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Birzeit University
Faculty of Engineering & Technology
Department of Civil & Environmental Engineering
Mechanics of Materials – ENCE233
Mid-Term Exam

P1	7
P2	25
P3	30
P4	30
Total (100)	92

Total Time Allowed = 90 minutes

Instructors: Dr. Nizar Assi, Dr. Khalil Qatu — Sunday (November 3, 2019)

Problem 1 (10 Points)

Circle the correct answer for each of the following sentences:

1) Modulus of rigidity is: G
   a) A geometrical property
   b) A material property
   c) The slope of the σ-ε diagram within the linear elastic region
   d) None of the above
   e) All of the above
2) Statically indeterminate members are those having:
   a) Number of reactions greater than number of equilibrium equations
   b) Number of reactions greater than number of compatibility equations
   c) Number of reactions greater than number of equilibrium and compatibility equations
   d) None of the above
   e) All of the above
3) If a material expands freely due to heating, it will develop:
   a) *Thermal stress*
   b) Compressive stress
   c) No stress
   d) None of the above
   e) All of the above
4) Principle of superposition is applied for:
   a) Small deformation analysis
   b) Indeterminate axially loaded members
   c) Materials within the linear-elastic behavior
   d) None of the above
   e) All of the above
5) Ductility is:
   a) A property of materials that show large deformation before fracture
   b) A property of materials that show minimal deformation before fracture
   c) The opposite of brittleness
   d) None of the above
   e) All of the above

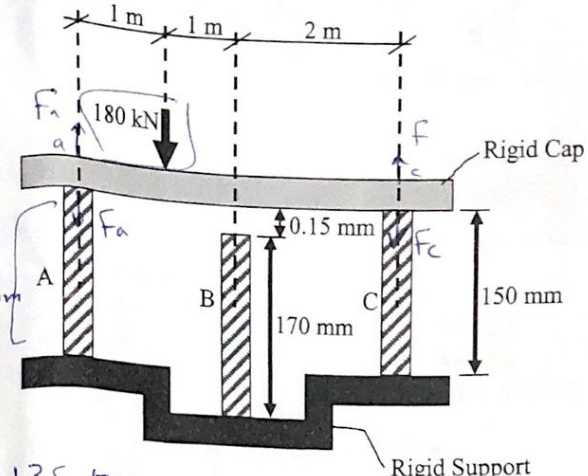
Fill your final answers in the table given below:

Number	1	2	3	4	5
Correct Answer	b	a	c	c	e

Problem 2 (30 Points)

Posts A, B and C are used to support a 180-kN load applied on the rigid cap as shown. Each post has a cross-sectional area of 500 mm^2 and has a modulus of elasticity E= 70 GPa. The original length of post B is 170 mm, whereas posts A and C have a length of 150 mm. determine:

a) The force in each post.
b) Change in length for each post.



$A_{Post} = 500\ mm^2$

$E = 70\ GPa$

Check:-

$\circlearrowleft + \Sigma M_a = 0$

$(-180\ kN) + F_c(4) = 0$

$F_c = \frac{+180}{4} = 45\ kN$

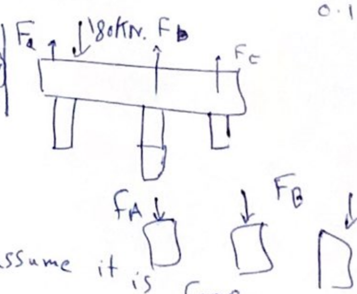
$\circlearrowleft \Sigma M_c = 0$

$(180)(3) - (4)F_a = 0 \Rightarrow F_a = 135\ kN$

$\delta_{Post\,C} = \frac{-F_c L_c}{EA} = \frac{-(45\ kN)(150\ mm)}{(70\ GPa)(500\ mm^2)} = -0.193\ mm$

$\delta_{Post\,A} = \frac{-(135)(150)}{(70)(500)} = -0.5786\ mm$ > Gap 0.15 mm

$\uparrow + \Sigma F_y = 0 \Rightarrow F_a + F_b + F_c = 180\ kN$ (1)



$\circlearrowleft \Sigma M_a = 0$: $(-180) + F_b(2) + F_c(4) = 0$

$2F_B + 4F_c = 180\ kN$ (2)

$\delta_B = \delta_C - 0.15 \Rightarrow \frac{(-)F_B L_B}{EA} = \frac{(-)F_c L_c}{EA} - 0.15$

$-F_B L_B = -F_c L_c - (0.15)(500)(70)$

$+F_B(170) = +F_c(150) + 5250$

$F_B = \left(\frac{150}{170}\right) F_c + 30.88$ → Sub. in (2):

