جامعة بيرزيت
BIRZEIT UNIVERSITY

Reinforced Concrete Design I
ENCE 335
Shear Design

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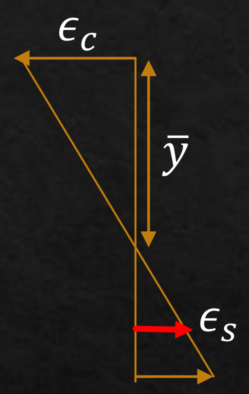
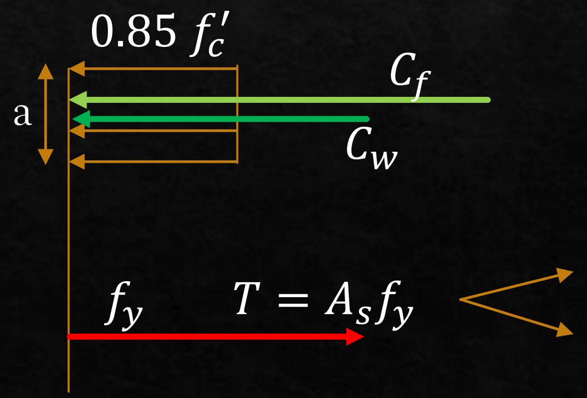
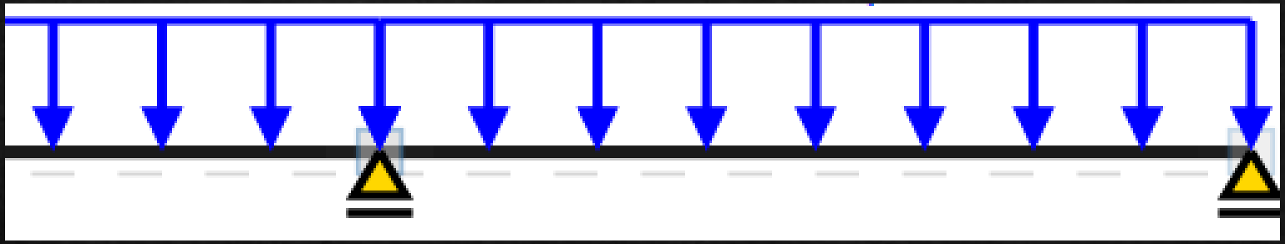
Previously on Design I

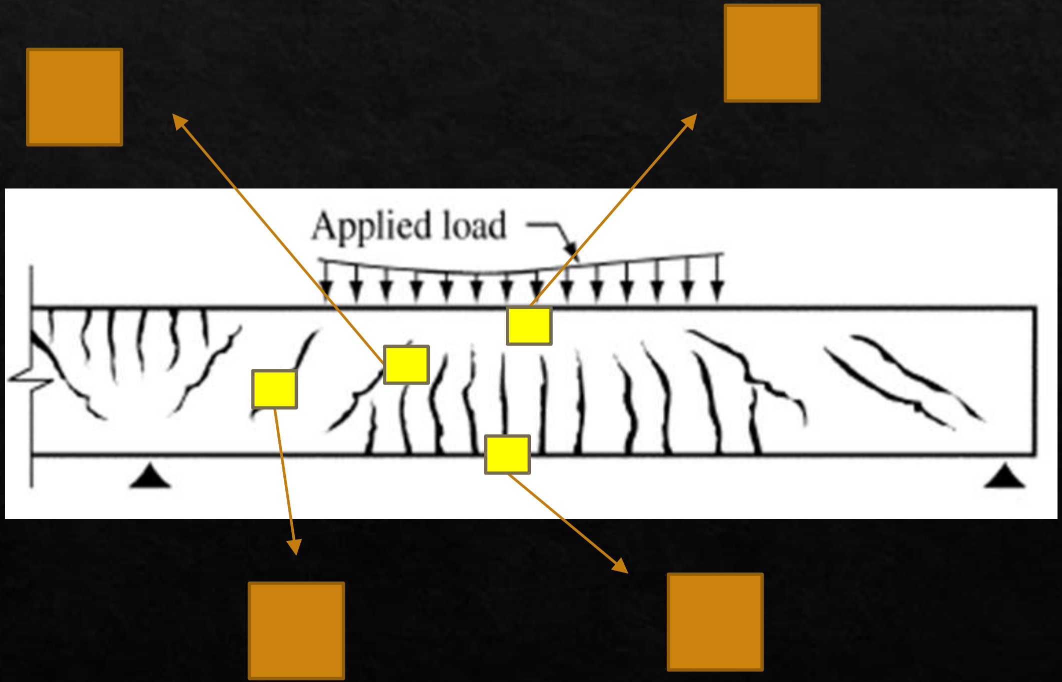
- We only designed for the internal Bending moment
  - Rectangular sections (Single, Double) reinforced.
  - T-sections
  - Arbitrary sections
    - In all cases we relied on stress and strain distributions
    - We need to comply with ACI code requirements for
      - Minimum spacing between bars (Width)
      - Minimum and maximum reinforcement ratios (maintain a min 0.004 tension strain)
      - Minimum depth

