only Single force

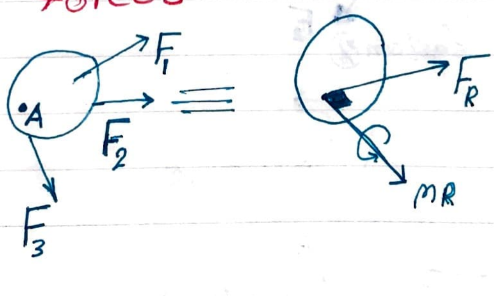
Conditions → ① Parallel forces.

② Concurent forces.

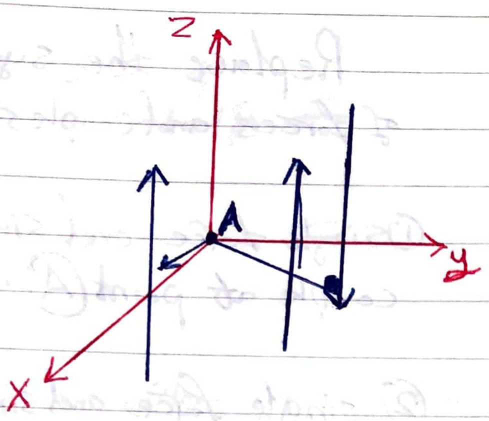
③ Co - planar forces.

$F_R \overset{?}{\perp} M_R$

① co-planar forces



② parallel forces



$F_R = R\mathbf{k}$

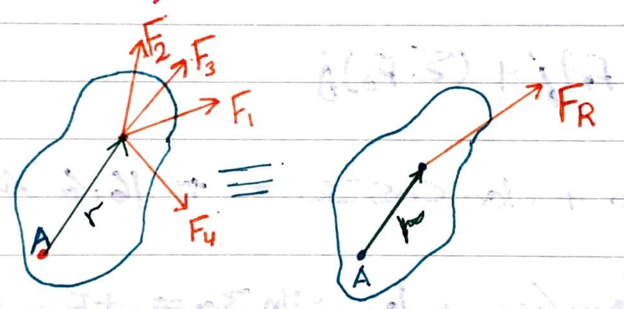
$\vec{r} = x\mathbf{i} + y\mathbf{j}$

$M_A^R = \Sigma(r \times F)$

$= \Sigma(x\mathbf{i} + y\mathbf{j}) \times (F\mathbf{k})$

$= \Sigma(Fx)\mathbf{j} + \Sigma(Fy)\mathbf{i}$

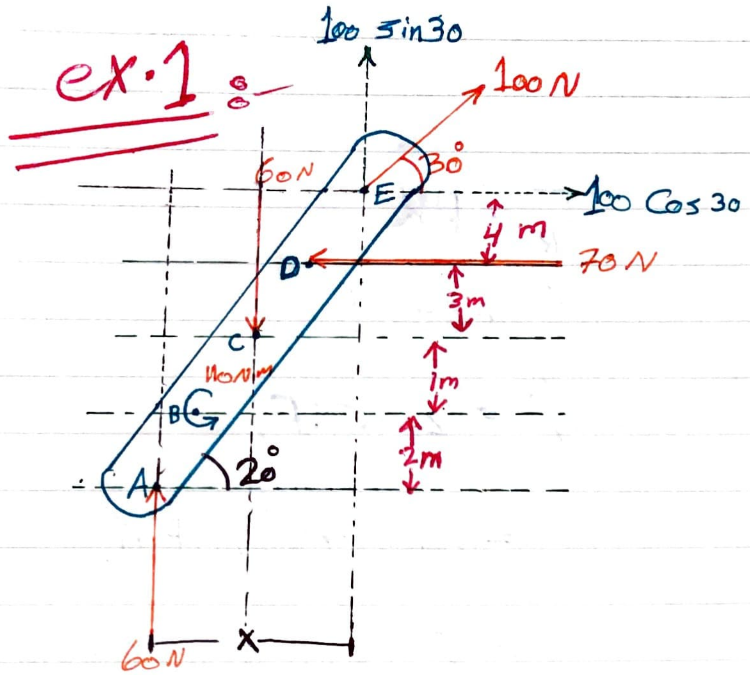
③ Concurrent forces



$M_A^R = \Sigma(r \times F)$

$= r \times F^R$

ex.1 :



Replace the system of forces and couples by

① single force and single couple at point Ⓐ.

② single force and single couple at B.

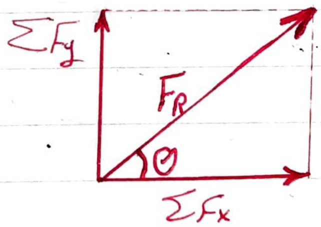
③ single force and locate it's line of action.

1) $F_R = \sum \vec{F} = (\sum F_x)\,i + (\sum F_y)\,j$

$\overset{+}{\rightarrow} \sum F_x = -70 + 100 \cos 30 = +16.6\ N\ (\longrightarrow)$

$+\uparrow \sum F_y = +60 - 60 + 100 \sin 30 = +50\ N\ (\uparrow)$

$F_R = \sqrt{(16.6)^2 + (50)^2} = 52.68\ N$

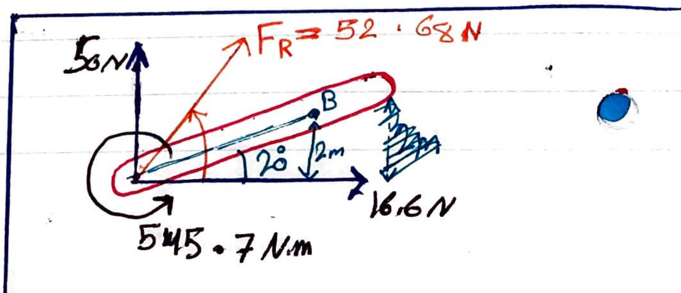


$\theta = \tan^{-1}\left(\frac{50}{16.6}\right) = 71.6°$

$\vec{F_R} = 16.6\,i + 50\,j\ (N)$

$\circlearrowleft +\ M_A^R = -(60)(8.2) + 110 + (70)(6) - (100 \cos 30)(10) + (100 \sin 30)(x)$

$= +545.7\ (N.m)$

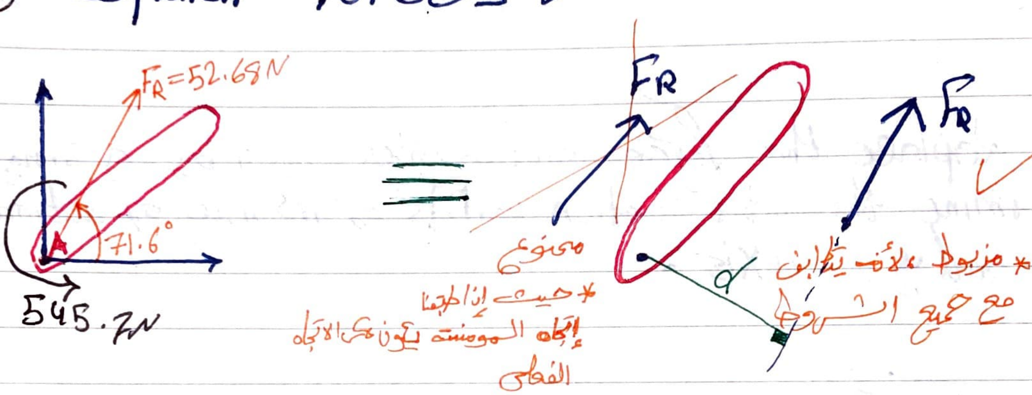


2) $F_R = 52.68\ N$ $\quad \theta = 71.6^\circ$

$\circlearrowright + \sum M_B = +545.7 + (16.6)(2) - (50)(2/\tan 20)$

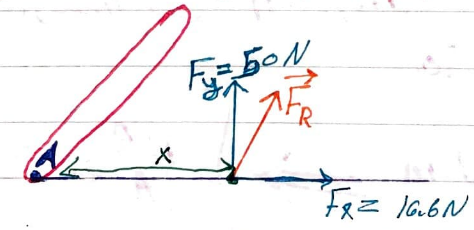
$= +304.1\ N.m$

③ Coplanar forces



$$d = \frac{M_A}{F_R} = \frac{545.7}{52.68} = 10.4\ m$$

additional question :- Find the distance between point A and the point at which the line of action of F_R intersects the horizontal

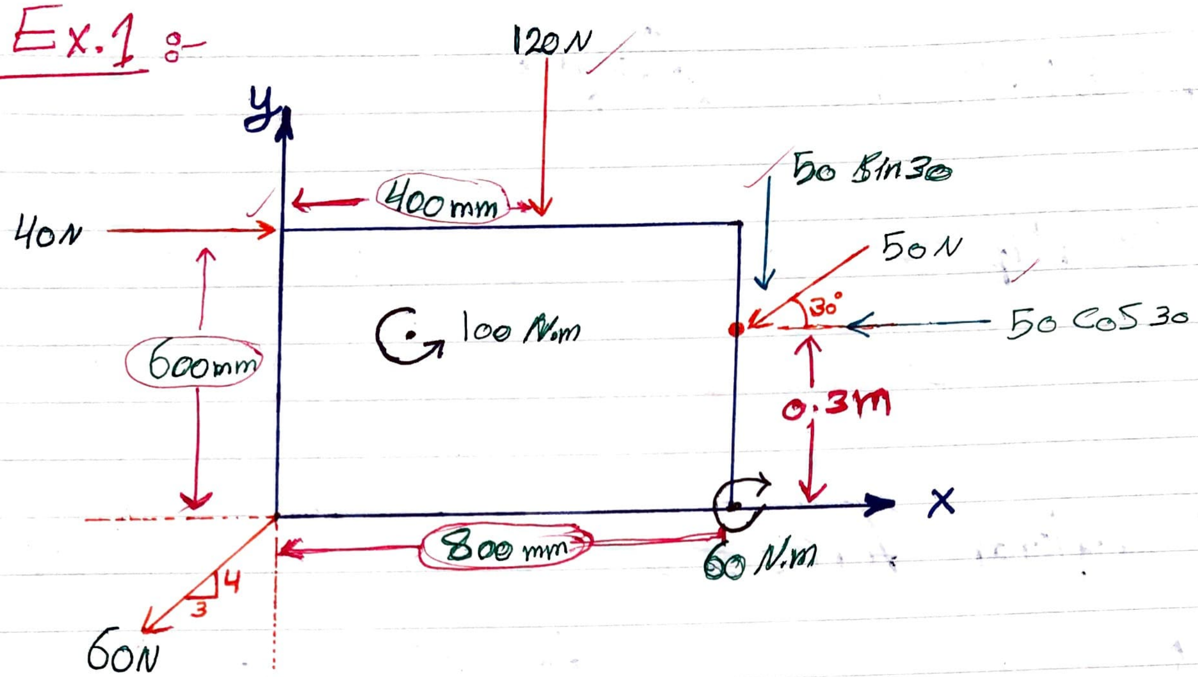


$M_A^R = x \cdot F_y \Rightarrow x = \frac{545.7}{50}$

$= \text{---}\ m$

#

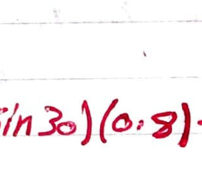
Ex.1 :-



* Replace the forces and couples shown by a single force and determine the intersection points of its line of action with the X and y axis.

$$\vec{F}_R = \sum \vec{F} = 40\,i - 120\,j - 50\sin 30\,j - 50\cos 30\,i - 60\left(\frac{3}{5}\right)i - 60\left(\frac{4}{5}\right)j$$

$$\vec{F}_R = -39.3\,i - 193\,j \ (N)$$

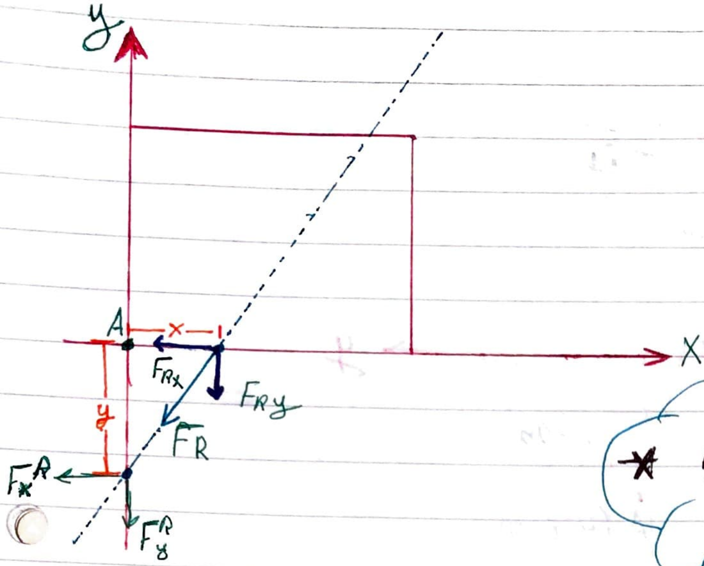


$$|\vec{F}_R| = \sqrt{(39.3)^2 + (193)^2} = 196.9\ N$$

$$\theta = \tan^{-1}\left(\frac{193}{39.3}\right) = 78.5^\circ$$

$$\circlearrowleft + \sum M_A^R = +(40)(0.6) - (120)(0.4) - (50\sin 30)(0.8) + (50\cos 30)(0.3) - 60 + 100$$

$$= -39\ N.m \implies \boxed{M_A^R = 39\ N.m \ \circlearrowright}$$

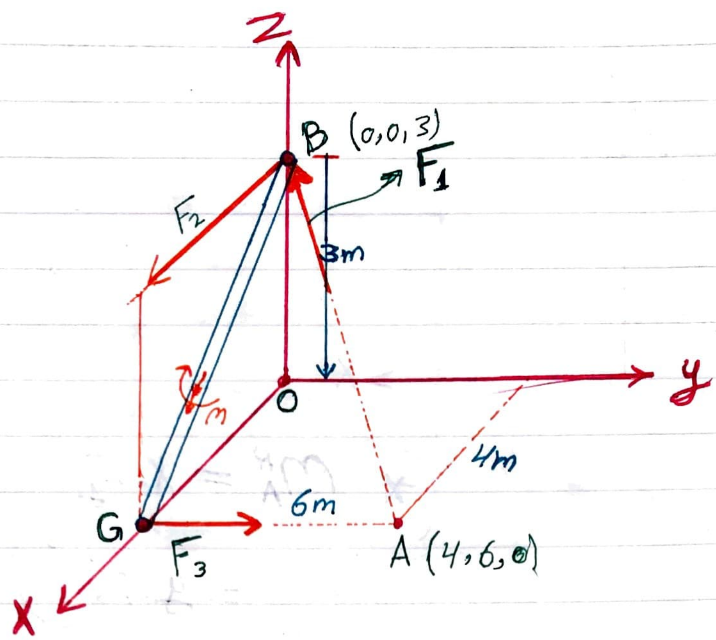


$$M_A^R = x \cdot F_y$$
$$= y \cdot F_x$$

$$x = \frac{39 \cdot N.m}{193\,N} = \ldots$$

$$y = \frac{39\ N.m}{39.3} = \ldots$$

Ex. 2 :-



Determine the equivalent force-couple system with a force acting at point G, where $F_1 = 100\,N$, $F_2 = 90\,N$, $F_3 = 120\,N$.

