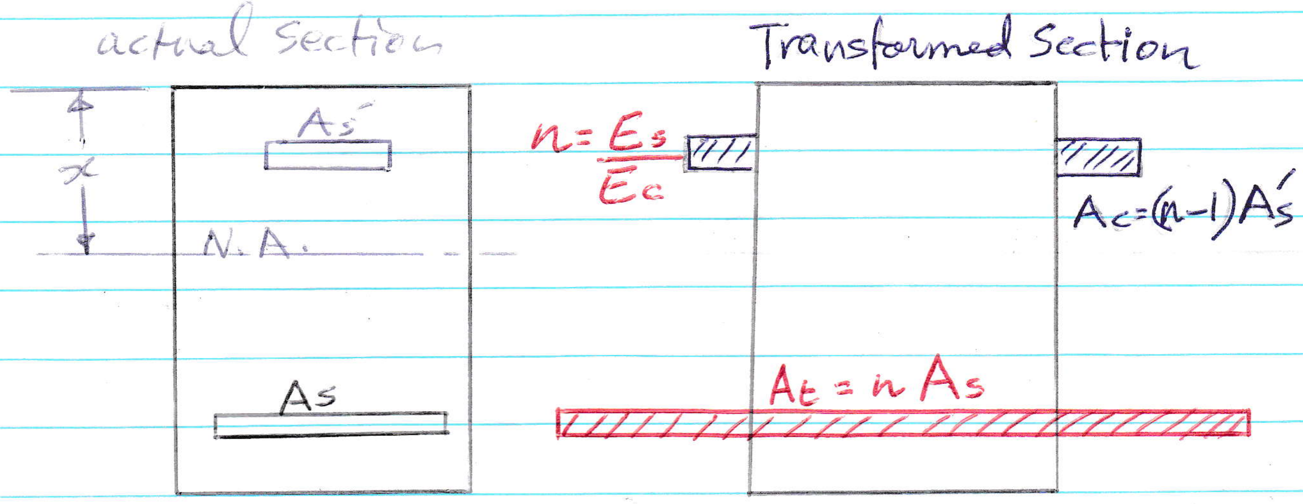
Deflections:

Transformed Section:



$$A_s f_s = A_t f_t$$
$$f_t = \frac{f_s}{n}$$

Example:

for $A_s' = 9.68\ cm^2$, $A_s = 45.68\ cm^2$

$f_c' = 30\ MPa$, $E_s = 200\,000\ MPa$

$E_c = 4700\sqrt{30} \approx 25\,000\ MPa$

$n = 8$, $b = 35\ cm$, $d = 55\ cm$, $d' = 6\ cm$

$h = 61\ cm$

Locate N.A.

$$A_t = 8(45.68) = 365\ cm^2$$
$$A_c = (8-1)(9.68) = 68\ cm^2$$

$$(35)(x)\left(\frac{x}{2}\right) + 68(x-6) = (365)(55-x)$$

solving for x, $x = 24\ cm$

$$I_{cr} = \frac{1}{3}(35)(24)^3 + 68(24-6)^2 + 365(55-24)^2 = 534{,}077\ cm^4$$

$$I_g\text{, ignoring steel, } = \frac{(35)(61)^3}{12} = 662{,}000\ cm^4$$

Effective Moment of Inertia, I_e

$$I_e = \left(\frac{M_{cr}}{M_{max}}\right)^3 I_g + \left[1 - \left(\frac{M_{cr}}{M_{max}}\right)^3\right] I_{cr} \leq I_g$$

(24.2.3.5 a)

$$M_{cr} = \text{cracking moment} = \frac{f_r I_g}{y_t} \quad (24.2.3.5\,b)$$

$f_r = \overset{0.62}{0.7}\sqrt{f'_c}$

y_t = distance from N.A. to extreme fiber of concrete in tension $(h/2)$

M_{max} = maximum service load moment acting at the condition for which deflection is computed

a) Instantaneous deflection is the immediate (elastic) deflection determined using analysis.

b) Long-term (creep and shrinkage) deflection is determined as follows:

$$\Delta_{cp+sh} = \lambda (\Delta i)_D$$

$$\lambda = \frac{\xi}{1+50\rho'} \qquad \rho' = \frac{A'_s}{bd}$$

$\xi = 2.0$ (≥ 5 years)
$= 1.4$ (1 year)
$= 1.2$ (6 months)
$= 1.0$ (3 months)

$(\Delta i)_D$ = instantaneous deflection due to all sustained loads (usually dead load).

Maximum permissible deflections are provided by the ACI- Code.

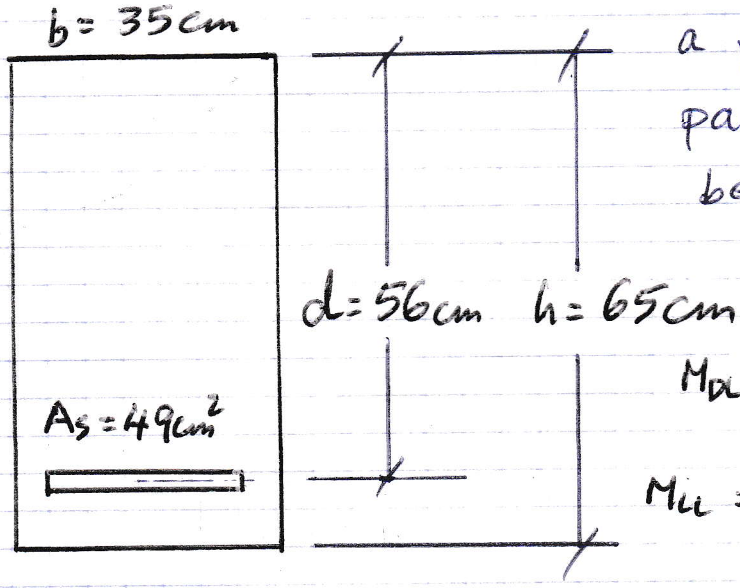
Example:

Investigate the deflection behavior for the beam shown. The beam is used on a 7.6-m simple span.

