BIRZEIT UNIVERSITY

Reinforced Concrete Design I
ENCE 335

Doubly Reinforced beams

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Doubly Reinforced beams

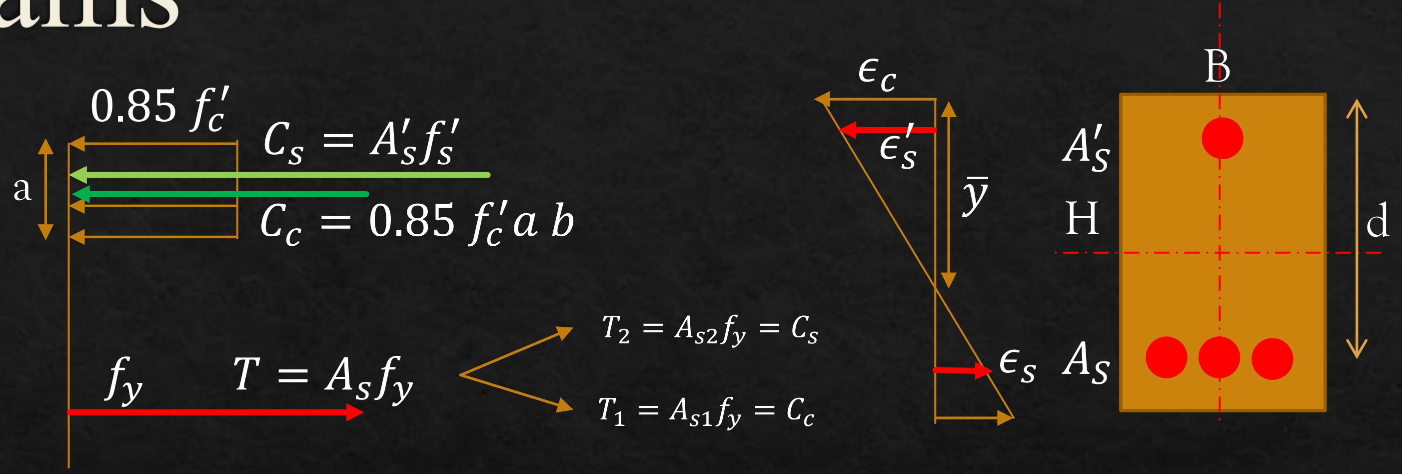
- What to do IF the reinforcement ratio exceeds the maximum allowed by ACI code ??
  - Increase Dimensions, Which one is more effective ? ?
  - If dimensions are limited?
  - Add compression steel to ensure concrete doesn’t fail
    - Develop an extra compression force to balance the required tension force without compression failure.

Doubly Reinforced beams

◈ Nominal strength:

TABLE 3.2
Minimum beam depths for compression reinforcement to yield

	$\epsilon_t = 0.004$		$\epsilon_t = 0.005$	
f_y, psi	Maximum d'/d	Minimum d for $d' = 65$ mm, mm	Maximum d'/d	Minimum d for $d' = 65$ mm, mm
40,000	0.23	282	0.20	325
60,000	0.13	500	0.12	542
75,000	0.06	1083	0.05	1300



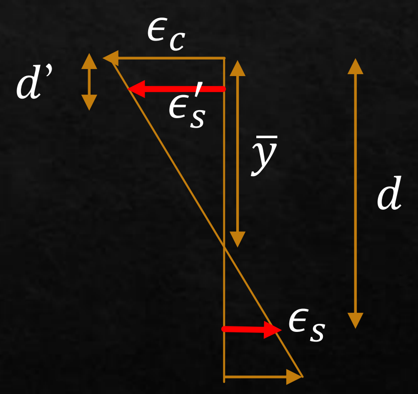
Nominal strength

- C_s coupled with part of T
  - Compression steel yields ($\epsilon_s' > \epsilon_y$) — $C_s = A_s f_y$ — $\phi M_{n_1} = \phi A_s' f_y (d - d')$
  - Compression steel doesn't yield ($\epsilon_s' < \epsilon_y$) — $C_s = A_s f_s$, $f_s' = E\epsilon_s'$ — $\phi M_{n_1} = \phi A_s' f_s' (d - d')$
- C_c coupled with rest of T
  - $T_1 = A_{s1} f_y$
  - $T_1 = 0.85 f_c' ab$ — $\phi M_{n2} = \phi T_2 \left(d - \frac{a}{2}\right)$
  - $T_1 = T - T_2$