$M = 200\ N.m$

$$\vec{F_R} = \sum \vec{F}$$

$$\vec{AB} = -4i - 6j + 3K\ (m)$$

$$|\vec{AB}| = 7.8\ m \Rightarrow \lambda_{AB} = \frac{\vec{AB}}{|\vec{AB}|} = \frac{-4i - 6j + 3K}{7.8}$$

$$\vec{F_1} = 100\,\lambda_{AB} = -51.22\,i - 76.82\,j + 38.41\,K\ (N)$$

$$\vec{F_2} = +90\,i\ (N)$$

$$\Rightarrow \vec{F_R} = +38.78\,i + 43.18\,j + 38.41\,K\ (N)$$

$$F_3 = +120\,j\ (N)$$

$$\lambda_{Bg} = \frac{+4}{5}\,i - \frac{3}{5}\,K$$

$$\vec{M} = 200\,(\lambda_{BG})$$

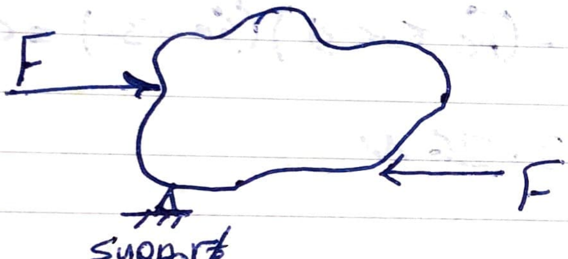
$$= 160\,i - 120\,K\ N.m$$

$$M_G^R = \vec{r}_{GB} \times \underbrace{(F_1 + F_2)}_{\text{at the same point}} + \vec{M}$$

$$= (-4i + 3K) \times (38.78i - 76.82j + 38.41K) + (160j - 120K)$$

$$= \begin{vmatrix} i & j & K \\ -4 & 0 & +3 \\ 38.78 & -76.82 & 38.41 \end{vmatrix} + (160i - 120K)$$

Ch.4 "Equilibrium of rigid bodies"



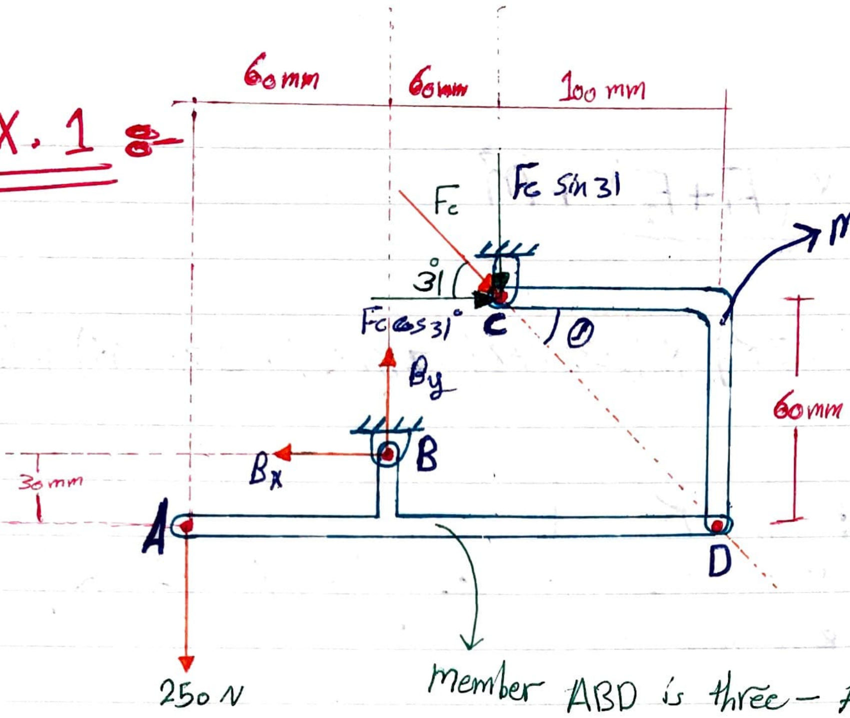
* No translation ⇒ $\Sigma F = 0$ "in any direction"

* No rotation ⇒ $\Sigma M = 0$ "about any axis"

Supports → prevent translation → Reaction Force.

// Rotation → Reaction moment.

Ex. 1 :-



→ member CD is two force body

$\theta = \tan^{-1}\left(\frac{60}{100}\right) = 31$

$\circlearrowleft + \sum M_B = 0 :$

$$+250(60) - (F_c \sin 31)(60) - (F_c \cos 31)(60-30) = 0$$

$$\Rightarrow F_c = 323.86\ N$$

$\xrightarrow{+} \sum F_x = 0 : -B_x + 323.86 \cos 31 = 0$

$$D_x = 277.6\ N$$

$+\uparrow \sum F_y = 0 : -250 + B_y - 323.86 \sin 31 = 0$

$$B_y = 416.8\ N$$

x راجع 10 صفحات للموضوع
التالي وليس هنا .

Ch4:- Equilibrium of rigid bodies

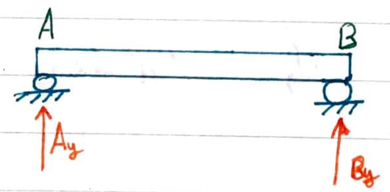
$\sum F = 0$

3D	2D
$\sum F_x = 0$	$\sum F_x = 0$
$\sum F_y = 0$	$\sum F_y = 0$
$\sum F_z = 0$	

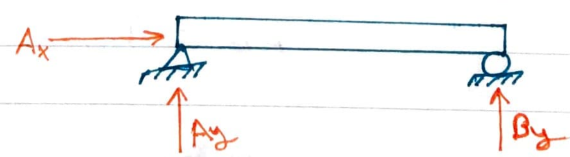
$\sum M = 0$

3D	2D
$\sum M_x = 0$	$\sum M_z = 0$
$\sum M_y = 0$	3 equations
$\sum M_z = 0$	
6 equilibrium equations	

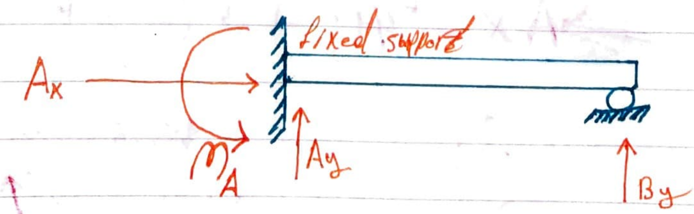
Equilibrium in 2D :-



2 Reactions < 3 equations
" unstable "

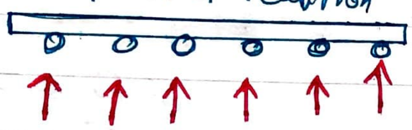


3 Reaction = 3 equations
"stable and determinate"

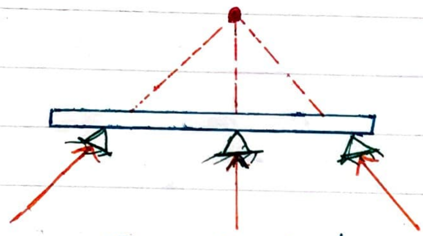


4 reactions ~~equation~~ > 3 equations
"stable and Indeterminate"

parallel reaction



unstable



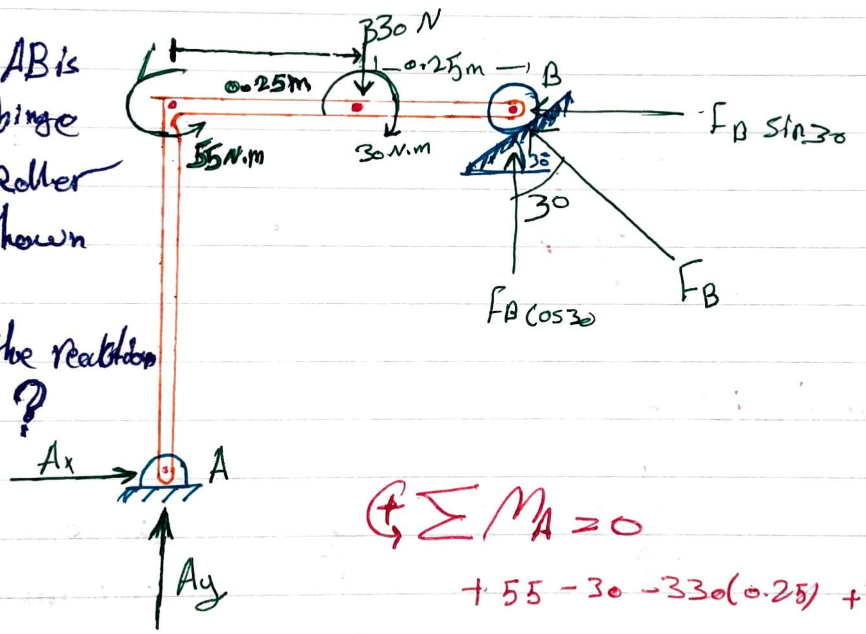
Concurrent reactions

$\sum M \neq 0$

Ex.

Rigid body AB is supported by hinge at A and Roller at B as shown

Determine the reaction at A and B ?



$\circlearrowleft + \sum M_A = 0$

$+55 - 30 - 330(0.25) + (F_B \cos 30)(0.5) + (F_B \sin 30)(0.3) = 0$

$\Rightarrow F_B = 98.63\ N$

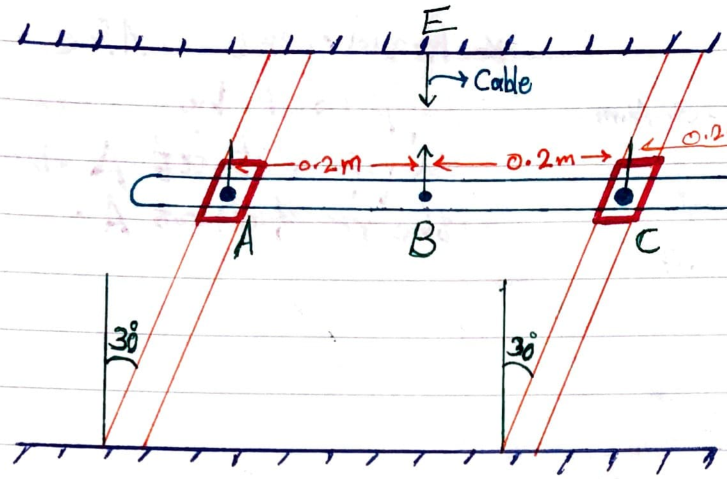
$\xrightarrow{+} \sum F_x = 0$: $+A_x - 98.63(\sin 30) = 0$

$\Rightarrow A_x = +49.31\ N\ (\longrightarrow) +$

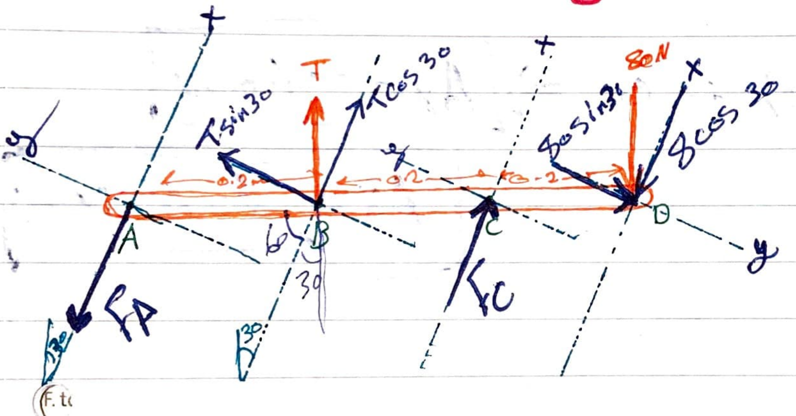
$\sum F_y = 0$, $+A_y - 330 + 98.63 \cos 30 = 0$

$\Rightarrow A_y = 244.58\ N\ (\uparrow)$

Ex.2 :- Determine the tension in cable BE and reactions at A and C.