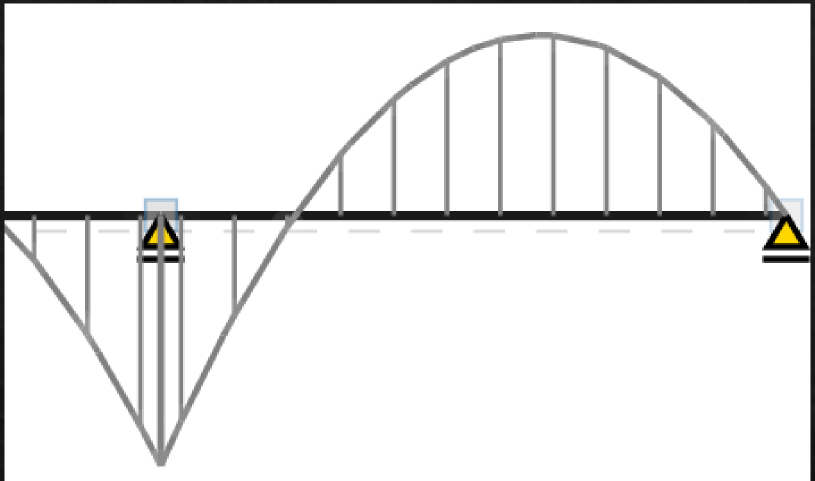


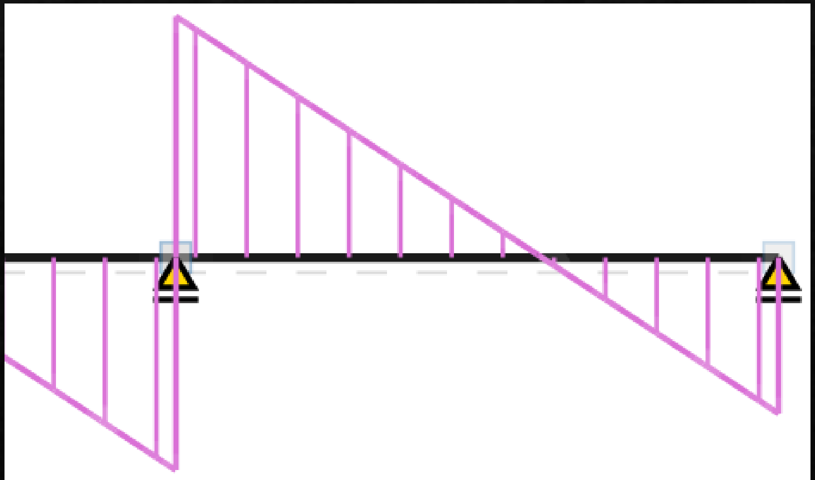


Cracks in Reinforced beams



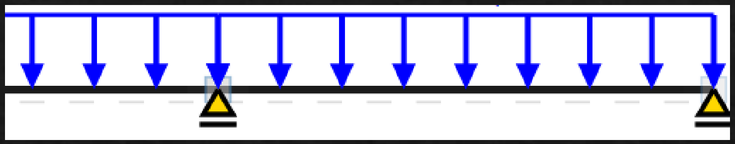


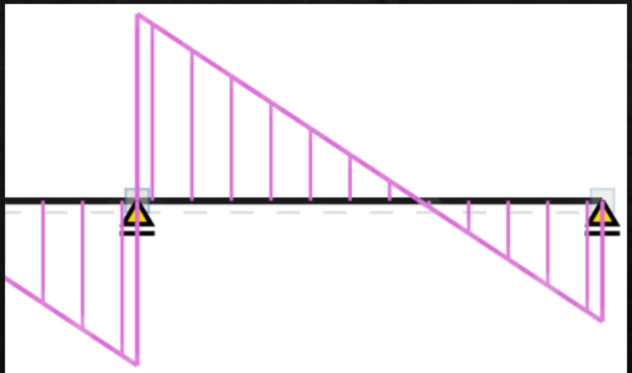




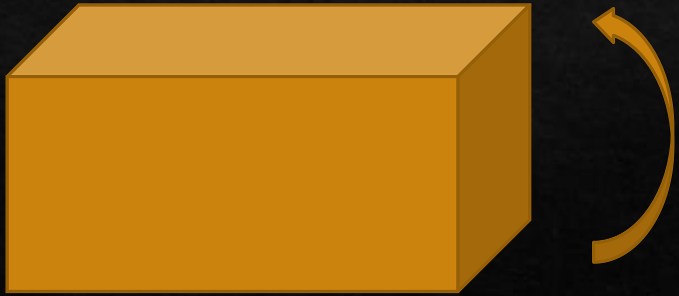
Cracks in Reinforced beams

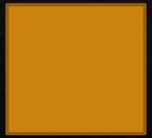
- Shear Cracks
