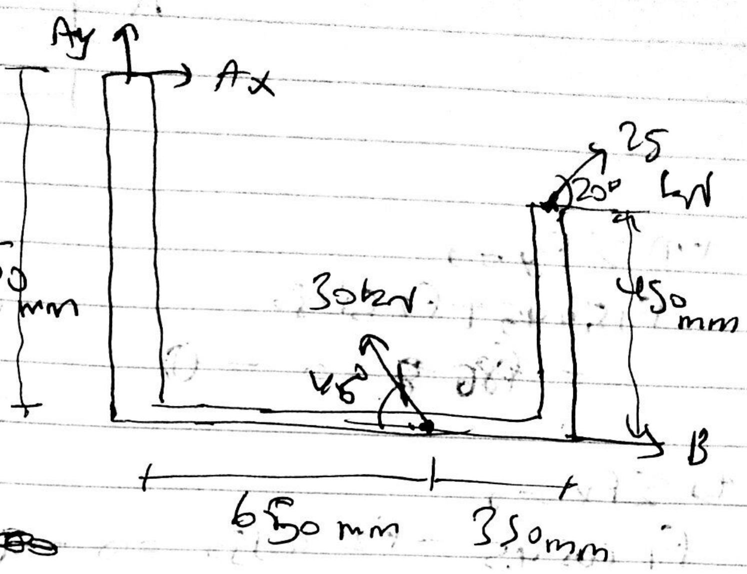
Statics ENCE 233 ; HW (2): Key Solution

Q. 1



a) $\circlearrowleft + \Sigma M_A = 0$

$25 \cos 20 \, (650 - 450)$
$+ 25 \sin 20 \, (650 + 350)$ +
$+ 30 \sin 45 \, (650)$
$- 30 \cos 45 \, (650)$
$+ B \, (650) = 0$

$\Rightarrow B = -20.38$ kN

* Note: One could notice that the force 30 kN 45° passes through point A. So, it makes no moment about A.

$\overset{+}{\rightarrow} \Sigma F_x = 0$

$A_x + 25 \cos 20 + B - 30 \cos 45 = 0$

$A_x = 22.66$ kN $\overset{+}{\rightarrow}$

$+\uparrow \Sigma F_y = 0$

$A_y + 25 \sin 20 + 30 \sin 45 = 0$

$A_y = -29.76$ kN $\rightarrow A_y = 29.76 \downarrow$ kN

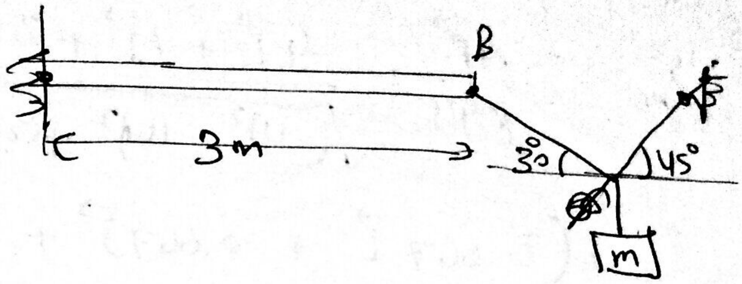
b) $\circlearrowleft + \Sigma M_B =$

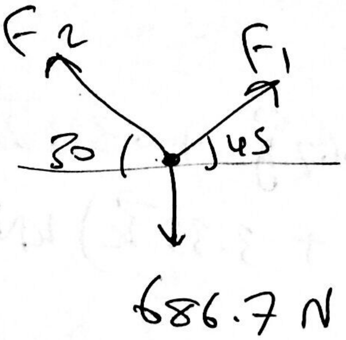
$-A_y(650 + 350) - A_x(650) \mp 25 \cos 20 \, (450)$

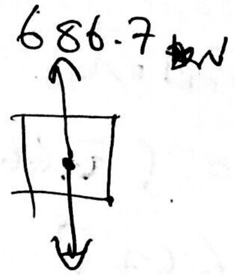
$- 30 \sin 45 \, (350) = -2.965$ kN.m.

