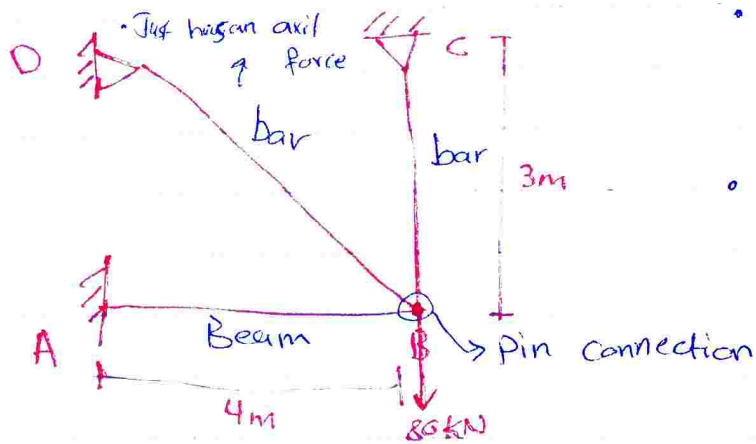


Force Method: - Combined system.

• Find member forces

FBD, FBC.

• Draw B.M diagram AB.



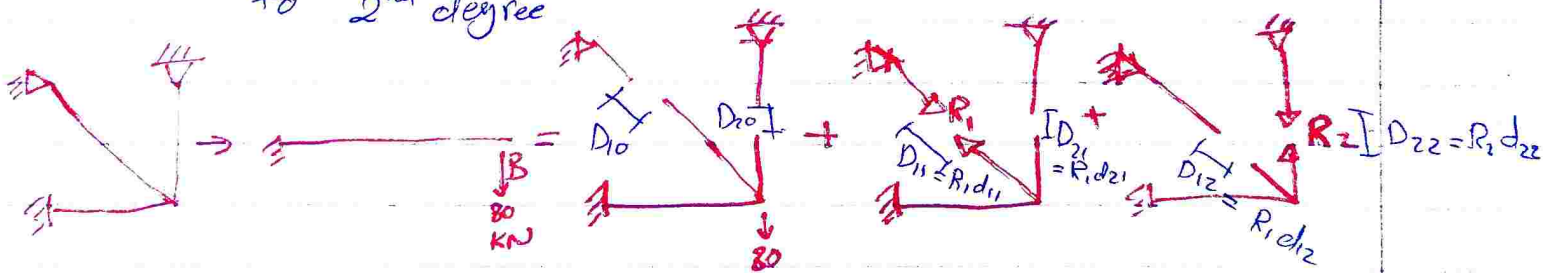
$$R_1 = F_{BD}$$

$$R_2 = F_{BC}$$

* unknowns * Eq

5 > 3

Statically Ind.
to 2nd degree



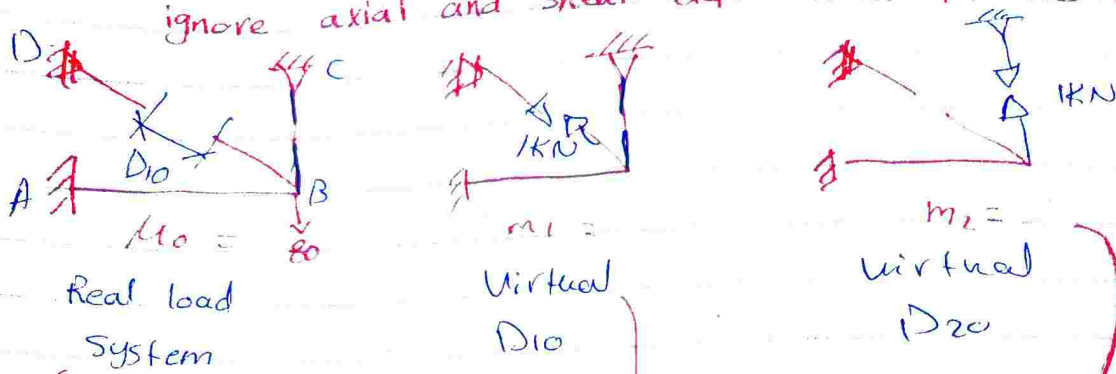
• Relative disp.
between cut edges
member BD.

$$0 = D_{10} + R_1 d_{11} + R_2 d_{12}$$

• Relative disp.
between cut edges
member BC

$$0 = D_{20} + R_1 d_{21} + R_2 d_{22}$$

→ In the calculation of the deformations ignore axial and shear deformations in the beam.