Doubly Reinforced beams

- Tension Reinforcement limits
  - Minimum reinforcement remains the same
  - Maximum reinforcement allowed by ACI code still maintains a tension strain in steel $\epsilon_t \geq 0.004$
    - We can calculate and compare the strain in the tension steel with the limit from the strain distribution

$$\epsilon_s = \epsilon_u \frac{d - \bar{y}}{\bar{y}} \geq 0.004$$



    - Or we can use this formula (derived from strain distribution and section equilibrium)

$$\bar{\rho}_{0.004} = \rho_{0.004} + \rho' \frac{f_s'}{f_y}$$

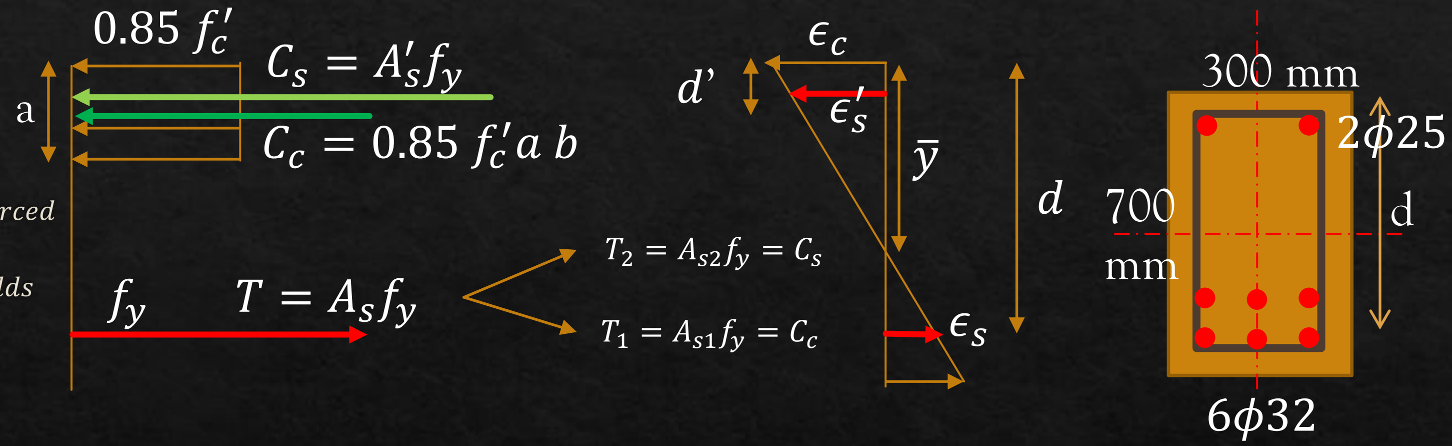
Doubly Reinforced beams

- Example: Moment Capacity

$f_y = 420\ MPa\ , f_c' = 35\ MPa$

Check: $\rho = \frac{A_s}{bd} = 0.027 > \rho_{max}$ $\rightarrow$ *This section was designed as doubly reinforced*

$check:\ yielding\ of\ compression\ steel \rightarrow \frac{d'}{d} = 0.104 < 0.13 \rightarrow comp.steel\ yields$



$we\ need\ to\ find\ location\ of\ N.A$

$T = C$

$A_s f_y = A_s' f_y + 0.85 f_c' a\ b \ \rightarrow a = 183.24mm \rightarrow \bar{y} = 229mm$