Draw free-body diagram :-



$\nwarrow^{+} \sum F_y = 0 : \quad +T \sin 30 - 80 \sin 30 = 0$

$$\boxed{T = 80\ N}$$

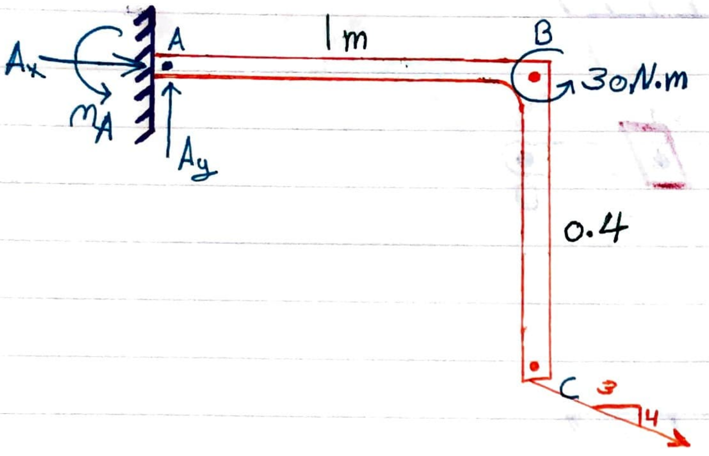
$\nearrow^{+} \sum F_x = 0 : \quad F_C - F_A + T \cancel{\cos 30} - 80 \cancel{\cos 30} = 0$

$$\boxed{F_C = F_A}$$

$\circlearrowleft^{+} \sum M = 0 , \quad -80(0.4) + F_C(0.4 \sin 60) = 0$

$$\boxed{F_C = F_A = 160\ N}$$

Ex.3:-



* Rigid member ABC is supported by fixed support at A. Determine the reactions at A.

$$(\xrightarrow{+}) \sum F_x = 0$$

$$+A_x + \frac{3}{5}(100) = 0 \Rightarrow \boxed{A_x = -60\,N}$$

$$\boxed{A_x = 60\,N\ (\leftarrow)}$$

$$(+\uparrow) \sum F_y = 0$$

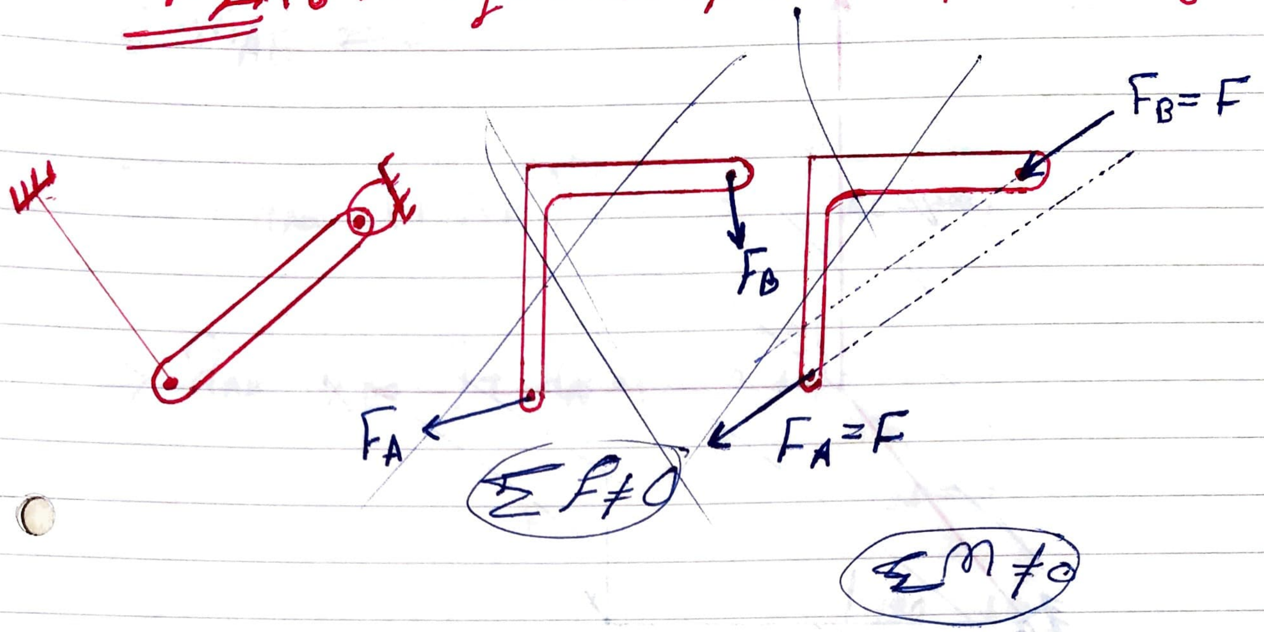
$$+A_y - \frac{4}{5}(100) = 0 \Rightarrow \boxed{A_y = {}^{+}80\,N\ \uparrow}$$

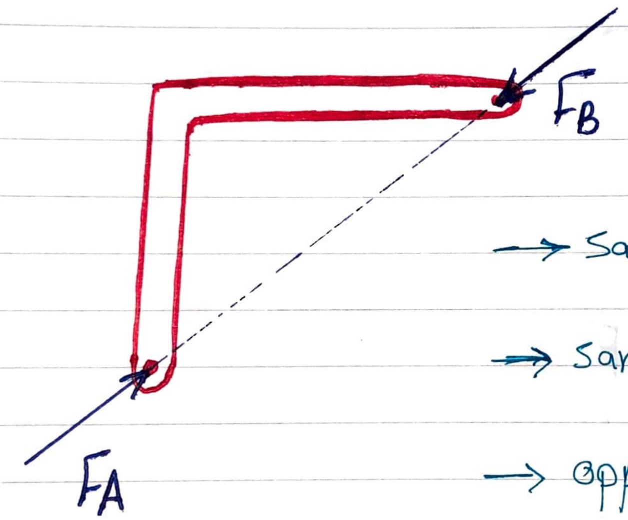
$$(\circlearrowleft +) \sum M_A = 0$$

$$+M_A + 30 - \frac{4}{5}(100)(1) + \frac{3}{5}(100)(0.4) = 0$$

$$M_A = +26\ N.m \circlearrowleft$$

4.2A: Equilibrium of a Two-force body



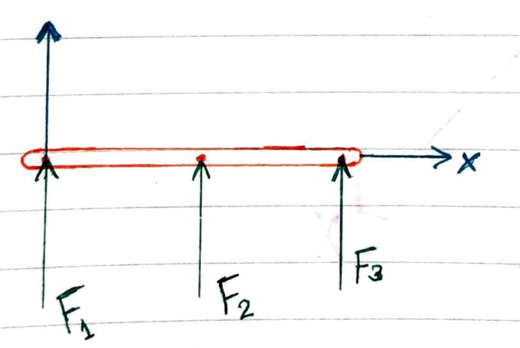


→ Same magnitude

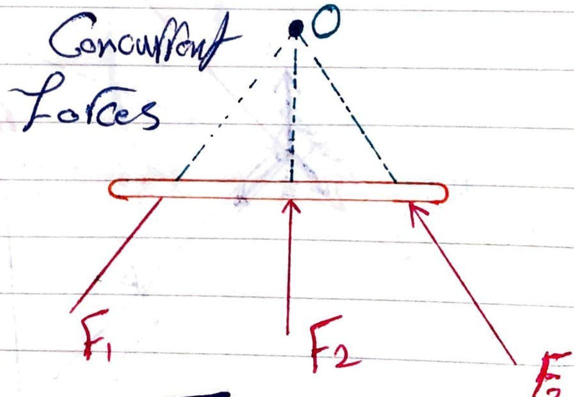
→ Same line of action (Co-linear)

→ opposite direction

4.2B: Equilibrium of a three-force body



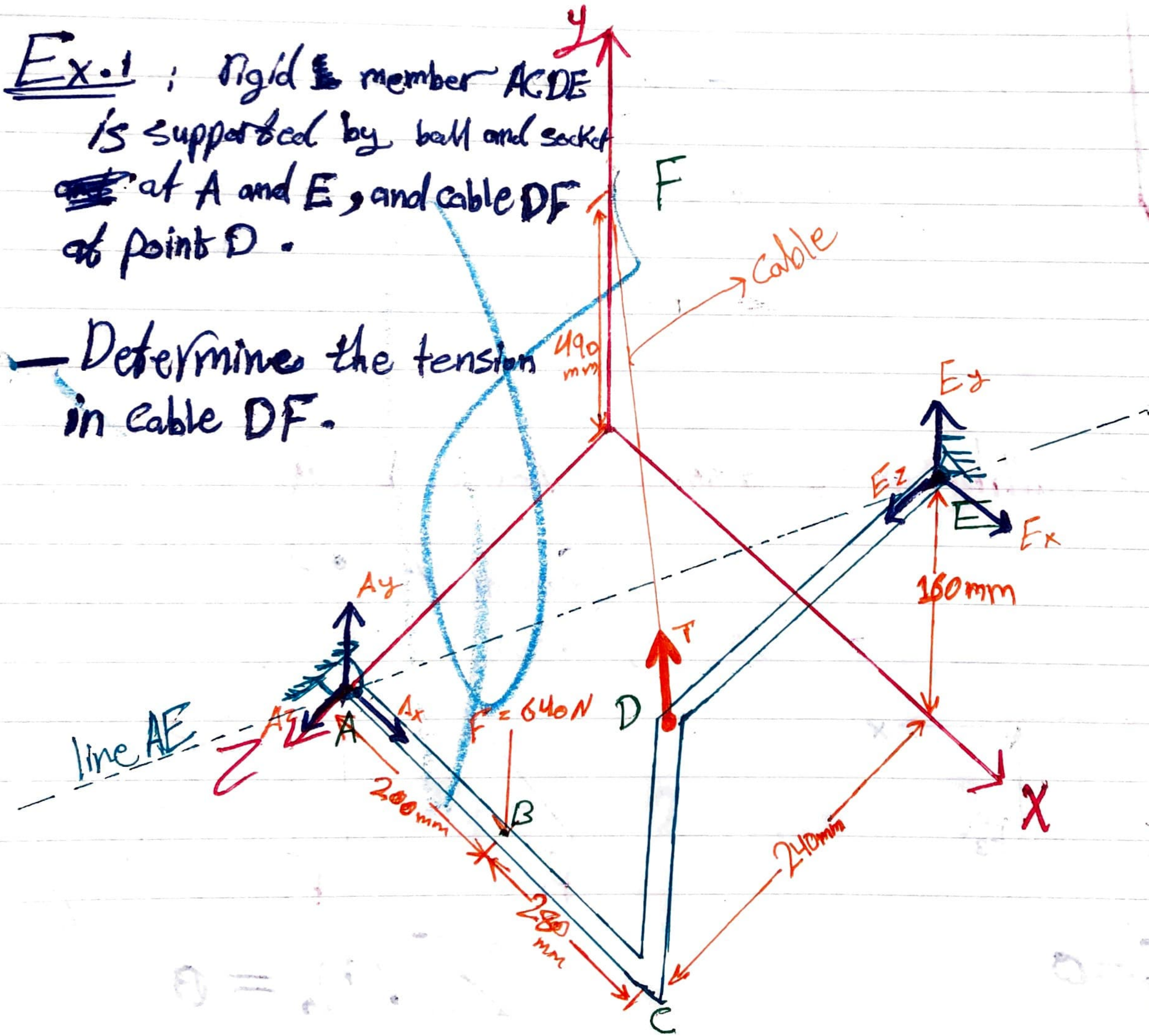
$$\Sigma F_x = 0$$



$$\Sigma M_o = 0$$

Ex.1 ; rigid member ACDE is supported by ball and socket at A and E, and cable DF at point D.

— Determine the tension in cable DF.



$$\Sigma M_{AE} = 0$$

$A(0,0,240)$

$E(480,160,0)$

$\lambda_{AE} = \frac{480}{560} i + \frac{160}{560} j - \frac{240}{560} k$ (mm)

$$\Sigma [\lambda_{AE} \cdot (\vec{r} \times \vec{F})] = 0$$

$F = -640 j$ (N)

$r_{AB} = +0.2 j$ (m)

$$\Rightarrow \lambda_{AE} \cdot (r_{AB} \times \vec{F}) + \lambda_{AE} \cdot (\vec{r}_{ED} \times \vec{T}) = 0.$$

$\lambda_{DF} = \frac{-480}{630} i + \frac{330}{630} j - \frac{240}{630} k$

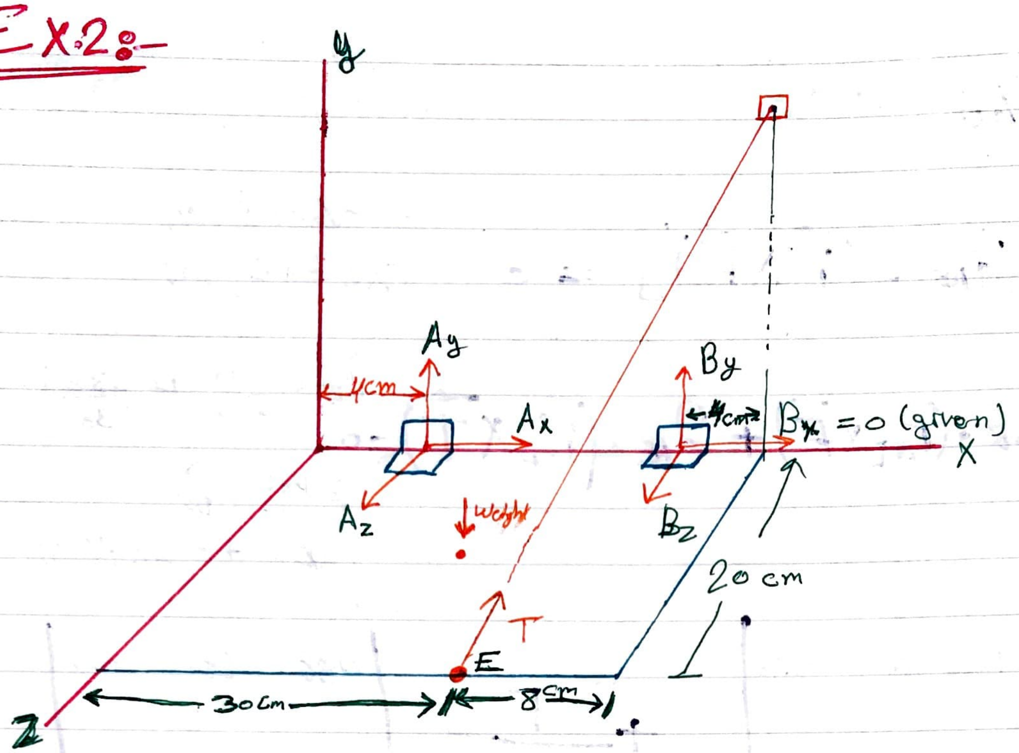
$T = |T| \vec{\lambda}_{DF}$

$r_{ED} = +0.24 k$ (m)

$$\frac{1}{560} \begin{vmatrix} 480 & 160 & -240 \\ +0.2 & 0 & 0 \\ 0 & -640 & 0 \end{vmatrix} + \frac{T}{(560)(630)} \begin{vmatrix} 480 & 160 & -240 \\ 0 & 0 & +0.24 \\ -480 & 330 & -240 \end{vmatrix} = 0$$

$$\boxed{T = 342.9 \text{ N}}$$

Ex2:-



- The plate shown is supported by hinges at A and B and by wire at E, Determine the tension in wire EF and reactions at A and B.