$$D_{10} = \int \frac{M m_1}{EI} dx + \sum n \frac{N L}{EA}$$

$$(N_0)_{BD} = 0$$

$$(N_0)_{BC} = 0$$

$$(m_1)_{BD} = 1$$

$$(m_1)_{BC} = 0$$

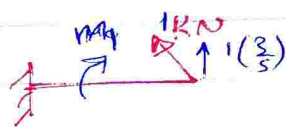
$$(m_2)_{BD} = 0$$

$$(m_2)_{BC} = 1$$



$$M_0 = -80x$$

$$0 \leq x \leq 4$$



$$m_1 = 0.6x$$

$$0 \leq x \leq 4$$



$$m_2 = x$$

$$0 \leq x \leq 4$$

$$\rightarrow D_{10} = \int_0^4 \frac{m_1 M_0}{EI} dx + \sum n \frac{N_0 L}{EA}$$

$$= \int_0^4 \frac{(0.6x)(-80x)}{EI} dx + \left[\frac{(1)(0)(5)}{EA} + \frac{(0)(0)(3)}{EA} \right]$$

$$= -\frac{1024}{EI}$$

$$\rightarrow D_{20} = \int_0^4 \frac{(x)(-80)x}{EI} dx + \left[\frac{(0)(0)(5)}{EA} + \frac{(1)(0)(3)}{EA} \right] = -\frac{1706.67}{EI}$$

$$\rightarrow d_{11} = \int_0^4 \frac{(0.6x)^2}{EI} dx + \left[\frac{(1)^2(5)}{EA} + \frac{(0)^2(3)}{EA} \right] = \frac{7.68}{EI} + \frac{5}{EA}$$

$$\rightarrow d_{22} = \int_0^4 \frac{(x)^2}{EI} dx + \left[\frac{0(5)^2}{EA} + \frac{(1)^2(3)}{EA} \right] = \frac{21.33}{EI} + \frac{3}{EA}$$

$$\rightarrow d_{21} = \int \frac{m_2 m_1}{EI} dx = \int \frac{(0.6x)(x)}{EI} = \frac{12.8}{EI}$$

$$\rightarrow d_{12} = \frac{12.8}{EI}$$

$$E = 200 \text{ GPa} \quad I_b = 200 \times 10^6 \text{ mm}^4 \quad A = 100 \text{ mm}^2$$

$$\begin{aligned} \bullet EI &= (200 \times 10^6 \text{ KN/m}^2) (200 \times 10^6 \text{ mm}^4) \frac{10^{-12} \text{ m}^4}{\text{mm}^4} \\ &= 4 \times 10^4 \text{ KN} \cdot \text{m}^2 \end{aligned}$$

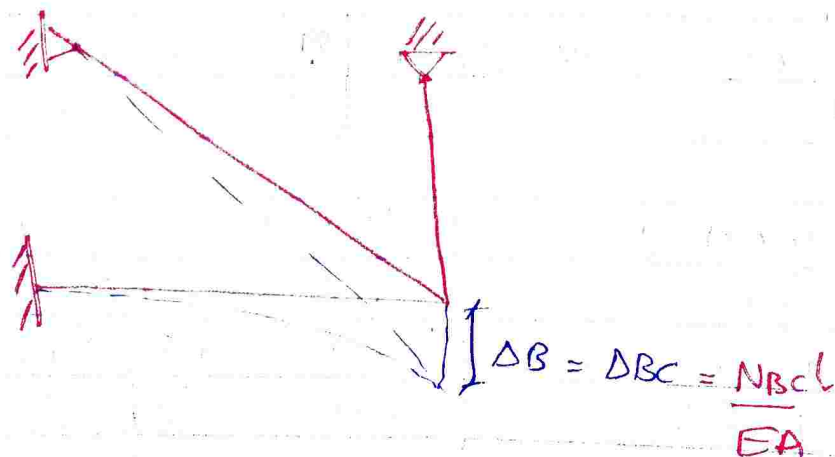
$$\begin{aligned} \bullet EA &= (200 \times 10^6 \text{ KN/m}^2) (100 \text{ mm}^2) \times \frac{10^{-6} \text{ m}^2}{\text{mm}^2} \\ &= 2 \times 10^4 \text{ KN} \cdot \text{m} \end{aligned}$$