Check strain in tension and comp. steel :$\epsilon_s' = 2.18 * 10^{-3} > \epsilon_y \rightarrow comp.Steel\ yields$ … … $\epsilon_s = 4.88 * 10^{-3} > 0.004 \ \rightarrow tension\ controlled$

$\epsilon_s = 4.88 * 10^{-3} < 0.005 \ \rightarrow \phi \neq 0.9 \ \rightarrow \phi = 0.89$

$$\phi M_n = \phi M_{n_1} + \phi M_{n2} = 0.89 * A_s' f_y\ (d - d') + 0.89 * \left(A_s f_y - A_s' f_y\right)\left(d - \frac{a}{2}\right) = 206.5 + 743 = 949.5\ kN.m$$

Doubly Reinforced beams

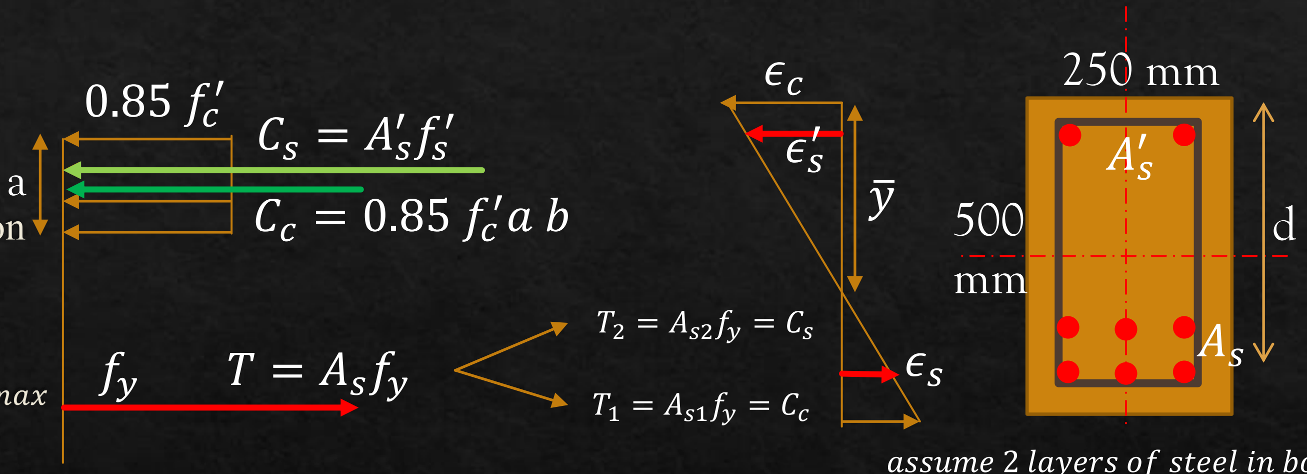
- Example: Design

  $f_y = 420\ MPa\ , f_c' = 28\ MPa, M_u = 290\ kN.m$

  Assume the section is to be designed and singly reinforced section

  Assume $\phi = 0.9$

  $R = \frac{M_u}{\phi b d^2} = 7.39\ \rightarrow Table\ A5 \rightarrow doesn't\ exist\ \rightarrow \rho_{req} > \rho_{max}$



assume 2 layers of steel in bottom

$d = 417\ mm\ \ldots\ldots d' = 60\ mm$

  The section should be designed as doubly reinforced section

  $PART\ 1: C_c\ coupled\ with\ T_{s_1} \rightarrow Take\ \rho = \rho_{0.005} \rightarrow R = 6.36\ MPa\ \rightarrow \phi M_{n1} = 248.8\ kN.m\ \ldots\ldots A_{s1} = \rho bd = 1887\ mm^2$