  - Shear forces are typically maximum near supports
    - This not always the case, depends on the loading pattern

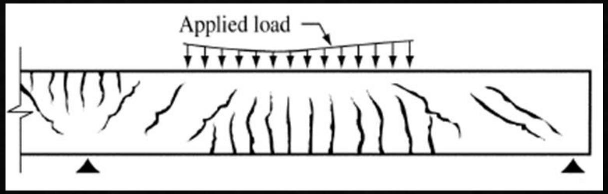




  - Shear stress distribution in the cross-section ?? (Mechanics)



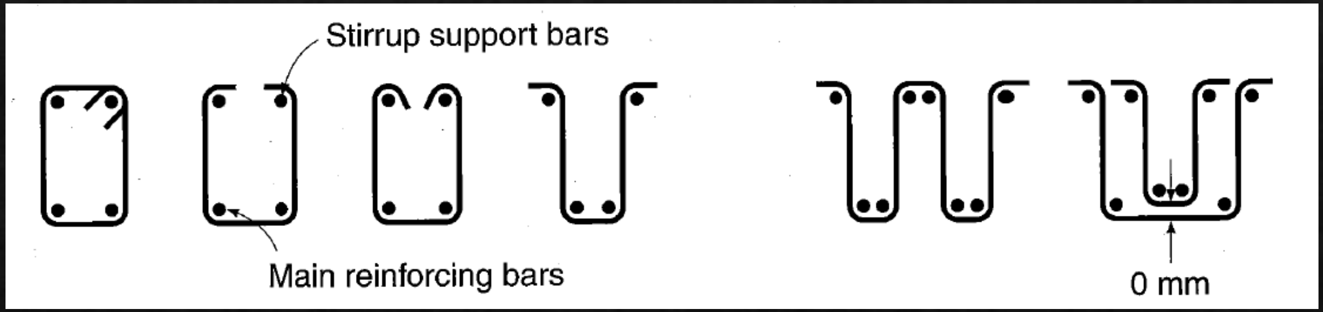




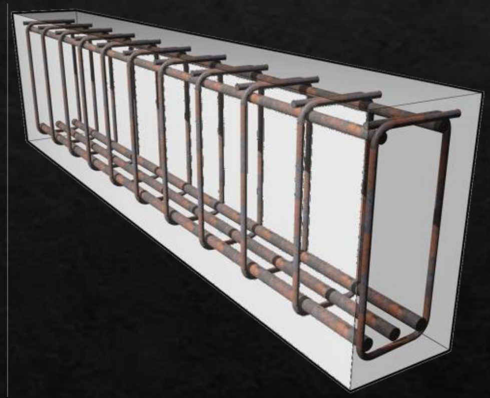
  - Where is the critical section in concrete ?? (Mohr Circle)