$$\rightarrow 0 = -\frac{1724}{EI} + R_1 \left[\frac{7.68}{EI} + \frac{\Sigma}{EA} \right] + R_2 \left[\frac{12.8}{EI} \right]$$

$$\rightarrow 0 = -\frac{1706.67}{EI} + R_1 \left[\frac{12.8}{EI} \right] + R_2 \left[\frac{21.33}{EI} + \frac{3}{EA} \right]$$

$$R_1 = F_{BD} = 14.2 \text{ KN (T)}$$

$$R_2 = F_{BC} = 53.4 \text{ KN (T)}$$



Influence line and Envelopes

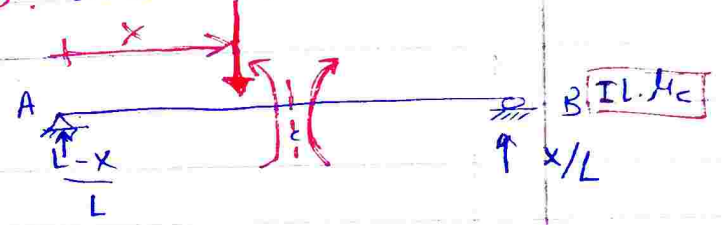
• **Influence line:** is a diagram shows the relation of an objective function (reaction force, shear, bending moment!) with the position of loading.

• **Why do we construct Influence lines?**

- live loads are variable in position (gravity loading)
- To decide the critical load cases when considering the live load, and develop load cases to get the required design values (V, R, M)

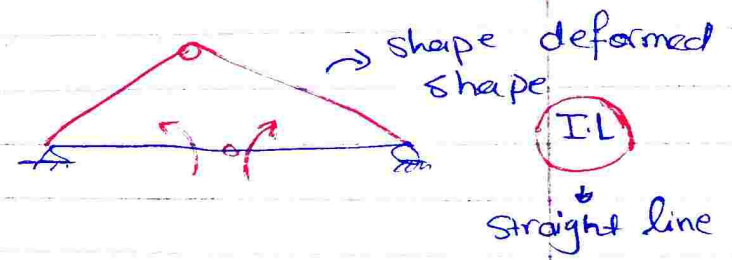
• **Influence line of det. systems?** can be determine By 3 ways:-

- 1 - point by point, $x = 1m \Rightarrow M_c = ?$
 $x = 2m \Rightarrow M_c = ?$



2 - Equations $\Rightarrow M_c(x)$

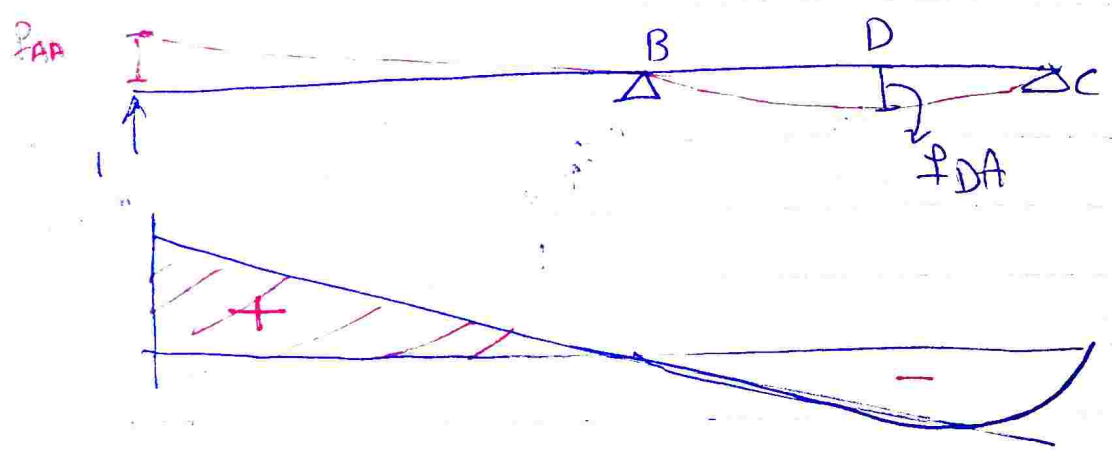
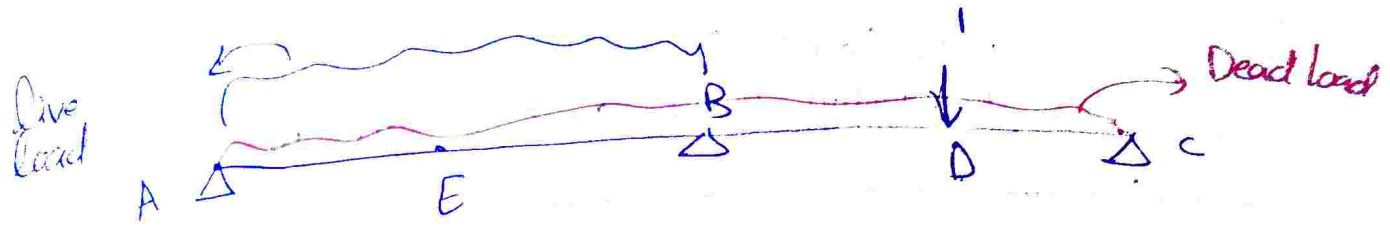
3 - Qualitative approach