$\Rightarrow \frac{300}{170} F_c + 61.7 + 4F_c = 180$

$5.76 F_c = 118.3 \Rightarrow F_c = 20.54\ kN$ (C) to Post C

from (2): $F_B = 48.9\ kN$ (C) to Post B $\Rightarrow F_A = 110.54\ kN$ (C) to Post A

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(b) $\delta_C = \dfrac{F_C \; L_C}{E \; A} = -0.088$ mm ~~-0.0238 mm~~

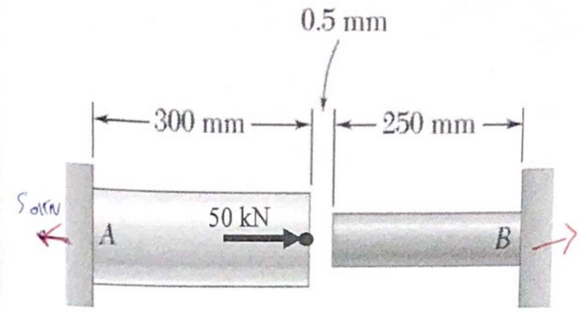
$\delta_B = \dfrac{F_B \; L_B}{E \; A} = -0.2375$ mm

$\delta_A = \dfrac{F_A \; L_A}{E \; A} = -0.4737$ mm

Problem 3 (30 Points)

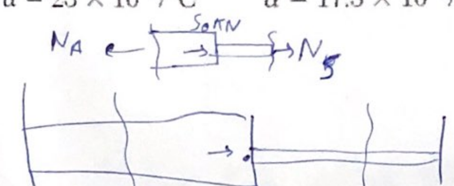
Aluminum and steel rods are fixed supported at A and B, respectively. At room temperature (20˚C) and with no external loads applied, a 0.5-mm gap exists between the ends of the two rods as shown. At a later time when the temperature reaches 93˚C and a 50-kN force is applied, determine:

a) the normal stress in the aluminum rod
b) the change in length of the aluminum rod.



Aluminum
$A = 2000\ mm^2$
$E = 75$ GPa
$\alpha = 23 \times 16^{-6}/°C$

Stainless steel
$A = 800\ mm^2$
$E = 190$ GPa
$\alpha = 17.3 \times 10^{-6}/°C$



$\Delta T = 73\ °C$

$\delta_A = \delta_T + \delta_{force}$

$= \Delta T\, \alpha\, L_o + \frac{N_A L_o}{E_{AL} A_{AL}}$

$= (73)(23 \times 10^{-6})(300) + \frac{(50\ kN)(300)}{(75)(2000)}$

$= 0.5037 + 0.1$

$= 0.6037\ mm > Gap.$

$\delta_{AB} = 0.5\ mm$

$\delta_A + \delta_B = 0.5$

$\Sigma F_x = 0 \Rightarrow N_s + 50 = N_A$

$\Delta T(\alpha_{AL})(L_A) + \frac{N_A L_{AL}}{E_{AL} A_{AL}} + \Delta T \alpha_s L_s + \frac{N_s L_s}{E_s A_s} = 0.5$

$0.5037 + N_A(2 \times 10^{-3}) + 0.315725 + N_s(1.645 \times 10^{-3}) = 0.5$

$0.5037 + 0.1 + (2 \times 10^{-3}) N_s + 0.315725 + (1.645 \times 10^{-3}) N_s = 0.5$

$(3.645) 10^{-3} N_s = -0.419425 \Rightarrow N_s = -115.07\ kN$

$\therefore N_A = 50 + N_s = -65.07\ kN.$

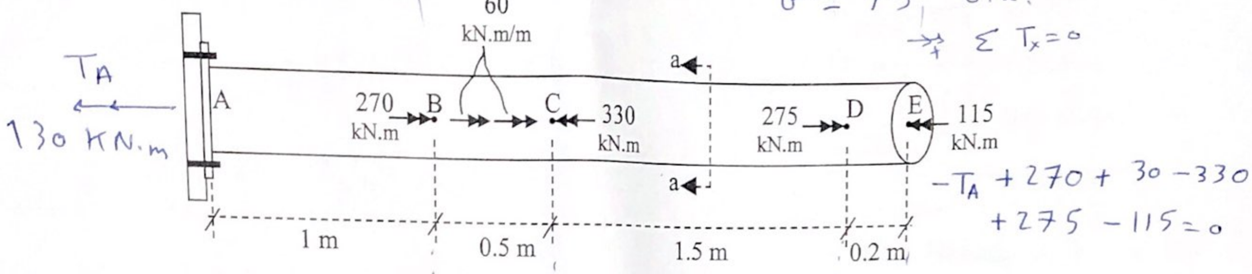
ⓐ $\sigma_{Al} = \frac{N_A}{A} = -0.032535$ GPa.