Shear reinforcement

- We need to put the steel in a way to bridge these shear cracks
  - Stirrups

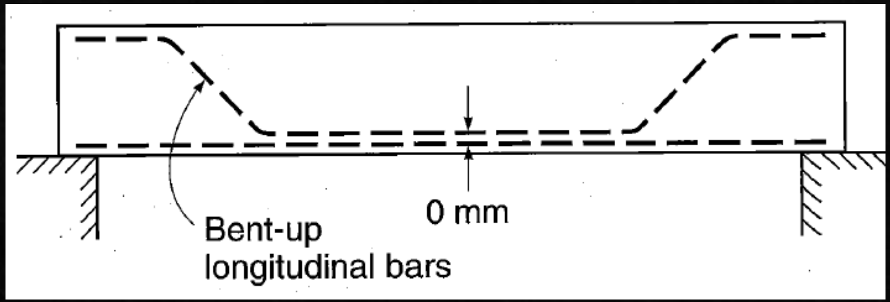


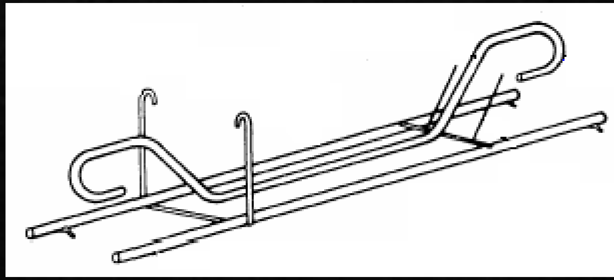
What steel area resist shear stress ???



Which one is more efficient ??

  - Bent Bars





Shear resistance components

- Concrete resistance for shear

$$V_c = 0.17\sqrt{f_c'}\,b_w d$$

- Shear reinforcement (steel stirrups)

$$V_s = \frac{A_v f_y d}{S} \ldots\ldots\ldots V_s = \frac{A_v f_y d\,(\sin\alpha + \cos\alpha)}{s}$$

$Where : A_v : steel\ area\ resisting\ shear\ stress$

$f_y : yeild\ stress\ of\ steel$

$d : effective\ dept$

$S : Spacing\ between\ reinforcement$

$\alpha : angle\ of\ bent\ bars$

ACI Code requirements

- When do we need shear reinforcement ??
  - If $V_u \leq \frac{\phi V_c}{2} \rightarrow$ No reinforcement is needed
  - If $\phi V_c > V_u \geq \frac{\phi V_c}{2} \rightarrow$ Minimum reinforcement is needed (Except ??)
    - In general, we choose the stirrups diameter and control the spacing between them
    - This means, Minimum shear reinforcement $\longrightarrow$ max spacing

Table 9.6.3.1—Cases where $A_{v,min}$ is not required if $V_u \leq \phi V_c$

Beam type	Conditions
Shallow depth	$h \leq 250$ mm
Integral with slab	$h \leq$ greater of $2.5t_f$ or $0.5b_w$ and $h \leq 600$ mm
Constructed with steel fiber-reinforced normalweight concrete conforming to 26.4.1.5.1(a), 26.4.2.2(i), and 26.12.7.1(a) and with $f_c' \leq 40$ MPa	$h \leq 600$ mm and $V_u \leq 0.17\phi\sqrt{f_c'}b_w d$
One-way joist system	In accordance with 9.8

$$S_{max} = \max \begin{cases} \frac{A_v f_y}{0.062\sqrt{f_c'}\, b_w} \leq \frac{A_v f_y}{0.35 b_w} \\ \frac{d}{2} \ldots\ldots \left(\frac{3}{4} d \text{ for bent bars}\right) \\ 600\ mm \end{cases}$$

- If $\phi V_c > V_u \rightarrow$ We Need to Calculate the required spacing for reinforcement (from previous slide) and compare it with max spacing
- If $V_s > 2V_c = 0.33\sqrt{f_c'}\, b_w d \rightarrow$ The max spacing is divided by 2

Where in the ACI Code ???

