Dr. Jamal Zalatimo

Doubly Reinforced Beams

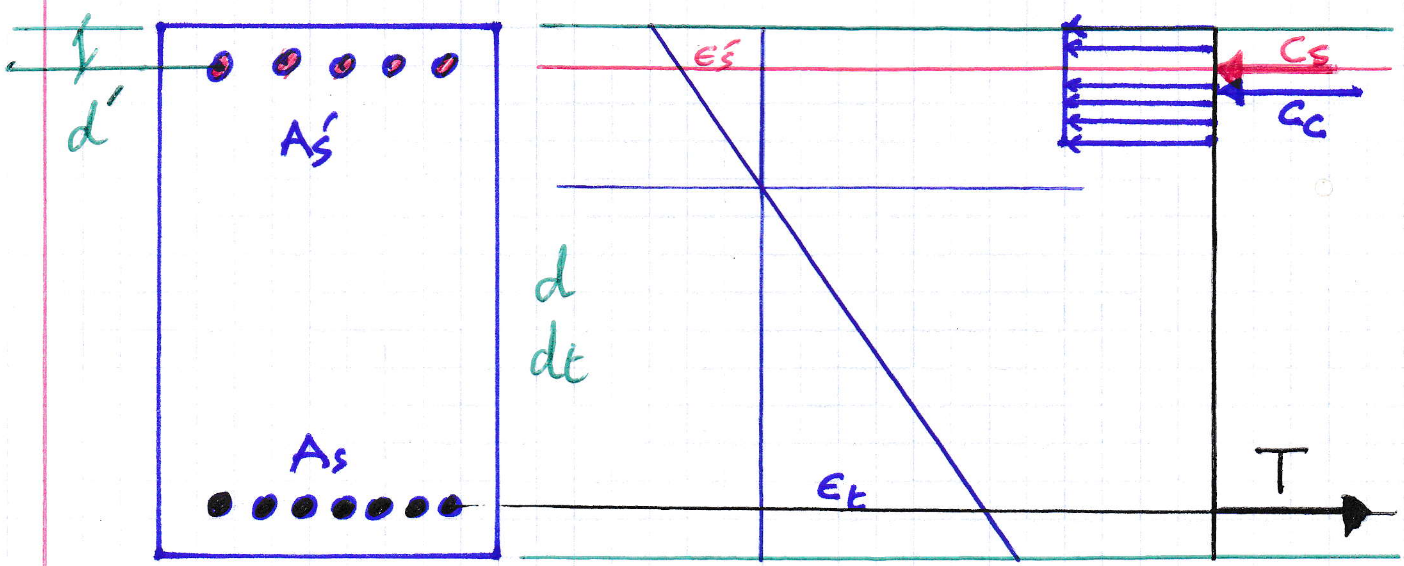
A_s : tension reinforcement

A_s' : compression reinforcement

Purposes :

* Increase capacity, however not in a direct manner but, by increasing the ability of the beam to take additional tension reinforcement