Q.3

$m = 70$ kg







$70 * 9.81 = 686.7$ kN

$\overset{+}{\rightarrow} \Sigma F_x = 0:$

$F_2 \cos 30 = F_1 \cos 45 \rightarrow F_2 = F_1 \dfrac{\cos 45}{\cos 30}$ ...(1)

$+\uparrow \Sigma F_y = 0.0:$

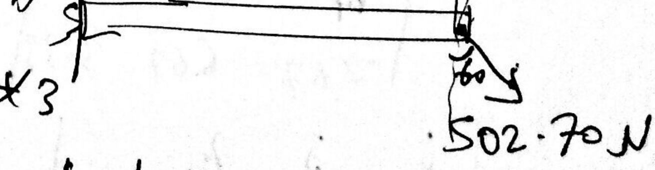
$F_2 \sin 30 + F_1 \sin 45 = 686.7 \text{ N} \rightarrow (2)$

$F_1 \dfrac{\cos 45}{\cos 30} \sin 30 + F_1 \sin 45 = 686.7 \text{ N}$

$\boxed{F_1 = 615.68 \text{ N}}$

$F_2 = 615.68 * \dfrac{\cos 45}{\cos 30} = 502.70$

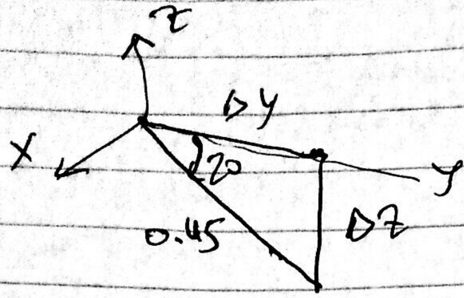
$\boxed{F_2 = 502.70}$ N.

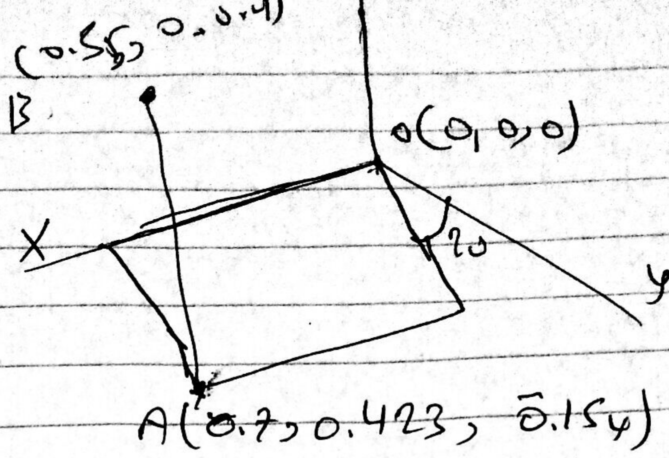


$M_A = 502.70 * \cos 60 * 3$

$= 754.05$ N.m clockwise.

Q.3





$\Delta z = 0.45 \sin 20 = 0.423$

$\Delta y = 0.45 \cos 20 = 0.154$

$\vec{r}_{AB} = -0.35 i - 0.423 j + 0.554 \vec{k}$

$T = 143.4\ N \qquad |\vec{r}_{AB}| = 0.78$

$$T_{AB} = \frac{143.4}{0.78}\left(-0.35 \vec{i} - 0.423 j + 0.554 \vec{k}\right)$$

$\vec{T}_{AB} = -64.2 \vec{i} - 77.8 \vec{j} + 101.9 \vec{k}$

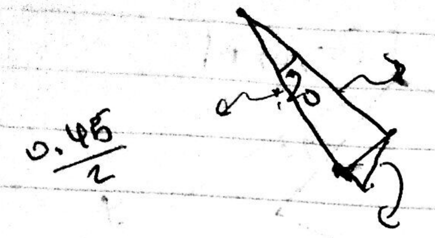
$\vec{r}_{OA} = 0.7 \vec{i} + 0.423 \vec{j} - 0.154 \vec{k}$

$$M_o = \begin{vmatrix} i & j & k \\ 0.7 & 0.423 & -0.154 \\ -64.2 & -77.8 & 101.9 \end{vmatrix} = 31.12 \vec{i} - 61.44 \vec{j} - 27.3 \vec{k}$$