ACI Code requirements

- Minimum spacing between stirrups: recommended not less than 100mm

- Strength reduction Factor: φ = 0.75

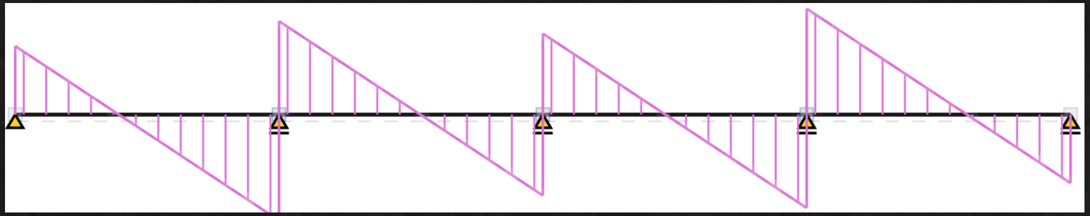
- $V_u: shear\ at\ distance\ d\ from\ the\ face\ of\ support\ V_{u@d}$

- If $V_s > 4V_c = 0.66\sqrt{f_c'}b_w d$ → Section geometry must be increased

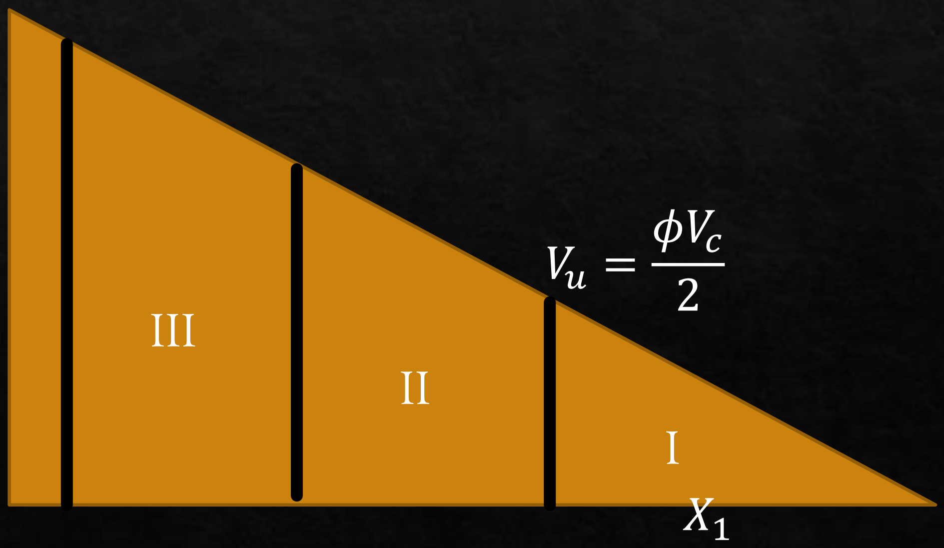
Table 21.2.1—Strength reduction factors φ

	Action or structural element	φ	Exceptions
(a)	Moment, axial force, or combined moment and axial force	0.65 to 0.90 in accordance with 21.2.2	Near ends of pretensioned members where strands are not fully developed, φ shall be in accordance with 21.2.3.
(b)	Shear	0.75	Additional requirements are given in 21.2.4 for structures designed to resist earthquake effects.
(c)	Torsion	0.75	—
(d)	Bearing	0.65	—
(e)	Post-tensioned anchorage zones	0.85	—
(f)	Brackets and corbels	0.75	—
(g)	Struts, ties, nodal zones, and bearing areas designed in accordance with strut-and-tie method in Chapter 23	0.75	—
(h)	Components of connections of precast members controlled by yielding of steel elements in tension	0.90	—
(i)	Plain concrete elements	0.60	—
(j)	Anchors in concrete elements	0.45 to 0.75 in accordance with Chapter 17	—