  $T_1 = C_c \rightarrow a = 133.2\ mm\ \rightarrow \bar{y} = 156.7\ mm$

  Remaining Moment to be carried by coupling between comp. steel and tension steel

  $\phi M_{n2} = M_u - \phi M_{n_1} = 290 - 248.8 = 41.2\ kN.m$

Doubly Reinforced beams

◈ Example: Design

$PART\ 2: \phi M_{n2} = 41.2\ kN.m$

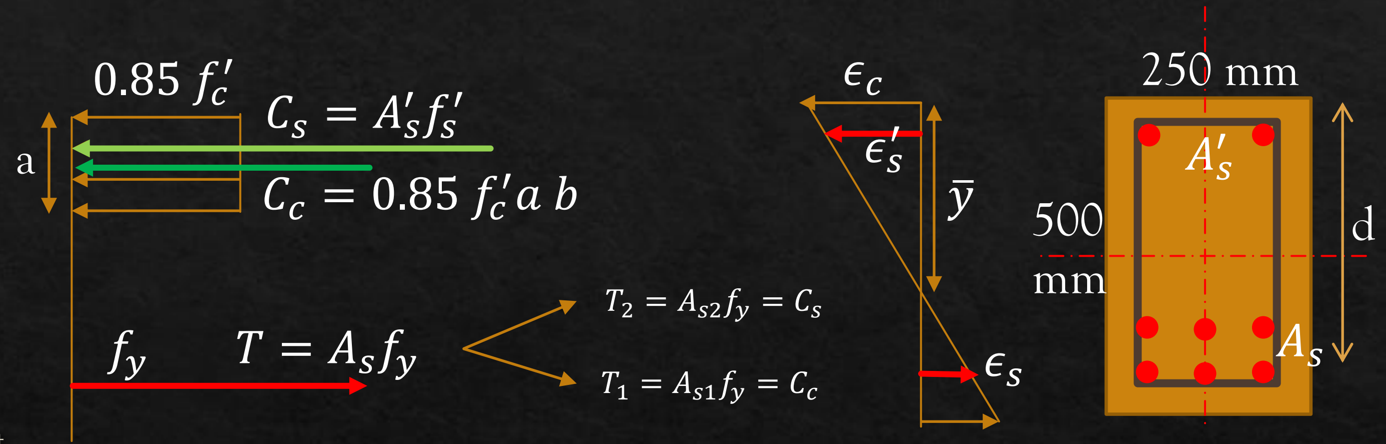
Check yielding in Comp. steel → $\bar{y} = 156.7\ mm \rightarrow \epsilon_s' = 1.85 * 10^{-3}$

$\epsilon_s' < \epsilon_y \rightarrow f_s' = 370\ MPa$

$\phi M_{n2} = 41.2\ kN.m = \phi A_s' f_s' \ (d - d') \rightarrow A_s' = 346.6\ mm^2$

We need to add an equivalent steel to the tension side to be coupled with it

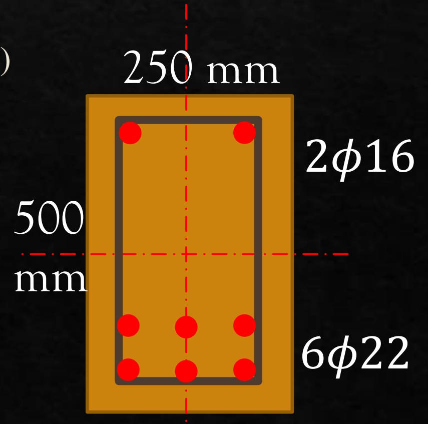
$C_s = T_2 \rightarrow A_s' f_s' = A_{s2} f_y \rightarrow A_{s2} = 304.8\ mm^2 \ \rightarrow A_s = A_{s1} + A_{s2} = 2191\ mm^2$



$assume\ 2\ layers\ of\ steel\ in\ bottom$
$d = 417\ mm\ \ldots\ldots d' = 60\ mm$

Go to Tables A2 and A7 to choose the most suitable compression steel bars ($A_s' = 346.6\ mm^2$) and tension steel bars ($A_s = 2191\ mm^2$)

Let's choose 2φ16 ($A_s' = 398\ mm^2$) for comp. bars and 6φ22 ($A_s = 2322\ mm^2$) for tension bars