$\lambda_x = \vec{i}$

$M_x = \vec{\lambda}_x \cdot \vec{M}_o$

$= 31.12 \vec{i}$

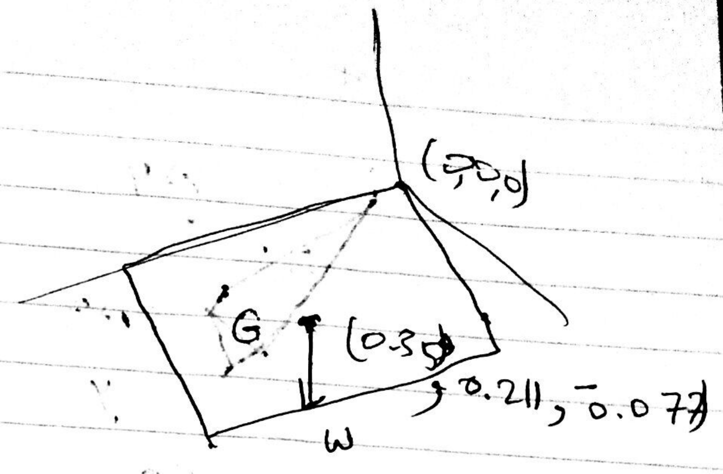


$\Delta y = \frac{0.45}{2} \cos 20$

$= 0.211$

$\Delta z = \frac{0.45}{2} \sin 20$

$= 0.077$



$W = 15 \times 9.81 = -147.15\ N\ \vec{k}$

$\vec{r}_{OG} = 0.35i + 0.211j - 0.077k$

$$M_O = \begin{vmatrix} i & j & k \\ 0.35 & 0.211 & -0.077 \\ 0 & 0 & -147.15 \end{vmatrix} = -31.04i + 51.5j$$

$$\vec{M}_x = \vec{\lambda}_x \cdot \vec{M}_O = -31.04$$

produce moment equal in magnitude and opposite in direction.

Since the moment of tension AB about point B is zero, then the moment of T_{AB} about line OB is zero

$$M_{OB} = \vec{\lambda}_{OB} \cdot \underbrace{(r_{BA} \times T_{AB})}_{\text{zero}}$$

Q.4

$$\vec{\lambda}_{AB} = \frac{\vec{AB}}{|AB|} = \frac{-4\vec{i} + 4\vec{j} + 2\vec{k}}{\sqrt{(-4)^2 + (4)^2 + (2)^2}}$$

$$= (-0.667\vec{i} + 0.667\vec{j} + 0.333\vec{k})\ m.$$

$$\vec{F}_{AB} = |F_{AB}| \cdot \vec{\lambda}_{AB}$$

$$= 10\,(-0.667\vec{i} + 0.667\vec{j} + 0.333\vec{k})$$

$$= (-6.67\vec{i} + 6.67\vec{j} + 3.33\vec{k})\ kN.$$

$$\vec{\lambda}_{AC} = \frac{\vec{AC}}{|AC|} = \frac{(2\vec{i} + 3\vec{j} - 6\vec{k})}{\sqrt{(2)^2 + (3)^2 + (-6)^2}}$$

$$= (0.2857\vec{i} + 0.4286\vec{j} - 0.8571\vec{k})\ m$$

$$\vec{F}_{AC} = |F_{AC}|\ \vec{\lambda}_{AC}$$

$$= 20\,(0.2857\vec{i} + 0.4286\vec{j} - 0.8571\vec{k})$$

$$= (5.71\vec{i} + 20.43\vec{j} - 17.14\vec{k})\ kN.$$

$$M_O^{AB} = \vec{r}_{OA} \times \vec{F}_{AB}$$

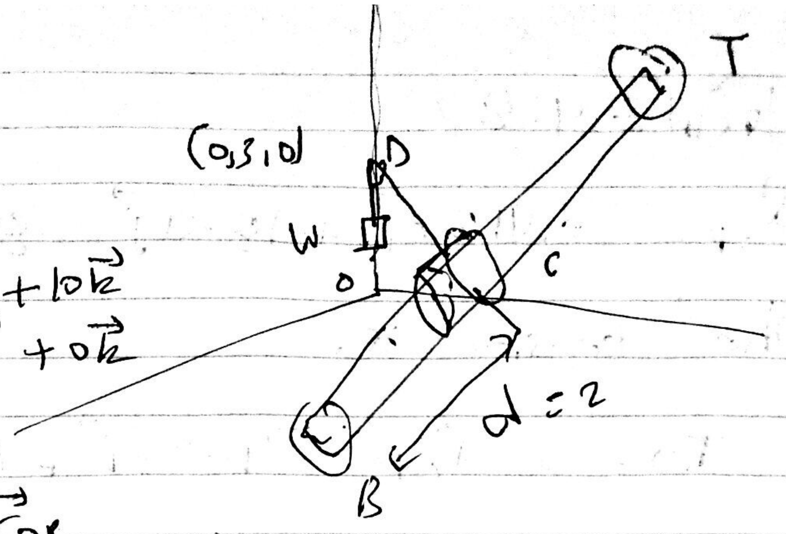
$$= \begin{vmatrix} i & j & k \\ 4 & 0 & 6 \\ -6.67 & 6.67 & 3.33 \end{vmatrix} = (-40.02\vec{i} - 53.34\vec{j} + 26.68\vec{k})\ kN.m$$