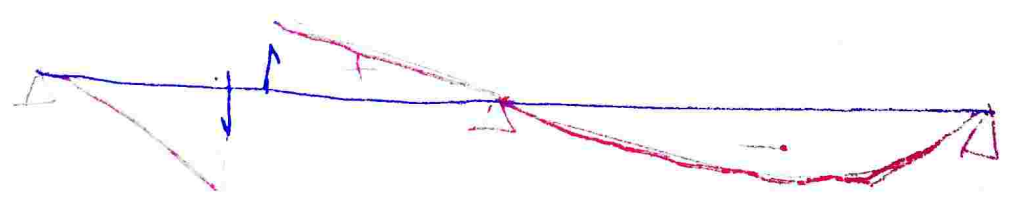
Qualitative approach to build influence lines - Muller Breslau principle.

Induce a unit deformation, the deformed shape due to unit deformation is the shape of the influenced.

Load case for max A_y . (Reaction force)



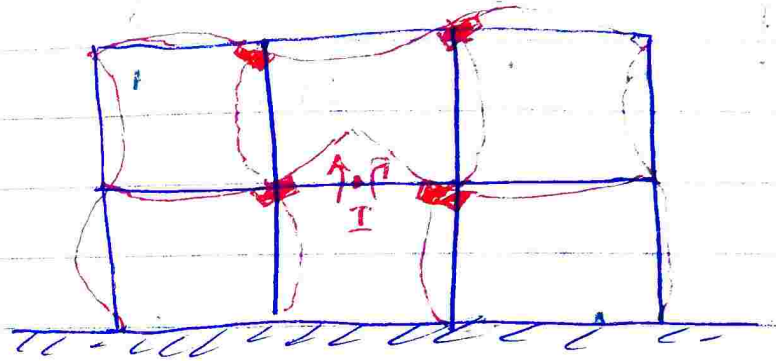
IL for Shear VE.



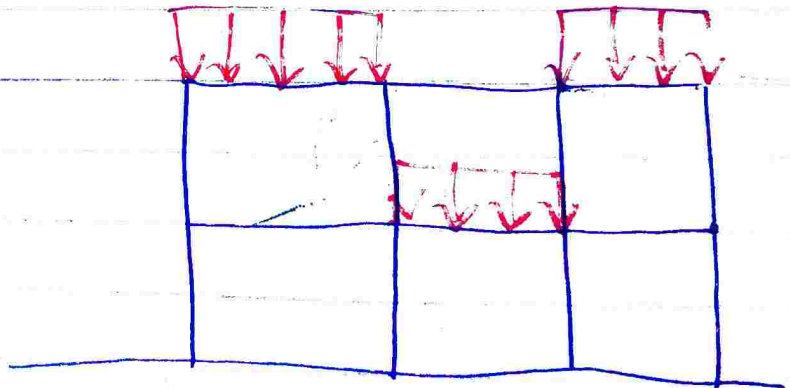
• IL (M_I)



⇒ I.L M_I



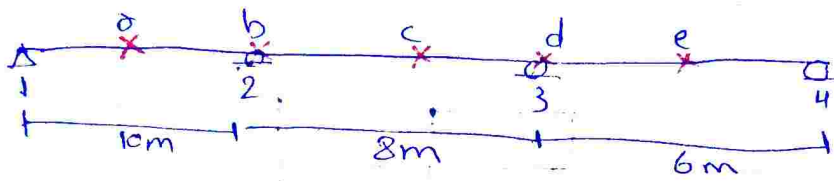
⇒ To get max M_I



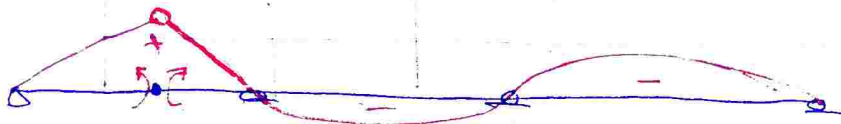
• Example: three beam span

• Draw qualitative IL for the Bending Moment of the beam, and develop the load cases.