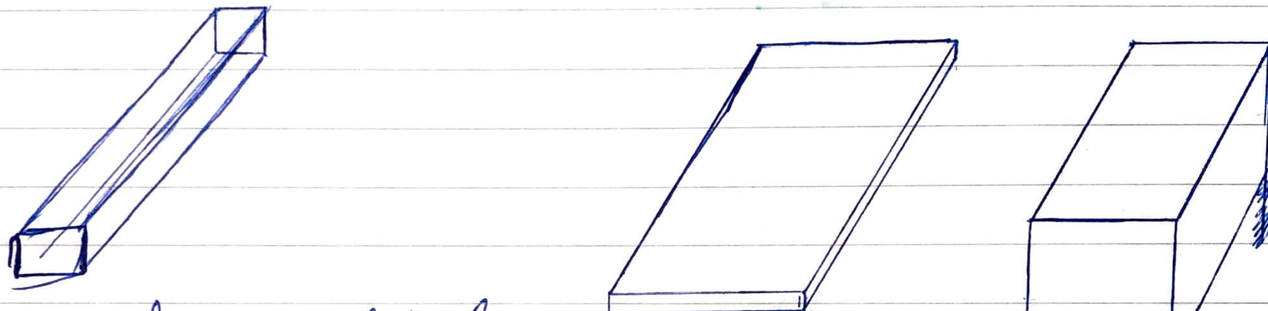
* Weight of plate = 75 N (the weight is at center of mass)

$$T = |T| \lambda_{EF} = \frac{8T}{33} i + \frac{25T}{33} j - \frac{20T}{33} k$$

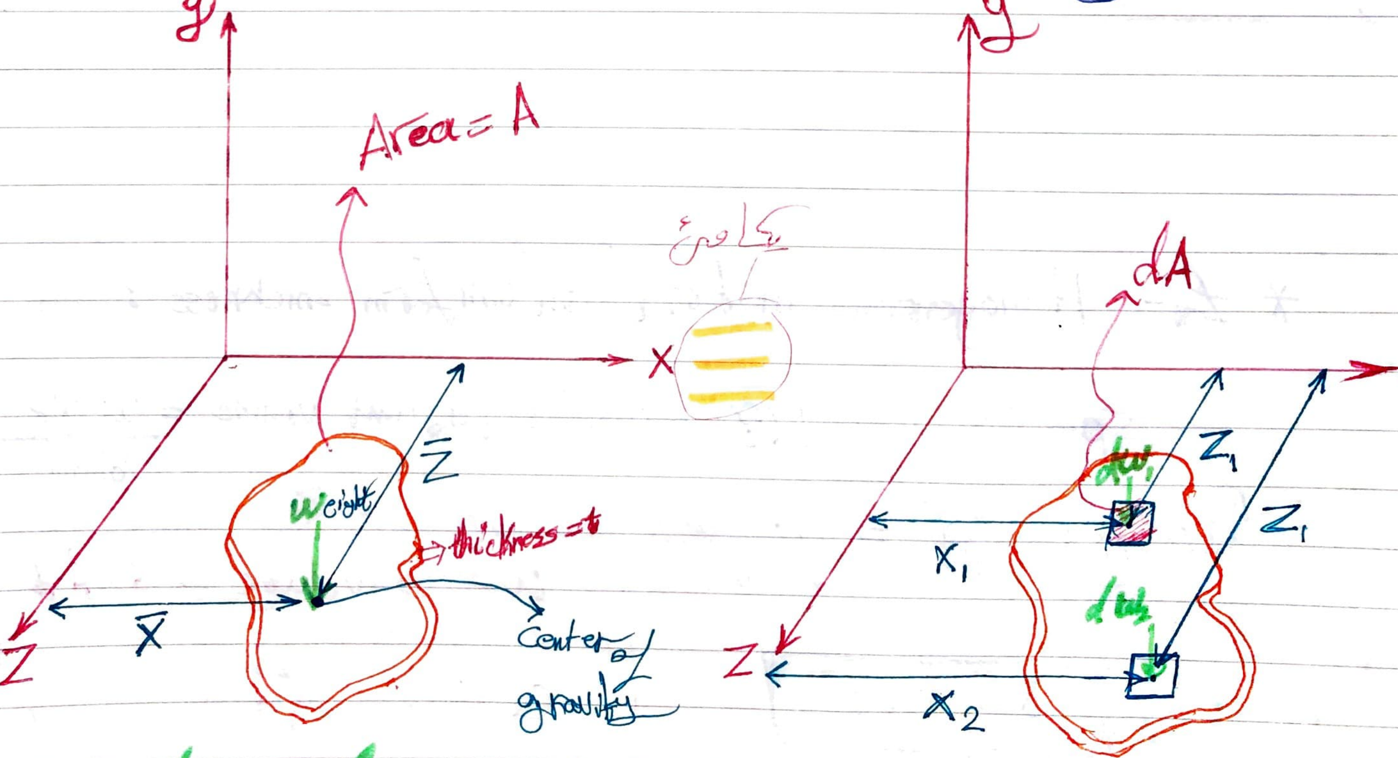
$$\sum M_{x\text{-axis}} = 0 \Rightarrow \sum \lambda_x \cdot (\vec{r} \times \vec{F}) = 0$$

Ch 5: "Distributed forces: Centroids and Center of Gravity"

5.1: "A center of gravity of a two-dimensional body".



one dimensional body 2D 3D



$w = dw_1 + dw_2 + dw_3$ — —

$w = \int dw$

$$\sum M_x = W \cdot \bar{Z} = dw_1 \cdot z_1 + dw_2 \cdot z_2 \ldots\ldots$$

$$W \cdot \bar{Z} = \int z \cdot dw$$

$$\bar{Z} = \frac{\int z \cdot dw}{\int dw}$$

$$\sum M_z = W \cdot \bar{x} = dw_1 \cdot x_1 + dw_2 \cdot x_2 \ldots\ldots$$

$$\bar{x} = \frac{\int x \cdot dw}{\int dw}$$

* for Homogeneous material and uniform thickness :

specific weight (γ) = weight per unit volume = $\frac{\text{weight } (w)}{\text{Volume } (V)}$

g/*

Density (ρ) = $\frac{\text{Mass } (m)}{\text{Volume } (V)}$

$$\boxed{\gamma = \rho \cdot g}$$

$W = \gamma \cdot \text{Volume} = \gamma \cdot A \cdot t$

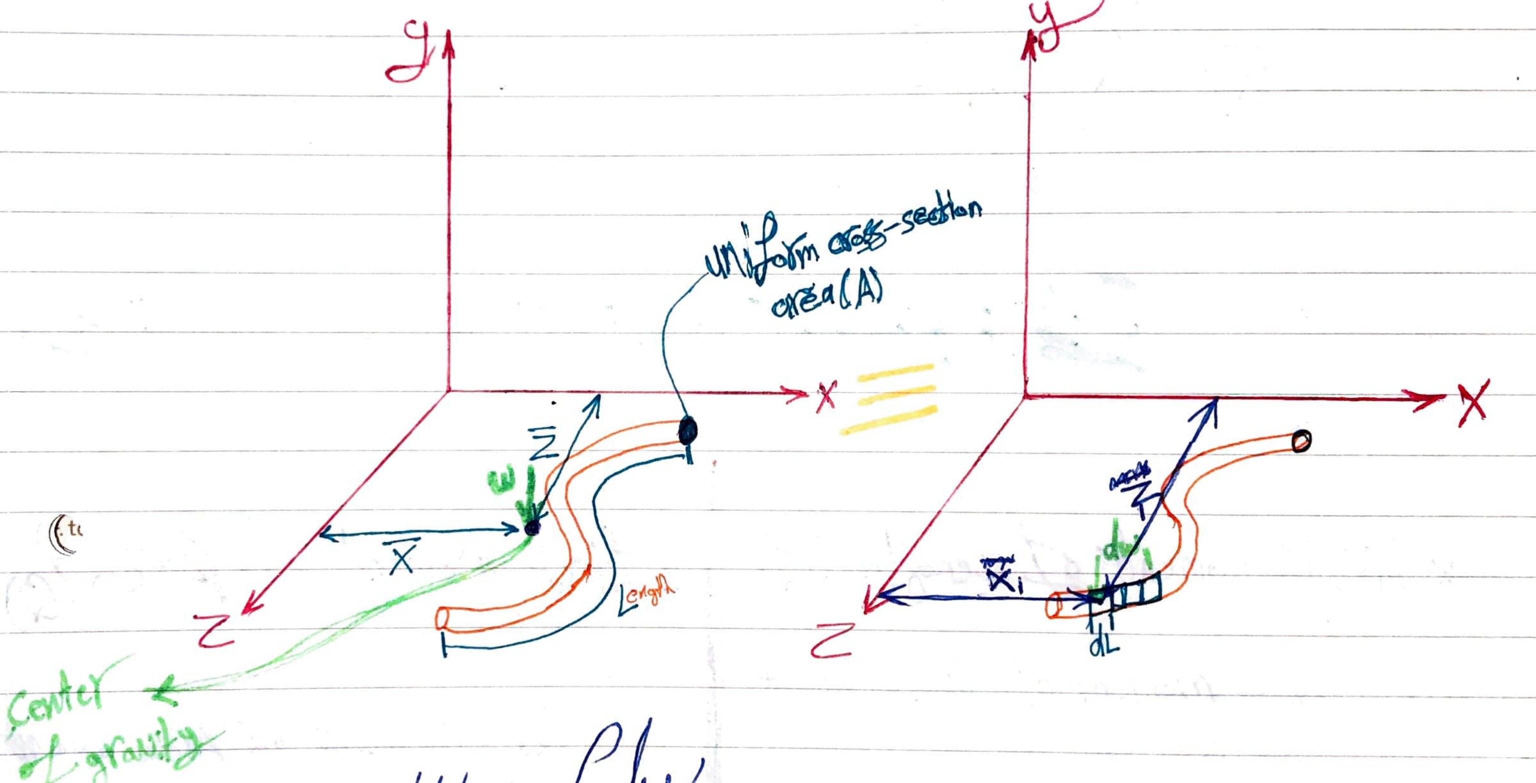
$dw = \gamma \cdot dA \cdot t$

تسبح في الصفحات التالية ....

$$\bar{Z} = \frac{\int Z \cdot dw}{\int dw} = \frac{\int Z \cdot dA \cdot \cancel{\gamma} \cdot \cancel{t}}{\int \cancel{\gamma} \cdot \cancel{t} \cdot dA} = \frac{\int Z \cdot dA}{\int dA}$$

as the same

$$\bar{x} = \frac{\int x \cdot dA}{\int dA}$$



$$W = \int dw$$

$$\sum M_{x\text{-axis}} = \sum Z \cdot dw = dw_1 \cdot Z_1 + dw_2 \cdot Z_2 + dw_3 \cdot Z_3 \ldots n$$

$$\bar{Z} = \frac{\int Z \cdot dw}{\int dw}$$

$$Volume = L \cdot A$$

$$W = \gamma \cdot L \cdot A$$

$$dw = \gamma \cdot dL \cdot A$$

$$\bar{Z} = \frac{\int z \cdot \gamma \cdot dL \cdot A}{\int \gamma \cdot dL A} = \frac{\int z \cdot dL}{\int dL}$$

$$\bar{X} = \frac{\int x \cdot dL}{\int dL}$$

Q_Z

5.1 : planar Centers of gravity

*** first moment of Length (Q)**

Q_x = Moment of lines about x - axis

$$Q_x = \int z \cdot dL$$

$$Q_z = \int x \cdot dL$$

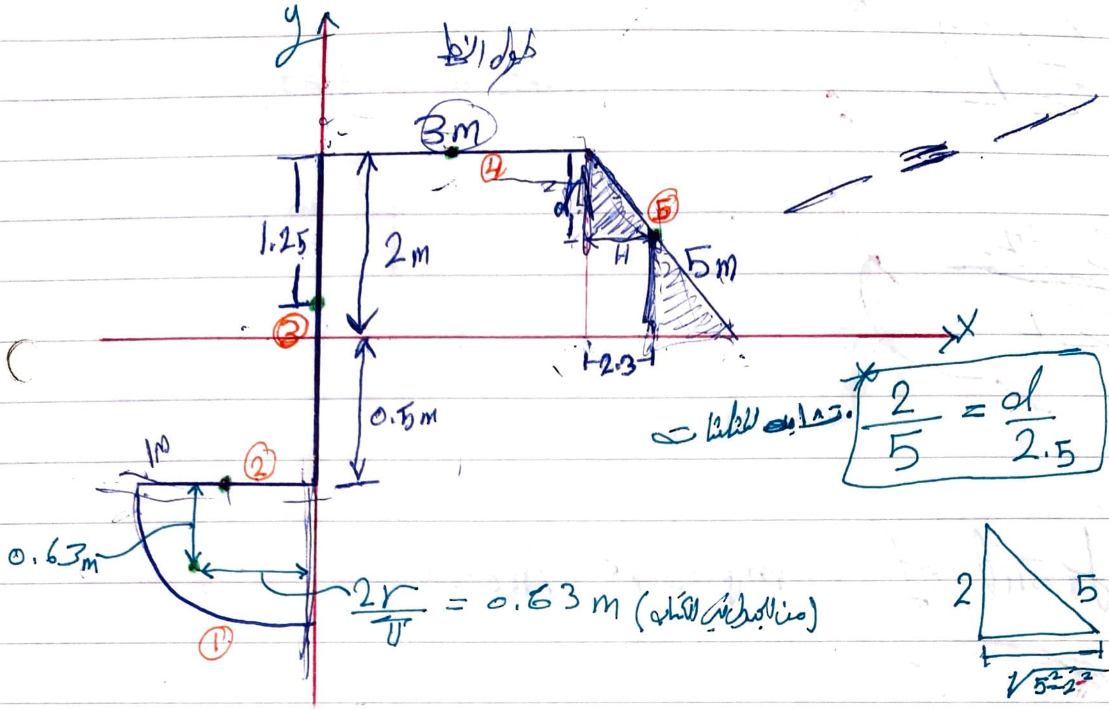
*** first Moment of Area (Q)**

Q_x = Moment of Area about x - axis.

$$Q_x = \int z \cdot dA$$

$$Q_z = \int x \cdot dA$$

Ex.1 :- Determine the centroid and first moment of Lines about x and y – axes for figure shown.