$$M_O^{AC} = \begin{vmatrix} i & j & k \\ 4 & 0 & 6 \\ 5.71 & 20.43 & -17.14 \end{vmatrix} = (122.58\vec{i} + 102.82\vec{j} + 81.72\vec{k})\ kN.m$$

$$(M_O = 82.56\vec{i} + 49.48\vec{j} + 108.4\vec{k})\ kN.m$$

Q.5

$W = 100$



$\vec{r}_{OB} = 3\vec{i} + 0\vec{j} + 10\vec{k}$
$\vec{r}_{OT} = 12\vec{i} + 10\vec{j} + 0\vec{k}$

$\vec{r}_{BT} = \vec{r}_{OT} - \vec{r}_{OB}$
$= 9\vec{i} + 10\vec{j} - 10\vec{k}$, $|\vec{r}_{BT}| = 16.763$

$\vec{\lambda}_{BT} = 0.536\,\vec{i} + 0.5965\,\vec{j} - 0.5965\,\vec{k}$

So, the position vector of point C relative to the Bottom of the bar:

$\vec{r}_{BC} = \underbrace{2}_{\text{magnitude } d=2}\vec{\lambda}_{BT} = 1.074\,i + 1.193\,j - 1.193\,\vec{k}$

the position vector of point C relative to the origin

$\vec{r}_{OC} = \vec{r}_{OB} + \vec{r}_{BC} = 4.074\,\vec{i} + 1.193\,\vec{j} + 8.807\,\vec{k}$

the position vector of point D is:

$\vec{r}_{OD} = 0\vec{i} + 3\vec{j} + 0\vec{k}$

the vector parallel to line CD:

$\vec{r}_{CD} = \vec{r}_{OD} - \vec{r}_{OC} = -4.074\,i - 8.807\,k + 1.807\,j$

$|\vec{r}_{CD}| = 9.87$

$\vec{\lambda}_{CD} = -0.4127\vec{i} + 18.31\vec{j} - 89.23\vec{k}$

the tension is:

$T_{CD} = 100\,\vec{\lambda}_{CD} = -41.27\vec{i} + 18.31\vec{j} + -89.23\vec{k}$

the magnitude of the moment about the z axis:

$|M| = \lambda_z \cdot (\vec{r}_{OC} \times T_{CD})$

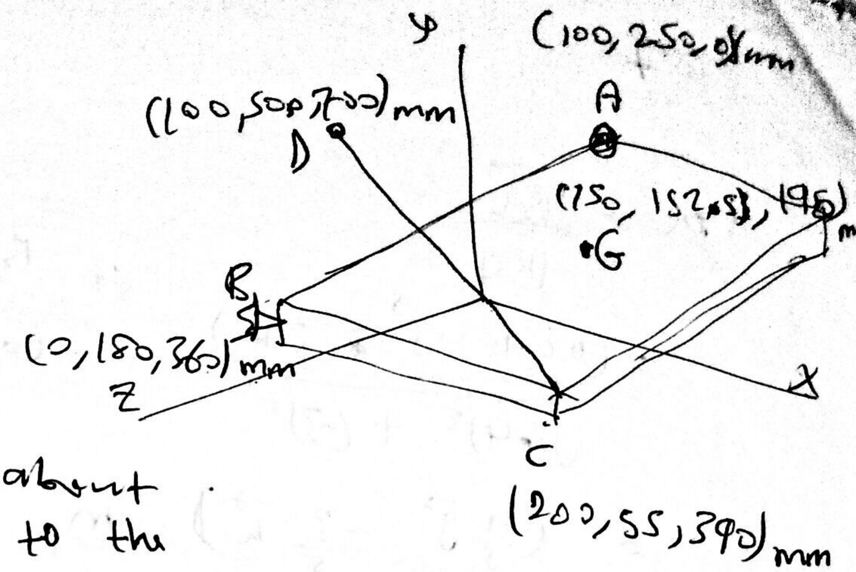
$$= \begin{vmatrix} 0 & 0 & 1 \\ 4.074 & 1.193 & 8.807 \\ -41.27 & 18.31 & -89.23 \end{vmatrix} = 123.83$$

Q.6

Weat G = 4 kg

$W = 4 * 9.81$

$= 39.24$ N.