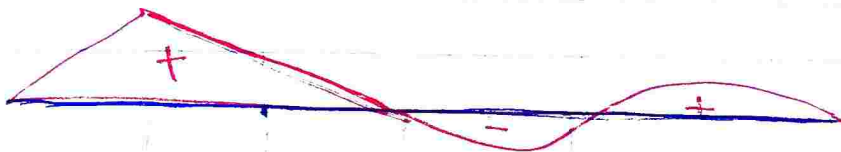
• choose midspans & Interior supports



(1) (a)

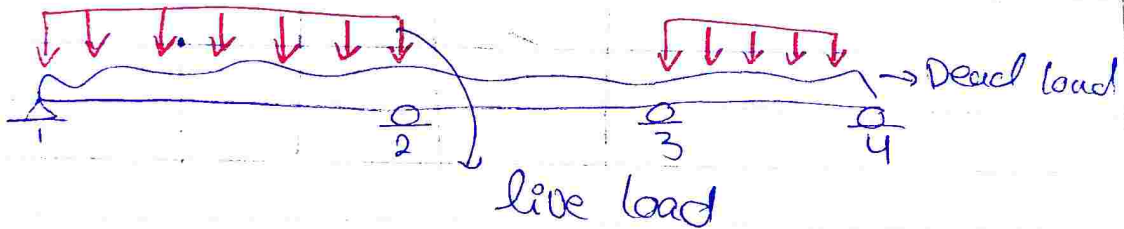


max M_a

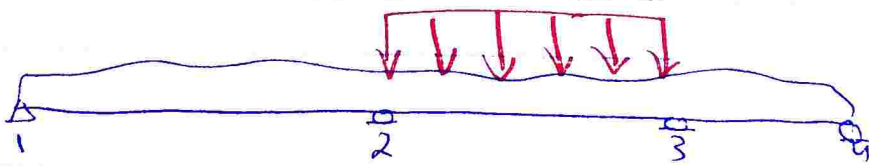


IL, M_a
- load cases

• Load Case (LC1) for max M_a



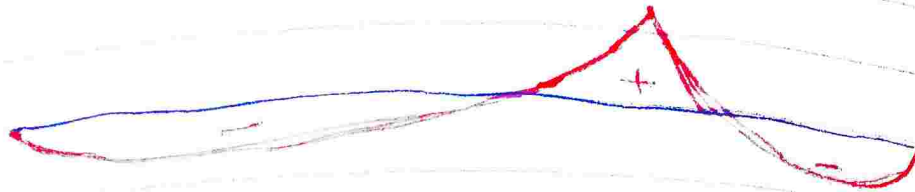
• Load Case (LC2) for min M_a



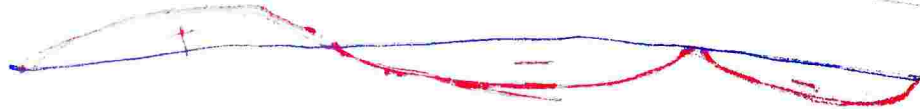
② b I.I.A^M



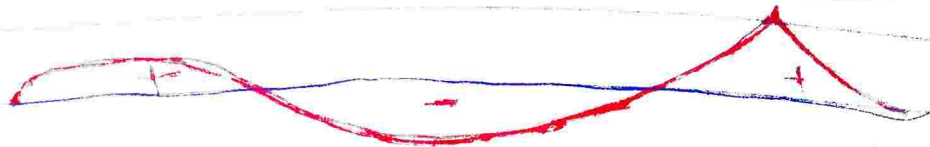
I.L.R^M



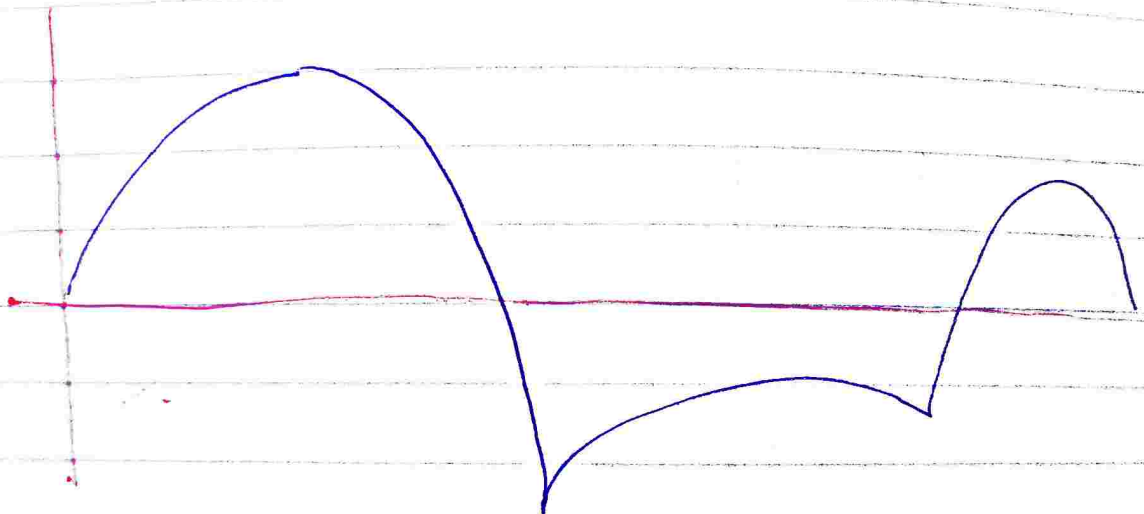