$\frac{2}{5} = \frac{d}{2.5}$ → $d = 1m$

$\bar{X} = \frac{\Sigma (X.L)}{\Sigma L}$, $\bar{y} = \frac{\Sigma (y.L)}{\Sigma L}$

# Lines	L (m)	X (m)	y (m)	X.L (m^2)	y.L (m^2)
1	$\frac{2\pi}{4} = 1.52$	-0.63	-1.13	-0.98	-1.77
2	1	-0.5	-0.5	-0.5	-0.5
3	2.5	0	+0.75	0	+1.875
4	3	+1.5	+2	+4.5	+6
5	5	+5.3	+1	+26.5	+5
Σ	+13.07			+29.52 = Q_y	+10.6 = Q_x

$\bar{X} = \frac{Q_y}{\Sigma L} = \frac{+29.52}{+13.07} = \text{-----} \ m$

$\bar{y} = \frac{Q_x}{\Sigma L} = \frac{10.6}{13.07} = \text{-----} \ m$

+0.6

0.6

Centroid of Lines :-

$$\bar{x} = \frac{\sum (x \cdot L)}{\sum L}$$

$$\bar{y} = \frac{\sum (y \cdot L)}{\sum L}$$

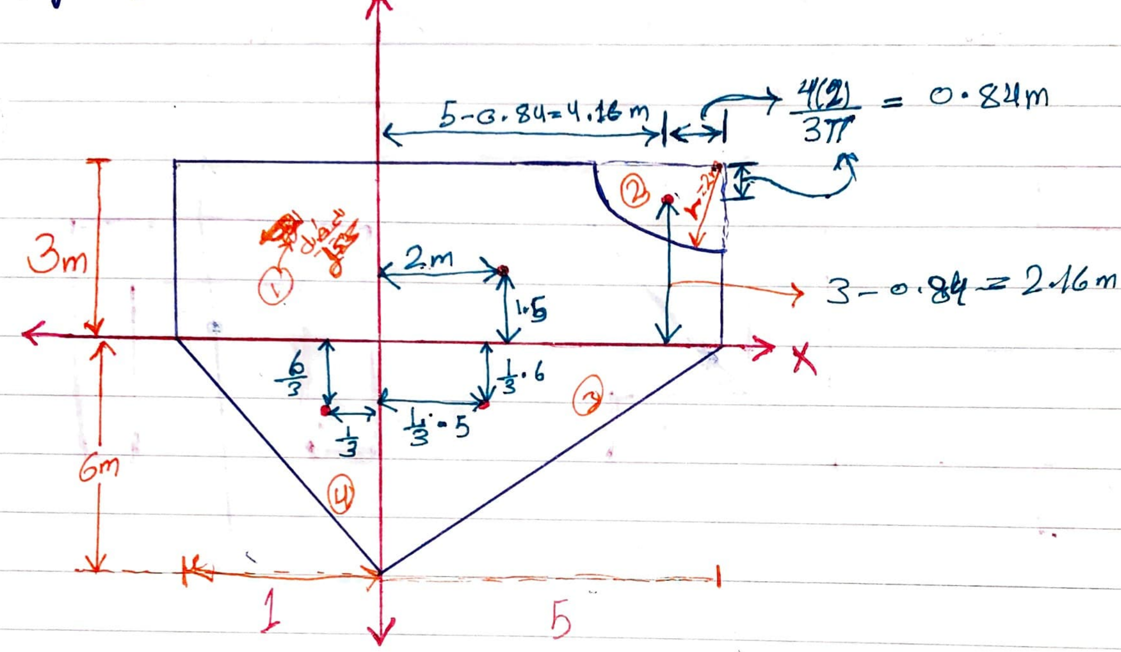
* Centroid of Composite Area :-

$$\bar{x} = \frac{\sum (x \cdot A)}{\sum A}$$

$$\bar{y} = \frac{\sum (y \cdot A)}{\sum A}$$

Ex. 1

① Determine the first moment of area about X and y axes. for the Composite Area shown and ② locate the centroid of the Composite area.

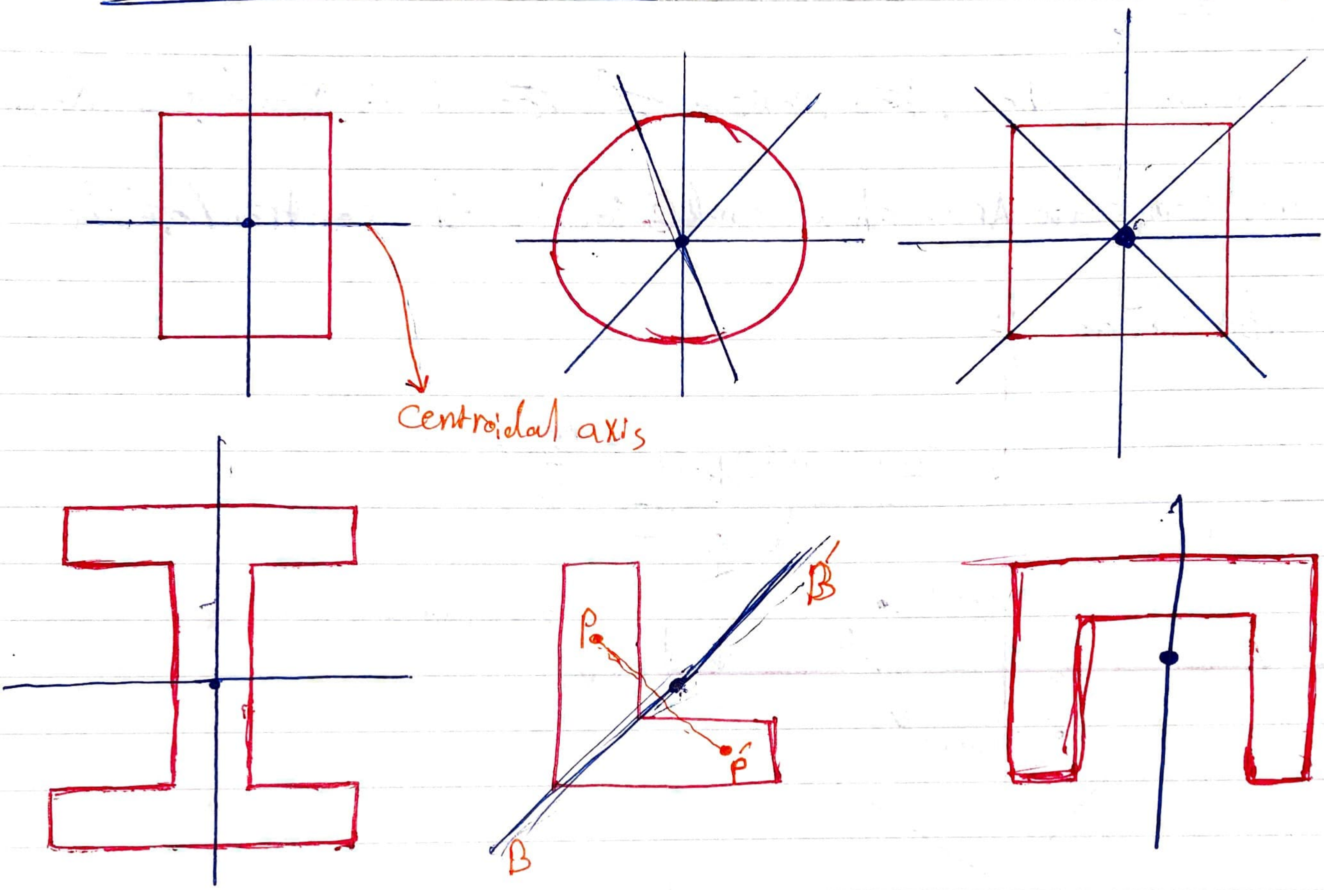


# Area	Area (m^2)	X (m)	y (m)	X·A (m^3)	y·A (m^3)
①	+ 3(6)	+2	+ 1.5	+ 36	+ 27
②	$-\frac{\pi(2)^2}{4}$	+4.16	+2.16	−13.06	− 6.78
③	$+\frac{1}{2}(5)(6)$	$+\frac{5}{3}$	$-\frac{6}{3}$	+25	−30
④	$+\frac{1}{2}(1)(6)$	$-\frac{1}{3}$	$-\frac{6}{3}$	−1	−6
Σ	$A=\Sigma A$			$Q_y=\Sigma x.A$ (+)	$Q_x=\Sigma y.A$ (−)

$$\bar{X} = \frac{\Sigma (X.A)}{\Sigma A} = + \text{---} \quad Q_y$$

$$\bar{y} = \frac{\Sigma (y.A)}{\Sigma A} = \text{--} \ \text{---} \quad Q_x$$

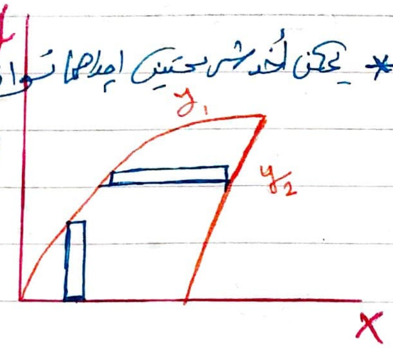
Section Symmetry :-



* An area is symetric with respect to an axis BB' if for every point "p" there exists a point p' such that pp' is perpendicular to BB' and is divided into two equal parts by BB'.

Q = Firs moment of area about the axis of symmetry equal Zero

* يمكن أخذ شريحتين إحداهما توازي y و x
لأنها تغطي الرسم كاملاً.



Centroid by integration Method :-

Area :-

$$\bar{x} = \frac{\int x \, dA}{\int dA} \qquad Q_y = \int x \, dA$$

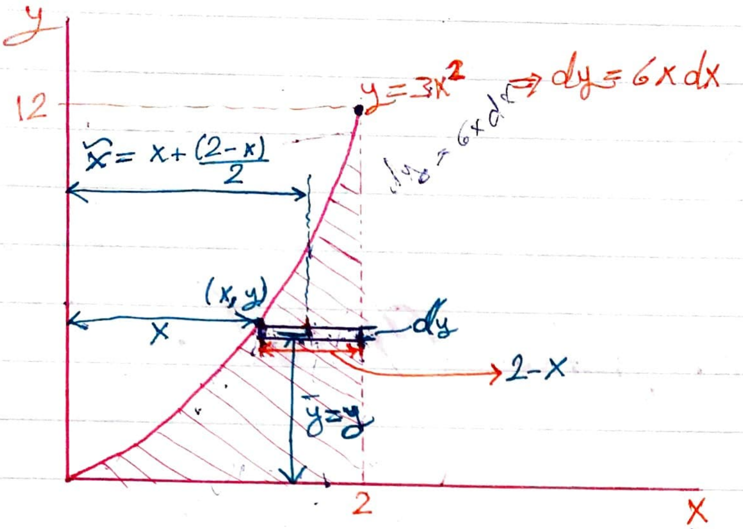
$$\bar{y} = \frac{\int y \, dA}{\int dA} \qquad Q_x = \int y \, dA$$

Lines :-

$$\bar{x} = \frac{\int x \, dL}{\int dL} \qquad Q_y = \int x \, dL$$

$$\bar{y} = \frac{\int y \, dL}{\int dL} \qquad Q_x = \int y \, dL$$

Ex. 1: locate the centroid of the area shown in figure below.



$$dA = (2-x)\,dy$$

$$\text{Area} = \int dA = \int (2-x)\,dy = \int_0^2 (2-x)(6x\,dx)$$

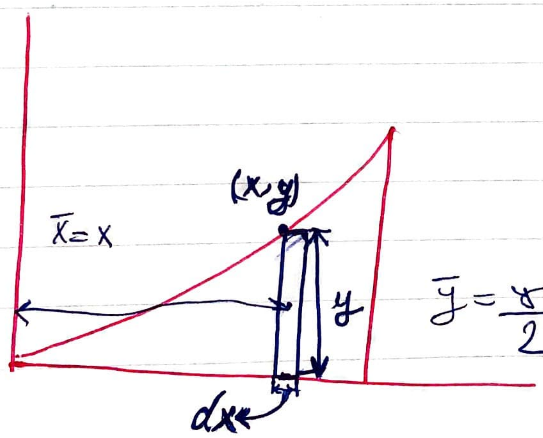
$$\bar{x} = \frac{\int \tilde{x}\,dA}{\int dA} = \frac{\int_0^2 [x+\frac{2-x}{2}][2-x]\,dy}{\int_0^2 (2-x)\,dy}$$

$dy = 6x\,dx$

$$\bar{y} = \frac{\int \tilde{y}\,dA}{\int dA} = \frac{\int y\,(2-x)\,dy}{\int (2-x)\,dy} = \frac{\int_0^2 (3x^2)(2-x)\,dy}{\int_0^2 (2-x)(6x\,dx)}$$