* Control deflection



$$T = C_c + C_s$$

$$M_n = C_c \left(d - a/2\right) + C_s \left(d - d'\right)$$

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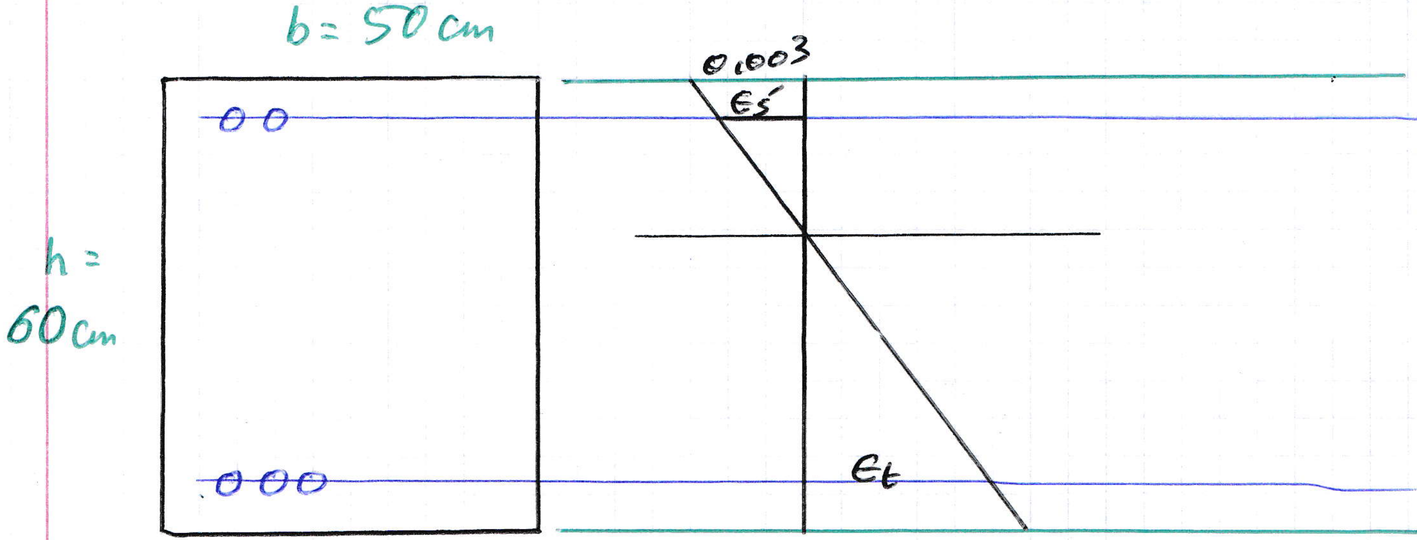
Example



$f_c' = 28$ MPa

$f_y = 420$ MPa

ϕ 30 tension bars $\quad d_t = 53.5$ cm

ϕ 10 stirrups

$M_u = 100$ t.m

Design the beam for flexure.

Note: Excessive deflection will not result in serviceability problems.

$$\rho_{max}\big|_{\varepsilon_t = 0.005} = 0.01806$$

$$A_{s\,max}\big|_{\varepsilon_t = 0.005} = \underline{48.31\ cm^2}$$

$$A_{s\,min} = \left[0.25\sqrt{28}/420\right](50)(53.5) \geq \left[\frac{1.4}{420}\right](50)(53.5)$$

$$= 8.42\ cm^2 \geq \underline{8.92\ cm^2}$$

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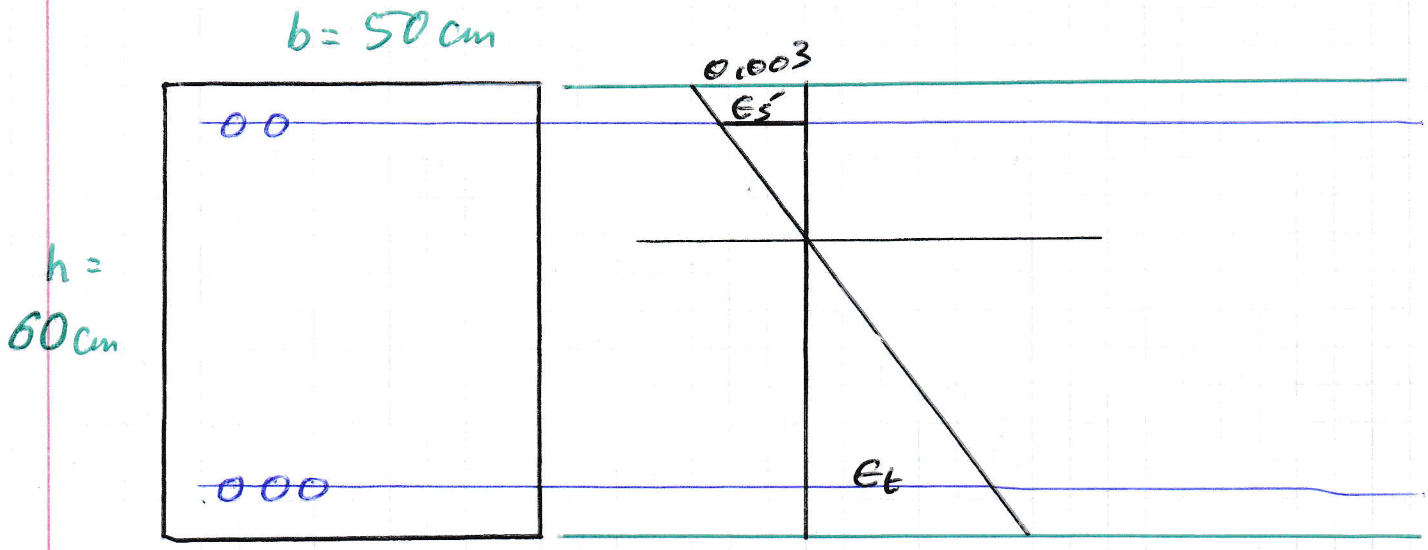
Example