I.L.D^M



I.L.M_c



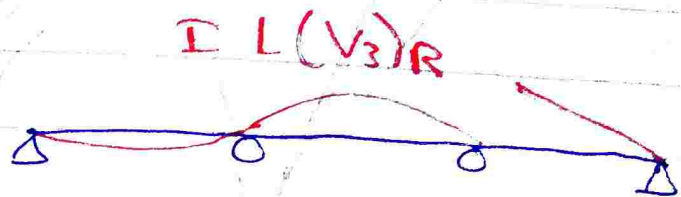
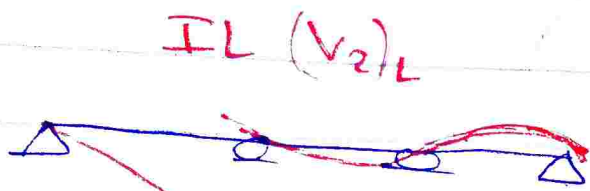
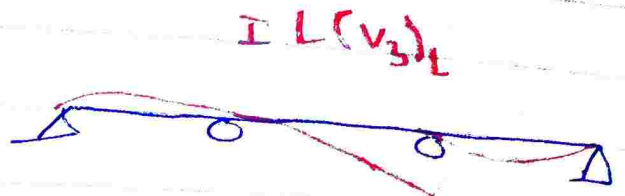
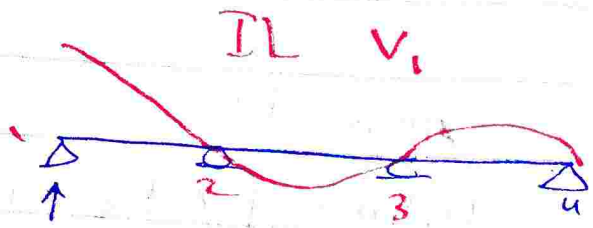
B.M.L.C



• Draw Bending Moment Diagrams for all Cases

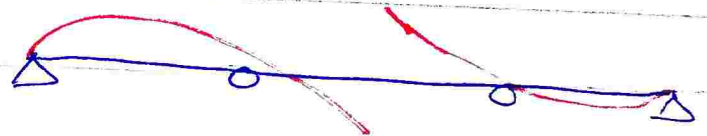
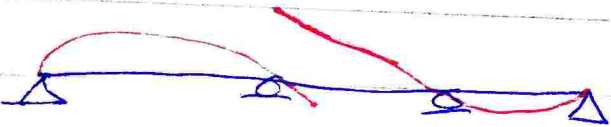
- then we take the max. value from all cases.

Influence Line for Shearing Forces of the Beam.



IL $(V_2)_R$

IL $(V_4)_L$



And so ...