$6x\,dx$

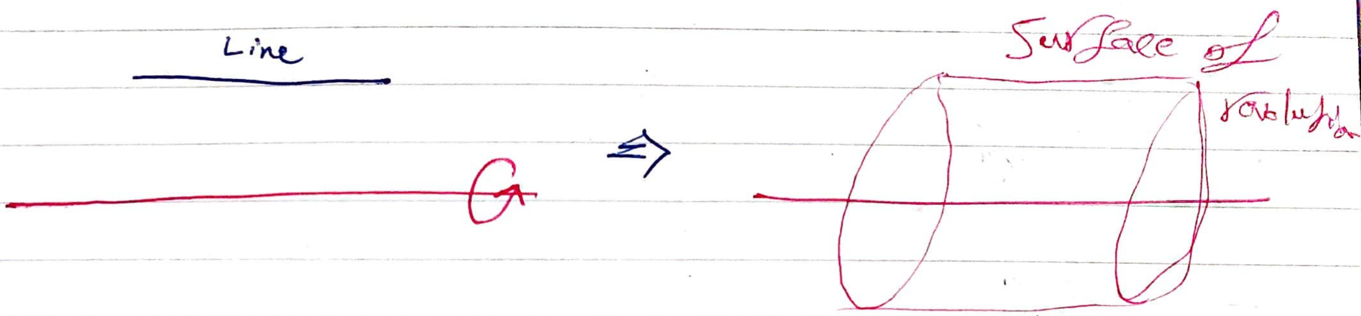
*

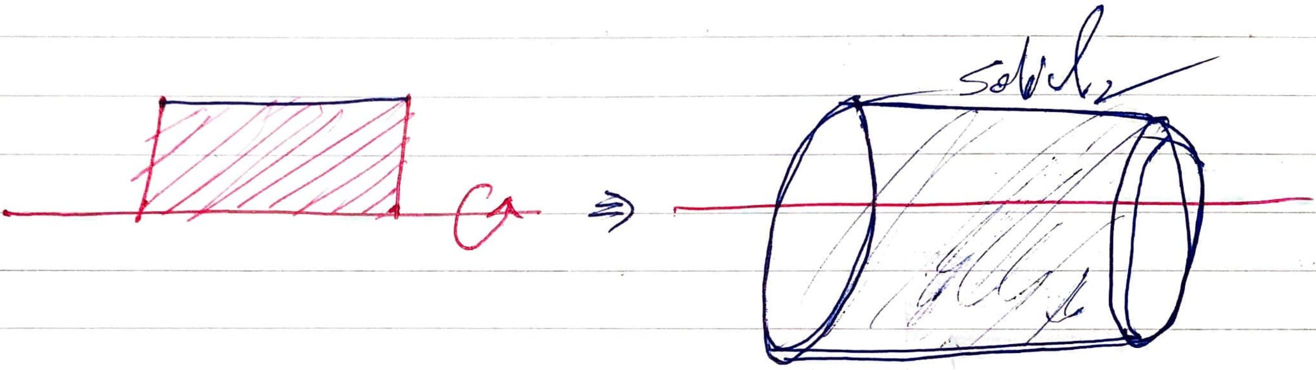


$$\tilde{y} = \frac{y}{2}$$

$$dA = y\,dx$$

"Volume of revolution"



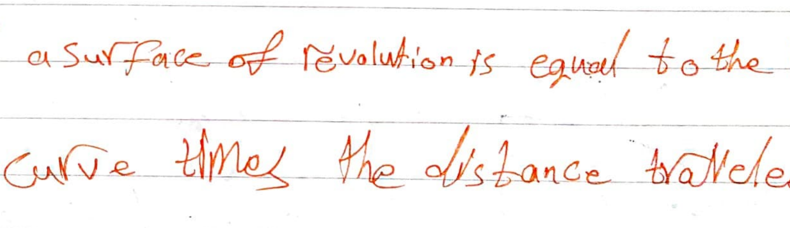


* Theorem of pappus – Guldinus :-

(.tot)

Th: The area of a surface of revolution is equal to the length of the generating curve times the distance traveled by the centroids of the curve while the surface is being generated.

$$A = L * 2\pi \bar{y}$$



*

*تكملة هذا الشابتر في آخر صفحات الدفتر.

موقع

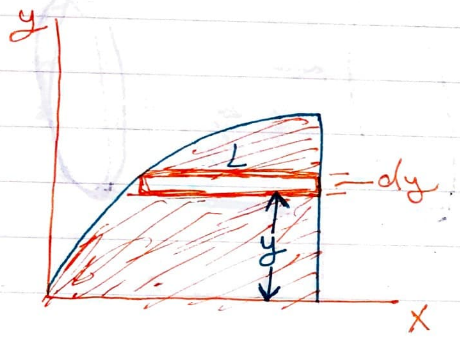
Centroid مسافة عمودية

$$* \quad I_x = \int y^2 \, dA$$

$$* \quad I_y = \int x^2 \, dA$$

$$J = \int \rho^2 \, dA$$

* choice of element of

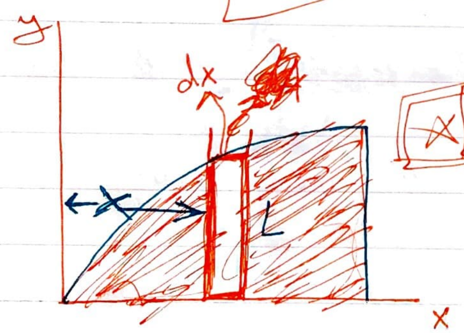


$$\boxed{* \; dA = L \, dy}$$

$$I_x = \int y^2 \, dA$$

حول x

أخذ شريحة توازي axis - x



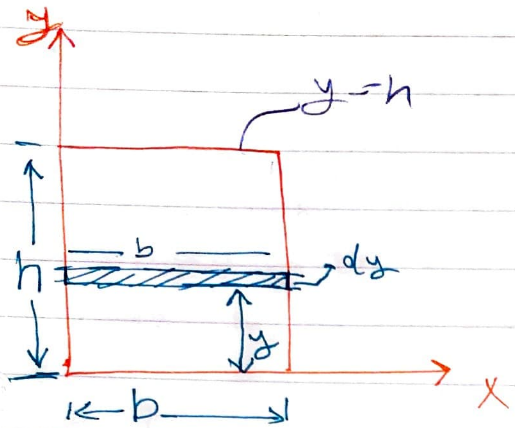
$$\boxed{* \; dA = L \, dx}$$

$$I_y = \int x^2 \, dA$$

ممكن زالتك ازداد العزوبة في الشي حول المحول Moment of inertia

exp:-

$\int y^2 dA = \int y^2 b\,dy = \frac{b y^3}{3} \Big|_0^h$



$\boxed{dA = b \cdot dy}$

* calculate the moment of inertia about x and y - axis.

$I_x = \int y^2 \cdot dA = \int_0^h y^2 \cdot b\,dy = \frac{bh^3}{3}$

ازا فرق نشد من صفر الى h

$\boxed{I_y = \frac{b^3 h}{3}}$

$I_x = \int y^2\, b\,dy$

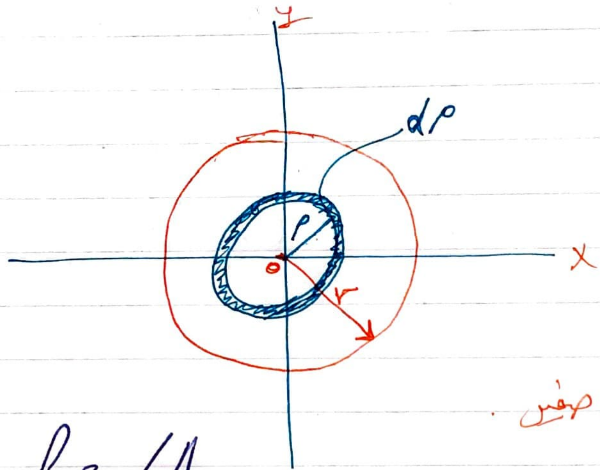
$= b \int_0^h y^2\,dy$

$\boxed{I_x = \int y^2\,dA}$

first moment of area = $Q_x = \int y^2 dA$

ex:

* calculate I_x and I_y.



هنا نأخذ polar moment of inertia

أي حول z-axis

الكروس سيكشن هي دائرة ذلك كله صفر.

$\int \rho^2 dA$

*

$J_o = I_x + I_y \Rightarrow * I_x = I_y = I$ أي تمر بالنقطة O

$$J = 2I \Rightarrow I = \cancel{\frac{2}{J_o}} \Rightarrow I = \frac{J_o}{2}$$

$$J = \int \rho^2 dA = \int_0^r \rho^2 (2\pi \rho \, d\rho)$$

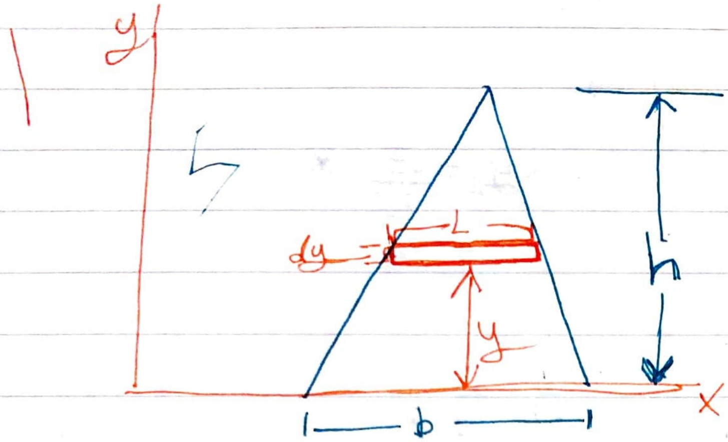
$$= \frac{\pi}{2} r^4$$

$J =$

$$I_x = I_y = \frac{\pi r^4}{4}$$

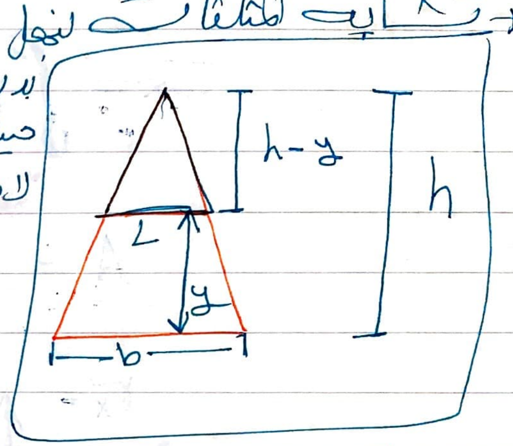
* Determine the moment of inertia about x axis

* general Triangle



$dA = L\,dy$

* تشابه المثلثات لنجد L بدلالة y حيث L متغير لا يصح أن تكون متغير.



$$\boxed{\frac{L}{b} = \frac{h-y}{h}}$$

* $L = \dfrac{bh - by}{h}$

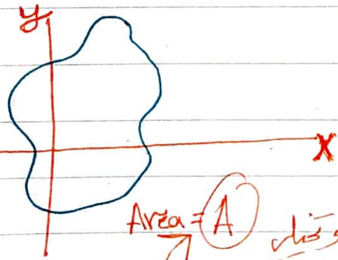
$$I_x = \int y^2\, dA$$

$$= \int_0^h y^2 \left(\frac{bh - by}{h}\right) dy$$

$$= \frac{bh^3}{3} - \frac{bh^{4}}{4h} = \frac{bh^3}{12}$$

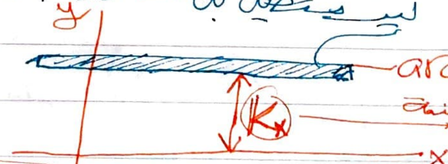
F. to

$\boxed{9.10}$ * Radius of generation of an areas



Area = A هو شكل

ليس مسطح بل خط رفيع جداً

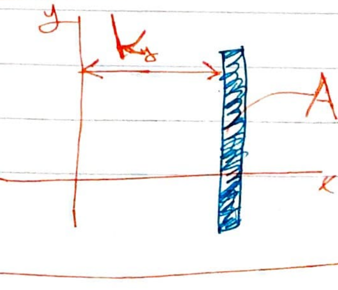


area = A

ما فيه ثابتة

K: radius of generation

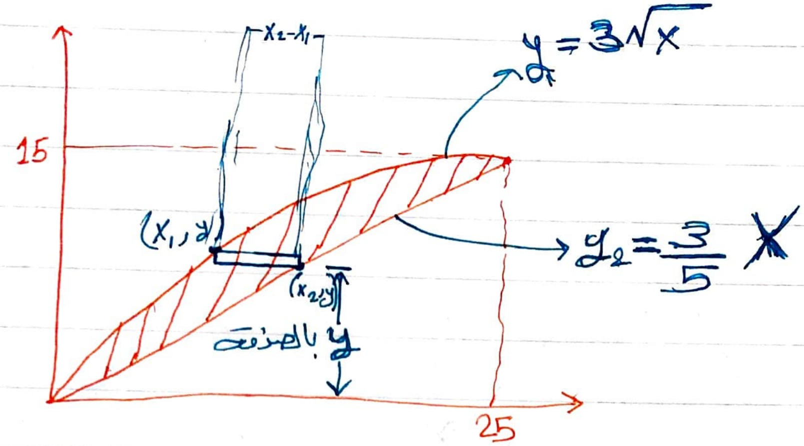
$$I_x = K_x^2 A$$



[illegible]

W depth X

ex :-

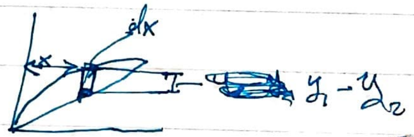


$$I_x = \int y^2\, dA = \int y^2 (x_2 - x_1)\, dy = \int y^2 \cdot \left(\frac{5}{3}y - \frac{y^2}{9}\right) dy$$

$$A = \int dA = \int (x_2 - x_1)\, dy = \int_0^{15} \left(\frac{5}{3}y - \frac{y^2}{9}\right) dy$$

$$r_x = \sqrt{\frac{I_x}{A}}$$

* $I_y = \int x^2\, dA = \int_0^{25} x^2 \cdot (y_1 - y_2)\, dx$



$w = \frac{N}{m}$

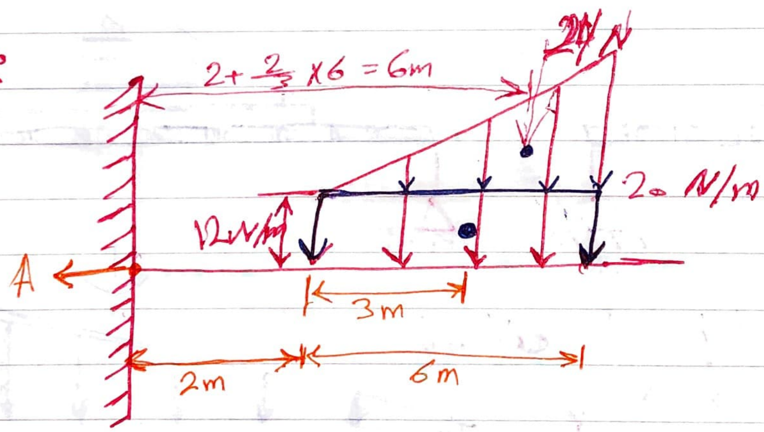
$w \times m = \ldots$

$\int x \cdot w \, dx = \bar{x} \cdot F_R$

$\bar{x} = \frac{\int x \cdot w \, dx}{\int w \cdot dx}$ = Centroid of area under load diagram

* we can replace the distributed load (w) by a concentrated force (W) equal in magnitude to the area A under the load diagram and passing through the centroid C of that area.