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$$Mn_{req} = \frac{100}{0.9} = 111.11 \text{ t.m}$$

$$Rn_{req} = \frac{11111}{(50)(53.5)^2} = 0.07764 \text{ t/cm}^2$$

$$m = 17.65$$

$$\rho_{req} = 0.02326 \quad (As_{req} = 62.22 \text{ cm}^2)$$

This amount of reinforcement exceeds the maximum.

If the beam is reinforced with As_{max} ($\epsilon_t = 0.005$),

$$T = 48.31 \times 4.2 = 202.9 \text{ t.m} \quad (= C_c)$$

$$a = 17.05 \text{ cm} \qquad (x = 20.06 \text{ cm})$$

$$Mn = 202.9 \left(\frac{53.5 - 17.05/2}{100} \right) = 91.25 \text{ t.m}$$

$(= Mnc)$

∴ Compression steel must be used

$$Mns = 111.11 - 91.25 = 19.86 \text{ t.m}$$

$$Cs_{req} = \frac{19.86 \times 100 \text{ t.cm}}{(53.5 - 6)} = 41.80 \text{ t}$$

$d' = 6$ cm assuming $\phi 20$ compression bars

Note that this requires the concrete in compression to work at its ultimate capacity, when $\epsilon_t = 0.005$

Therefore, $$\epsilon_s' = \frac{20.06 - 6}{20.06}(0.003) = 0.00210$$

Note:

$$x = \frac{17.05}{0.85} = 20.06 \text{ cm}$$

which is exactly equal to x_{max} $\epsilon_t = 0.005$

$$= \frac{3}{8}(53.5)$$

Also, $\epsilon_s' = 0.00210 = \epsilon_y$

$\therefore f_s' = 4.20 \text{ t/cm}^2$

However, if ϵ_s' was less than ϵ_y,

$f_s' = \epsilon_s' \times E_s$

$E_s = 200 \text{ GPa} = 2000 \text{ t/cm}^2$

There is no requirement for ϵ_s' to equal or exceed ϵ_y

$C_s = A_s' (f_s' - 0.85 f_c')$

$0.85 f_c'$ is subtracted from f_s' so that C_c may be calculated for the entire portion of the section in compression, without having to subtract A_s' for $(a \times b)$

Therefore,

$$A'_{s\,req} = \frac{41.80\,t}{[4.2 - 0.85(0.28)]} = 10.55\,cm^2$$

using ϕ 20 bars, 3.4 bars are required

∴ Use 4 ϕ 20 compression bars.