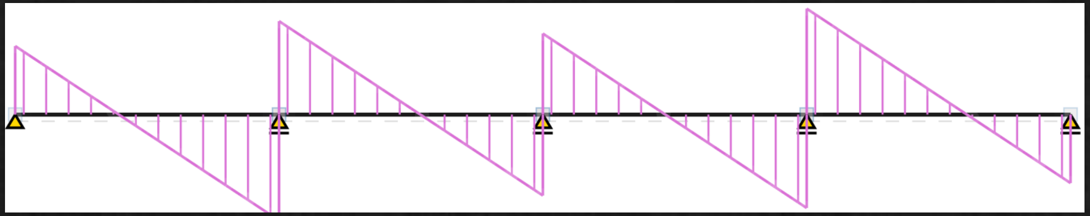
Summary



- Draw shear diagram → and get a triangle (if it is a triangular shape)
  - Since shear is changing, we need to change the spacing accordingly
  - We divide the shear diagram into three regions
    - Region I: No shear reinforcement is needed
      - We need to find where on the shear diagram $V_u = \frac{\phi V_c}{2}$



Summary



- Draw shear diagram → and get a triangle (if it is a triangular shape)
  - We divide the shear diagram into three regions
    - Region II: Max spacing is needed
      - For this we need to calculate the shear force to be carried by steel

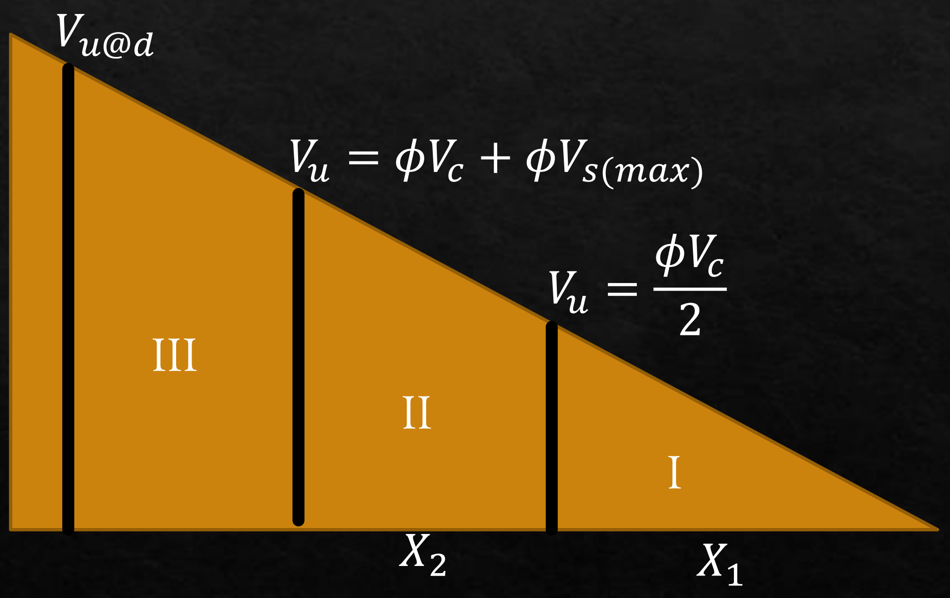
$$V_s = \frac{V_{u@d} - \phi V_c}{\phi}$$

      - Compare V_s with $2V_c = 0.33\sqrt{f_c'}b_w d$
        - $V_s \leq 2V_c = 0.33\sqrt{f_c'}b_w d$ → The max spacing remains the same
        - $V_s > 2V_c = 0.33\sqrt{f_c'}b_w d$ → The max spacing is divided by 2
        - $V_s > 4V_c = 0.66\sqrt{f_c'}b_w d$ → Change Section geometry
      - Calculate the shear force stirrups can support with max spacing

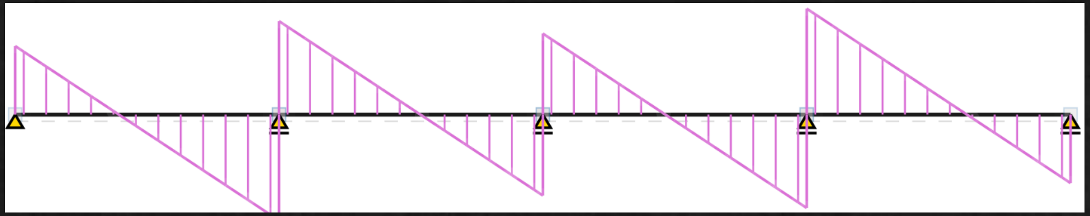
$$\phi V_{s(max)} = \frac{\phi A_v f_y d}{S_{max}}$$