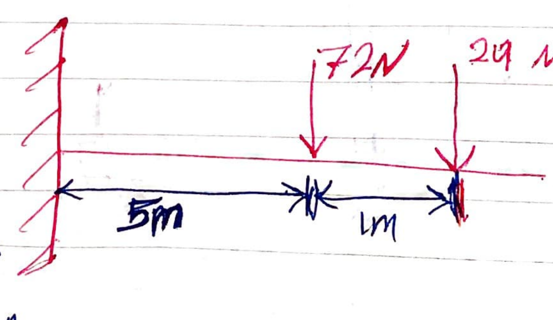
ex1:



Replace the distributed load shown by an equivalent single force, and determine reaction at point A

$F_1 = \frac{1}{2}(6)(8) = 24\ N$

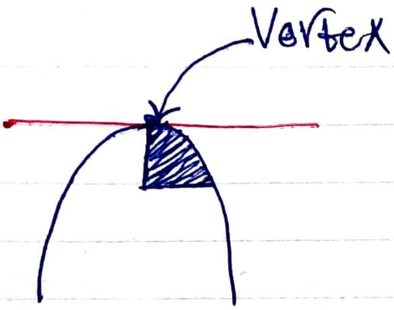
$F_2 = (6)(12) = 72\ N$

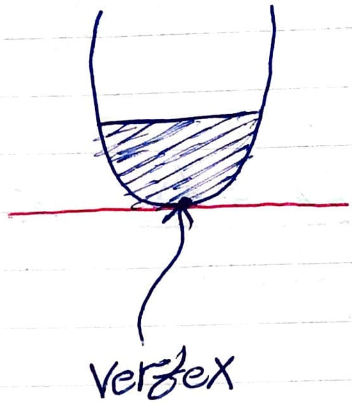


$A_x = 0$

$M_A = 72 \times 5 + 24 \times 6$

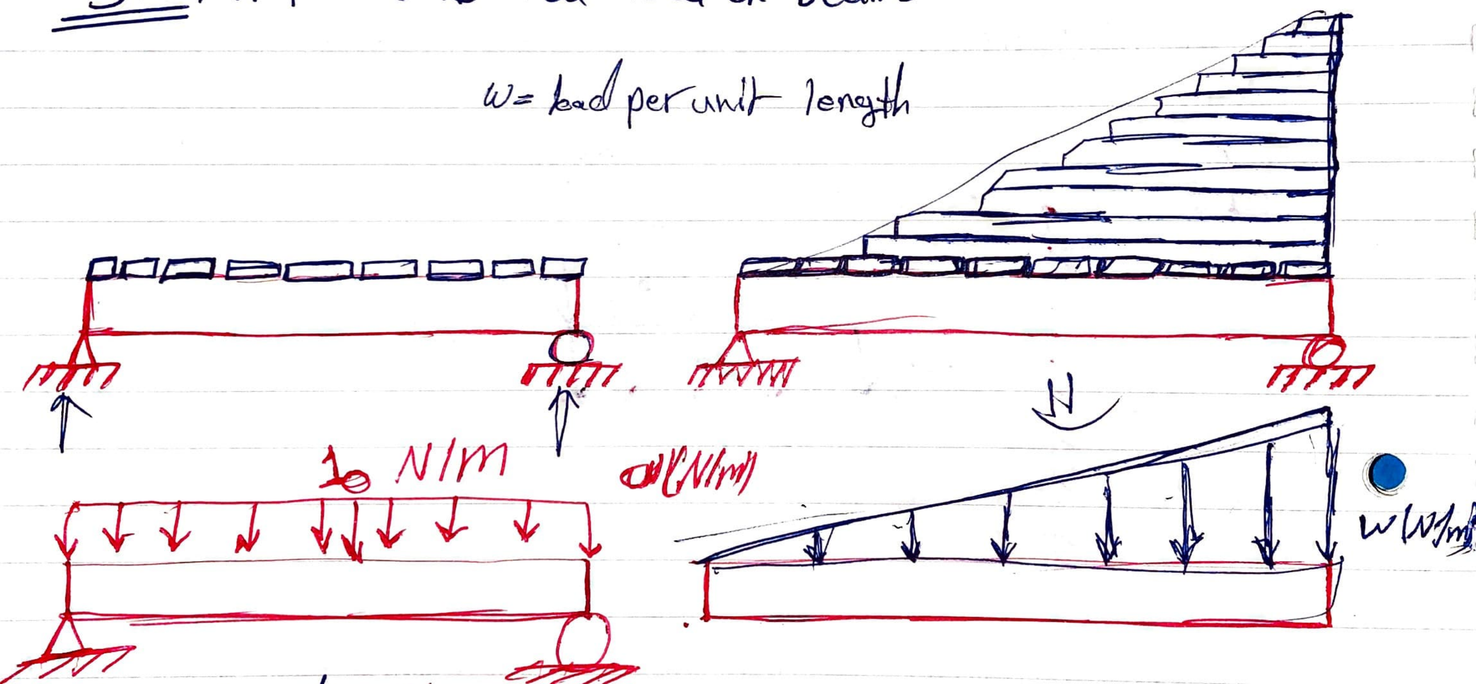
$A_y = 96\ N$

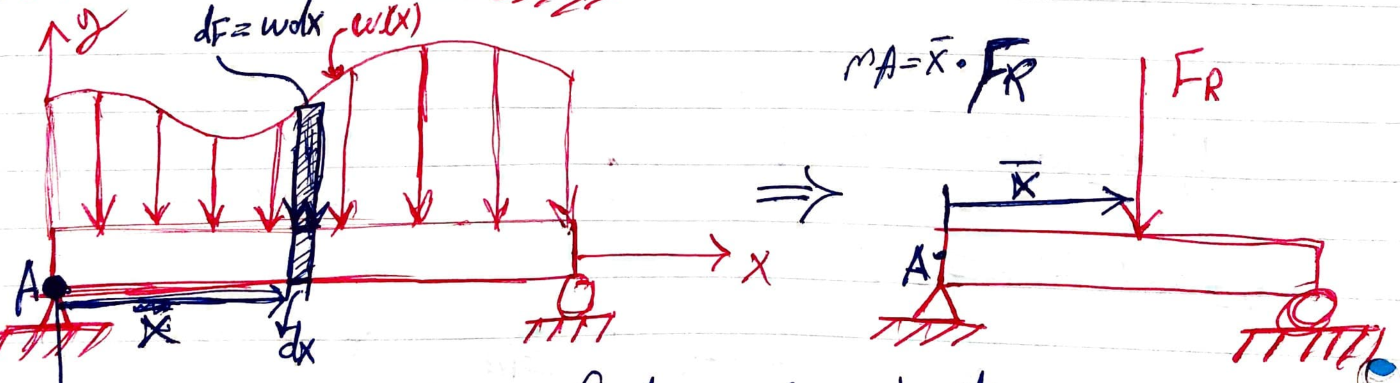




5.3: A: distributed load on beams

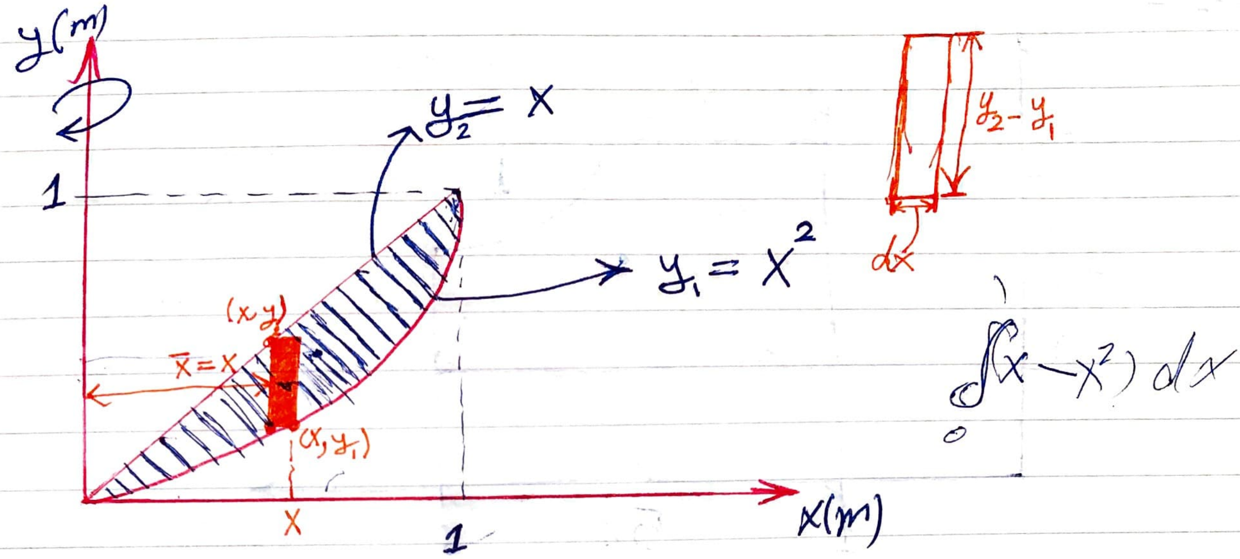
w = load per unit length





$$F_R = \int dF = \int w\,dx = \text{area under diagram}$$

$$\sum M_A = \int x \cdot w\,dx = \int dF \cdot x$$



* The shaded area is revolved about the y-axis.

Determine the volume of revolution.

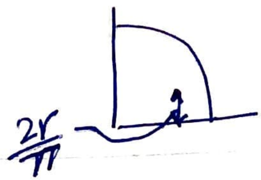
$$A = \int dA = \int_0^1 (x - x^2)\,dx = \left(\frac{x^2}{2} - \frac{x^3}{3}\right)\Big|_0^1 = \frac{1}{2} - \frac{1}{3} = \frac{1}{6}\ m^2$$

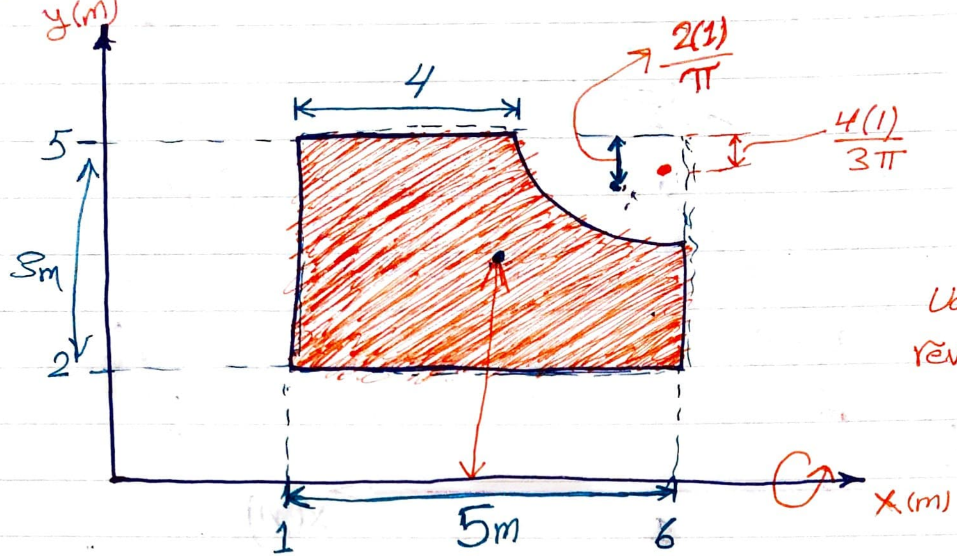
$$\bar{x} = \frac{\int x \cdot dA}{\int dA} = \frac{\int_0^1 x \cdot (x - x^2)\,dx}{\left(\frac{1}{6}\right)} = 6\int_0^1 (x^2 - x^3)\,dx$$

$$= 0.5\ m$$

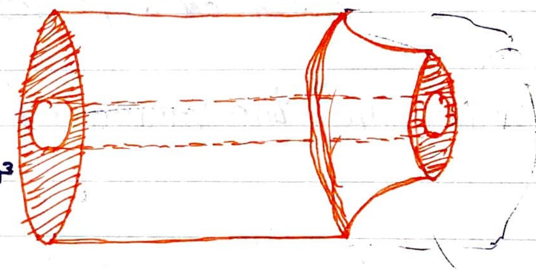
$$\text{Volume} = A \cdot 2\pi\bar{x} = \left(\frac{1}{6}\right)(2\pi * 0.5)\ m^3$$

Ex. 2





volume of body of revolution about x-axis

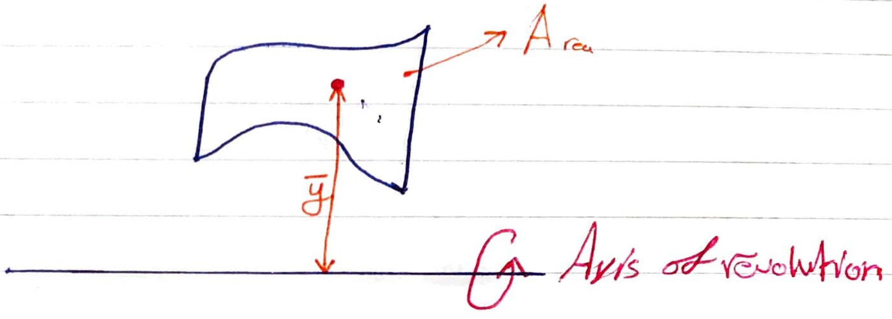


$$V = A \cdot 2\pi \bar{y}$$

$$= (5\times 3)(2\pi \cdot 3.5) - \left(\frac{\pi(1)^2}{4}\right)(5-0.42)\ m^3$$

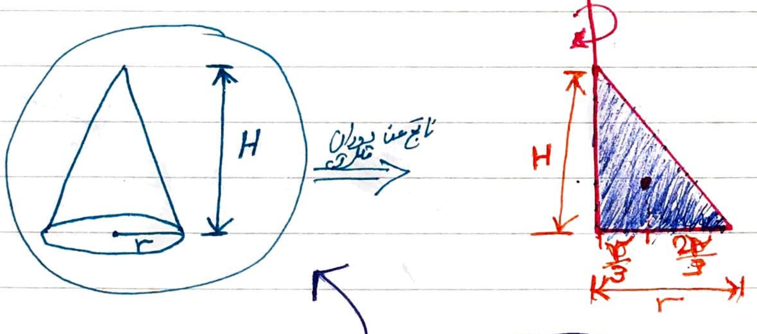
$$\text{surface area} = (4)(2\pi * 5) + \frac{(2\pi(1))}{4}\left(2\pi * \left[5 - \frac{2(1)}{\pi}\right]\right.$$