NOW we need to check our initial assumptions and the moment capacity of the chosen cross-section

Doubly Reinforced beams

⟐ Example: Design

Let's choose 2φ16 ($A_s' = 398\ mm^2$) for comp. bars

and 6φ22 ($A_s = 2322\ mm^2$) for tension bars

check our initial assumptions and the moment capacity of the chosen cross-section

⟐ Check comp. steel : Location of N.A. → $T = C$

$A_s f_y = A_s' f_s' + 0.85\ f_c' a\ b$ (two unknowns f_s' & a)

itr1: assume $f_s' = 370\ MPa\ \rightarrow a = 139.16\ mm\ \rightarrow \bar{y} = 163.71\ mm$

check $f_s' \rightarrow \epsilon_s' = 1.94 * 10^{-3} \rightarrow f_s' = 387.42\ MPa$

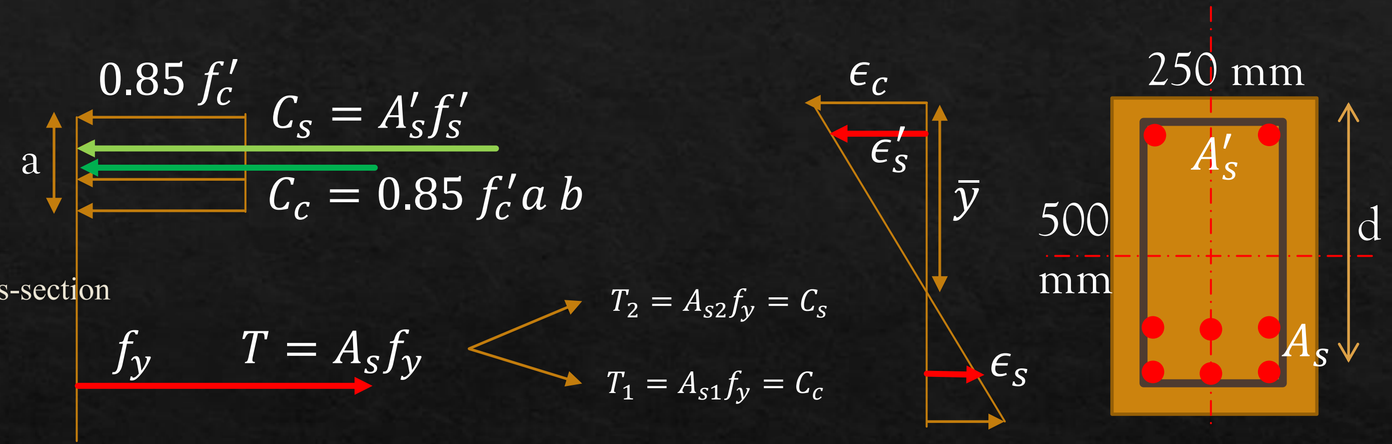
itr 2: assume $f_s' = 387\ MPa\ \rightarrow a = 138.02\ mm\ \rightarrow \bar{y} = 162.14\ mm$

check $f_s' \rightarrow \epsilon_s' = 1.93 * 10^{-3} \rightarrow f_s' = 385.7\ MPa$ …… *OK*

⟐ Check tension steel $\epsilon_s = 4.68 * 10^{-3} > 0.004\ \rightarrow$ *tension controlled* … OK

$\epsilon_s = 4.68 * 10^{-3} < 0.005\ \rightarrow \phi \neq 0.9\ \rightarrow \phi = 0.873$

⟐ $\phi M_n = \phi M_{n_1} + \phi M_{n2} = 0.873 * A_s' f_s'\ (d - d') + 0.873 * (A_s f_y - A_s' f_s')\left(d - \frac{a}{2}\right) = 47.9 + 248.6 = 296.5\ kN.m > M_u$ … OK



2 layers of steel in bottom
$d = 415.5\ mm$ …… $d' = 58\ mm$