$$A_{s\,req} = \frac{202.9 + 41.8}{4.2} = 58.26\ cm^2$$

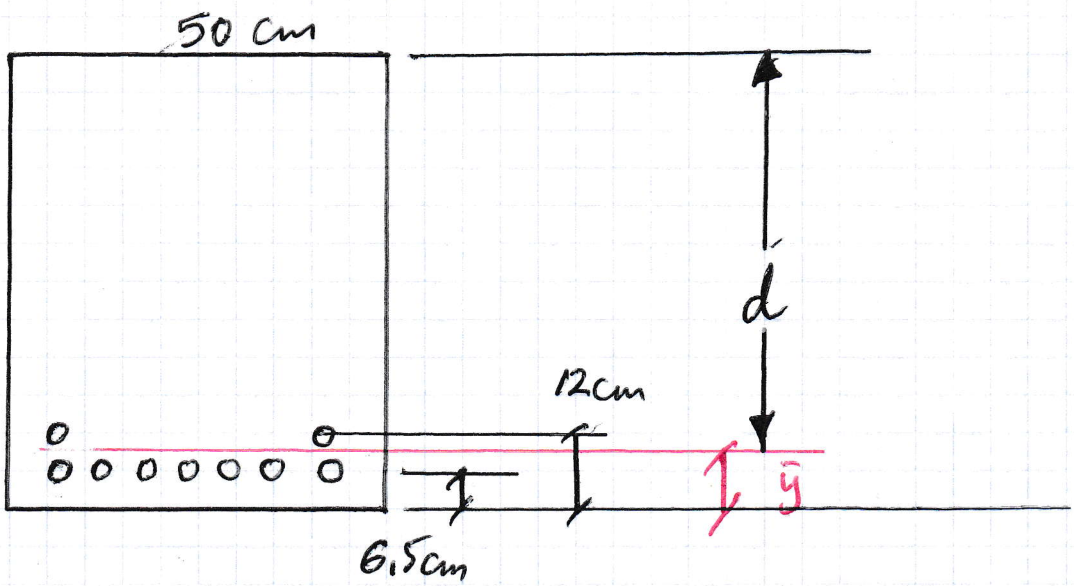
8.24 ϕ 30 bars

Use 9 ϕ 30 bars.

These will not fit in one layer
(see Table 2)

Only 7 ϕ 30 will fit within b = 50 cm

∴ Two layers must be used



$$\bar{y} = \frac{(7\times 6.5) + (2\times 12)}{9} = 7.72\ cm$$

$$d = 52.28\ cm$$

which is less than $d_f = 53.5\ cm$ which was used in the calculations

Check the final capacity

$$T = (9\times 7.07)*(4.2) = 267.2\ t$$

$$A_{s\ actual} = 63.62\ cm^2$$

$$T = C_c + C_s$$

Assuming that the compression steel yields,

$$C_s = (4\times 3.14)\left[4.2 - 0.85(0.28)\right] = 49.79\ t$$

$$A_s' = 12.57\ cm^2$$

$$C_c = 0.85(0.28)(50)(0.85x) = 10.12x$$

$$267.2 = 10.12x + 49.79$$

$$\therefore\ x = 21.52\ cm \quad (a = 18.29\ cm)$$

Check the assumption,

$$\epsilon_s' = \left[\frac{21.52 - 6}{21.52}\right](0.003) = 0.00216 > \epsilon_y$$

$\therefore$ the assumption is correct

$\therefore C_c = 10.12(21.52) = 217.8\,t$

$C_s = 49.8\,t$

$C_c + C_s = 267.6\,t = T$

$\therefore$ Calculations are correct

$$M_n = 217.8\left(\frac{52.28 - 18.29/2}{100}\right)$$

$$+ 49.8\left(\frac{52.28 - 6}{100}\right)$$

$$= 93.9 + 23.1$$

$$= 117.0\ t.m$$

However, ϕ must be checked,

$\phi M_n \geq M_u$

$$\epsilon_t = \frac{53.5 - 21.52}{21.52}(0.003)$$

$$= 0.004458 < 0.005$$

$\therefore \phi = 0.855$

$\phi M_n = 100.0\ t.m$

$\therefore$ the Design is Perfect !!

However, excessive deflection is expected.

Repeat the design of the beam, however,

excessive deflection must be prevented.