Th ② : The Volume of a body of revolution is equal to the generating area times the distance traveled by the Centroids of the area while the body is being generated.



$$V = A * 2\pi \bar{y}$$

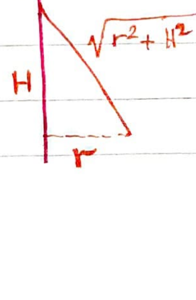
Cone :-



$$\boxed{\text{Volume} = \left(\tfrac{1}{2} r H\right)\left(2\pi \cdot \tfrac{r}{3}\right)}$$

$$= \frac{\pi r^2 H}{3}$$

Surface Area $= \left[\sqrt{r^2 + H^2}\right]\left[\frac{2\pi * r}{2}\right]$

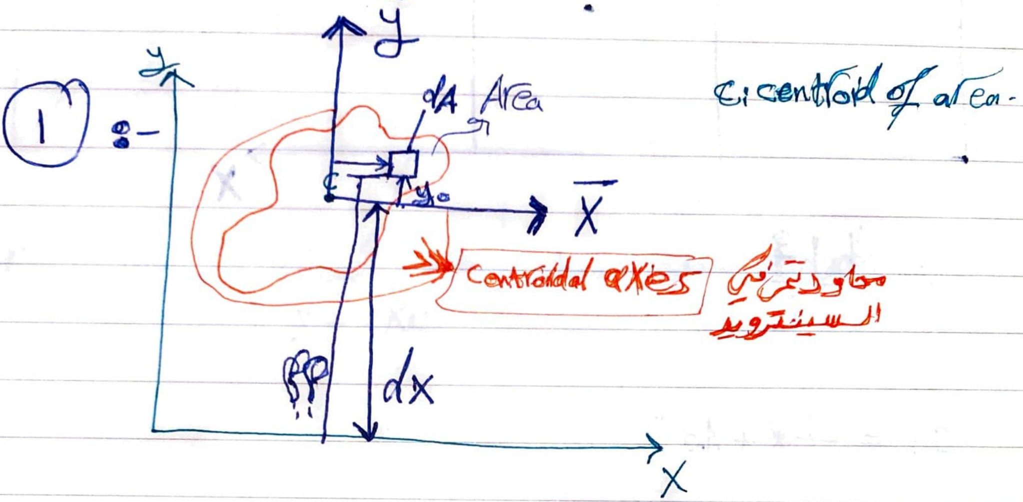


Moment of inertia "second moment of area"

9.2 : parallel axis - theorem and Composit areas.

(1)

(1) :-



$$I_x = \int y^2\, dA = \int (y_0 + d_x)^2\, dA$$

$$= \int (y_0^2 + 2y_0\, d_x + d_x^2)\, dA$$

$$= \int y_0^2\, dA + 2y_0\, d_x\, dA + d_x^2\, dA$$

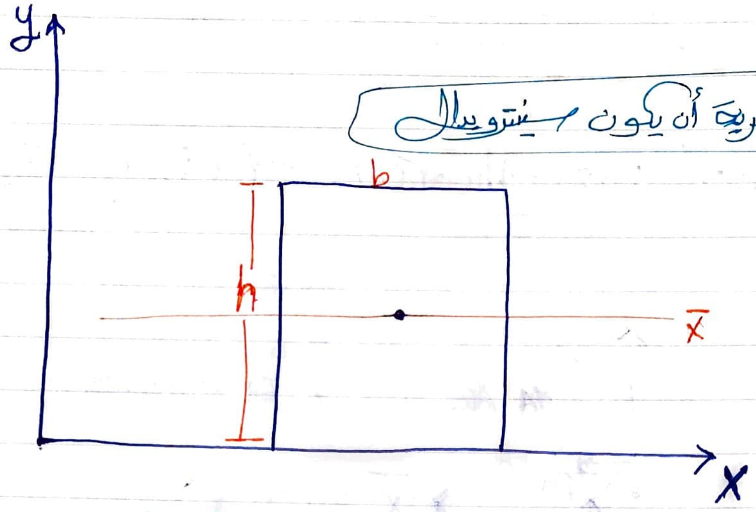
∵ لأن العزم الأول لأي جزء من الجسم عند السينترويد = صفر .

$$= \int y_0^2\, dA + 2d_x \int y_0\, dA + d_x^2 \int dA$$

تساعد هذه النظرية في إيجاد Moment of inertia لجسم حول محور غير السينترويدي إذا كان موازي لمحور آخر centroidal axis.

$$= \boxed{I_{\bar{x}} + d_x^2 \cdot A}$$

ex.1



* يشترط في النظرية أن يكون سنترويدال

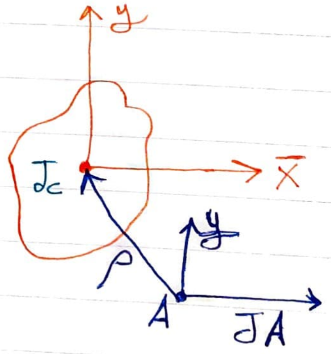
$$I_x = \frac{bh^3}{3}$$

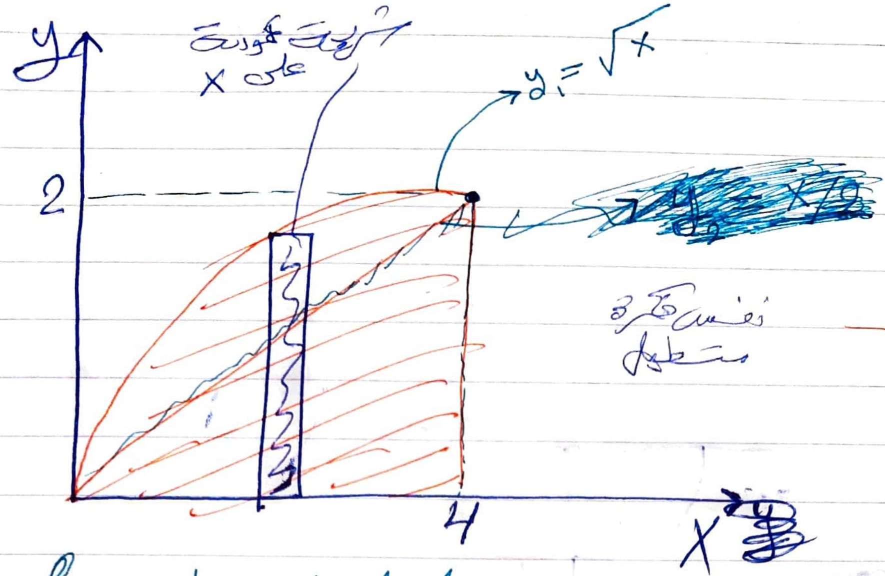
$$I_x = I_{\bar{x}} + A d^2$$

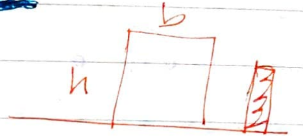
$$\frac{bh^3}{3} = I_{\bar{x}} + \left[(b \cdot h)\,\tfrac{1}{2}h\right]^2$$

$$I_{\bar{x}} = \frac{bh^3}{12}$$

$$J_A = J_c + A \cdot \rho^2$$







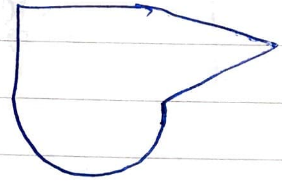
$$I_{a-a} = \frac{bh^3}{3}$$

for the shaded area shown, calculate the moment of inertia about x-axis

$$I_x = \int dI_x = \int \frac{(dx)(y^3)}{3} = \int_0^4 \frac{(\sqrt{x})^3}{3}\, dx$$

* استخدم قانون الشريحة على المربع لإيجاد I_x
حيث اعتبر الشريحة مستطيل

Composit area =

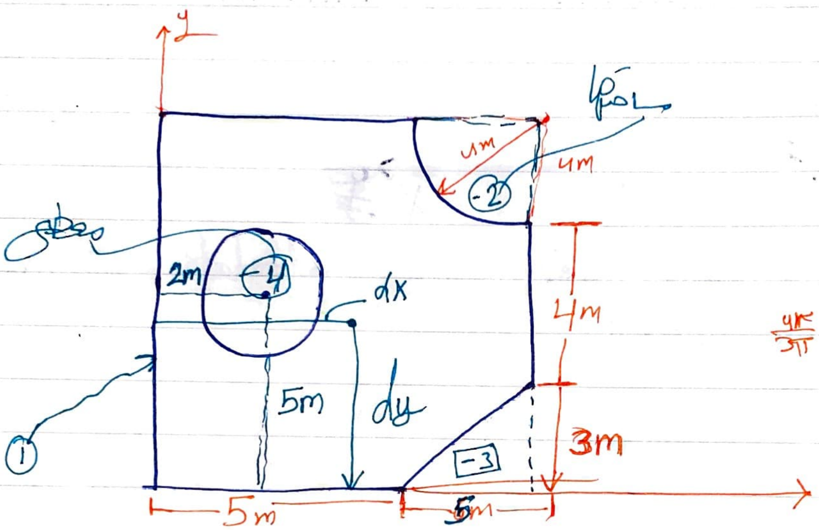


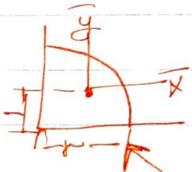
مركبة من عدة أشكال

ex 1: for the composite area shown, calculate

① I_x, r_x.

② I_y, r_y.





اعتمدنا انا

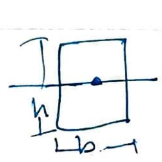
تركيبنا الجدول الي

number

~~Area #~~	~~$A(m)^2$~~	~~X(m)~~	~~y(m)~~	~~X.A~~	~~y.A~~

Fig 9.12

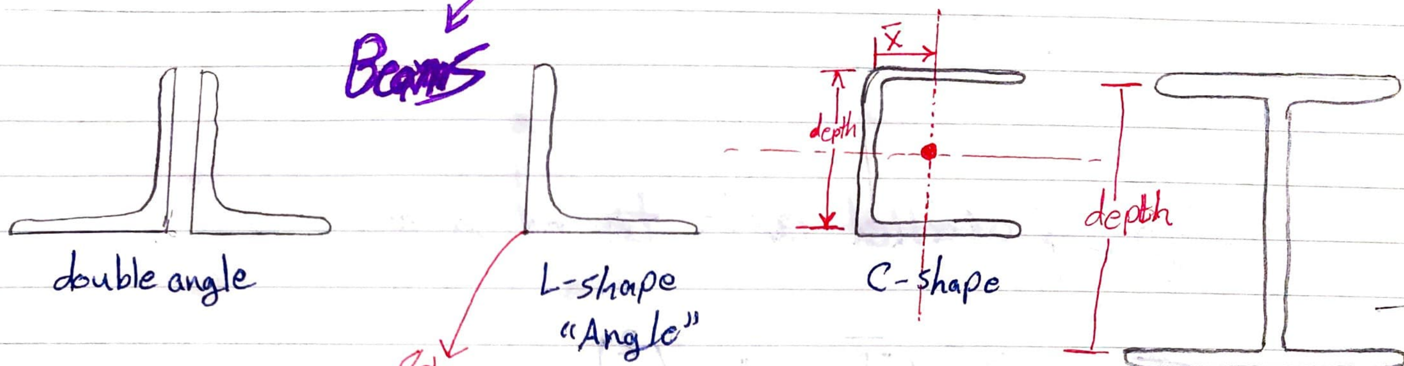
Area #	$A(m^2)$	X(m)	y(m)	X.A	y.A	$I_{\bar{x}}$	$I_{\bar{y}}$	dx	dy	$A\cdot dx^2$ (m^4)	$A\cdot dy^2$ (m^4)
1	(11)(10) = 110					$\frac{(10)(11)^3}{12}$	$\frac{(10)^3(11)}{12}$	5	5.5	$(110)(5)^2$	$(110)(5.5)^2$
2	$-\frac{(4)^2\pi}{4}$					-	-				
3	-					-	-				
4	-					-	-				



$$I = \frac{bh^3}{12}$$

" Steel structures "

Beams



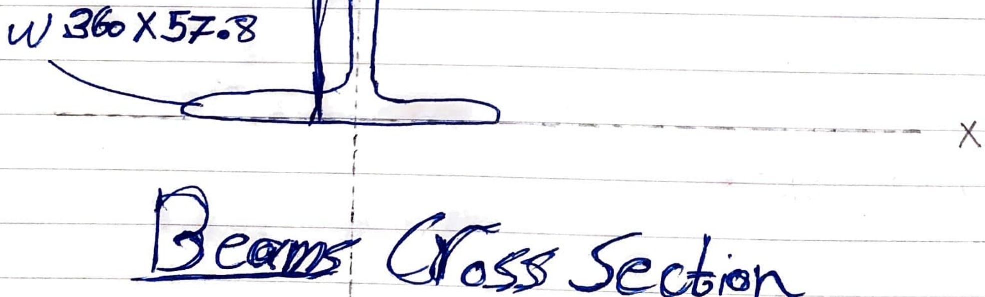
يرجى البحث عن المعلومات الخاصة به

C 310 x 30.8

W 140 X 113
depth
mass per unit length
"113 Kg/m"

* Fig 9.13 "B" SI (mm, m, N) — هنا المطلوب

exp: Locate the centroid of the Composite Area shown and determine the moment of inertia about X-axis.



*look at fig 9.13 B

Beams Cross Section

$$\bar{y} = \frac{\sum (A \cdot y)}{\sum (A)} = \frac{(220 \times 18)(358 + \frac{18}{2}) + 7230 (\frac{358}{2})}{220 \times 18 + 7230} = \text{---} \ m$$

Using parallel axis theorem :-

$$I_x = \sum (\bar{I} + A \cdot d^2)$$

$$= \frac{(220)(18)^3}{12} + (220)(18)(358 + \frac{18}{2})^2$$

$$+ 160 \times 10^6 + (7230)(358)^2 = \text{---} \ mm^4$$