Service DL = 3.2 t/m , LL = 2.9 t/m

use $f_c' = 28$ MPa , $f_y = 300$ MPa

$E_c = 24870$ MPa



The beam supports a floor with several partitions that will be damaged by large deflections.

$$M_{DL} = \frac{3.2(7.6^2)}{8} = 23.1 \text{ t.m}$$

$$M_{LL} = \frac{(2.9)(7.6)^2}{8} = 20.9 \text{ t.m}$$

From ACI- Table 9.3.1.1 , for $f_y = 300$ MPa ,

$$h_{min} = \frac{l}{16}\left(0.4 + \frac{f_y}{700}\right) = \frac{760}{16}\left(0.4 + \frac{300}{700}\right)$$

$$= 39.4 \text{ cm}$$

(Note that h = 65 cm is used, which considerably exceeds the specified 39.4 cm.

$$W_u = 1.2(3.2) + 1.6(2.9) = 8.48 \text{ t/m}$$

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$$M_u = \frac{(8.48)(7.6)^2}{8} = 61.22 \text{ t.m}$$

$$M_{n,req} = \frac{61.22}{0.9} = 68.03 \text{ t.m}$$

$m = 12.61$

$$R_{n,req} = \frac{6803}{(35)(56)^2} = 0.0620 \text{ t/cm}^2$$

$\rho_{req} = 0.02443$, $A_{s,req} = 47.88 \text{ cm}^2$, $A_{s,actual} = 49 \text{ cm}^2$

$\rho_{max} = 0.31875\left(\frac{28}{300}\right)(0.85) = 0.0253$, $A_{s,max} = 49.57 \text{ cm}^2$

$\frac{\rho_{req}}{\rho_{max}} = \cancel{0.96}$ 0.99 (~~The required is very close to the maximum~~)

Although a depth of 65 cm is used, the high reinforcement ratio strongly indicates that a deflection problem is likely to occur.

$$I_g = \frac{(35)(65^3)}{12} = 800990 \text{ cm}^4$$

for the cracked section, locate the neutral axis under service loads,

$$n = \frac{200\,000 \text{ MPa}}{24\,870 \text{ MPa}} = 8.04$$

$$(35)(x)(x/2) = (49 \times 8.04)(56 - x)$$

$$\therefore x = 26.0 \text{ cm}$$

$$I_{cr} = \frac{1}{3}(35)(26.0)^3 = 204\,970 \text{ cm}^4$$

$$+ (49 \times 8.04)(56-26)^2 = 354\,560 \text{ cm}^4$$

$$I_{cr} = 559\,530 \text{ cm}^4$$

$$M_{cr} = \frac{f_r I_g}{y_t} \quad (24.2.3.5b)$$

$$f_r = 0.62 \lambda \sqrt{f_c'} \quad (19.2.3.1) = 3.28 \text{ MPa}$$

$$M_{cr} = \frac{(3.28/100)\, t/cm^2 \, (800990\, cm^4)}{32.5\, cm} \times \frac{1\, m}{100\, cm}$$

$$= 8.09 \text{ t.m}$$

For DL deflection, $\frac{M_{cr}}{M_{max}} = \frac{8.09}{23.1} = 0.35$

$$(I_e)_{DL} = (0.35)^3 (800990) + \left[1 - (0.35)^3\right](559530)$$

$$= 569880 \text{ cm}^4 < I_g$$

For total load deflection, $\frac{M_{cr}}{M_{max}} = \frac{8.09}{44} = 0.184$

$$(I_e)_{TL} = (0.184)^3 (800990) + \left[1 - (0.184)^3\right](559530)$$

$$= 561030 \text{ cm}^4 < I_g$$

Immediate Deflections:

$$(DL): \; (\Delta_i)_{DL} = \frac{5 w l^4}{384 EI} = \frac{(5)(3.2)(7.6^4)(100)^3}{(384)(248.7)(569880)}$$

$$= 0.98 \text{ cm}$$

$$(TL): \; (\Delta_i)_{TL} = \frac{(5)(6.1)(7.6)^4 (100)^3}{(384)(248.7)(561030)} = 1.90 \text{ cm}$$

$$\therefore (\Delta_i)_{LL} = 1.90 - 0.98 = 0.92 \text{ cm}$$

Long-term Deflections:

$$\lambda_\Delta = 2.0 / 1 + 50(0) = 2.0$$

$\Delta_{long\ term} = (2.0)(0.98) = 1.96\ cm$

From Table 24.2.2, the maximum permissible calculated deflection,

$$(\Delta i)_{LL} + (\Delta)_{long\ term} \leq l/480$$

$$0.92 + 1.96 = 2.88\ cm$$

$$l/480 = \frac{760}{480} = 1.58\ cm$$

which clearly indicates that a deflection problem exists as was expected due to the high reinforcement ratio.

Note that Table 9.3.1.1 requirements would not be sufficient to prevent deflection problems. The reinforcement ratio is a much better indicator.