      - Calculate the shear force the section can support with max spacing and locate it on the shear diagram

$$V_u = \phi V_c + \phi V_s$$



Summary



- Draw shear diagram → and get a triangle (if it is a triangular shape)
  - We divide the shear diagram into three regions
    - Region III: Specially designed
      - For this we need to calculate the shear force to be carried by steel

$$V_s = \frac{V_{u@d} - \phi V_c}{\phi}$$

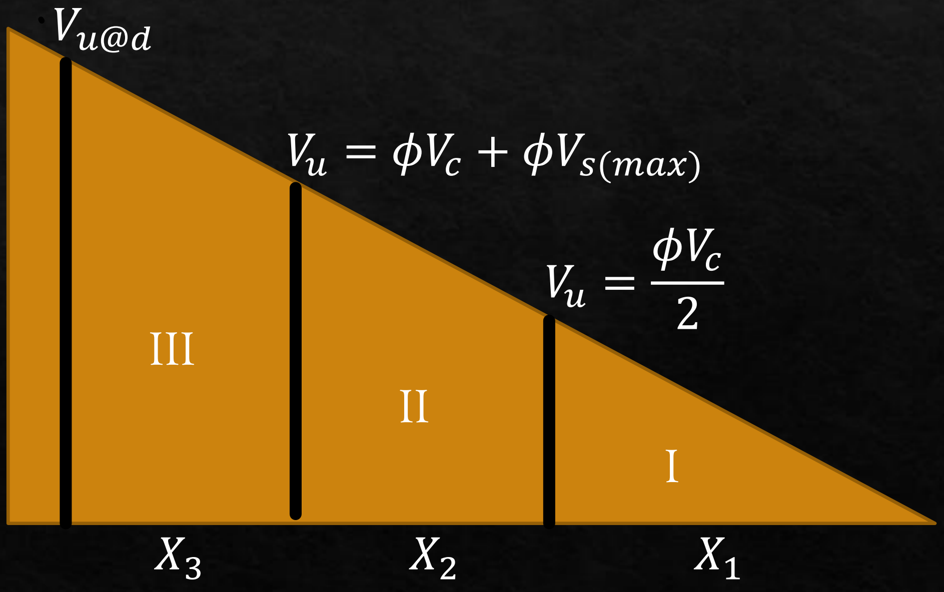
      - Calculate the spacing required to support the shear force

$$S = \frac{\phi A_v f_y d}{V_s}$$

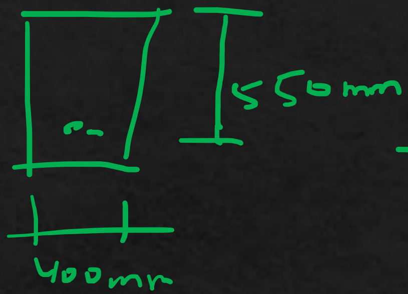
- Calculate the total # of stirrups needed

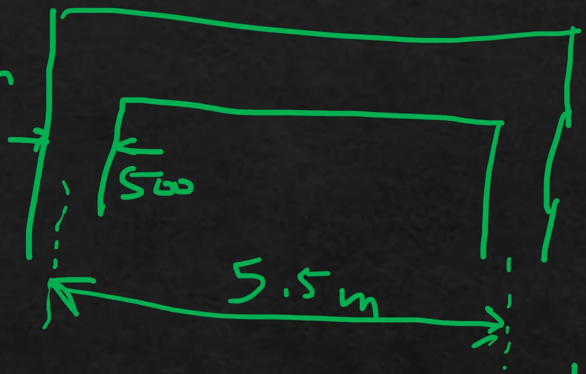
$$\#stirrups = \frac{X_3}{S} + \frac{X_2}{S_{max}} + 1$$

Total # of stirrups for the beam ????



Example





Design the following beam:

Simply supported beam with 5.5m span (center to center)

B=400 mm … d=550 mm

Column width 500 mm

$$w_D = 60\frac{kN}{m}\ldots\ldots w_L = 60\frac{kN}{m}$$

$$f_c' = 28\ MPa\ \ldots\ldots f_y = 420\ MPa$$

Analysis

$W_u = 1.2(60) + 1.6(60) = 168\ kN/m$