ⓑ $\delta_{AL} = \delta_T + \delta_{NA} = 0.5037 + \frac{(-65.07)(300)}{(2000)(75)}$

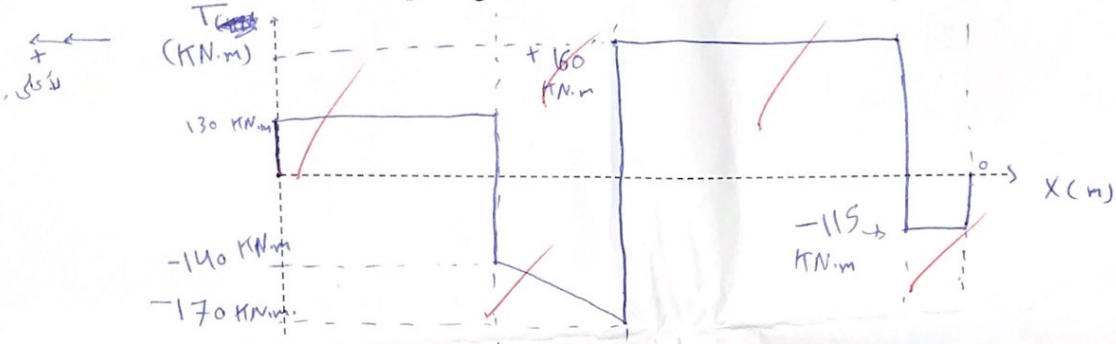
$= 0.37356\ mm$

Problem 4 (30 Points)

The solid circular shaft (ABCDE) is subjected to concentrated torques (at B, C, D, and E) and uniformly distributed torque (over region BC only) as shown. The shaft is fixed supported at A and has a modulus of rigidity (G) of 75 GPa.



a) Draw the internal torque diagram.



b) If the material of the shaft has an allowable shear stress of 150 MPa, determine the required diameter of the shaft.

$\tau_{allow.} = 150$ MPa $\quad \tau_{max} = \frac{T_{max}\, r_{req.}}{J}$

KN.m → 10^6 N.mm

$150 \text{ MPa} = \frac{(170 \times 10^3 (N.mm) \times 10^3)(r_{mm})}{\frac{\pi}{2} r^4_{mm}} = \frac{(170 \times 10^6)}{\frac{\pi}{2} r^3}$

$\Rightarrow r^3 = \left(\frac{170}{150}\right)\left(\frac{2}{\pi}\right) \times 10^6$

$r^3 = 0.7219 \times 10^6$

$r = 89.7$ mm

c) If the angle of twist of end E relative to end A is limited so that it does not exceed 0.05 rad, determine the required diameter of the shaft.

$\phi = \frac{TL}{JG}$

$\phi_E = 0.05$ rad

$= \phi_{ED} + \phi_{D/C} + \phi_{C/B} + \phi_{B/A}$

$= \frac{(-115 \text{ KN.m})(0.2 \text{ m})}{JG} + \frac{(160 \text{ KN.m})(1.5 \text{ m})}{JG} + \int_1^{1.5} \frac{T(x)\, dx}{JG} + \frac{(130)(1)}{JG}$

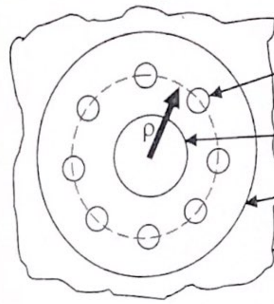
$= \frac{1}{JG}\left[-23 + 240 - 70 - 7.5 + (130)\right]$

$0.05 = \frac{269.5 \times 10^6 \text{ KN.mm}}{J\,(75)_{GPa}} \Rightarrow J = 71.8667 \times 10^6 \ (mm^4)$

$= \frac{\pi}{2}[r^4]$

$r = 82.254$ mm

d) If eight bolts, each has a diameter of 30 mm and a failure stress of τ_{fail} =200 MPa, are used to fix the base of the shaft to rigid wall at A as shown below, determine the smallest radial distance ρ from the center of shaft to the center of each bolt so that the bolts will not fail. Apply a factor of safety of F.S = 1.5.



$\tau_{fail} = 200\ MPa$

$\tau_{allow} = \frac{\tau_{fail}}{F.S} = \frac{200}{1.5} = 133.33\ MPa.$

$\tau_{allow} = \frac{F}{A_{bolt}} = \frac{16.25}{P} \times 10^6 \div \frac{4}{\pi \cdot (30)^2}$

$200\ MPa = \frac{23000.7}{P}$

$P = 115\ mm$

$T = 130 \times 10^6\ N \cdot mm$

$= n\ P\ F$

$10^6 \times 130 = (8)\ P\ F$

$F = \frac{16.25 \times 10^6}{P}$ --- (1)

e) For section a-a within region CD, qualitatively draw the variation of shear stress along the radial line, shown below, and show the stress at point P on a volume element.