this may be indirectly achieved by limiting x_{actual} to about $0.75 x_{max}$

$\epsilon_t = 0.005$

$$x \approx 0.75(20.06) = 15.05 \text{ cm}$$

$$a = 12.79 \text{ cm}$$

$$C_c = 152.2 \text{ t}$$

$$M_{nc} = 152.2 \left(\frac{53.5 - 12.79/2}{100}\right) = 71.69 \text{ t.m}$$

$$M_{ns\,req} = 111.11 - 71.69 = 39.42 \text{ t.m}$$

$$\epsilon_s' = \frac{15.05 - 6}{15.05}(0.003) = 0.001804$$

$< \epsilon_y$

$$\therefore f_s' = 0.001804(2000) = 3.61 \text{ t/cm}^2$$

$$C_{s\,req} = \frac{3942 \text{ t.cm}}{53.5 - 6} = 82.99 \text{ t}$$

$$A_{s\,req}' = \frac{82.99}{3.61 - 0.85(0.28)} = 24.61 \text{ cm}^2$$

$7.83 \Rightarrow 8\phi20$ bars in one layer

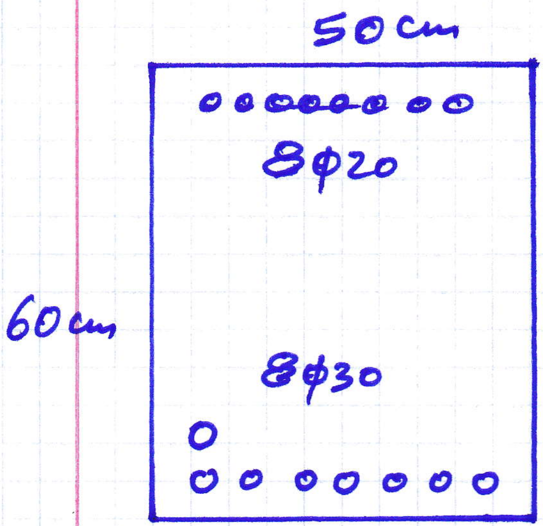
Compression steel $A_s' = 25.13 \text{ cm}^2$

$$As_{req} = \frac{152.2 + 83.0}{4.2}$$

$$= 56.0\ cm^2$$

7.9 bars $\Rightarrow$ use 8 ϕ 30 tension bars.

$As_{actual} = 56.56\ cm^2$



$$\bar{y} = \frac{7 \times 6.5 + 1 \times 12}{8}$$

$$= 7.19\ cm$$

$$d = 52.81\ cm$$

$$d_t = 53.5\ cm$$

Check Capacity,

Assume that the compression steel yields, (note that in this case it is obvious that the assumption is wrong, however, just to see in a general case)

$C_s = 25.13(4.2 - 0.85(0.28)) = 99.57t$

$C_c = 10.12\,x$

$T = 56.56 \times 4.2 = 237.6t$

$$237.6 = 10.12x + 99.57$$

$$x = 13.63 \text{ cm}$$

$$\epsilon_s' = \frac{13.63 - 6}{13.63}(0.003)$$

$$= 0.00168 < \epsilon_y$$

∴ the assumption is wrong!

f_s' must be written as a function of x

$$\epsilon_s' = \left(\frac{x-6}{x}\right)(0.003)$$

$$f_s' = 6\left(\frac{x-6}{x}\right)$$

$$C_s = 25.13\left(6 - \frac{36}{x} - 0.85(0.28)\right)$$

$$= 144.8 - \frac{904.7}{x}$$

$$237.6 = 144.8 - \frac{904.7}{x} + 10.12x$$

$$10.12x^2 - 92.8x - 904.7 = 0$$

$$x = 15.09 \text{ cm}$$

$$(a = 12.83 \text{ cm})$$

$$(\epsilon_t = 0.00764)$$

Note:

$$\varepsilon_s' = \frac{15.09 - 6}{15.09}(0.003) = 0.001807 < \varepsilon_y$$

$C_s = 84.85\, t$

$C_c = 152.7\, t$

$C_c + C_s = 237.6\, t = T$

$\therefore$ Calculations are correct,

$$M_n = 152.7\left(\frac{52.81 - 12.83/2}{100}\right) + 84.85\left(\frac{52.81 - 6}{100}\right)$$

$$= 70.85\ t.m + 39.72\ t.m$$

$$= 110.6\ t.m$$

$$\phi M_n = (0.9)(110.6) = 99.54\ t.m$$

Exercise:

Repeat with $A_s = 9\phi 30$

$A_s' = 8\phi 20$

Note:

When $4\phi 20$ compression bars were used,

$$x_{max} \underset{\epsilon_t = 0.005}{=} \frac{3}{8} \times 53.5 = 20.06 \text{ cm}$$

$$C_{c_{max}} = 202.9 \, t$$

$$\epsilon_s' = 0.002103, \quad f_s' = f_y = 4.2 \, t/cm^2$$

$$C_s = 12.57\left(4.2 - 0.85(0.28)\right) = 49.79 \, t$$

$$A_{s\,max} \underset{\substack{\epsilon_t = 0.005 \\ A_s' = 4\phi 20}}{=} \frac{202.9 + 49.8}{4.2}$$

$$= 60.16 \text{ cm}^2$$

Note that when $9\phi 30$ were used in part 1 of the example,

$A_{s\,actual} = 63.62 \text{ cm}^2$ exceeded this limit, and ϕ was reduced,

The same result was obtained by determining ϵ_t.

Exercise:

Repeat for $A_s' = 8\phi 20$ compression bars

Determine $A_{s\,max}$ ($\epsilon_t = 0.005$, $A_s' = 8\phi 20$)