Calculate the moment about the line BA due to the two forces:

$$\vec{\lambda}_{BA} = \frac{\vec{BA}}{|BA|} = \frac{0.1\vec{i} + 0.07\vec{j} - 0.36\vec{k}}{\sqrt{0.1445}}$$

$$= 0.2631\vec{i} + 0.1841\vec{j} - 0.9470\vec{k}$$

$$\vec{r_1} = (0.2\vec{i} - 0.125\vec{j}\ 0.03\vec{k})$$

$$F_1 = T_{CD}\frac{(0.1\vec{i} + 0.445\vec{j} + 0.03\vec{k})}{\sqrt{0.204125}} =$$

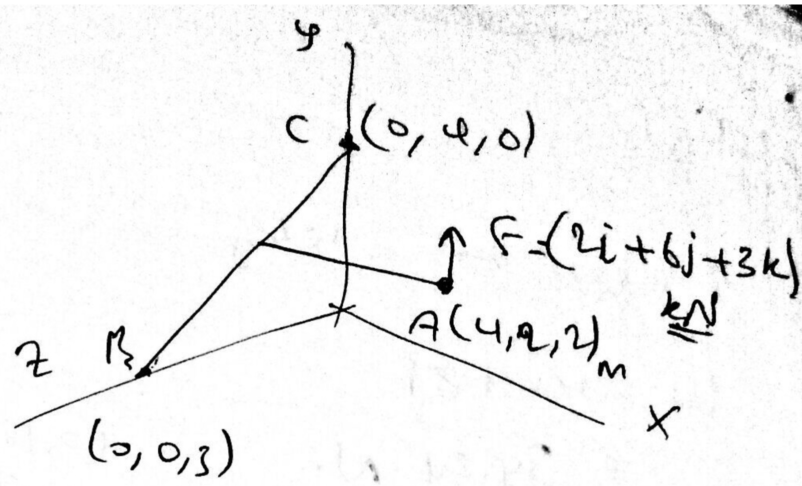
$$r_2 = (0.15\vec{i} - 0.0275\vec{j} - 0.165\vec{k})\ m$$

$$F_2 = -(4\ kg)(9.81)\ j$$

$$M_{BA} = \vec{\lambda}_{BA} \cdot (\vec{r_1} \times \vec{F_1} + \vec{r_2} \times \vec{F_2})$$

$$M_{BA} = 3.871 - 0.17793\ T_{CD} = 0 \rightarrow T_{CD} = 21.8\ N.$$

Q.7



$$\lambda_{BC} = \frac{\vec{BC}}{|BC|}$$

$$= \frac{(0\vec{i} + 4\vec{j} - 3\vec{k})}{\sqrt{(-4)^2 + (-3)^2}}$$

$$= \left(\frac{4}{5}\vec{j} - \frac{3}{5}\vec{k}\right) \text{ m}$$

$$M_B = \vec{r}_{BA} \times \vec{F}$$

$$\vec{r}_{BA} = 4\vec{i} + 2\vec{j} - 1\vec{k}$$

$$\vec{M}_B = (4\vec{i} + 2\vec{j} - 1\vec{k}) \times (2\vec{i} + 6\vec{j} + 3\vec{k})$$

$$= \begin{vmatrix} i & j & k \\ 4 & 2 & -1 \\ 2 & 6 & 3 \end{vmatrix} = (2\times3 + 6)i - (4\times3 + 2)\vec{j} + (24 - 4)\vec{k}$$

$$= (12\vec{i} - 14\vec{j} + 20\vec{k}) \text{ kN·m}$$

$$M_{AC} = \vec{\lambda}_{BC} \cdot \vec{M}_B = \begin{vmatrix} i & j & k \\ 0 & \frac{4}{5} & \frac{-3}{5} \\ 12 & -14 & 20 \end{vmatrix} = \left(\frac{4}{5} * 20 - \frac{3 * 14}{5}\right) - \left(\frac{+3}{5} * 12\right) + \left(\frac{-4}{5} * 12\right)$$

$$= -9.2 \text{ kN·m}$$